Relational Contracts and Property Rights*

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Abstract

We propose a simple general framework for comparing different ownership structures with respect to creating appropriate incentives for cooperative behavior (efficient investment) in long-run specific relationships, and introduce the notion *relational efficiency* (long term efficiency), based on Abreu's (1986, 1988) definition of optimal punishment. We identify relational efficient ownership structures and find that the short term efficient ownership structure in the tradition of Grossman, Hart and Moore is generally not relational efficient, although it tends to be *constrained* efficient (the constraint being grim trigger strategies). We generalize models by Garvey (1995) and Halonen (2002), and reconsider Baker, Gibbons and Murphy (2001, 2002), confirming some of their results but not others.

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1 Introduction

The property rights theory of the firm, as pioneered by Coase (1937), Williamson (1975, 1986) and Klein, Crawford, and Alchian (1978), and formally developed by Grossmann and Hart (1986) and Hart and Moore (1990), has been a tremendous breakthrough in our understanding of the potential effects of ownership and analogous contractual rights on parties' incentives to undertake non-contractible investments, and on the economic process in general. This literature, however, has focused mainly on individual, isolated transactions and formal, explicit contracts. Macaulay (1963), Klein and Leffler (1981) and Telser (1981) have stressed early that a large fraction of economic transactions are not isolated anonymous exchanges but rather episodes of a history of exchanges, and that one can hardly build a complete theory of the firm without taking into account the informal agreements that regulate long-term relationships.

Complete contracts, covering all conceivable contingencies, are absent from our world, and relational contracts, flexible self-enforcing implicit arrangements that complete the rigid, incomplete explicit contracts are indispensable to "have things running smoothly" in organizations.[1] A sign of this is that in developed countries with detailed employment contracts and efficient law enforcement, "working to rule" – i.e. following literally what prescribed by the explicit contract – is one of the tougher traditional weapons in the hands of employees when bargaining for higher wages. Relational contracts are also very important between organizations, in particular for the governance of specific supply relations, where crucial aspects of the transactions are hard to contract upon.[2]

Understanding the interaction between the rigid and incomplete explicit contracts enforced by courts, in particular of property rights on assets instrumental for production, and the relational contracts that complete them is therefore crucial for the theory of organizations. The aim of this paper is to provide a general framework for analyzing and comparing different ownership structures (and potentially other explicit contracts) with respect to their effects on parties' ability to sustain the relational contracts necessary to achieve productive efficiency in long-run business relationships.

The formal theory of implicit incentive contracts is well developed since Bull (1987) and MacLeod and Malcomson (1989) and has been considerably extended by Levin (2003). Building on this knowledge, authors like Garvey (1995), Halonen (2002) and Baker, Gibbons and Murphy (BGM, 2001, 2002) have begun to formally analyze how asset ownership and implicit,

[1] We do not enter the debate on why contracts are incomplete. The interested reader may e.g. look at the special issue in RES 66 (1999); we just note that, independent of complexity considerations, the mere presence of (typically very high) costs of court enforcement justify the limited use of detailed explicit contracts often observed in reality.

[2] Fehr, Brown, and Falk (2003) provide striking experimental evidence of the overwhelming importance of relational contracts when there are unobservable/uncontractable aspects involved in the transactions.

relational contracts interact within dynamic models of ongoing organizations.[3] A message common to these papers is that in a dynamic framework where relational contracts are important, ownership still matters (when self-enforcing constraints bind), but for somewhat different reasons than in the Grossmann-Hart-Moore (GHM) framework, so that the optimal ownership structure may differ from the one identified by GHM.[4]

Apart from this common message, the results of these contributions differ substantially and are not easy to compare as the authors use different specifications and assumptions. Moreover, these authors restrict focus to special strategies, "grim trigger" or Nash reversion (play cooperatively; if a defection takes place, revert for ever to the static equilibrium, with or without ownership renegotiation along the punishment path). These strategies are familiar and simple to analyze but are somewhat *ad hoc.* They are generally not optimal for the agents (in the sense of Abreu 1986, 1988), nor robust to ex post renegotiation (in the sense of Farrell and Maskin, 1989), nor robust to trembles and mistakes (in the sense of Segerström, 1988). The exclusive focus on these strategies makes it therefore hard to assess how robust or general the conclusions of these models are, while others argue that the optimal ownership structure tend to be the same in a static and a dynamic framework.[5]

In this paper we try to clarify this issue with the help of established results in the theory of repeated and dynamic games. A key element of our analysis, in contrast to the previous literature, will be *not to make any specific assumption on strategies.*

First we propose a simple but general framework where the models of Garvey and Halonen are contained as subcases, which allows us to highlight strengths and weaknesses of their results. Then we introduce efficiency measures for short term (one-shot) behavior and for long run (relational) behavior with and without the possibility to renegotiate both strategies and property rights. In particular, a "short term efficient" ownership structure maximizes joint payoffs in the static equilibrium, hence coincides with the optimal ownership structure identified by GHM. An ownership structure is called "relational efficient" if it supports a maximal range of discount parameters such that cooperation or first best behavior is an equilibrium path of the repeated game. This concept is closely related to Abreu's (1986, 1988) concept of *optimal punishment*, since to establish the "relational efficient" ownership structure we allow agents

[3] The work of Bernheim and Whinston (1998) and Rosenkrantz and Schmitz (2001) should also be mentioned, although their two-stage models fall somewhat short of depicing a long-term relation.

[4] The result is in the same spirit of Che and Yoo (2001), who showed that in a dynamic framework team-based incentives may be optimal where they are not in a static one (see also Spagnolo, 1999). A different message comes from Rajo (2003) and Rajo and Levin (2003), who show that concentrating residual claims on profit streams and control (discretion) tends to be relational efficient.

[5] See e.g. Hart (2001), and Bragelien (2002) who extends BGM's framework to the case of two investing parties, and maintaining their assumptions on strategies shows that short and long term efficient ownership structures tend to be similar.

to choose optimal strategies.[6] We also take into account the "constrained" form of relational efficiency considered in previous work (Garvey 1996; BGM 2001, 2002), i.e. relational efficiency under the constraint that agents use grim trigger strategies and renegotiate ownership before the punishment phase.

We then compare different constellations of property rights on assets with respect to their relative "relational efficiency", that is, with how easily can parties sustain efficient behavior in equilibrium under each constellation. We follow the tradition in the theory of repeated and dynamic games by adopting the minimum discount factor $\underline{\delta}$ at which efficiency can be achieved in subgame perfect Nash equilibrium (with and without renegotiation) as an index of relational efficiency.[7]

Our main finds are that in general short term efficiency and relational efficiency do not coincide, and that in contrast to short term efficiency the possibility to renegotiate does not influence the relational efficient ownership structure. Hence, by our results we should expect that the short term efficient ownership structure identified by GHM should in general not emerge in long-term relationships where parties consider future consequences of their present behavior as essential elements of their strategic environment. These results confirm and generalize the message common to Halonen, Garvey and BGM, that ownership does matter in dynamic environments, but for somewhat different reasons than in the GHM's framework. However, we also find that GHM's short term efficient structure is often constrained relational efficient, and we identify specific situations in which it is also relational efficient.

As for more specific results, we reformulate Garvey's (1995) and Halonen's (2002) models as sub-specifications of our model and re-examine them without assumptions on strategies. Halonen's result that joint ownership may be optimal in a dynamic environment is confirmed and generalized to the case when renegotiation is possible.[8] On the contrary, Garvey's result that relational contracting requires more symmetric ownership structures breaks down once his somewhat special assumption on transfers is dropped. Curiously enough, with optimal transfer and strategies in his model ownership is irrelevant. Finally, we reconsider BGM's model without assumptions on strategies, confirming and extending the main result in BGM (2002), but not that in BGM (2001) on the impossibility to "bring the market inside the firm":

[6] A punishment is called optimal if any defector obtains minimal continuation payoffs in the subsequent subgame. The optimal punishment is maximal in the sense that no lower continuation payoff can be supported by a subgame perfect equilibrium of the continuation game.

[7] In future work we plan to introduce other measures for the stability of co-operative agreements in dynamic games, those related to risk dominance (see Blonski and Spagnolo, 2002).

[8] Halonen (2002) has been criticized for assuming away the possibility to (costlessly) renegotiate ownership after cooperation breaks down, a possibility that in her framework destroys her result (see e.g. Bragelien (2002)). We show here that this objection, although justified in itself, does not change the results when agents are free to choose the strategies by which to support efficient investments.

we find that when parties are free to choose strategies optimally, patient firms can mimic the market allocation.

The paper unfolds as follows. Section 2, describes the investment stage game. Section 3 describes the dynamic/repeated game, defines long term (relational) efficiency and states the main results. Section 4 applies the general framework to the model specifications of Garvey (1995) and Halonen (2002). Section 5 reformulates and analyzes BGM (2001, 2002) in the light of our theory, and Section 6 concludes. All proofs are in the Appendix.

2 Investment stage game

Two parties $i = 1, 2$ play the following investment stage game. In the first substage both parties decide simultaneously on a non-contractible costly action $e_i \in E_i$, with cost $C_i(e_i)$, that can be interpreted as investment. In the second substage agents bargain over the jointly created "cake" $Q(e_1, e_2) \geq 0$. The size of this cake depends on both actions. Let

$$e = (e_1, e_2) \in E := E_1 \times E_2$$

denote an action profile. We assume for simplicity that there exists a unique action profile $e^c = (e_1^c, e_2^c) \in E$ that maximizes the size of the joint surplus given as

$$S(e) = Q(e) - C_1(e_1) - C_2(e_2)$$

and call $e_i^c \in E_i$ the "cooperative" or "first-best" action of agent i. In the tradition of Grossmann-Hart-Moore agents' actions not only determine the joint surplus but also bargaining positions. To model this, introduce the threat point or disagreement (or status quo) payoffs

$$(P_1(e, \omega), P_2(e, \omega)) \in \left\{(x_1, x_2) \in R^2 \,|x_1 + x_2 \leq Q(e) \right\}.$$

Agent i's disagreement payoff $P_i(e, \omega)$ depends on parameter $\omega \in \Omega$ – interpreted as "ownership structure" – and on action profile e. A possible interpretation is that agent i obtains $P_i(e, \omega)$ by using his assets for the next best alternative outside the relationship[9].

Call $Q(e) - P_1(e, \omega) - P_2(e, \omega)$ the *value* of the relationship or the "net-cake". We assume that the value is positive $Q(e) - P_1(e, \omega) - P_2(e, \omega) \geq 0$ and stick to the original assumption that within the relationship parties split equally the "net-cake" $Q(e) - P_1(e, \omega) - P_2(e, \omega)$, i.e. agree on the Nash-Bargaining-Solution (NBS) yielding payoff

$$u_i(e, \omega) = \frac{1}{2}\left[Q(e) + P_i(e, \omega) - P_{-i}(e, \omega)\right] - C_i(e_i) \tag{1}$$

[9] We allow explicitly for the case that the disagreement payoff depends on both parties' actions. For example some assets may be worthless without outside expertise.

to agent i.[10] Note that the joint surplus $S(e) = u_1(e,\omega) + u_2(e,\omega)$ and in particular the first best joint surplus

$$S^* \equiv u_1(e^c, \omega) + u_2(e^c, \omega) = Q(e_1^c, e_2^c) - C_1(e_1^c) - C_2(e_2^c) \tag{2}$$

does not depend on ω. For any $\omega \in \Omega$ the payoffs $u_i(e,\omega)$ define a game with simultaneous choice of actions in the first substage denoted by $\Gamma(\omega)$ and called subsequently the "reduced form stage game".

The salient topic of the literature on ownership rights relates to the observation that maximization of individual payoffs $u_i(e,\omega)$ and of joint surplus $S(e) = u_1(e,\omega) + u_2(e,\omega)$ create different incentives. In order to relate our results to this literature we focus on structures such that it is not in both agents' individual short term interest to act cooperatively.

Definition 1 *A family of stage games $\{\Gamma(\omega)\}_{\omega\in\Omega}$ is called a "holdup structure" iff*

(i) for every $\omega \in \Omega$ each agent $i = 1,2$ has a unique best response strategy denoted by $e_i^b(e_{-i}, \omega)$ where $e_i^b(e_{-i}^c, \omega) \neq e_i^c$ for all $\omega \in \Omega$ and

(ii) $\Gamma(\omega)$ has a unique (pure strategy) Nash equilibrium called "holdup equilibrium" denoted by $e^d(\omega) = (e_1^d(\omega), e_2^d(\omega))$. This implies $e_i^d(\omega) = e_i^b(e_{-i}^d, \omega) \neq e_i^c$.

To keep notation suggestive and simple we introduce shortcut variables for $i = 1,2$

$$c_i(\omega) = u_i((e^c), \omega) \text{ for "Cooperation payoff"}, \tag{3a}$$
$$d_i(\omega) = u_i\left(e^d(\omega), \omega\right) \text{ for "Defection payoff", holdup equilibrium payoff}, \tag{3b}$$
$$b_i(\omega) = u_i\left(\left(e_i^b(e_{-i}^c, \omega), e_{-i}^c\right), \omega\right) \text{ for "Betray payoff"}, \tag{3c}$$
$$a_i(\omega) = u_i\left(\left(e_i^c(\omega), e_{-i}^b(e_i^c, \omega)\right), \omega\right) \text{ for "Afflicted payoff"}, \tag{3d}$$
$$S^* = c_1(\omega) + c_2(\omega) \text{ joint first best surplus}, \tag{3e}$$
$$D(\omega) = d_1(\omega) + d_2(\omega) \text{ joint holdup equilibrium payoffs}, \tag{3f}$$
$$B(\omega) = b_1(\omega) + b_2(\omega) \text{ joint betray payoffs}. \tag{3g}$$

Capital letters D, B stand for aggregates. In contrast to the "first best" cooperative action profile e^c the holdup equilibrium $e^d(\omega)$ depends on ownership structure ω. Therefore, we can compare different ownership structures with respect to the sum of the generated (transferable) equilibrium utilities.

[10] It is well known that allowing for "outside options" may change the conclusions of the Grossmann-Hart-Moore approach (De Meza and Lookwood, 1998; Chiu, 1998). Extending the present model to outside option bargaining is potentially interesting but left to future work.

Definition 2 *Call $\omega^* \in \Omega$ "short term efficient" if it maximizes the sum of equilibrium payoffs*

$$D(\omega^*) \geq D(\omega) \quad \forall \omega \in \Omega. \tag{4}$$

The corresponding set of short term efficient ownership structures is denoted by Ω^*. Further, denote by $d_i^*(\omega) = u_i\left(e^d(\omega^*), \omega^*\right)$ the short term efficient holdup equilibrium payoff to party i and by $D^* = d_1^*(\omega) + d_2^*(\omega)$ the joint short term efficient holdup equilibrium payoffs.

Agents who recognize that cooperation is not sustainable have an incentive to renegotiate ownership whenever the initial ownership structure ω was inefficient and the cost of renegotiation is negligible. If the total cost of renegotiating/reallocating ownership is $z \geq 0$, agents' payoff increases by

$$T = \frac{1}{2}(D^* - D(\omega) - z)$$

if agents again apply Nash bargaining (split the pie).

3 Relational efficiency

Suppose parties play repeatedly (with positive probability) the investment stage game described in the previous section. We assume complete information, i.e. at each decision node both players can observe the entire back history of the game. In the language of contract theory, we assume that investments are observable but not verifiable. At the beginning of each period agents can renegotiate and change the ownership structure before playing the investment game. Call $\Gamma(\delta, \Omega)$ the resulting dynamic game with joint discount factor δ generated by the repeated play of a game $\Gamma(\omega)$ with $\omega \in \Omega$, so that ω is a state variable for $\Gamma(\delta, \Omega)$.

To support the efficient level of investment on the equilibrium path of this dynamic game – to maximize joint payoffs, "the pie" – agents may need to split the pie in a different way than specified by the payoffs of $\Gamma(\omega)$. We assume that to optimally adjust the shares of the pie, agents can choose a monetary transfer $\theta_1 = \theta = -\theta_2$, so that $u_i(e^c, \omega) + \theta_i$ goes to agent i if both agents cooperate.[11]

The generic period-t stage of the dynamic game $\Gamma(\delta, \Omega)$ has the following three-step structure

1	2	3
ownership structure ω^t is chosen	$\Gamma(\omega^t)$ is played (simultaneous investments)	θ^t is paid (profits are split)

.

[11] In the background of this formulation are efficient equilibria of a model where parties can negotiate over the transfer payment path before playing the supergame. This motivates our language that agents can choose a transfer. The monetary transfer could in principle be contracted upon, but in equilibrium it will only be part of the relational contract since by assumption investments are non-contractible.

In this section we want to identify ownership structures that are most supportive for efficient, cooperative behavior in this dynamic game. In particular, we introduce the following efficiency criterium:

Definition 3 *Ownership structure ω is called "relational efficient" or "long term efficient" if it minimizes the lower bound $\underline{\delta}$ on discount factors such that for all $\delta \geq \underline{\delta}$ there exists a subgame-perfect equilibrium supporting indefinite cooperation with ownership structure ω on its equilibrium path for $\Gamma(\delta, \Omega)$. Accordingly, call the respective equilibrium and the equilibrium strategies "relational efficient". The set of relational efficient ownership structures is denoted by Ω^{**}.*

Negotiations on ownership structure at the beginning of the dynamic game and in any other period may involve transfers between agents and take place at some cost $z \geq 0$. When $z = 0$, we would expect agents to renegotiate ownership structure to one that is relational efficient for the continuation game, both at the beginning of the dynamic game and at each of the nodes reached thereafter.

Which ownership structure is more efficient in a relation may depend on the strategies agents are allowed to use to support cooperative behavior (efficient investment). Previous work has focused on two specific cases.

On the one hand, Garvey (1995) and BGM (2001, 2002) assume that agents support cooperation through grim trigger strategies and that $z = 0$. In their framework, after a defection cooperation breaks down, but agents are still able to renegotiate ownership. At the beginning of the infinite punishment phase ownership structure is renegotiated to the short-term efficient one, the gains from renegotiation being split according to symmetric Nash bargaining.

On the other hand, Halonen (2002) also assumes that agents support efficient investments through grim-trigger strategies, but she assumes z large enough, so that ownership structure is not renegotiated when cooperation breaks down.[12]

While both assumptions $z = 0$ and z large can be defended and are worth being studied, the exclusive focus on grim-trigger strategies appears restrictive. The theory of discounted repeated games offers several criteria for the choice of reasonable strategies, and grim trigger performs rather poorly on several grounds. First, as mentioned, grim-trigger strategies are generally not optimal (in the sense made precise by Abreu (1986, 1988)), and relying exclusively on suboptimal strategies does not seem the best way to look for an optimal ownership structure. Second, since grim trigger strategies prescribe inefficient play forever after a defection, they are somewhat fragile with respect to trembles or mistakes. As convincingly argued by Segerström (1988), if agents make mistakes with positive probability, they will tend not to choose strategies

[12] Halonen (2002) also considers an example with renegotiation of ownership, but does not derive general results for that case.

by which cooperation breaks down forever after a mistake occurs. Third, because they prescribe play of an inefficient equilibrium of the continuation game, grim trigger strategies are not robust to renegotiation of strategies (Farrell and Maskin, 1989). The restriction to these strategies, therefore, is particularly troublesome in models that allow for renegotiation of ownership. Without further assumptions, it is not clear what would prevent agents from renegotiating strategies while they are renegotiating ownership.

In what follows, we will consider both cases of z large and $z = 0$, but we will let agents free to choose strategies optimally, and we will pay attention to their ability to renegotiate strategies, besides ownership.

3.1 No renegotiation

To establish the first benchmark we assume:

Assumption 1.

Renegotiation of ownership ω and of strategies is not possible (z is large).

Under Assumption 1 the dynamic game $\Gamma(\delta, \Omega)$ described above degenerates into a standard discounted repeated game $\Gamma(\delta, \omega)$. Abreu (1988) has shown that under mild regularity conditions for discounted repeated games there exist optimal punishments.[13] Therefore the optimal punishment continuation payoff for player i is well defined and is denoted by $U_i(\omega)$. We use the notation $v_i(\omega) = (1 - \delta) U_i(\omega)$ and $V(\omega) = v_1(\omega) + v_2(\omega)$ to simplify exposition. We can then state the following.

Theorem 1 *Assumption 1 implies:*

1. *The range of supporting discount factors is maximal by paying the transfer*

$$\theta^{**}(\omega) = \frac{(b_1(\omega) - c_1(\omega))(b_2(\omega) - v_2(\omega)) - (b_2(\omega) - c_2(\omega))(b_1(\omega) - v_1(\omega))}{B(\omega) - V(\omega)};$$

2. *An ownership structure ω is relational efficient iff*

$$\omega \in \Omega^{**} := \left\{\omega \left| \underline{\delta}^{**}(\omega) \leq \underline{\delta}^{**}(\omega') \right. \ \forall \omega' \in \Omega\right\}$$

where $\underline{\delta}^{**}(\omega) = \frac{B(\omega) - S^*}{B(\omega) - V(\omega)}$.

Since in general minimizing $\underline{\delta}^{**}(\omega) = \frac{B(\omega) - S^*}{B(\omega) - V(\omega)}$ and maximizing $D(\omega)$ leads to different results, a direct implication of Theorem 1 is that the relational efficient ownership structure is generally different from the short term efficient structure. Borrowing terminology from Compte et al. (2002), minimizing $\underline{\delta}^{**}(\omega)$ implies taking care of the "deviation concern" (minimizing

[13] Action spaces E_i have to be compact topological spaces and payoff functions u_i must be continuous.

short-run gains from deviating from the contract) and of the "punishment concern" (maximizing the sanctions against deviators). The interaction between these two concerns has in general little in common with GHM's concern of maximizing payoffs in the static Nash equilibrium of the stage game.

The following example, inspired by Halonen (2002), illuminates this discrepancy for a simple case.

Example 1 *Consider a stage game with a binary action space* $E_i = \left\{e_i^c, e_i^d\right\}$ *where* $\Gamma(\omega)$ *is a Prisoner's Dilemma. Let* $Q\left(e^d\right) = P_i\left(e^d, \omega\right) + P_{-i}\left(e^d, \omega\right)$*, i.e. the defective action profile* e^d *yields the same joint payoff as quitting the relationship. Hence, the minimal subgame perfect continuation payoff is given by playing defect forever, and* $V(\omega) = D(\omega)$*. Let further ownership structures be given as* $\Omega = \{\omega_{SO}, \omega_{JO}\}$ *interpreted as "single ownership" and "joint ownership" with similar aggregated betray payoffs* $B(\omega_{SO}) = B(\omega_{JO})$ *but a big difference outside the relationship* $D(\omega_{SO}) > D(\omega_{JO})$*. In that case single ownership is short term efficient* $\Omega^* = \{\omega_{SO}\}$ *since it maximizes* $D(\omega)$ *and joint ownership is relational efficient* $\Omega^{**} = \{\omega_{JO}\}$ *since it minimizes* $D(\omega)$ *and therefore minimizes* $\underline{\delta}^{**}(\omega)$*.*

3.2 Renegotiation

As a second benchmark assume that parties can renegotiate costlessly. Once agents are allowed to renegotiate the ownership structure after cooperation breaks down, it becomes natural to assume that they can renegotiate strategies as well if they like (modifying agents' strategies should arguably be cheaper than renegotiating the ownership structure).

Assumption 2.

Renegotiation of ownership structure and/or of strategies is possible and costless ($z = 0$)*.*

In this section we assume that after any defection parties can renegotiate strategies and ownership structure, but assume nothing on strategies. We show that in a model with monetary transfers the possibility to renegotiate ownership does not alter optimal punishment continuation payoffs.

Note first that although the literature on renegotiation in repeated games did not reach a definitive agreement so far, and the issue remains open to debate, the *strong perfect equilibrium* proposed earlier by Rubinstein (1980) satisfies all weaker properties – if it exists.[14]

[14] The debate on renegotiation in repeated games includes work by Farrell and Maskin (1989), Bernheim and Rey (1989), van Damme (1989), Pearce (1987), Asheim (1992), Bergin and MacLeod (1993), and Abreu, Pearce and Stacchetti (1993). An equilibrium is strong if no coalition of players can gain by a joint deviation. An equilibrium is a strong perfect equilibrium if this is the case for all subgames. A strong perfect equilibrium is weakly and strongly renegotiation proof since these concepts only compare to a subset of outcomes – i.e. to subgame equilibria of the given equilibrium (for weak renegotiation proofness) or to other renegotiation proof equilibria (for strong renegotiation proofness). While the weak renegotiation proof equilibrium always exists,

We can now state the following.

Theorem 2 *Assumption 2 implies:*

1. *The possibility to renegotiate does not affect the set of relational efficient ownership structures* Ω^{**}.
2. *For* $\omega \in \Omega^{**}$ *there exist optimal punishment equilibria which are (i) efficient on each punishment path, or "strongly perfect" (Rubinstein, 1980); and (ii) robust against mistakes as defined by Sergerström (1988).*

The main conclusion beyond theorem 1 is that the set of relational efficient ownership structures Ω^{**} does not depend on what can be renegotiated, but only depends on optimal punishments and is again given by

$$\Omega^{**} := \left\{\omega \left| \underline{\delta}^{**}(\omega) \leq \underline{\delta}^{**}\left(\omega'\right) \ \forall \omega' \in \Omega \right.\right\},$$

as stated in Theorem 1.

The intuition behind the result is as follows. Our contracting environment inherited the standard, natural assumption made in the literature that players can exchange monetary transfers. Levin (1998, 2003) noted (for repeated games with incomplete information) that when monetary transfers are possible, optimal punishments can take the simple, natural form of a "fine" levied against a player that defects, with play remaining on the equilibrium path in the continuation subgame. This last feature, together with the asymmetry in payoffs these strategies generate, guarantees that if the original cooperation equilibrium is on the Pareto frontier, then it is a strong perfect equilibrium.[15] In addition, by keeping play on the Pareto efficient path, these strategies are as robust as feasible with respect to mistakes. Our model is not a repeated game as the possibility to renegotiate and change ownership structure in each period makes it a fully dynamic game. Still, it turns out that natural, optimal asymmetric punishment strategies based on monetary fines can be constructed easily in our framework, with effects summarized by Theorem 2. Renegotiation of ownership never takes place in the punishment path as the original ownership structure and equilibrium are efficient, so that no renegotiation (of ownership, strategies or both) can bring the Pareto improvement necessary to have both players' agreeing.

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the strong renegotiation proof equilibrium and thereby also the strong perfect equilibrium may fail to exist.

[15] The ability of asymmetric punishments to ensure renegotiation was first noted by Farrell and Maskin (1989) and van Damme (1989).

3.3 Renegotiation of ownership only

Even though all results until now point at the efficiency criterium identified by Theorem 1 as the most relevant to real world agents, we mentioned that there is no general consensus yet in the profession on renegotiation in repeated and dynamic games. One could defend Garvey's and BGM's choice of allowing for renegotiation of ownership but not of strategies by noting that renegotiating away from the punishment phase of grim trigger strategies to a new cooperation equilibrium would entail building again trust between parties after trust has been unilaterally broken, and that this may be harder to achieve than a reallocation of ownership (which requires no trust given that ownership changes can be implemented contractually). This line of reasoning, though, is irrelevant if grim trigger strategies are not optimal. Moreover, it is hard to believe that real players would bargain symmetrically after one has been cheated upon and the other has defected unilaterally.

Still, to be able to compare the change of assumptions, we also investigate the consequences of "constrained relational efficiency" (and the related optimal transfer) relevant under the assumption that only ownership can be renegotiated and agents are restricted to grim trigger strategies. That is, to clarify the effect of (i) restricting renegotiation possibilities to ownership structure and (ii) restricting focus to grim trigger punishment, we follow Garvey (1995) and BGM (2001, 2002) in assuming the following.

Assumption 3.

(a) *Renegotiation of ownership structure is possible and costless ($z = 0$);*

(b) *Renegotiation of strategies is not possible.*

(c) *If any party defects, all parties play the stage game equilibrium forever after.*

When agents use grim-trigger strategies to support efficient investment, the possibility to renegotiate ownership (at zero or low cost) to the Pareto-optimal level at each node down the game tree makes the efficient investment harder to sustain in equilibrium. This is of course because punishments are weaker, since renegotiation of ownership implies a lower bound on time-average payoffs during the punishment phase, namely the payoffs obtained from non-cooperative investment under the "short-term efficient" ownership structure ω^*.

When agents use grim-trigger strategies and in each period can costlessly renegotiate ownership to the Pareto-optimal level, the efficient investment can be supported in equilibrium in the dynamic game $\Gamma(\delta, \Omega)$ for the same range of discount factors at which it can be supported in equilibrium in a corresponding discounted repeated game. For this latter game the incentive compatibility condition to cooperate supported by grim-trigger-strategies is given by

$$\frac{1}{1-\delta}\left(c_i(\omega) + \theta_i\right) \geq b_i(\omega) + \frac{\delta}{1-\delta}\left(d_i(\omega) + S\right),$$

where $c_i(\omega) + \theta_i$ goes to agent i if both agents cooperate. The new sharing rule θ_1 depends now on per-period gains from renegotiation denoted by S. One can then state the following.

Theorem 3 *Under Assumption 3, if agents support cooperation by means of grim-trigger strategies, then:*

1. *The range of supporting discount factors is maximal by paying the transfer*

$$\theta^{CRE} = \frac{(b_1(\omega) - c_1(\omega))(b_2(\omega) - d_2(\omega) - S) - (b_2(\omega) - c_2(\omega))(b_1(\omega) - d_1(\omega) - S)}{b_1(\omega) - d_1(\omega) + b_2(\omega) - d_2(\omega) - 2S}.$$

   where $S = \frac{1}{2}(D^* - D(\omega))$*;*

2. *Ownership structure ω is "*constrained *relational efficient" for the repeated interaction iff it minimizes aggregated betray payoffs, that is*

$$\omega \in \Omega^{CRE} = \left\{\omega \left| B(\omega) \leq B(\omega') \;\; \forall \omega' \in \Omega \right.\right\}.$$

We use the lable "constrained relational efficient" (CRE) – the constraint being grim trigger strategies – because grim-trigger strategies are generally not optimal. Stronger punishments can easily be build that enlarge the set of discount factors at which the cooperative level of investment can be supported in equilibrium. In practice, assumption 3b sterilizes all potential effects of ownership on the "punishment concern" (on the punishment phase), leaving only the effects on the "deviation concern" (short run gains from defections).

Note that the "deviation concern" tends to be related to the size of the hold-up problem: a short term efficient ownership structure minimizes the hold-up problem $(e_i^c - e_i^d(\omega))$, and a small hold-up tends to reduce incentives to defect in the corresponding relation $(u_i\left(\left(e_i^d(\omega), e_j^c\right), \omega\right) - u_i(e_i^c, e_j^c))$. Therefore, the sterilization of the "punishment concern" through assumption 3b makes the (constrained) relational efficient structure appear more similar to the short term efficient one. Because of this, in the specific examples in section 4.1, when only ownership can be renegotiated and agents use grim-trigger strategies short term efficient ownership structures are also constrained relational efficient.[16]

4 Ownership Rights and Specific Models

Our main object of interest in this article are ownership structures. So far we have not assumed any structure on property rights. Therefore, the basic structure of this theory holds for any exogenous parameter that influences bargaining positions (or the threat point if bargaining

[16] This sterilization led Bragelien (2002) – who maintains BGM's assumptions of ownership renegotiation and grim trigger strategies – to conclude that there is a tendency for short term optimal ownership structures to be also optimal for relational contracts.

breaks down). This could be any other part of the legal framework or environmental and technological conditions etc. In this section, however, we want to be more explicit about ownership rights, since in many real situations "ownership rights" can be decomposed into "asset ownership" as promoted by Hart and Moore (1990).

Let A denote a set of nonhuman assets (machines, buildings, land, client lists, patents, copy rights, etc.).

Definition 4 *A partition $\omega = (A_1, A_2, A_{12})$ is called* "two-party-ownership-structure"*. The subset A_i are privately owned assets of party i and A_{12} are jointly owned assets.*

Our interpretation of ownership follows the tradition of Hart and Moore. Ownership of an asset is defined as veto power over the use of the asset. Joint ownership means that every owner has veto power, i.e. a jointly owned asset can only be used by consent of all owners. In contrast to agents' actions that are observable but not verifiable ownership structure ω is assumed observable and verifiable in court. The following definition differentiates several cases.

Definition 5 *We will call a two party ownership structure $\omega = (A_1, A_2, A_{12})$:*

1. Joint Ownership (J), *if all assets are owned jointly $\omega^J = (\emptyset, \emptyset, A)$ or $A_{12} = A$ and $A_i = \emptyset$ for $i = 1, 2$;*
2. Integration (I), *if one party owns all assets $\omega^I = (A, \emptyset, \emptyset)$ or $A_1 = A$ and $A_2 = A_{12} = \emptyset$;*
3. Outsourcing (O), *if there are no jointly owned assets and both parties own assets $\omega^O = (A_1, A_2, \emptyset)$ or $A_{12} = \emptyset, A_i \neq \emptyset$ for $i = 1, 2$;*
4. Mixed Ownership (M), *if there are privately owned assets for at least one party, say 1, and jointly owned assets $\omega^M = (A_1, A_2, A_{12})$, and $A_1, A_{12} \neq \emptyset$.*

In a sense mixed ownership is the generic case since every other ownership structure can be approximated by a converging sequence of mixed ownership structures. The remainder of this article will be devoted to the question which of the previously defined ownership structures are relational efficient under different specifications of the model.

4.1 Garvey and Halonen

Garvey (1995) and Halonen (2002) are subspecifications of the following more general structure:

$$
\begin{aligned}
E_i &= \mathbb{R}_+ \\
Q(e_1, e_2) &= q_1 e_1 + q_2 e_2 \\
P_i(e, \omega) &= p_i e_i + r_{-i} e_{-i} \text{ with } p_i + r_i \leq q_1 + q_2 \\
p_i, q_i, r_i &\in \mathbb{R}_+
\end{aligned}
$$

and thereby

$$u_i\left(e,(\lambda_1,\lambda_2)\right)=\frac{1}{2}\left(\left(q_i+\lambda_i\right)e_i+\left(q_{-i}-\lambda_{-i}\right)e_{-i}\right)-C_i\left(e_i\right)$$

where $\lambda_i \equiv p_i - r_i \in [-q_i, q_i]$ for $i = 1, 2$. The boundary case $(\lambda_1, \lambda_2) = (q_1, q_2)$ implies $p_i = q_i$ and $r_i = 0$. In this formulation this is the unique case where the hold-up problem disappears and there is no gain in forming a relationship or $Q(e_1, e_2) - P_1(e, \omega) - P_2(e, \omega) = 0$. Cost functions are power functions given as

$$C_i\left(e_i\right)=k_i e_i^{\gamma}$$

with $\gamma > 1$. Ownership structures Ω are parametrized as subsets of 2-vectors

$$\omega=(\lambda_1,\lambda_2)\in\Omega\subset[-q_1,q_1]\times[-q_2,q_2]\subset\mathbb{R}^2.$$

Note that in this formulation ownership structures at the same time determine the value of the relationship $Q(e_1, e_2) - P_1(e, \omega) - P_2(e, \omega)$ because they are directly defined by its consequences on disagreement payoffs $P_i(e, \omega)$. Clearly, it is not realistic to assume that all $(\lambda_1, \lambda_2) \in [-q_1, q_1] \times [-q_2, q_2]$ are available in practice. In order to design efficient ownership, however, it is an interesting exercise to see which constellation of disagreement payoffs induces relational efficient ownership.

Proposition 1 *Consider the set of ownership structures given by the 2-dimensional parameter space* $\Omega = [-q_1, q_1] \times [-q_2, q_2] \subset \mathbb{R}^2$.

1. *The relational efficient ownership structure* $(\lambda_1, \lambda_2)^{**}$ *generally differs from the short term efficient* $(\lambda_1, \lambda_2)^{*}$ *and depends on parameter values, in particular on the cost function parameter* γ. *Numerical computations suggest a pattern where*

$$\begin{aligned}(\lambda_1,\lambda_2)^{**} &= (\lambda_1,\lambda_2)^{*} \text{ for } \gamma<2\\ (\lambda_1,\lambda_2)^{**} &\neq (\lambda_1,\lambda_2)^{*} \text{ for } \gamma>2.\end{aligned}$$

2. *If the cost function is a power function the short term efficient ownership structure and the* constrained *relational efficient ownership structure coincide. Formally,* $\forall\gamma > 0$, $(\lambda_1, \lambda_2)^{*} = (\lambda_1, \lambda_2)^{CRE}$.

3. *For quadratic cost functions ownership is irrelevant, that is all ownership structures* $(\lambda_1, \lambda_2) \in \Omega$ *including the whole set of mixed ownership structures are relational efficient. Formally,* $\gamma = 2 \Rightarrow \Omega^{**} = \Omega$.

For the first claim one has to identify cases where relational efficienty and short term efficiency do not single out the same ownership structure. We do this by numerical examples

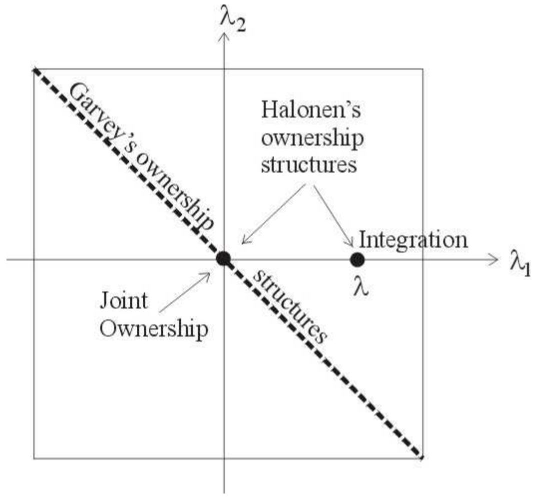


Figure 1: Garvey's and Halonen's ownership structures in a λ_1, λ_2-diagram.

(see appendix). These computations suggest that the pattern based on the cost function parameter γ identified by Halonen (2002) is rather general. Claim 2 shows that in this simple framework sterilizing the effects of ownership on punishments ensures that short term efficient structures are also constrained relational efficient (BGM 2001 show that this is not the case when moral hazard is present; see Section 5). Finally, Claim 3 tells us that quadratic cost functions have very special (non-generic) implications, which makes it unfortunate that Garvey and BGM focus much of their analysis on this case.

The set of ownership structures we investigated in proposition 1 is larger than the one in the original papers, where a finite (Halonen) or one-dimensional (Garvey) subset of $[-q_1, q_1] \times [-q_2, q_2]$ is considered. Figure 1 illuminates Garvey's and Halonen's ownership specifications within this formulation. A restricted set of feasible ownership structures may lead to different results (since the here identified efficient ownership structures may then be unavailable). In the next subsections we briefly investigate the effect of restricting the choice of ownership structures to the sets originally considered by the authors.

4.1.1 Garvey's Model

The subspecifications in Garvey's model are

$$\begin{aligned} Q(e_1, e_2) &= e_1 + e_2 \\ C_i(e_i) &= \frac{1}{2\alpha_i} e_i^2 \text{ with } \alpha_1 = \alpha, \alpha_2 = 1 - \alpha \\ P_i(e, \omega) &= \rho_i (e_1 + e_2) \text{ with } \rho_1 = \rho, \rho_2 = 1 - \rho \\ \rho &\in \Omega = [0, 1] \text{ continuum of ownership structures} \end{aligned}$$

that is $q_1 = q_2 = 1$, $p_1 = r_2 = \rho$, $p_2 = r_1 = 1 - \rho$, $\lambda_1 = -\lambda_2 = 2\rho - 1$ and for the cost function $k_1 = \frac{1}{2\alpha_i}, \gamma = 2$. Applying our theory (theorems 1, 2, and 3) yields that Garvey's results are not valid. Instead we obtain:

Proposition 2 *(i) In Garvey's specification, independent of what can be renegotiated (Assumptions 1 or 2), the ownership structure is irrelevant for relational efficiency:* $\Omega^{**} = \Omega$.

(ii) If instead in Garvey's specification agents are restricted to grim trigger strategies and only ownership can be renegotiated (Assumption 3), then the short term efficient ownership structure and the constrained relational efficient ownership structure coincide:

$$\Omega^{*} = \Omega^{CRE} = \{\alpha\}.$$

A remark is here in order.

Remark 1 *As already emphasized, in our view the relevant notion is relational efficiency. If, however, we subscribe to Assumption 3 we expect to replicate Garvey's results. Why does our reformulation contradict the original paper by Garvey? The trouble in Garvey's formulation is that he assumes for the transfer* θ *to be exogenously defined by*

$$\begin{aligned} c_1 + \theta &= \rho\left(c_1 + c_2\right) \text{ or} \\ \theta &= \rho c_2 - (1-\rho)\, c_1 \neq \theta^{CRE}. \end{aligned}$$

This additional restriction is relaxed in our model and is the reason behind Garvey's mistaken conclusion: His exogenously given transfer payment is not relational efficient, and we found no special reason to single it out.

4.1.2 Halonen's Model

In contrast to Garvey, Halonen only compares joint ownership ω^J with full integration ω^I (see figure 1). Her specifications within this formulation are

$$\begin{aligned} \omega &\in \Omega = \left\{\omega^J, \omega^I\right\} = \{0, \lambda\} \\ \omega^J &= 0 = \text{Joint Ownership} \\ \omega^I &= \lambda = \text{Integration with } \lambda \in [0,1] \\ Q\left(e_1, e_2\right) &= e_1 + e_2 \\ P_1\left(e, \omega\right) &= \omega e_1 \\ P_2\left(e, \omega\right) &= 0, \end{aligned}$$

that is $q_1 = q_2 = 1$, $p_2 = r_1 = r_2 = 0$, $p_1 = \omega$ and for the cost structure $k_1 = 1, \gamma > 1$ or

$$C_i\left(e_i\right) = e_i^{\gamma} \text{ with } \gamma > 1$$

Lemma 1 *The short term efficient ownership structure defined by (4) is given by full integration:* $\omega^* = \omega^I$.

Applying our theory to Halonen's specification generalizes Halonen's observation that joint ownership may be optimal in a dynamic investment relation.

Proposition 3 *(i) Suppose agents can choose optimally equilibrium strategies. Then independent of what can be renegotiated (Assumptions 1 or 2), in Halonen's specification the relational efficient ownership structure is*

$$\Omega^{**} = \begin{cases} \omega^I & \text{for} \quad \gamma \in (1,2) \\ \{\omega^J, \omega^I\} & \text{for} \quad \gamma = 2 \\ \omega^J & \text{for} \quad \gamma > 2 \end{cases},$$

where $C_i(e_i) = e_i^\gamma$ *with* $\gamma > 1$.

(ii) If instead agents are restricted to use grim trigger strategies and ownership can be renegotiated (Assumption 3), then the constrained relational efficient ownership structure coincides with the short term efficient ownership structure, and is full integration: $\Omega^{CRE} = \{\omega^I\}$.

4.2 Baker, Gibbons and Murphy's Model

BGM's model is not exactly an example of our theory since they assume moral hazard (i.e. player's actions are not directly observable, only their stochastic consequences are). Therefore, parts of our theory do not apply without adjustment. However, our criticism of the restriction to grim trigger strategies carries over to BGM's model. In this section we shortly investigate the dependence of BGM's (2001) and (2002) results with respect to this restriction, and show that our earlier conclusions are not affected by the presence of moral hazard.

BGM only compare outsourcing ω^O with full integration ω^I (there named "employment"). To be able to compare BGM's formulation to the previous setup we add joint ownership ω^{JO} that is $\Omega = \{\omega^O, \omega^I, \omega^{JO}\}$. In their specification only one party (interpreted as upstream party) can invest in the joint project. We can now confirm and extend the main proposition in BGM (2002).

Proposition 4 *In BGM's model when agents are free to choose strategies optimally (Assumption 2) asset ownership affects the parties' temptations to renege on a relational contract and at the same time the maximal punishment they may be subject to, and hence it affects whether a given relational contract is feasible. Moreover, joint ownership* ω^{JO} *dominates (is never less relational efficient than) outsourcing* ω^O *although it may be less efficient than integration, depending on further specification.*

The proof in the appendix contains a reformulation of BGM in our notation. On the other hand, BGM's (2001) result on the "impossibility for firms to mimic spot markets", as they formulate it, turns out to rely heavily on their restriction on strategies and is not valid in a more general formulation where parties can choose strategies optimally.

Proposition 5 *"Bringing the market inside the firm". Consider integration ownership structure $\omega = \omega^I$ interpreted as a firm. The relational contract that replicates the payoffs of the stage game under outsourcing (interpreted as market outcome), also satisfies BGM's necessary and sufficient incentive constraints if players punish optimally and are sufficiently patient.*

5 Conclusion

We developed a model for the analysis of optimal allocations of property rights in long term relations, where investment levels are not contractible and must be sustained in equilibrium. Applying elements of repeated game analysis to our framework reveals two weaknesses of the previous literature on the subject. First, previous work's restriction to grim trigger strategies, while technically convenient, is not only objectionable from a theoretical viewpoint because such strategies are not robust against mistakes and renegotiation: our analysis shows that this restriction may turn conclusions upside down. Second, in line with the prevailing tradition in contract theory previous research has devoted much attention to renegotiation of ownership, but no attention at all to the possibility to renegotiate strategies. Opening the analysis to optimal punishment strategies that are robust against mistakes and renegotiation ironically reinvigorates results of models that exclude renegotiation (e.g. Halonen 2002), since we find that the same ownership structures are relational efficient without any renegotiation and with renegotiation of both, ownership and strategies. Our results confirm that in a dynamic world the optimal allocation of property rights does not in general coincide with the static one identified by the Grossman-Hart-Moore paradigm, but we identify situations where it does.

6 Appendix: Proofs

Theorem 1. **Proof.** Incentive compatibility is given by

$$\begin{aligned}\frac{1}{1-\delta}\left(c_i\left(\omega\right)+\theta_i\right) &\geq b_i\left(\omega\right)+\frac{\delta}{1-\delta}v_i\left(\omega\right) \Leftrightarrow \\ c_i\left(\omega\right)+\theta_i &\geq \left(1-\delta\right)b_i\left(\omega\right)+\delta v_i\left(\omega\right)\end{aligned}$$

or

$$\underline{\delta}_i=\frac{b_i\left(\omega\right)-c_i\left(\omega\right)-\theta_i}{b_i\left(\omega\right)-v_i\left(\omega\right)}.$$

Incentive compatibility is satisfied for both players if no player has a discount factor below

$$\underline{\delta}=\max\left\{\underline{\delta}_1,\underline{\delta}_2\right\}.$$

The "optimal sharing rule" θ^{**} minimizes $\underline{\delta}$ and hence satisfies

$$\frac{b_1\left(\omega\right)-c_1\left(\omega\right)-\theta^{**}}{b_1\left(\omega\right)-v_1\left(\omega\right)}=\frac{b_2\left(\omega\right)-c_2\left(\omega\right)+\theta^{**}}{b_2\left(\omega\right)-v_2\left(\omega\right)}$$

this yields

$$\theta^{**}\left(\omega\right)=\frac{\left(b_1\left(\omega\right)-c_1\left(\omega\right)\right)\left(b_2\left(\omega\right)-v_2\left(\omega\right)\right)-\left(b_2\left(\omega\right)-c_2\left(\omega\right)\right)\left(b_1\left(\omega\right)-v_1\left(\omega\right)\right)}{b_2\left(\omega\right)-v_2\left(\omega\right)+b_1\left(\omega\right)-v_1\left(\omega\right)} \tag{5}$$

and

$$\underline{\delta}^{**}\left(\omega\right)=\frac{b_1\left(\omega\right)-c_1\left(\omega\right)+b_2\left(\omega\right)-c_2\left(\omega\right)}{b_1\left(\omega\right)-v_1\left(\omega\right)+b_2\left(\omega\right)-v_2\left(\omega\right)} \tag{6}$$

or

$$\underline{\delta}^{**}\left(\omega\right)=\frac{B\left(\omega\right)-S^*}{B\left(\omega\right)-V\left(\omega\right)}. \tag{7}$$

■

Theorem 2. **Proof.** The proof proceeds (i) to define strategies as in Levin (1998, 2003), and then (ii) to show that in our model these strategies satisfy the required properties.

Definition 6 *Strategies:*[17]

Start playing from Phase 1.

Phase 1:

Invest efficiently e_i^c *and pay the equilibrium transfer* θ*; if an agent* i *deviates, start Phase 2.*

[17] The strategies are inspired by – but slightly different from (due to monetary transfers) – by those discussed in van Damme (1989), Farrell and Maskin (1989) and Segerström (1988). "Asymmetry" refers to the different behavior of the "defector" and the "afflicted party" off equilibrium, in contrast to grim trigger strategies where both parties (also the defector) punish. Asymmetry does not mean that players in the same role (e.g. defector) behave differently.

Phase 2:

Agent $j \neq i$: If at the beginning of the period you receive transfer F^{ij} from agent i, go back to Phase 1; otherwise choose $e_j = 0$ and start again Phase 2.

Agent i : Pay fine F^{ij} to agent j and go back to Phase 1.

If a player deviates in Phase 2, re-starts Phase 2 against that player.

We first show that these strategies with maximal F^{ij} are a SPNE and constitute an optimal punishment. First note that $v_i(\omega) = P_i\left(e_i^b(0), e_j = 0, \omega\right)$ and that defecting during Phase 2 does not increase agent i's continuation payoff. Now note that for player j, defecting in Phase 2 is not profitable if

$$(1-\delta)\, P_j\left(e_i^b(0), e_j = 0, \omega\right) + \delta S^* \geq (1-\delta)\, P_j(e_i^b(0), e_j^b(e_i^b(0)), \omega),$$

which is always satisfied when $\delta > \underline{\delta}^{**}$, which implies that if the strategies constitute an equilibrium, the equilibrium is subgame perfect.

Now note that for any given equilibrium transfer θ, if agent i defects unilaterally from restitution strategies with $F^{ij} = \frac{c_i - v_i(\omega) + \theta}{1-\delta}$, he expects $b_i(\omega)$ from the period of the defection and $\delta U_i(\omega) = \frac{\delta}{1-\delta} v_i(\omega)$ from the rest of the game. Hence, a defection is deterred when $c_i(\omega) + \theta \geq (1-\delta)\, b_i(\omega) + \delta v_i(\omega)$, which is exactly the condition relevant with optimal punishments and no renegotiation of ownership, hence these strategies are optimal.

