Math 142 Lecture Notes
Section 2.1 – Polynomial and Rational Functions

☆Polynomial Functions:

Polynomial Function A function that can be written in the form:

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ where n is a nonnegative integer, called the degree of the polynomial. It is the highest exponent present in the term. The domain is the set of real numbers. $a_n \neq 0$

Important points:

1) The shape is connected to the degree of the polynomial.

2) The sign (+/-) of the leading coefficient describes the end-line behavior.

3) Examples of even and odd polynomials:

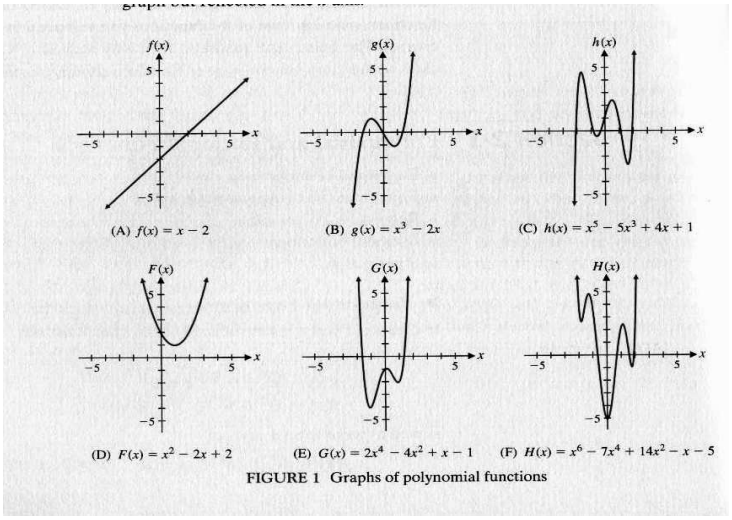

FIGURE 1 Graphs of polynomial functions

from: pg 80 of Calculus for Business, Economics, Life Sciences and Social Sciences
by: Barnett, Ziegler and Byleen

4) Even functions :
start high and end high, or start low and end low depending on the sign of the leading coefficient

5) Odd functions :
start high and end low, or start low and end high depending on the sign of the leading coefficient

6) Graphs of polynomials are **continuous.** No holes or breaks, and **NO** sharp corners.

7) **Turning point:** a place on the graph where the curve changes from increasing to decreasing or vice versa.

Theorem 1 The graph of a polynomial function of positive degree n can have at most *n - 1* turning points. It can cross the x-axis at most n times.

☆**Answer the following questions:**

1. What is the least number of turning points an odd-degree polynomial function can have?

2. What is the least number of turning points an even-degree polynomial function can have?

3. What is the maximum number of x-intercepts for a polynomial of n^{th} degree?

4. What is the least number of x-intercepts for a polynomial of n^{th} degree,
   a. if the polynomial has odd degree?
   b. if the polynomial has even degree?

5. What is the least number of real solutions of a polynomial function
   a. of odd degree?
   b. of even degree?

☆**Regression Polynomials:**

1. Estimate the value of the car after by finding a quadratic function which relates it's age and value:

Age in yrs	0	2	3	5	7	8	9
Value in $'s	42,120	39,575	36,500	28,455	18,750	12,575	4,580

A, value after 4 yrs: _____
B. value after 10 yrs: _____

C. What is the quadratic function you used to find the value of the car? _____

Rational Functions

A **rational function** is any function that can be written in the form

$$f(x)=\frac{n(x)}{d(x)}, d(x)\neq 0$$ and **n(x) and d(x)** are polynomials.

☆**Domains:** Check for values that make the denominator zero, or any values that would make an even-indexed radical negative, or if logarithms are present, the values needed to keep the expression positive, and if it's a word problem, check to see what values make sense in the problem.

A value which would make the denominator zero is ***point of discontinuity.***

☆**Intercepts:** Let x=0 and solve for f(x), then let f(x)=0 and solve for x.

☆**Asymptotes:** A line the graph approaches. Asymptotes are graphed with *dotted* lines.

Vertical asymptotes If *a* is a real number, such that *d(a)=0*, then **x=a** is a vertical asymptote.

Horizontal asymptotes Compare the degrees of the numerator and the denominator.

$$f(x)=\frac{ax^n+\ldots+c}{bx^d+\ldots+f}$$

Note: in this example the degree of the numerator is n, and the degree of the denominator is d.

1) if n>d, then no horizontal asymptote exist
2) if n=d, then a horizontal asymptote exist at $y=\frac{a}{b}$
3) if n<d, then a horizontal asymptote exist at y=0

Note: You can always divide every term in the numerator and the denominator, by the highest power present in the denominator, to change it into a simplier form and then determine the asymptotes.

Oblique asymptotes If the degree of the numerator is ONE more than the degree of the denominator, then an oblique (or slanted) asymptote exists. Discussed more in Ch 4

Graph: $f(x)=\frac{4-4x}{2x-10}$

1) Intercepts:
2) Vertical asymptote:
3) Horizontal asymptote:
4) Graph: