Meson Form Factors and the BaBar Puzzle

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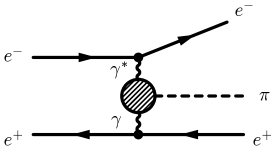
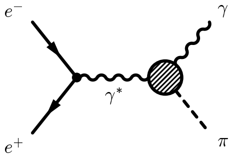
Séminaire Particule Orsay 17.November 2011

$\pi(\eta^{(')}) \to \gamma^*\gamma$-transition form factors

$$\int d^4x\, e^{iq_1\cdot x}\langle P(p)|\mathrm{T}\,\{j_\mu(x) j_\nu(0)\}\,|0\rangle = i\, e^2 \epsilon_{\mu\nu\alpha\beta}\, q_1^\alpha\, q_2^\beta\, F_{\gamma^*\gamma^*\to P}(q_1^2, q_2^2)$$

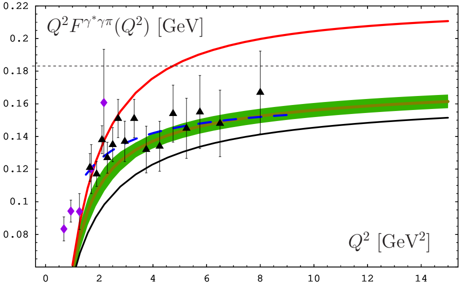
- related to axial anomaly for $q_1^2 = q_2^2 = 0$ $\qquad F(0,0) = \frac{1}{4pi^2 f_\pi}$
- theoretically cleanest case: both photons virtual $q_1^2 \neq 0$, $q_2^2 \neq 0$



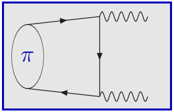


- experimentally easier: one real photon $q_2^2 = 0$, $q_1^2 = -Q^2 < 0$

Good Old Times



- asymptotic limit from handbag diagram



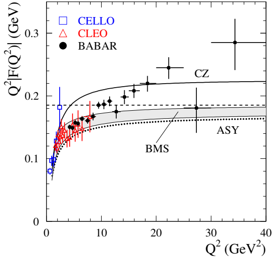
$\longrightarrow$

$$\lim_{Q^2 \to \infty} Q^2 F_{\pi\gamma\gamma^*}(Q^2) = 2 f_\pi$$

Brodsky, Lepage

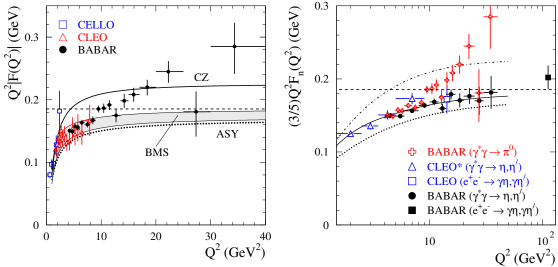
- collinear QCD seemed to describe main part of FF
- asymptotic regime reached for $Q^2 \sim$ few GeV^2 ?

The BaBar-Puzzle



- BaBar-Puzzle part I: experimental results exceed asymptotic limit for the π^0 form factor

The BaBar-Puzzle



- BaBar-Puzzle part I: experimental results exceed asymptotic limit for the π^0 form factor
- BaBar-Puzzle part II:
  - $\eta^{(\prime)}$ form factors behave as expected
  - assume flavour mixing scheme

$$|n\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle), \quad |s\rangle = |\bar{s}s\rangle$$

$$|\eta\rangle = \cos\phi|n\rangle - \sin\phi|s\rangle, \quad \eta' = \sin\phi|n\rangle + \cos\phi|s\rangle$$

  - for similar DAs the difference between $F_{\pi\gamma^*\gamma}$ and $F_{|n\rangle\gamma^*\gamma}$ factor $\frac{3}{5}$

The BaBar puzzle II

- **Fixed-order NLO QCD calculation with $\mu^2 = Q^2$ does not work:**

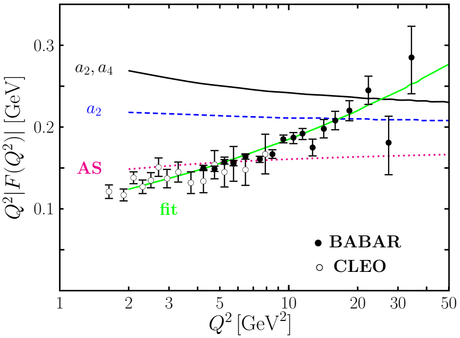


Figure: The fixed-order NLO QCD calculation

Input parameters at 1 GeV:

magenta:	$a_0 = 1$,	
blue:	$a_0 = 1$,	$a_2 = 0.39$,
black:	$a_0 = 1$,	$a_2 = 0.39$,

- **Changing pion distribution amplitude does not help at all**
- ? **Power-suppressed effects** $\sim 1/Q^p$ **??**

The general picture

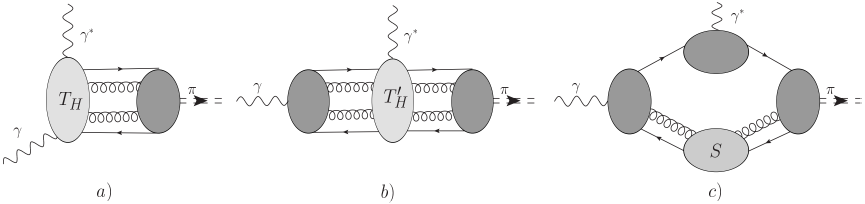


Figure: Schematic structure of the QCD factorization for the $F_{\gamma^*\gamma\to\pi^0}(Q^2)$ formfactor.

A : hard subgraph that includes both photon vertices $\quad \frac{1}{Q^2} + \frac{1}{Q^4} + \ldots$

B : real photon is emitted at large distances $\quad \frac{1}{Q^4} + \ldots$

C : Feynman Mechanism: soft quark spectator $\quad \frac{1}{Q^4} + \ldots$

- Contributions of regions A, B, C are additive
- All other possibilities lead to exponentially small corrections $\exp[-Q^2]$ not seen in OPE

Region A: $\frac{1}{Q^2}$-Terms

- leading term of OPE from $\boxed{T\{j_\mu(x) j_\nu(0)\}}$
- can be written in factorised form:

$$F_{\gamma\gamma^*\to\pi}(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int dx \, T_H(x, Q^2, \mu, \alpha_s(\mu)) \, \phi_\pi(x, \mu)$$

- T_H known to NLO in $\overline{MS}$ scheme and NNLO in conformal scheme
- ϕ_π: leading twist distribution amplitude

$$\sqrt{2} f_\pi \, p_\mu \int_0^1 dx \, e^{ixp\cdot z} \, \phi(x, \mu) = \langle \pi(p) | \bar{u}(z) \gamma_\mu [z, 0] u(0) | 0 \rangle_{z^2=0}$$

- ER-BL evolution implies expansion in Gegenbauer-polynomials

$$\boxed{\phi_\pi(x, \mu) = 6x(1-x) \sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_n^{(0)}/2\beta_0} a_n(\mu_0) C_n^{3/2}(2x-1) \,, \qquad a_0(\mu) = 1}$$

expect $1 = a_0 > a_2 > a_4 > a_6 > \cdots$

$a_2[1\text{ GeV}] = 0.30 \pm 0.15$, $\quad a_4[1\text{ GeV}] \sim \begin{cases} 0.1 & \text{B-decays} \\ -0.1 & \text{NLC SR [BMS-model]} \end{cases}$, $\quad a_{n>4}[1\text{ GeV}]$ unconstrained

Region A: Twist 4 terms

- twist 4 term from OPE of $\boxed{T\{j_\mu(x)j_\nu(0)\}}$

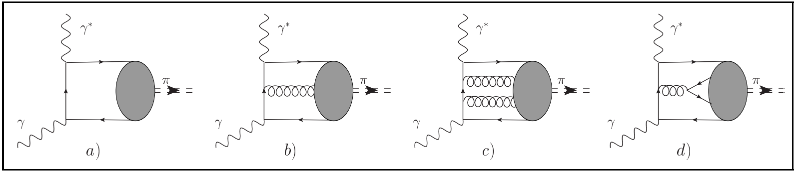


Figure: Twist-4 corrections to the pion transition form factor

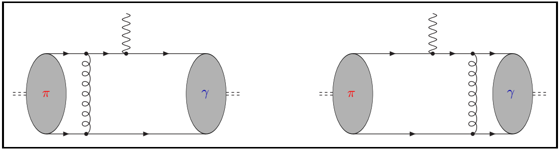
- involves twist-4 quark-gluon pion distribution amplitudes

$$F_{\gamma^*\gamma\to\pi^0}(Q^2) \quad = \quad \frac{\sqrt{2}f_\pi}{Q^2}\left(\frac{1}{3}\int\frac{dx}{x}\phi_\pi(x) - \frac{80}{27}\frac{\delta_\pi^2}{Q^2}\right) \qquad \delta_\pi^2 \simeq 0.2\ \mathrm{GeV}^2$$

- Might be significant at $Q^2 \sim 1-5$ GeV^2 but does not change high Q^2 behaviour

Region B: Photon Emission From Large Distances

- hard scattering kernel convoluted with twist three pion and photon DA



- results in

$$F^{(B)}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{\sqrt{2}f_\pi}{3}\,\frac{16\pi\alpha_s\chi\langle\bar{q}q\rangle^2}{9f_\pi^2Q^4}\int_0^1 dx\,\frac{\phi^p_{3;\pi}(x)}{x}\int_0^1 dy\,\frac{\phi_\gamma(y)}{\bar{y}^2}$$

- infrared divergent $\longrightarrow$ overlap with region C
- regularised result

$$\boxed{F_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{\sqrt{2}f_\pi}{Q^2}\left(\frac{1}{3}\int\frac{dx}{x}\phi_\pi(x) + \frac{0.2\ \mathrm{GeV}^2}{Q^2}\cdot\ln^2\frac{Q^2}{\mu^2_{IR}}\right)}$$

- might be significant up to $Q^2 \sim 5\ \mathrm{GeV}^2$

Region C: Feynman Mechanism

- truly non-perturbative
- one quark carries almost all momentum
- overlap integral of wave functions
- use e.g. Drell-Yan representation as convolution of light-cone WFs (Brodsky-Lepage)

$$(\varepsilon_\perp \times q_\perp) F^{\bar{q}q}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{f_\pi}{4\pi^3\sqrt{3}} \int_0^1 dx \int d^2k_\perp \frac{(\varepsilon_\perp \times (xq_\perp + k_\perp))}{(xq_\perp + k_\perp)^2 - i\epsilon} \Psi_{\bar{q}q}(x, k_\perp)$$

- has to be calculated in some model

Needs approaches that go geyond this picture.

Sudakov Suppression

Kroll; Li, Sterman; Botts, Sterman

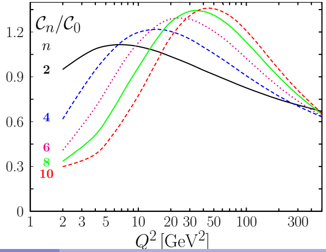
- general idea: keep $k_\perp$ dependence in hard scattering kernel

$$\frac{1}{xQ^2} \longrightarrow \frac{1}{xQ^2 + k_\perp^2}$$

- ... and in wave function
- double logs from collinear and soft regions exponentiate

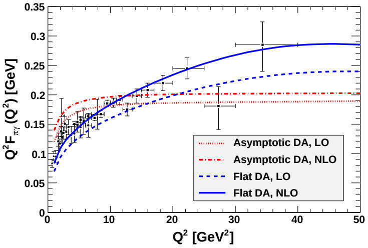
$$F_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{\sqrt{2}f_\pi}{3}\int dx \int \frac{d^2b}{2\pi}\widetilde{T}_H(x,Q^2,b,\mu,\alpha_s(\mu))\, e^{-\mathcal{S}}\, \phi_\pi(x,b_0/b)$$

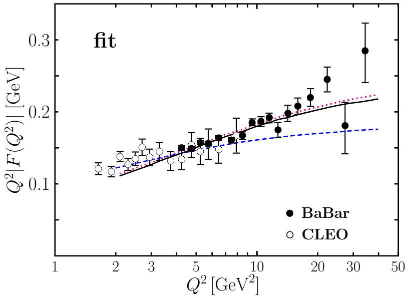
- Sudakov factor suppresses region of large b
- soft contributions modelled by wave function
- hard scattering kernel and Sudakov factor suppress higher Gegenbauer moments



State-of-the-art calculations in $k_\perp$ factorization

$k_\perp$ factorization





Li, Mishima, arXiv:0907.0166

- flat: $a_2 = 0.39$, $a_4 = 0.24$
- $\int dx \int d^2k_t |\Psi_{\bar{q}q}(x, k_\perp)|^2 = \infty$

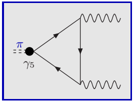
P. Kroll, arXiv:1012.3542

- fit: $a_2 = 0.25$, $a_4 = 0.07$

needs separate fit for $\eta \to \gamma\gamma^*$

Quark Models

Dorokhov



- simple interal over quark loop
- with constituent quark mass $M_q \approx 135$ MeV reproduces BaBar data for $F_{\gamma\gamma^* \to \pi}$
- caveat: f_π in same model divergent
- regularised model does not reproduce data...

no Pion in QCD, no γ_5-vertex,
wrong chiral limit $M_q \to 0,\ m_\pi \to 0$
similar model from PCAC by Pham, Pham

- quark models imply flat DA for light quarks, more peaked DAs for heavier quarks
- introducing

dynamical constituent mass	$m(k^2) = M_q\, e^{-\Lambda k^2}$
nonlocal quark photon vertex	$\Gamma_\mu = -ie_q(\gamma_\mu + \Delta\Gamma_\mu)$
pion quark vertex $\sim$ LCWF	$\gamma_5\, F(k_1^2, k_2^2)$

- can describe $\gamma\gamma^* \to \pi^0,\ \eta^{(\prime)},\ \eta_c$ data with three different M_q
- physical justification for $m(k^2)$?

models soft contribution...

Musatov-Radyushkin Model I

- Use Drell-Yan representation as convolution of light-cone WFs (Brodsky-Lepage)

$$(\varepsilon_\perp \times q_\perp) F^{\bar{q}q}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{f_\pi}{4\pi^3\sqrt{3}} \int_0^1 dx \int d^2k_\perp \frac{(\varepsilon_\perp \times (xq_\perp + k_\perp))}{(xq_\perp + k_\perp)^2 - i\epsilon} \Psi_{\bar{q}q}(x, k_\perp)$$

with a model wave function

$$\Psi_{\bar{q}q}(x, k_\perp) = \frac{4\pi^2}{\sigma\sqrt{6}} \frac{\phi_\pi(x)}{x\bar{x}} \exp\left(-\frac{k_\perp^2}{2\sigma x\bar{x}}\right)$$

to get

$$\boxed{F^{\mathrm{MR}}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{\sqrt{2}f_\pi}{3} \int_0^1 \frac{dx\,\phi_\pi(x)}{xQ^2} \left[1 - \exp\left(-\frac{xQ^2}{2\bar{x}\sigma}\right)\right]}$$

- using $\sigma = 0.53\ \mathrm{GeV}^2$ and flat pion DA $\phi_\pi(x) = 1$ can fit the BABAR data !

caveat: $\int dx \int d^2k_t |\Psi_{\bar{q}q}(x, k_\perp)|^2 = \infty$, ?!
$F_{\gamma^*\gamma\to\pi}(0) \sim \int_0^1 dx \frac{\phi_\pi(x)}{\bar{x}} = \infty$, ?!

Musatov-Radyuskin Model II

- correction in Musatov-Radyushkin model is exponentially suppressed

absent in OPE

$$F^{\mathrm{MR}}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1 \frac{dx\;\phi_\pi(x)}{xQ^2}\left[1 - \exp\left(-\frac{xQ^2}{2\bar{x}\sigma}\right)\right]$$

- for flat DA and large Q^2 numerically very similar to

$$\frac{\sqrt{2}f_\pi}{3}\int_0^1 dx\,\frac{\phi_\pi(x)}{xQ^2+M^2}, \qquad \begin{aligned} &M^2 \approx 0.6 GeV^2 \\ &\sigma \approx 0.53 \end{aligned}$$

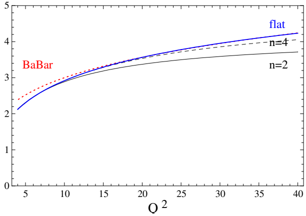
- average $k_\perp^2$, $\langle k_\perp^2\rangle = \frac{\sigma}{3} = (0.42\,\mathrm{GeV})^2$
- close to folklore value $\sqrt{k_\perp^2} \approx 300$ MeV
- flat DA does not evolve for Photon-Pion form factor

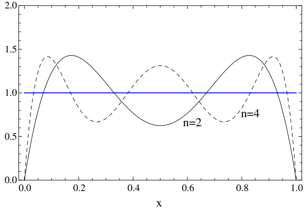
Flat Distribution Amplitude?

- flat distribution amplitude would force us to reconsider pQCD predictions e.g.

$$F_\pi^{as\,(\mathrm{pQCD})}(Q^2) = \frac{8\pi\alpha_s}{9Q^2}\int_0^1 dx \int_0^1 dy\, \frac{\phi_\pi(x)\,\phi_\pi(y)}{x\,y\,Q^2} \to \infty$$

- flat DA really necessary in MR-model?
- **Answer:**





alternatively, check how much is contributed by each successive Gegenbauer polynomial:

$$\mathcal{F}^{MR}_{\mathrm{flat}}(Q^2 = 20) \quad = \quad 3.56513 = \underset{n=0}{2.72402} + \underset{n=2}{0.648618} + \underset{n=4}{0.16226} + \underset{n=6}{0.027945} + \cdots$$

First Summary

- shape of Pion distribution amplitude
  - The Gegenbauer expansion for the form factor calculated with flat DA converges very fast
  - At $Q^2 < 10 - 20$ GeV^2 using $n = 4$ truncation is sufficient
  - End-point behavior of a “true” pion DA is irrelevant
- soft corrections are modelled by different approaches

physical (QCD) interpretation not always clear...

Systematic calculation of soft effects possible?
Dispersion relations.