We now show that the equilibria in restitution strategies with maximal F^{ij} are strong perfect equilibria. Suppose $\omega \in \Omega^{**}$ (defined in theorem 3). After a defection by i, before the punishment phase starts i can propose j either to play alternative equilibrium strategies with the same ownership structure $\omega \in \Omega^{**}$, or to modify ownership structure, or to modify both ownership structure and equilibrium strategies. However, in the subgame after agent i defects from the given equilibrium, agent j's subgame equilibrium payoff at the beginning of phase 2 is the entire remaining value of the relationship (net surplus from the relation with efficient investment). Since $\omega \in \Omega^{**}$, this is strictly greater than any payoff he could obtain by renegotiating strategies and/or ownership using Nash bargaining, hence – even though $z = 0$ – renegotiation of ownership or/and of strategies cannot occur.

Finally, note that these strategies are robust against mistakes since they prescribe a transfer between agents to compensate for the damages caused by an eventual mistake, but do not switch to inefficient play. This concludes the proof. ■

Theorem 3. Proof. Incentive compatibility for trigger strategies

$$\frac{1}{1-\delta}\left(c_i(\omega) + \theta_i\right) \geq b_i(\omega) + \frac{\delta}{1-\delta}\left(d_i(\omega) + T\right)$$

yields

$$\underline{\delta}_i = \frac{b_i(\omega) - c_i(e^c, \omega) - \theta_i}{b_i(\omega) - d_i(\omega) - T}$$

and

$$\underline{\delta} = \max\{\underline{\delta}_1, \underline{\delta}_2\}$$

The "optimal sharing rule" θ^{CRE} minimizes $\underline{\delta}$ and hence satisfies

$$\frac{b_1(\omega) - c_1(\omega) - \theta^{CRE}}{b_1(\omega) - d_1(\omega) - T} = \frac{b_2(\omega) - c_2(\omega) + \theta^{CRE}}{b_2(\omega) - d_2(\omega) - T}$$

this yields

$$\theta^{CRE} = \frac{(b_1(\omega) - c_1(\omega))(b_2(\omega) - d_2(\omega) - T) - (b_2(\omega) - c_2(\omega))(b_1(\omega) - d_1(\omega) - T)}{b_1(\omega) - d_1(\omega) + b_2(\omega) - d_2(\omega) - 2T}$$

and

$$\begin{aligned}
\underline{\delta}^{CRE} &= \frac{b_1(\omega) - c_1(\omega) - \theta^{CRE}}{b_1(\omega) - d_1(\omega) - T} \\
&= \frac{b_1(\omega) - c_1(\omega) - \frac{(b_1(\omega)-c_1(\omega))(b_2(\omega)-d_2(\omega)-T)-(b_2(\omega)-c_2(\omega))(b_1(\omega)-d_1(\omega)-T)}{b_1(\omega)-d_1(\omega)+b_2(\omega)-d_2(\omega)-2T}}{b_1(\omega) - d_1(\omega) - T} \\
&= \frac{b_1(\omega) - c_1(\omega) + b_2(\omega) - c_2(\omega)}{b_1(\omega) - d_1(\omega) + b_2(\omega) - d_2(\omega) - 2T} \\
\underline{\delta}^{CRE} &= \frac{b_1(\omega) - c_1(\omega) + b_2(\omega) - c_2(\omega)}{b_1(\omega) - d_1^* + b_2(\omega) - d_2^*}
\end{aligned}$$

or

$$\underline{\delta}^{CRE}(\omega) = \frac{B(\omega) - S^*}{B(\omega) - D^*}. \tag{8}$$

$\underline{\delta}^{CRE}(\omega)$ strictly increases with $B(\omega)$ since $S^* > D^*$. This proves theorem 3. ■

Proposition 1. **Proof.** This specification yields $e_i^c = \left(\frac{q_i}{\gamma k_i}\right)^{\frac{1}{\gamma-1}}$, $e_i^d = \left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}}$ and

$$\begin{aligned}
c_i(\lambda_1, \lambda_2) &= \frac{1}{2}\left((q_i + \lambda_i) e_i^c + (q_{-i} - \lambda_{-i}) e_{-i}^c\right) - k_i (e_i^c)^\gamma \\
&= \frac{1}{2}\left((q_i + \lambda_i)\left(\frac{q_i}{\gamma k_i}\right)^{\frac{1}{\gamma-1}} + (q_{-i} - \lambda_{-i})\left(\frac{q_{-i}}{\gamma k_{-i}}\right)^{\frac{1}{\gamma-1}}\right) - k_i\left(\frac{q_i}{\gamma k_i}\right)^{\frac{\gamma}{\gamma-1}}
\end{aligned}$$

and

$$\begin{aligned}
S^* &= q_1\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}} + q_2\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}} - k_1\left(\frac{q_1}{\gamma k_1}\right)^{\frac{\gamma}{\gamma-1}} - k_2\left(\frac{q_2}{\gamma k_2}\right)^{\frac{\gamma}{\gamma-1}} \\
&= q_1\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}} + q_2\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}} - \frac{q_1}{\gamma}\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}} - \frac{q_2}{\gamma}\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}} \\
&= \frac{(\gamma-1)}{\gamma}\left[q_1\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}} + q_2\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}}\right] \\
d_i(\lambda_1, \lambda_2) &= \frac{1}{2}\left((q_i + \lambda_i) e_i^d + (q_{-i} - \lambda_{-i}) e_{-i}^d\right) - k_i\left(e_i^d\right)^\gamma \\
&= \frac{1}{2}\left((q_i + \lambda_i)\left(\frac{q_i + \lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}} + (q_{-i} - \lambda_{-i})\left(\frac{q_{-i} + \lambda_{-i}}{2\gamma k_{-i}}\right)^{\frac{1}{\gamma-1}}\right) - k_i\left(\frac{q_i + \lambda_i}{2\gamma k_i}\right)^{\frac{\gamma}{\gamma-1}}
\end{aligned}$$

Further, we obtain

$$
\begin{aligned}
D(\lambda_1,\lambda_2) &= V(\lambda_1,\lambda_2) = d_1(\omega) + d_2(\omega) = \\
&= q_1\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}} - k_1\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{\gamma}{\gamma-1}} + q_2\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{1}{\gamma-1}} - k_2\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{\gamma}{\gamma-1}} \\
&= \left(q_1 - \frac{q_1+\lambda_1}{2\gamma}\right)\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}} + \left(q_2 - \frac{q_2+\lambda_2}{2\gamma}\right)\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{1}{\gamma-1}}
\end{aligned}
$$

For the short term efficient ownership structure first order conditions yield

$$
\begin{aligned}
\frac{\partial D(\lambda_1,\lambda_2)}{\partial \lambda_i} &= \frac{\partial}{\partial \lambda_i}\left(q_1\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}} - k_1\left(\frac{q_i+\lambda_1}{2\gamma k_1}\right)^{\frac{\gamma}{\gamma-1}}\right) \\
&= \frac{q_i-\lambda_i}{2(\gamma-1)(q_i+\lambda_i)}\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}} = 0 \text{ for} \\
\lambda_i &= q_i.
\end{aligned}
$$

Next, calculate

$$
\begin{aligned}
b_i(\lambda_1,\lambda_2) &= \frac{1}{2}\left((q_i+\lambda_i)e_i^d + (q_{-i}-\lambda_{-i})e_{-i}^c\right) - k_i\left(e_i^d\right)^\gamma \\
&= \frac{1}{2}\left((q_i+\lambda_i)\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}} + (q_{-i}-\lambda_{-i})\left(\frac{q_{-i}}{\gamma k_{-i}}\right)^{\frac{1}{\gamma-1}}\right) - k_i\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{\gamma}{\gamma-1}} \\
&= (\gamma-1)k_i\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{\gamma}{\gamma-1}} + \frac{1}{2}(q_{-i}-\lambda_{-i})\left(\frac{q_{-i}}{\gamma k_{-i}}\right)^{\frac{1}{\gamma-1}}
\end{aligned}
$$

and

$$
\begin{aligned}
B(\lambda_1,\lambda_2) &= b_1(\omega) + b_2(\omega) \\
&= (\gamma-1)k_1\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{\gamma}{\gamma-1}} + \frac{1}{2}(q_2-\lambda_2)\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}} \\
&\quad + (\gamma-1)k_2\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{\gamma}{\gamma-1}} + \frac{1}{2}(q_1-\lambda_1)\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}} \\
&= (\gamma-1)\frac{q_1+\lambda_1}{2\gamma}\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}} + \frac{1}{2}(q_1-\lambda_1)\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}} \\
&\quad + (\gamma-1)\frac{q_2+\lambda_2}{2\gamma}\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{1}{\gamma-1}} + \frac{1}{2}(q_2-\lambda_2)\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}}
\end{aligned}
$$

To calculate the constrained relational efficient ownership structure apply theorem 3 and calculate first order conditions:

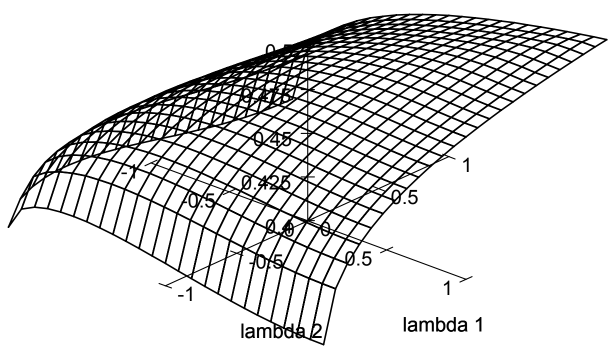
$$\begin{aligned}
\frac{\partial B(\lambda_1,\lambda_2)}{\partial \lambda_i} &= \frac{\partial}{\partial \lambda_i}\left((\gamma-1)\frac{q_i+\lambda_i}{2\gamma}\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}}+\frac{1}{2}(q_i-\lambda_i)\left(\frac{q_i}{\gamma k_i}\right)^{\frac{1}{\gamma-1}}\right) \\
&= \frac{1}{2}\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}}-\frac{1}{2}\left(\frac{q_i}{\gamma k_i}\right)^{\frac{1}{\gamma-1}} \\
&= \frac{1}{2}\left(\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}}-\left(\frac{q_i}{\gamma k_i}\right)^{\frac{1}{\gamma-1}}\right)=0 \text{ for} \\
\frac{q_i+\lambda_i}{2\gamma k_i} &= \frac{q_i}{\gamma k_i} \Leftrightarrow \lambda_i = q_i.
\end{aligned}$$

