MATH 152, Spring 2013

Exam III – Form A

Last name: ________________________ First name: ________________________

Signature: ____________________________________

Instructor: ____________________________________

Section number: ______

INSTRUCTIONS

1. In **Part 1** (Problems 1 – 14), mark the correct choice on your ScanTron form using a No. 2 pencil. **Record your choices on your exam. Scantrons will not be returned.**

2. Be sure to write your **name** and **section** number on your ScanTron.

3. In **Part 2** (Problems 15 – 18), present your solutions in the space provided. **Show all your work** neatly and concisely and **box your final answer**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading to it.

4. THE USE OF CALCULATORS, BOOKS OR NOTES OF ANY SORT IS NOT PERMITTED IN THIS EXAMINATION. TURN OFF and HIDE YOUR CELL PHONES.

(4 points each)

1. Consider the following pair of series:

$$\text{(I)} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{3/2}} \qquad\qquad \text{(II)} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1/2}}$$

Which of the following statements is true?

(a) Both series are divergent.

(b) Both series are convergent but not absolutely convergent.

(c) Both series are absolutely convergent.

(d) (I) is absolutely convergent, (II) is convergent but not absolutely convenrgent.

(e) (I) is convergent but not absolutely convergent, (II) is absolutely convergent.

2. Find all values of x such that vectors $\mathbf{a} = \langle x+2, x, x\rangle$ and $\mathbf{b} = \langle x-2, x+1, x\rangle$ are orthogonal.

(a) $x = -\frac{1}{6}$

(b) $x = -\frac{3}{2}, x = -1$

(c) $x = -1$

(d) $x = -\frac{4}{3}, x = 1$

(e) $x = 0$

3. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\dfrac{n-2}{n^2+2}$

(a) converges by the Alternating Series Test but not absolutely convergent.

(b) diverges by comparison with $\sum_{n=1}^{\infty}\dfrac{1}{n}$.

(c) converges absolutely by the Ratio Test.

(d) diverges by the Ratio Test.

(e) converges absolutely by the Alternating Series Test.

4. Find the coordinates of the center of the sphere whose equation is

$$x^2 + y^2 + z^2 - 2x + 4z = 1$$

(a) $(-1, 0, 2)$

(b) $(1, 0, -2)$

(c) $(1, 1, 1)$

(d) $(0, 1, -2)$

(e) $(1, -2, 0)$

5. Compute the sum of the infinite series $\sum_{n=1}^{\infty}\dfrac{x^{3n}}{n!}$

(a) e^{x^3}

(b) $e^{x^3} - 1$

(c) xe^{x^3}

(d) $xe^{x^3} - 1$

(e) $x\left(e^{x^3} - 1\right)$

6. The Maclaurin series for the function $f(x) = x\cos(x^2)$ is

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n}}{(2n)!}$

(b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n)!}$

(c) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+1}}{(4n+1)!}$

(d) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$

(e) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+1}}{(2n)!}$

7. The vector projection of the vector $\langle 1,2,3\rangle$ onto the vector $\langle 1,1,1\rangle$ is

(a) $\langle 2,4,6\rangle$

(b) $\left\langle \frac{3}{7},\frac{6}{7},\frac{9}{7}\right\rangle$

(c) $\left\langle \frac{3}{7},\frac{3}{7},\frac{3}{7}\right\rangle$

(d) $\langle 2,2,2\rangle$

(e) $\frac{3}{\sqrt{7}}$

8. If we express the function $f(x) = \dfrac{1}{3+4x}$ as a power series centered at 0, what is the corresponding radius of convergence?

(a) $\frac{1}{4}$

(b) ∞

(c) 0

(d) $\frac{4}{3}$

(e) $\frac{3}{4}$

9. Suppose that the power series $\sum_{n=0}^{\infty} c_n x^n$ has the radius of convergence 4. Consider the following pair of series:

$$\text{(I)} \sum_{n=0}^{\infty} c_n 3^n \qquad\qquad \text{(II)} \sum_{n=0}^{\infty} c_n 7^n$$

Which of the following statements is true?

(a) Neither series is convergent.

(b) Both series are convergent.

(c) (I) is convergent, (II) is divergent.

(d) (I) is divergent, (II) is convergent.

(e) No conclusion can be drawn about either series.

10. Find the power series expansion for the function $\dfrac{1}{(1-x)^2}$ centered at 0.

(a) $\displaystyle\sum_{n=1}^{\infty} x^{2n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n n x^{n-1}$

(c) $\displaystyle\sum_{n=1}^{\infty} n x^{n-1}$

(d) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$

11. For which of the series below is the Ratio Test inconclusive?

(a) $\sum_{n=1}^{\infty} \frac{n^2 - 3}{n!}$

(b) $\sum_{n=1}^{\infty} \frac{1}{2^n}$

(c) $\sum_{n=1}^{\infty} \frac{(-5)^n}{n^2 + 3}$

(d) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

(e) $\sum_{n=1}^{\infty} \frac{n!}{2^n}$

12. Consider the third partial sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ as an approximation for the sum of the series. Use the Alternating Series Estimating Theorem to find an upper bound of the absolute value of the error.

(a) $\frac{1}{64}$

(b) $\frac{1}{81}$

(c) $\frac{1}{216}$

(d) $\frac{1}{95}$

(e) $\frac{1}{125}$

13. Find the second degree Taylor polynomial of $f(x) = \sin x$ centered at $a = \dfrac{\pi}{4}$.

(a) $T_2(x) = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}\left(x - \dfrac{\pi}{4}\right) + \dfrac{\sqrt{2}}{4}\left(x - \dfrac{\pi}{4}\right)^2$

(b) $T_2(x) = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}\left(x - \dfrac{\pi}{4}\right) - \dfrac{\sqrt{2}}{2}\left(x - \dfrac{\pi}{4}\right)^2$

(c) $T_2(x) = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}\left(x - \dfrac{\pi}{4}\right) + \dfrac{\sqrt{2}}{2}\left(x - \dfrac{\pi}{4}\right)^2$

(d) $T_2(x) = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}x - \dfrac{\sqrt{2}}{4}x^2$

(e) $T_2(x) = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}\left(x - \dfrac{\pi}{4}\right) - \dfrac{\sqrt{2}}{4}\left(x - \dfrac{\pi}{4}\right)^2$

14. Let $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{n+1}{2^n}(x-3)^n$. Find $f^{(10)}(3)$, the 10th derivative of $f(x)$ at $x = 3$.

(a) $\dfrac{11!}{2^{10}}$

(b) $\dfrac{10!}{2^{10}}$

(c) $\dfrac{11!}{2^{11}}$

(d) $\dfrac{11}{2^{10}}$

(e) $\dfrac{11}{2^{11}}$

Partial credit is possible. No credit for unsupported answers will be given.

15. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{e^{-\frac{1}{n}}}{n^2}$.

(a) (8 pts) Use the Integral Test to prove convergence.

(b) (4 pts) Estimate the error if we approximate the series by the 9th partial sum.

16. (10 pts) Find the Taylor series for $f(x) = xe^x$ centered at $a = 3$.

17. (10 pts) Find the radius and the interval of convergence for $\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^n \sqrt{n+1}}$. Be sure to test the endpoints for convergence.

18. (a) (8 pts) Find the power series representation for $f(x) = \ln(4 - x)$ centered at 0 and its radius of convergence.

(b) (4 pts) Use the series in part (a) to evaluate the indefinite integral $\int x^2 \ln(4 - x)\, dx$ as a power series about 0.

DO NOT WRITE BELOW!

1–14.	15.	16.	17.	18	Total

Last name: ______________________ First name: ______________________ Section: ______