The Method I

Khodjamirian

- The QCD result satisfies an unsubtracted dispersion relation

$$F^{\mathrm{QCD}}_{\gamma^*\gamma^*\to\pi^0}(Q^2,q^2) = \frac{1}{\pi}\int_0^\infty ds\,\frac{\mathrm{Im}F^{\mathrm{QCD}}_{\gamma^*\gamma^*\to\pi^0}(Q^2,-s)}{s+q^2}\,.$$

- hadronic sum looks like

$$F_{\gamma^*\gamma^*\to\pi^0}(Q^2,q^2) = \frac{\sqrt{2}f_\rho F_{\gamma^*\rho\to\pi^0}(Q^2)}{m_\rho^2+q^2} + \frac{1}{\pi}\int_{s_0}^\infty ds\,\frac{\mathrm{Im}F_{\gamma^*\gamma^*\to\pi^0}(Q^2,-s)}{s+q^2}\,.$$

- Duality: assume that above a certain threshold

$$\int \mathrm{Im}F_{\gamma^*\gamma^*\to\pi^0}(Q^2,-s) \quad = \quad \int \mathrm{Im}F^{QCD}_{\gamma^*\gamma^*\to\pi^0}(Q^2,-s) \qquad \text{for} \;\; s > s_0$$

- Asymptotic freedom: QCD expression must be correct at $q^2 \to -\infty$, therefore

$$\sqrt{2}f_\rho F_{\gamma^*\rho\to\pi^0}(Q^2) = \frac{1}{\pi}\int_0^{s_0} ds\,\mathrm{Im}F^{\mathrm{QCD}}_{\gamma^*\gamma^*\to\pi^0}(Q^2,-s)\,.$$

- Duality sum rules: use this result to correct the QCD calculation

The Method II

correlation function

$$F(q_1^2, q_2^2) = i \int d^4x\, e^{iq_1 x} \langle \pi(p) \,|\, T\{ j_\mu(x)\, j_\nu(0) \} \,|\, 0 \rangle$$

hadronic dispersion relation

operator-product-expansion

$$F^{(had)}(Q^2, q^2) = \frac{\sqrt{2}\, f_\rho\, f^{\rho\pi}(Q^2)}{m_\rho^2 + q^2} + \int_{s_0}^{\infty} ds \frac{\rho(s)}{s + q^2}$$

$$F^{(OPE)}(Q^2, q^2) = \frac{1}{\pi} \int_0^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(Q^2, s)}{s + q^2}$$

quark-hadron-duality

$$\int_{s_0}^{\infty} ds \frac{\rho(s)}{s + q^2} \approx \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(Q^2, s)}{s + q^2}$$

$$\frac{\sqrt{2}\, f_\rho\, f^{\rho\pi}(Q^2)}{(m_\rho^2 + q^2)} = \frac{1}{\pi} \int_0^{s_0} ds \frac{\mathrm{Im}\, F^{(OPE)}(Q^2, s)}{s + q^2}$$

Borel-transformation

sum rule for $\lim q^2 \to 0$

$$F^{\gamma\pi}(Q^2) = \frac{1}{\pi\, m_\rho^2} \int_0^{s_0} ds\, \mathrm{Im}\, F^{(OPE)}(s)\, e^{(m_\rho^2 - s)/M^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(s)}{s}$$

main uncertainties

▷ universal input

The Method II

correlation function

$$F(q_1^2, q_2^2) = i \int d^4x \, e^{iq_1 x} \langle \pi(p) \,|\, T\{ j_\mu(x) \, j_\nu(0) \} \,|\, 0 \rangle$$

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$$F^{(OPE)}(Q^2, q^2) = \frac{1}{\pi} \int_0^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(Q^2, s)}{s + q^2}$$

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main uncertainties

- ▷ universal input
- ▷ truncated OPE

The Method II

correlation function

$$F(q_1^2, q_2^2) = i \int d^4x\, e^{iq_1 x} \langle \pi(p) \,|\, T\{j_\mu(x)\, j_\nu(0)\} \,|\, 0 \rangle$$

hadronic dispersion relation

operator-product-expansion

$$F^{(had)}(Q^2, q^2) = \frac{\sqrt{2}\, f_\rho\, f^{\rho\pi}(Q^2)}{m_\rho^2 + q^2} + \int_{s_0}^{\infty} ds \frac{\rho(s)}{s + q^2} \qquad F^{(OPE)}(Q^2, q^2) = \frac{1}{\pi} \int_0^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(Q^2, s)}{s + q^2}$$

quark-hadron-duality

$$\int_{s_0}^{\infty} ds \frac{\rho(s)}{s + q^2} \approx \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(Q^2, s)}{s + q^2}$$

$$\frac{\sqrt{2}\, f_\rho\, f^{\rho\pi}(Q^2)}{(m_\rho^2 + q^2)} = \frac{1}{\pi} \int_0^{s_0} ds \frac{\mathrm{Im}\, F^{(OPE)}(Q^2, s)}{s + q^2}$$