This is identical with the short term efficient ownership structure and hence shows claim 2 of proposition 1. To analyze relational efficiency theorems 1 and 2 yield

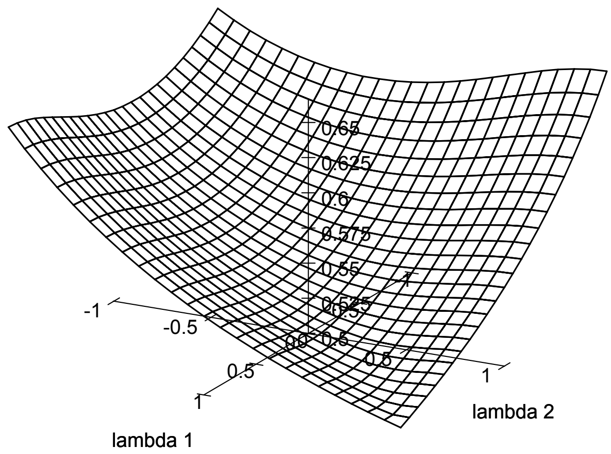
$$\underline{\delta}^{**}(\lambda_1,\lambda_2)=\frac{B(\lambda_1,\lambda_2)-S^*}{B(\lambda_1,\lambda_2)-V(\lambda_1,\lambda_2)}=$$

$$=\frac{\frac{(\gamma-1)(q_1+\lambda_1)}{\gamma}\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}}+\frac{2q_1-\gamma(q_1+\lambda_1)}{\gamma}\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}}+\frac{(\gamma-1)(q_2+\lambda_2)}{\gamma}\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{1}{\gamma-1}}+\frac{2q_2-\gamma(q_2+\lambda_2)}{\gamma}\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}}}{(q_1-\lambda_1)\left(\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}}-\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}}\right)+(q_2-\lambda_2)\left(\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}}-\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{1}{\gamma-1}}\right)}$$

To show claim 1. note that maximization of $D(\lambda_1,\lambda_2)$ and minimization of $\frac{B(\lambda_1,\lambda_2)-S^*}{B(\lambda_1,\lambda_2)-V(\lambda_1,\lambda_2)}$ may yield different results. The following pictures show $\underline{\delta}^{**}(\lambda_1,\lambda_2)=\frac{B(\lambda_1,\lambda_2)-S^*}{B(\lambda_1,\lambda_2)-V(\lambda_1,\lambda_2)}$ for two salient cost function specifications and $q_1=q_2=1$.



$\underline{\delta}^{**}(\lambda_1,\lambda_2)$ for cost functions $C_i(e_i)=2e_i^{2.5}$



$\underline{\delta}^{**}(\lambda_1, \lambda_2)$ for cost functions $C_i(e_i) = 2e_i^{1.5}$

Finally, for claim 3 the special case $\gamma = 2$ expressions simplify to

$$
\begin{aligned}
e_i^c &= \frac{q_i}{2k_i} \\
e_i^d &= \frac{q_i + \lambda_i}{4k_i} \\
c_i(\lambda_1, \lambda_2) &= \frac{1}{2}\left((q_i + \lambda_i)\frac{q_i}{2k_i} + (q_{-i} - \lambda_{-i})\frac{q_{-i}}{2k_{-i}}\right) - k_i\left(\frac{q_i}{2k_i}\right)^2 \\
S^* &= \frac{q_1^2}{4k_1} + \frac{q_2^2}{4k_2} \\
d_i(\lambda_1, \lambda_2) &= \frac{1}{2}\left((q_i + \lambda_i)\frac{q_i + \lambda_i}{4k_i} + (q_{-i} - \lambda_{-i})\frac{q_{-i} + \lambda_{-i}}{4k_{-i}}\right) - k_i\left(\frac{q_i + \lambda_i}{4k_i}\right)^2 \\
&= \frac{(q_i + \lambda_i)^2}{16k_i} + \frac{q_{-i}^2 - \lambda_{-i}^2}{8k_{-i}}
\end{aligned}
$$

and thereby

$$
\begin{aligned}
D(\lambda_1, \lambda_2) &= q_1\frac{q_1 + \lambda_1}{4k_1} + q_2\frac{q_2 + \lambda_2}{4k_2} - k_1\left(\frac{q_1 + \lambda_1}{4k_1}\right)^2 - k_2\left(\frac{q_2 + \lambda_2}{4k_2}\right)^2 \\
&= \frac{(q_1 + \lambda_1)^2}{16k_1} + \frac{q_2^2 - \lambda_2^2}{8k_2} + \frac{(q_2 + \lambda_2)^2}{16k_2} + \frac{q_1^2 - \lambda_1^2}{8k_1} \\
&= \frac{3q_1^2 + 2q_1\lambda_1 - \lambda_1^2}{16k_1} + \frac{3q_2^2 + 2q_2\lambda_2 - \lambda_2^2}{16k_2}
\end{aligned}
$$

and

$$\begin{aligned}
b_i(\lambda_1,\lambda_2) &= \frac{(q_i+\lambda_i)^2}{8k_i} + (q_{-i}-\lambda_{-i})\frac{q_{-i}}{4k_{-i}} - k_i\left(\frac{q_i+\lambda_i}{4k_i}\right)^2 \\
&= \frac{(q_i+\lambda_i)^2}{16k_i} + (q_{-i}-\lambda_{-i})\frac{q_{-i}}{4k_{-i}} \\
B(\lambda_1,\lambda_2) &= \frac{(q_1+\lambda_1)^2}{16k_1} + (q_2-\lambda_2)\frac{q_2}{4k_2} + \frac{(q_2+\lambda_2)^2}{16k_2} + (q_1-\lambda_1)\frac{q_1}{4k_1} \\
&= \frac{(q_1+\lambda_1)^2 + 4q_1(q_1-\lambda_1)}{16k_1} + \frac{(q_2+\lambda_2)^2 + 4q_2(q_2-\lambda_2)}{16k_2} \\
&= \frac{5q_1^2 - 2q_1\lambda_1 + \lambda_1^2}{16k_1} + \frac{5q_2^2 - 2q_2\lambda_2 + \lambda_2^2}{16k_2}.
\end{aligned}$$

Now plug in and obtain

$$\begin{aligned}
\frac{B(\lambda_1,\lambda_2)-S^*}{B(\lambda_1,\lambda_2)-V(\lambda_1,\lambda_2)} &= \frac{\frac{(q_1+\lambda_1)^2}{8k_1} + \frac{2q_1-2(q_1+\lambda_1)}{2}\frac{q_1}{2k_1} + \frac{(q_2+\lambda_2)^2}{8k_2} + \frac{2q_2-2(q_2+\lambda_2)}{2}\frac{q_2}{2k_2}}{(q_1-\lambda_1)\left(\frac{q_1}{2k_1}-\frac{q_1+\lambda_1}{4k_1}\right) + (q_2-\lambda_2)\left(\frac{q_2}{2k_2}-\frac{q_2+\lambda_2}{4k_2}\right)} \\
&= \frac{\frac{(q_1+\lambda_1)^2-4q_1\lambda_1}{8k_1} + \frac{(q_2+\lambda_2)^2-4q_2\lambda_2}{8k_2}}{\frac{2(q_1-\lambda_1)^2}{8k_1} + \frac{2(q_2-\lambda_2)^2}{8k_2}} \\
&= \frac{1}{2}
\end{aligned}$$

which does not depend on (λ_1,λ_2). This proves the last claim of the proposition. ■

Proposition 2. Proof. (i) The utility function (1) becomes

$$u_i(e,\rho) = \rho_i(e_1+e_2) - \frac{1}{2\alpha_i}e_i^2$$

This yields $e_i^c = \alpha_i, e_i^d = \rho_i\alpha_i$ and for Garvey's specification the crucial parameters (3a to 3g) of the stage game are given by

$$\begin{aligned}
b_i &= \frac{1}{2}\rho_i^2\alpha_i + \rho_i(1-\alpha_i) && \text{(9a)} \\
c_i &= \rho_i - \frac{1}{2}\alpha_i && \text{(9b)} \\
d_i &= \frac{1}{2}\rho_i^2\alpha_i + \rho_i(1-\rho_i)(1-\alpha_i) && \text{(9c)} \\
S^* &= \frac{1}{2} && \text{(9d)} \\
B(\rho) &= \frac{1}{2}\left(1+\alpha+\rho^2\right) - \rho\alpha && \text{(9e)} \\
D(\rho) &= V(\rho) = \frac{1}{2}\left(1-\alpha-\rho^2\right) + \rho\alpha && \text{(9f)}
\end{aligned}$$

As we have seen in the proof of proposition 1 plugging in parameters (9a - 9f) into equation (7) we obtain

$$\begin{aligned}\underline{\delta}^{**}(\rho) &= \frac{B(\rho)-S^*}{B(\rho)-D(\rho)} \\ &= \frac{1}{2}\end{aligned}$$

which implies the first claim, $\Omega^N = \Omega$. Apply definition (4) and theorem 3 to parameters (9a - 9f), and the second claim (ii) obtains. ■

Lemma 1. Proof. Halonen's specification yields

$$D(\omega) = \left(\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}} + \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right) + \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}} + \frac{1-\omega}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}\right)\gamma^{\frac{1}{\gamma-1}}.$$

The lemma follows from $D(\omega)$ being increasing with ω:

$$\begin{aligned}D'(\omega) &= \frac{d}{d\omega}\left(\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}} + \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right) + \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}} + \frac{1-\omega}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}\right)\gamma^{\frac{1}{\gamma-1}} \\ &= \gamma^{\frac{1}{\gamma-1}}\left(\frac{1}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}} - \frac{1}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}} + \frac{1}{2}\frac{1-\omega}{2}\frac{1}{\gamma-1}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}-1}\right) \\ &= \gamma^{\frac{1}{\gamma-1}}\left(\frac{1}{2}\frac{1-\omega}{2}\frac{1}{\gamma-1}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}-1}\right) \\ &> 0 \text{ for } \omega < 1\end{aligned}$$

