Chapter 2A Solving Equations

Solving Linear Equations

Linear equations are equations involving only polynomials of degree one.

Examples include $2t+1=7$ and $25x+16=9x-4$

A solution is a value or a set of values that yield a true statement when substituted for the variable in an equation.

Solve the equations above on your own.

You should find that the solutions are $t=3$ and $x=-\frac{5}{4}$, respectively.

(If you are unable to solve these equations, you should come see me for help.)

Solving Quadratic Equations

Quadratic equations are equations of the form $ax^2+bx+c=0$ where $a\neq 0$.

$a, b, c \in \mathbb{R}$. a, b, c are called the constants. x is called the variable.

If the instructions say, "Solve $x^2-1=0$" it means you are to find all values for x that make the equation true.

$x=1$ is a solution because ____

$x=-1$ is a solution because ____

Important ideas to understand:

If $ab=1$, must either $a=1$ or $b=1$? ______________________________

If $ab=0$, must either $a=0$ or $b=0$? ______________________________

This is called the Zero-Product Principle.

Solve by Factoring

1. Solve

	a) $1-2x^2=-x$
Put in standard form	
Factor	
Set each factor equal to zero	

	b) $6x^2=x+15$
Put in standard form	
Factor	
Set each factor equal to zero	

Solve by completing the square.

This is an important technique that we will use in other types of problems throughout the semester. There will be a problem on exam 1 in which I ask you to solve a quadratic equation by completing the square. Students who solve the equation correctly with methods other than completing the square will not receive credit for their solution.

2. Solve the following quadratic equations by completing the square:

a) $x^2 = 36$

b) $(x-2)^2 = 25$

c) $t^2 + 2t + 1 = 16$

d) $a^2 + 6a = 1$

e) $x^2 + 9x + 1 = 0$

f) $2x^2 + x - 8 = 0$

g) $ax^2 + bx + c = 0$

If $ax^2 + bx + c = 0$, then x = ___________________________________

This is called ____

3. Solve using the quadratic formula: $2x^2 = 6x - 3$

Equations in quadratic form

4. Solve

a) $x^4 - 5x^2 - 6 = 0$

b) $x^{10} + 2x^5 + 1 = 0$

c) $x^{\left(\frac{4}{3}\right)} - 5x^{\left(\frac{2}{3}\right)} + 4 = 0$

d) $4x^{12} - 9x^6 + 2 = 0$

Solving Rational Equations

To solve a rational equation, move everything to one side of the equation and remember

that a fraction is zero when____

5. Solve

a) $\frac{1}{2x} - \frac{1}{10x} = 2$

b) $\frac{5x}{x+2} + \frac{2}{x} = 5$

c) $\frac{x^2 - 6x + 9}{x+2} \bullet \frac{x^2 - x - 12}{x-3} = 0$

d) $\frac{4x-8}{x^2-6x+8}-\frac{3x-3}{x^2+x-2}=1$

Radical Equations

To Solve Radical equations:

I. Isolate one radical.

II. Raise both sides of the equation to the appropriate power to remove the radical.

III. Repeat the process until all radicals have been removed

IV. Check or extraneous solutions!

REMEMBER: $(a+b)^2 \neq a^2+b^2$

$(a+b)^2 =$ ____________________

6. Solve

a) $3 = x + \sqrt{2x - 3}$

b) $3\sqrt{x} + \sqrt{9x - 45} = 3$

c) $\sqrt{8x + 17} - \sqrt{2x + 8} = 3$

Equations with Absolute Values

7. Solve

a) $|x| = 9$

b) $|x-3| = 2$

c) $8-|x+2| = 7$

d) $|3x+1|+2 = 1$

e) $|x^2-3x-12| = 6$

Equations with several variables

Equations in science and engineering often include many variables, and it is useful to be able to solve for one of the variables in terms of the others.

8. The following equation comes from the physics of circuits $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Solve for R_{eq} .

9. $T_s = 2\pi\sqrt{\frac{m}{k}}$ Solve for k .

Chapter 2B - Solving Inequalities

True or False: 4 < 4 True or False: 4 ≤ 4

Also though 3 < 4 , -4 < -3 which could also be written -3 > -4.

Written more generally, if $a < b$, then ____________________

This leads to the rule that when you multiply both sides of an inequality by a negative number, you change the direction of the inequality.

1. Solve

a) $1 - x > 2$

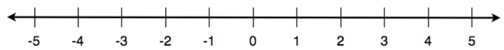
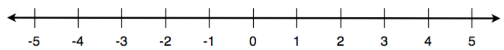
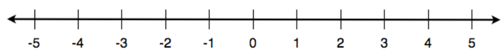
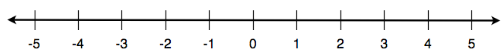
b) $1 - 2x < 17 - 4x \leq 8 - x$

c) $4x + 1 < 3x + 2 < x + 3$

d) $4x + 1 \leq 4 - 2x \leq 3 - x$

Absolute Value Inequalities

Determine the solution sets for the following inequalities

	Number Line	Inequality	Interval Notation
$\|x\| \le 3$			
$\|x\| > 3$			
$\|x\| > -3$			
$\|x\| < -3$			

2. Solve

a) $|x+2|<1$

b) $|2x-4|\ge 3$

c) $|5-2x|>4$

Nonlinear Inequalities

Consider $a,b \in \mathbb{R}$ $ab>0$, then either

or

If $ab<0$ then either

or

To solve nonlinear inequalities:

I. Move every term to one side (make one side zero).

II. Factor the nonzero side.

III. Find the critical values. (Critical values make the expression zero or undefined.)

IV. Let the critical numbers divide the number line into intervals.

V. Determine the sign of each factor in each interval.

VI. Use the sign of each factor to determine the sign of the entire product or quotient.

3. Solve

a) $(x+1)(x-2) > 0$

b) $\frac{2}{x} < 1$

c) $x^4 \le x^2$

d) $\frac{2x+3}{x+3} > \frac{6x+1}{1-x}$