Borel-transformation

main uncertainties

- ▷ universal input
- ▷ truncated OPE
- ▷ quark-hadron duality

sum rule for lim $q^2 \to 0$

$$F^{\gamma\pi}(Q^2) = \frac{1}{\pi\, m_\rho^2} \int_0^{s_0} ds\, \mathrm{Im}\, F^{(OPE)}(s)\, e^{(m_\rho^2 - s)/M^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\mathrm{Im}\, F^{(OPE)}(s)}{s}$$

Leading order example

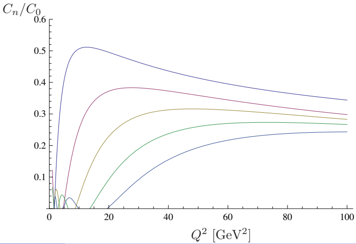
- QCD calculation

$$F^{\mathrm{QCD}}_{\gamma^*\gamma^*\to\pi^0}(Q^2,q^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1 \frac{dx\,\phi_\pi(x)}{xQ^2+\bar{x}q^2}\,.$$

- LCSR

$$\mathrm{Im}_s\,\frac{1}{xQ^2-\bar{x}s} \longrightarrow \frac{\pi}{\bar{x}}\delta\left(s-\frac{x}{\bar{x}}Q^2\right)$$

$$F^{\mathrm{LCSR}}_{\gamma^*\gamma\to\pi^0}(Q^2) \quad = \quad \frac{\sqrt{2}f_\pi}{3}\left\{\int_{x_0}^1 \frac{dx\,\phi_\pi(x)}{xQ^2} + \int_0^{x_0}\frac{dx\,\phi_\pi(x)}{\bar{x}m_\rho^2}\right\}, \quad x_0=\frac{s_0}{s_0+Q^2}$$

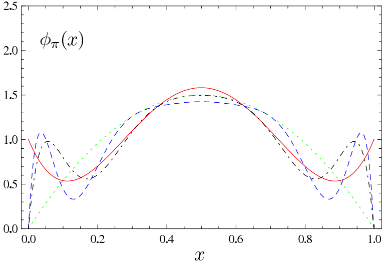


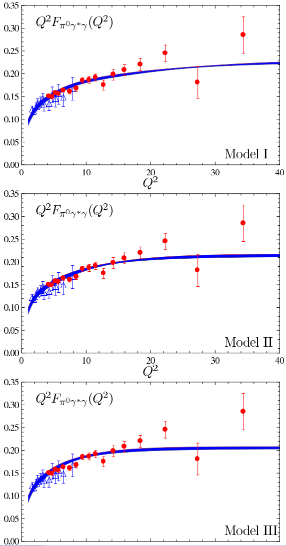
- The difference is a soft correction that suppresses higher Gegenbauer-moments
- qualitative picture stays the same after inclusion of NLO corrections

Results I

Agaev et al.

- Three models with $a_{n>4} \neq 0$





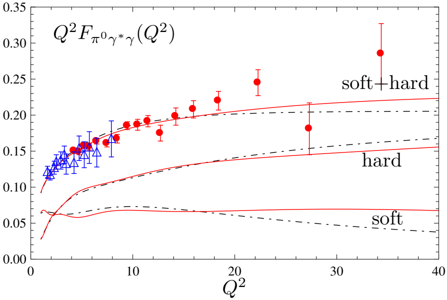
$\boxed{a_2 < a_4}$

$\boxed{a_2[1\ \text{GeV}] \sim 0.15, \quad a_4[1 GeV] \sim 0.2}$

Results II

Agaev et al.

- comparison of soft and hard contributions in LCSRs

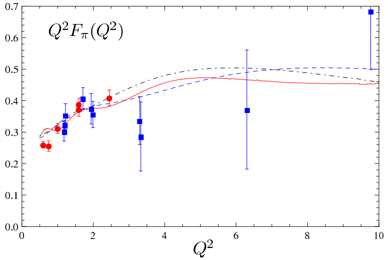


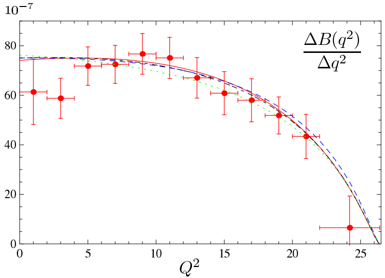
- soft part still $\sim$ 25% at $Q^2 \approx 40$ GeV^2
- asymptotic regime starts later than assumed?!

Results III

Agaev et al.

- The same models describe pion form EM factor and $B \to \pi \ell \nu_\ell$ width





- NLO LCSRs including twist up to 6 and up to 4, respectively
- What about η, η'?

$\eta \leftrightarrow \eta'$-mixing

- singlet-octet scheme

$$\langle 0|J^i_{\mu 5}|P(p)\rangle = i\, f^i_P\, p_\mu \quad (i = 1, 8;\ P = \eta,\, \eta')$$

$$\begin{aligned} f^8_\eta &= f_8 \cos\theta_8, & f^1_\eta &= -f_1 \sin\theta \\ f^8_{\eta'} &= f_8 \sin\theta_8, & f^1_{\eta'} &= f_1 \cos\theta \end{aligned}$$

- flavour scheme

$$J^q_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d), \quad J^s_{\mu 5} = \bar{s}\gamma_\mu\gamma_5 s$$

$$\langle 0|J^r_{\mu 5}|P(p)\rangle = i\, f^r_P\, p_\mu \quad (r = q,\, s)$$

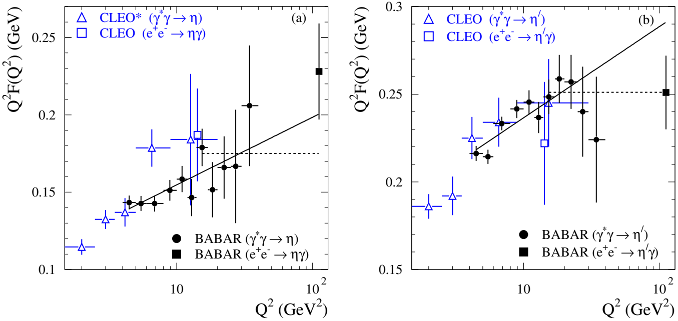
$$\begin{aligned} f^q_\eta &= f_q \cos\phi_q, & f^s_\eta &= -f_s \sin\phi_s \\ f^q_{\eta'} &= f_q \sin\phi_q, & f^s_{\eta'} &= f_s \cos\phi_s \end{aligned}$$

neglect difference $\phi_q - \phi_s$

$$\boxed{\phi_q \approx \phi_s \approx 41^\circ}$$

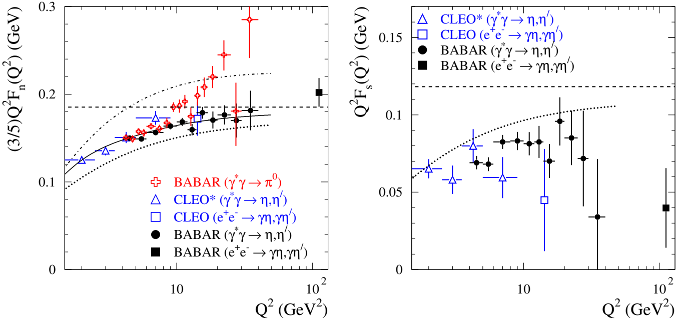
include η_c and G into mixing?

BaBar-Measurement



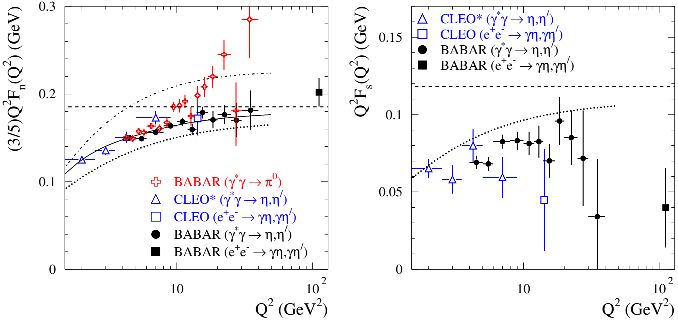
- BaBar measured $\eta^{(\prime)} \to \gamma\gamma^*$ and $e^+e^- \to \eta^{(\prime)}\gamma$ form factors

BaBar-Measurement



- BaBar measured $\eta^{(\prime)} \to \gamma\gamma^*$ and $e^+e^- \to \eta^{(\prime)}\gamma$ form factors
- used flavour scheme to translate to light quark and strange quark content
  - $|n\rangle$ FF does not rise as pion FF
  - $|s\rangle$ FF falls short even of prediction with asymptotic DA

BaBar-Measurement



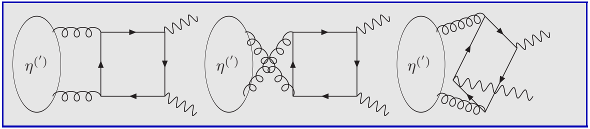
- BaBar measured $\eta^{(\prime)} \to \gamma\gamma^*$ and $e^+e^- \to \eta^{(\prime)}\gamma$ form factors
- used flavour scheme to translate to light quark and strange quark content
  - $|n\rangle$ FF does not rise as pion FF
  - $|s\rangle$ FF falls short even of prediction with asymptotic DA
- Can additional contributions solve this?

Additional Corrections for the $\eta^{(\prime)}$ form factors

Agaev et al.
work in progress

- gluonic content

include three Gluon or Twist 4 contribution?



- mass corrections due to $m^2_{\eta^{(\prime)}} \neq 0$
- $SU(3)$-breaking due to additional twist 3 corrections for massive strange quark

$$\sim \frac{m_s\,\mu_{\eta^{(\prime)}}}{Q^2}\,\frac{\sqrt{2}f_\eta^{(\prime)}}{3}\left\{\int_{x_0}^{1}\frac{dx}{xQ^2}\,\frac{d\phi^{\sigma}_{\eta^{(\prime)},3}(x)}{dx}+\int_0^{x_0}\frac{dx}{\bar{x}m_\rho^2}\,\frac{d\phi^{\sigma}_{\eta^{(\prime)},3}(x)}{dx}\right\} \qquad x_0=\frac{s_0+m_s^2}{s_0+Q^2}$$

- even though $m^2_{|n\rangle} > m^2_\pi$

larger effect if η_c is taken into account

- unlikely to cure discrepancy of $F_{\gamma\gamma^*\to\pi} \leftrightarrow F_{\gamma\gamma^*\to|n\rangle}$

Conclusions/Summary

- picture still rather confusing
- some important issues
- LCSR fits generally prefer a small value $a_2(1\text{ GeV}) \simeq 0.13 - 0.16$ compared to $a_2(1\text{ GeV}) \simeq 0.35 \pm 0.15$ from lattice calculations

  $\hookrightarrow$ higher precision lattice data needed
  $\hookrightarrow$ BES data?

- BABAR data in the $Q^2 = 10 - 20\text{ GeV}^2$ range require large $a_4(1\text{ GeV}) \simeq 0.25$; older data/other reactions not sensitive because of lower effective Q^2

- No natural explanation for the difference $\gamma^*\gamma \to \pi$ and $\gamma^*\gamma \to \eta$

  $\hookrightarrow$ more experimental data needed (KEK?)

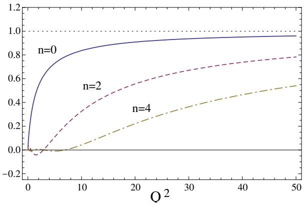
Method	$\mu = 1$ GeV	$\mu = 2$ GeV	Reference
LO QCDSR, CZ model	0.56	0.38	CZ 1981
QCDSR	$0.26^{+0.21}_{-0.09}$	$0.17^{+0.14}_{-0.06}$	Khodjamirian et al. 2004
QCDSR	0.28 ± 0.08	0.19 ± 0.05	Ball et al. 2006
QCDSR, NLC	0.19 ± 0.06	0.13 ± 0.04	BMS 91, 98, 01
$F_{\pi\gamma\gamma*}$, LCSR	0.19 ± 0.05	0.12 ± 0.03 ($\mu = 2.4$)	Schmedding, Yakovlev 99
$F_{\pi\gamma\gamma*}$, LCSR	0.32	0.20 ($\mu = 2.4$)	BMS 02
$F_{\pi\gamma\gamma*}$, LCSR, R	0.44	0.30	BMS 05
$F_{\pi\gamma\gamma*}$, LCSR, R	0.27	0.18	Agaev 05
F_{π}^{em},LCSR	$0.24 \pm 0.14 \pm 0.08$	$0.16 \pm 0.09 \pm 0.05$	Braun 99, Bijnens 02
F_{π}^{em},LCSR, R	0.20 ± 0.03	0.13 ± 0.02	Agaev 05
$F_{B\to\pi\ell\nu}$, LCSR	0.19 ± 0.19	0.13 ± 0.13	Ball 05
$F_{B\to\pi\ell\nu}$, LCSR	0.16	0.10	Duplancic 08
LQCD, $N_f = 2$, CW	0.329 ± 0.186	0.201 ± 0.114	QCDSF/UKQCD 06
LQCD,$N_f = 2+1$, DWF	0.382 ± 0.143	0.233 ± 0.088	RBS/UKQCD 07

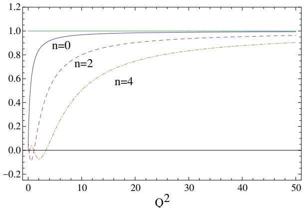
Region C: LCSR vs. MR model

- Separate contributions of different Gegenbauer polynomials

$$Q^2 F_{\gamma^*\gamma\to\pi^0}(Q^2) = \sqrt{2} f_\pi \left\{ f_0(Q^2) + a_2 f_2(Q^2) + a_4 f_4(Q^2) + \ldots \right\}$$

- …and compare the coefficients $f_n(Q^2)$





MR Model **LCSR**

- A qualitative agreement

Convincing evidence for strong suppression of end-point regions alias contributions of higher Gegenbauer polynomials in pion DA