■

Proposition 3. Proof. For Halonen's parameters equation (7) becomes

$$\begin{aligned}\underline{\delta}^{**}(\omega) &= \frac{B(\omega)-S^*}{B(\omega)-D(\omega)} \\ &= \frac{\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}} + \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right) + \frac{2-\omega}{2} - \frac{2(\gamma-1)}{\gamma}}{\frac{2-\omega}{2} - \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}} - \frac{1-\omega}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}}\end{aligned}$$

Figure 2 (and the corresponding projections for $\gamma < 2, \gamma = 2, \gamma > 2$) show $\underline{\delta}^{**}(\omega,\gamma)$ and confirm the first result. For the second statement, apply theorem 3 and verify that $B(\omega)$ decreases with ω:

$$\begin{aligned}B'(\omega) &= \frac{d}{d\omega}\left(\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}} + \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right) + \frac{2-\omega}{2}\right)\gamma^{\frac{1}{\gamma-1}} \\ &= \frac{1}{2}\gamma^{\frac{1}{\gamma-1}}\left(\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}} - 1\right) \\ &< 0 \text{ for } \omega < 1.\end{aligned}$$

■

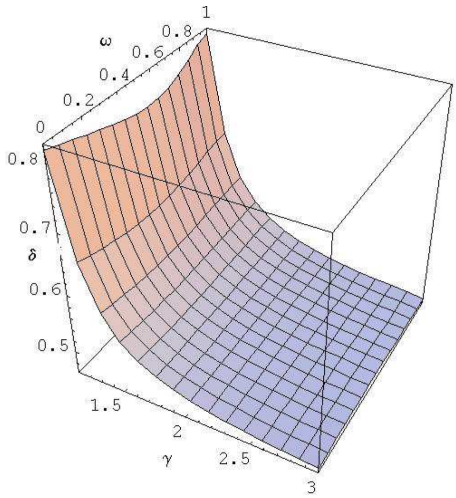


Figure 2: $\underline{\delta}^{**}(\omega, \gamma)$

Proposition 4. **Proof.** BGM's specifications in our notation are

$$\begin{aligned}
e_1 &= \left\{ \begin{array}{c} \mathbf{a} = (a_1, ..., a_n) \in \mathbb{R}^n_+, \text{ a vector} \\ \text{interpreted as"multi-task actions"} \end{array} \right\} \text{upstream party} \\
e_2 &= \emptyset \text{ downstream party has no action in the investment stage} \\
C_1(\mathbf{a}) &= \text{upstream party cost function} \\
C_2 &= 0 \\
Q(a) &= \left\{ \begin{array}{ccc} Q_L + \Delta Q & \text{with probability} & q(\mathbf{a}) \\ Q_L & \text{with probability} & 1 - q(\mathbf{a}) \end{array} \right. \\
P_1(a, \omega) &= \left\{ \begin{array}{ccccc} P_L + \Delta P & \text{with probability} & p(\mathbf{a}) & \text{for} & \omega = \omega^O \\ P_L & \text{with probability} & 1 - p(\mathbf{a}) & \text{for} & \omega = \omega^O \\ 0 & \text{for} & \omega = \omega^I & & \end{array} \right.
\end{aligned}$$

with $\Delta Q = Q_H - Q_L > 0$ and $\Delta P = P_H - P_L > 0$. Although the upstream partie's actions are not observable the downstream party can observe the realizations of P_1 and Q. Hence, the transfer payment may depend on them. In this setup the transfer or the so called *relational compensation contract* is

$$\theta_1 = -\theta_2 = s + b_{jk}$$

where s denotes a fixed salary and b_{jk} is a bonus payment that depends on the realizations of $Q = Q_j$ and $P = P_k$ with $j, k = H, L$. Corresponding to our notation let $S^* = Q_L + q(e^c)\Delta Q - C_1(e^c)$ be the *expected* first best surplus that can be achieved by the first best action e^c. As without moral hazard also in this setup the relational efficient ownership structure crucially depends on optimal punishment continuation payoffs denoted as before by $U_i(\omega) = \frac{v_i(\omega)}{1-\delta}$.

These differ[18] from those considered by BGM and are given by

$$\left(v_1\left(\omega\right), v_2\left(\omega\right)\right) = \begin{cases} \left(0, Q_L\right) & \text{for} \quad \omega^I \\ \left(\hat{P}, 0\right) & \text{for} \quad \omega^O \\ \left(0, 0\right) & \text{for} \quad \omega^{JO} \end{cases}$$

where $\hat{P} := \arg\max_{\mathbf{a}} P_L + p\left(\mathbf{a}\right)\Delta P - C_1\left(\mathbf{a}\right)$ is what the upstream party can guarantee himself (minmax payoff) if he owns the asset. Integration means that the downstream party owns the asset. Since the downstream party cannot invest the maximum she can guarantee herself is Q_L forever. That is, for ω^O, ω^I the asset can be used by the party that owns it. In contrast, under joint ownership ω^{JO} the optimal punishment continuation payoffs do not reflect any further use of the asset since the asset can only be used by consent. Optimal punishment alters the non-deviation constraints in BGM which become

$$\begin{array}{cc} \begin{array}{c} b_{jk} + \frac{\delta}{1-\delta}\left(c_1\left(\omega^I\right) + b_{jk}\right) \geq 0 \\ -b_{jk} + \frac{\delta}{1-\delta}\left(c_2\left(\omega^I\right) - b_{jk}\right) \geq \frac{\delta}{1-\delta} Q_L \end{array} & \text{for} \quad \omega^I \\ \begin{array}{c} b_{jk} + \frac{\delta}{1-\delta}\left(c_1\left(\omega^O\right) + b_{jk}\right) \geq \frac{\delta}{1-\delta}\hat{P} \\ Q_j - b_{jk} + \frac{\delta}{1-\delta}\left(c_2\left(\omega^O\right) - b_{jk}\right) \geq 0 \end{array} & \text{for} \quad \omega^O \\ \begin{array}{c} b_{jk} + \frac{\delta}{1-\delta}\left(c_1\left(\omega^{JO}\right) + b_{jk}\right) \geq 0 \\ Q_j - b_{jk} + \frac{\delta}{1-\delta}\left(c_2\left(\omega^{JO}\right) - b_{jk}\right) \geq 0 \end{array} & \text{for} \quad \omega^{JO}. \end{array}$$

If these inequalities hold for all j and k they must hold for the largest and smallest values of b_{jk}. This yields

$$\begin{array}{cc} \begin{array}{c} -\min_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(c_1\left(\omega^I\right) + b_{jk}\right) \\ \max_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(c_2\left(\omega^I\right) - b_{jk}\right) - \frac{\delta}{1-\delta} Q_L \end{array} & \text{for} \quad \omega^I \\ \begin{array}{c} -\min_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(c_1\left(\omega^O\right) + b_{jk}\right) - \frac{\delta}{1-\delta}\hat{P} \\ \max_{j,k=H,L}\left(b_{jk} - Q_j\right) \leq \frac{\delta}{1-\delta}\left(c_2\left(\omega^O\right) - b_{jk}\right) \end{array} & \text{for} \quad \omega^O \\ \begin{array}{c} -\min_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(c_1\left(\omega^I\right) + b_{jk}\right) \\ \max_{j,k=H,L}\left(b_{jk} - Q_j\right) \leq \frac{\delta}{1-\delta}\left(c_2\left(\omega^O\right) - b_{jk}\right) \end{array} & \text{for} \quad \omega^{JO}. \end{array}$$

Summing up over both parties and using $S^* = c_1\left(\omega\right) + c_2\left(\omega\right)$ the crucial necessary and sufficient conditions[19] for a self enforcing relational contract become

$$\begin{array}{ll} \max_{j,k=H,L} b_{jk} - \min_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(S^* - Q_L\right) & \text{for} \quad \omega^I \\ \max_{j,k=H,L}\left(b_{jk} - Q_j\right) - \min_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(S^* - \hat{P}\right) & \text{for} \quad \omega^O \\ \max_{j,k=H,L}\left(b_{jk} - Q_j\right) - \min_{j,k=H,L} b_{jk} \leq \frac{\delta S^*}{1-\delta} & \text{for} \quad \omega^{JO}. \end{array}$$

[18] Here BGM's specific assumptions on limiting the set of strategies and renegotiation possibilities come into play.

[19] The following inequalities go back to McLeod and Malcomson (1989) and correspond to (10) and (16) in the formulation of BGM (2002) who only consider two ownership structures. See also there for a more detailed explanation and interpretation how to derive them in this context..

This shows that the left side (the "maximum total reneging temptation" *and* the right side (the punishment concern) depend on the ownership structure. In particular, since $\hat{P} \geq 0$ outsourcing ω^O is weakly dominated with respect to relational efficiency by joint ownership. ■

Proposition 5. Proof. BGM show that the relational contract, that replicates the payoffs of the "market" or the stage game payoffs under outsourcing ω^O are given by $s = 0, b_{jk} = -\frac{1}{2}(Q_j + P_k)$. This implies $\max_{j,k=H,L} b_{jk} - \min_{j,k=H,L} b_{jk} = \frac{1}{2}(Q_H + P_H) - \frac{1}{2}(Q_L + P_L)$. To choose δ sufficiently high let

$$\begin{aligned} \delta &\geq \frac{\frac{1}{2}(Q_H + P_H) - \frac{1}{2}(Q_L + P_L)}{\frac{1}{2}(Q_H + P_H) - \frac{1}{2}(Q_L + P_L) + S^* - Q_L} \Leftrightarrow \\ \max_{j,k=H,L} b_{jk} - \min_{j,k=H,L} b_{jk} &\leq \frac{\delta}{1-\delta}(S^* - Q_L) \end{aligned}$$

which by the previous proof of proposition 4 corresponds to BGM's necessary and sufficient incentive constraint for integration ω^I with optimal punishment. Since for this δ the neccessary and sufficient condition is satisfied under these assumptions this contradicts BGM's claim that it is "impossible to bring the market inside the firm". ■

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