UNIVERSITY OF SASKATCHEWAN Department of Mathematics & Statistics Mathematics 101.3 Solutions to Practice Quiz #1

Mathematics 101.3 Solutions to Practice Quiz #1									
Early October, 1999			Time:50	minutes	Instructor: Doug MacLean				
CLOSED BOOK — NO CALCULATORS PERMITTED Each question is worth 4% The possible answers to all questions are the digits in the ANSWER SET :									
(A) ((B) 1	(C) 2 (D) 3	3 (E) 4	(F) 5	(G) 6	(H) 7	(I) 8	(J) 9	
If $\frac{7}{12} - \frac{3}{5}$ is written in its simplest form as $\frac{-a}{10b+c}$, where <i>a</i> , <i>b</i> , and <i>c</i> are digits, then									
Solut	ion: $\frac{7}{12}$ -	$-\frac{3}{5} = \frac{7}{12}\frac{5}{5} -$	$-\frac{3}{5}\frac{12}{12} = \frac{35}{5}$	$\frac{-36}{60} = \frac{-1}{60}$	<u>-</u>)				
(1) $a = 1$			(2	2) <i>b</i> = 6		(3) $c = 0$			

 $x^{2} + 8x + 20$ is to be written in the form $a(x + b)^{2} + c$ by completing squares. We must have:

Solution:
$$x^2 + 8x + 20 = x^2 + 2(4)x + 4^2 - 4^2 + 20 = (1)(x + 4)^2 + 4$$

(4) $a = 1$ (5) $b = 4$ (6) $c = 4$

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 $3x^2 + 12x + 4$ is to be written in the form $a(x + b)^2 - c$ by completing squares. We must have:

Solution:
$$3x^2 + 12x + 4 = 3\left(x^2 + 4x + \frac{4}{3}\right) = 3\left(x^2 + 2(2)4x + 2^2 - 2^2 + \frac{4}{3}\right) = 3\left((x + 2^2(2) - 4 + \frac{4}{3}\right) = 3\left((x + 2^2(2) - \frac{12}{3} + \frac{4}{3}\right) = 3\left((x + 2^2(2) - \frac{8}{3}\right) = 3(x + 2)^2 - 3\frac{8}{3} = 3(x + 2)^2 - 8$$

(7) $a = 3$

(8) $b = 2$

(9) $c = 8$

(10) If -a is the slope of the line through the points (5, -3) and (-1, 3), then *a* is:

Solution:
$$-a = \frac{3 - (-3)}{-1 - 5} = \frac{6}{-6} = -1$$
, so $a = 1$

(11) Let
$$f(x) = \frac{x^3}{(x+3)^3}$$
. Find **Correction:** $-f(-2)$

Solution:
$$-f(-2) = -\frac{(-2)^3}{((-2)+3)^3} = -\frac{-8}{1} = 8$$

The roots of $x^2 - 8x + 16 = 0$ in their simplest form are $\frac{A \pm B\sqrt{C}}{D}$. We must have:

Solution: The roots are
$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(16)}}{2(1)} = \frac{8 \pm \sqrt{64 - 64}}{2} = \frac{8 \pm \sqrt{0}}{2}$$

(12) $A = 8$ (13) $B = 1$ (14) $C = 0$ (15) $D = 2$

The polynomial $3x^4 + x^3 - 3x^2 - x$ can be factored in the form x(x - a)(x + b)(cx + d), where *a*, *b*, *c*, and *d* are digits. Their values are:

Solution: $p(x) = 3x^4 + x^3 - 3x^2 - x = x(3x^3 + x^2 - 3x - 1)$. The only possible integer roots of p(x) are -1 and 1. It is easily computed that both numbers are roots of p(x), so we have x + 1 and x - 1 are factors of p(x). Dividing them both into $3x^3 + x^2 - 3x - 1$, we find that 3x + 1 is the eremaining factor. Thus p(x) = x(x - 1)(x + 1)(3x + 1)

(16) a = 1 (17) b = 1 (18) c = 3 (19) d = 1

 $\frac{4-2\sqrt{2}}{2-\sqrt{2}}$ can be simplified to the expression $a - b\sqrt{c}$, where a, b, and c are digits. Their values are:

Solution: Use conjugates:
$$\frac{4-2\sqrt{2}}{2-\sqrt{2}}\left(\frac{2+\sqrt{2}}{2+\sqrt{2}}\right) = \frac{(4-2\sqrt{2})(2+\sqrt{2})}{2^2-2} = \frac{8+4\sqrt{2}-4\sqrt{2}-2(2)}{4-2} = \frac{4}{2} = 2 - 0\sqrt{2}$$

If we solve the inequality $\left|\frac{2-x}{3}\right| \le 2$, the solution is an interval of the form [a, b]. The values of *a* and *b* are:

Solution: $\left|\frac{2-x}{3}\right| \le 2 \Leftrightarrow \frac{|2-x|}{|3|} \le 2 \Leftrightarrow \frac{|2-x|}{3} \le 2 \Leftrightarrow 3\frac{|2-x|}{3} \le 3(2) \Leftrightarrow |2-x| \le 6$ so the inequality is satisfied by all numbers within 6 units of 2, so the solution is (-4, 8)

$$(23) a = -4 (24) b = 8$$

(25) The slope of the line perpendicular to the line $\gamma = -\frac{x}{9}$ which passes through the point (0,0) is:

