Chapter 1E - Complex Numbers

$\sqrt{-16}$ exists! So far the largest (most inclusive) number set we have discussed and the one we have the most experience with has been named the real numbers.

And $\forall\ x \in \mathbb{R}$, $x^2 \geq 0$

But there exists a number (that is not an element of $\mathbb{R}$) named i and $i^2 =$ _____

Since $i^2 =$ _____, $\sqrt{-1} =$ __________, so $\sqrt{-16}$ = ________________________

i is not used in ordinary life, and humankind existed for 1000's of years without considering i. i is, however, a legitimate number. i, products of i, and numbers like $\sqrt{2}+i$ are solutions to many problems in engineering. So it is unfortunate that it was termed "imaginary!"

i and numbers like $4i$ and $-3i$ are called ____________________________

Numbers like $\sqrt{2}+i$ and $\frac{17}{3}-5i$ are called ____________________________

More formally $\mathbb{C}$ is the set of all numbers ______________________________

When a complex number has been simplified into this _________ form, it is called

____

1. Put the following complex numbers into **standard form**:

a) $\left(16+\sqrt{-9}\right)+\left(-13-\sqrt{-100}\right)$ = ____

b) $(1+2i)-(3-4i)$ = ____

c) $(2+3i)(4+5i)$ = ____

d) $(5+4i)(3-2i)$ = ____

e) i^3 = ____________ f) i^4 = ____________ g) i^5 = ____________

Graphing Complex Numbers

When we graph elements of $\mathbb{R}$, we use ________________________________

When we graph elements of $\mathbb{C}$, we use the complex plane which will seem a lot like the Cartesian plane.

In the Cartesian plane, a point represents __________________________________

In the complex plane, a point represents ___________________________________

The horizontal axis is the ____

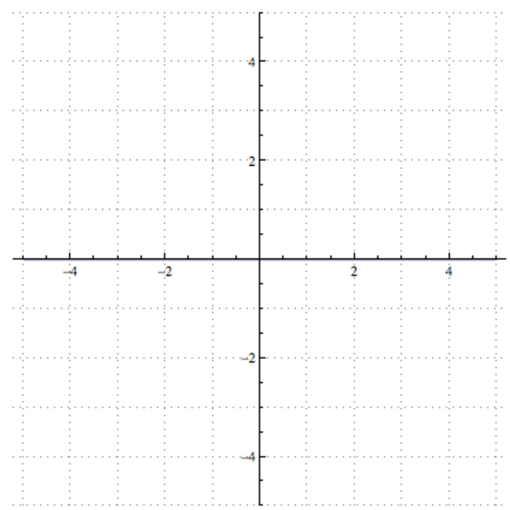
The vertical axis is the ____

2. Graph and label the following points on the complex plane:

A $3+4i$ B $-2+i$

C $\frac{3}{2}-3i$ D $3i$

E -4 F $-1-2i$



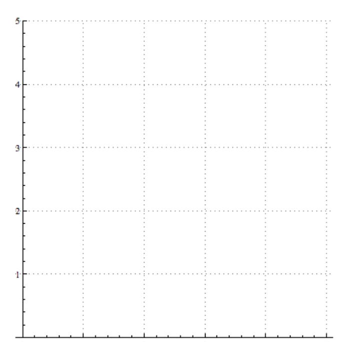
Absolute Value of a Complex Number:

If $z \in \mathbb{C}$ then $|z|$ is defined as its

____

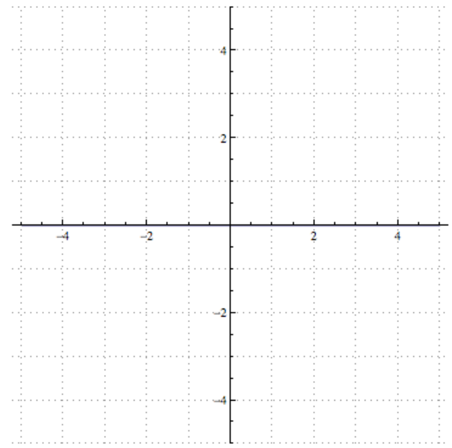
Calculate $|3+4i|$ (How far is $3+4i$ from the origin?)

$|3+4i|=$ _____________



Consider $a+bi$, an arbitrary element of $\mathbb{C}$.

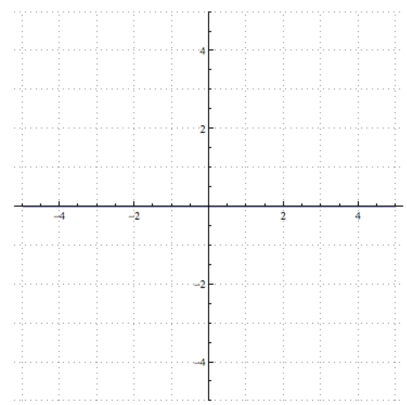
What is $|a+bi|$? (How far is $a+bi$ from the origin?)



In general $|z|$ = ______________________________

Notice that $|z|$ is a real number. It is z's distance from the origin. We did not use i to calculate $|z|$.

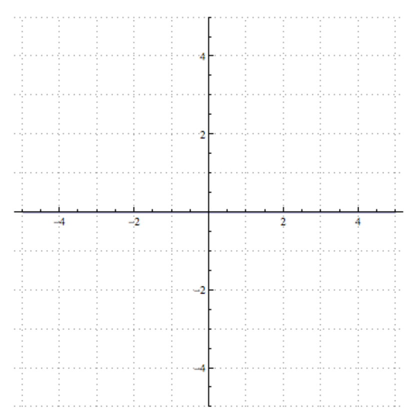
3. Calculate $|-5+i|$. (How far is $-5+i$ from the origin?)



$|-5+i|$ **=** ____________________

Complex Conjugate

If $z=3+4i$, then $\bar{z}$ = _________________ .



If $z=-1-2i$, then $\bar{z}$ = __________ .

Graphically the complex conjugate is the

_________________________ of the number through the

____

More generally if $z=a+bi$, then $\bar{z}$ = _______________________

4. Put the following complex numbers into standard form.

a) $\overline{2-3i}$ = ________________ b) $\overline{4i}$ = ____________________

c) $\overline{5}$ = _____________________ d) $\overline{3i-2}$ = ____________________

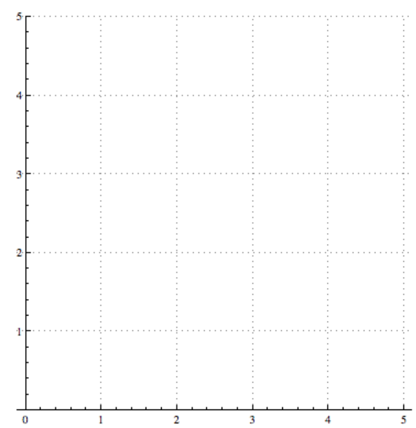
Finally notice that $(3+4i)(3-4i)=$ ____

More generally $(a+bi)(a-bi)=$ ____

in other words $z \bullet \bar{z}=$ ____

Distance Between Two Complex Numbers

Plot and label two points in the complex plane
$z_1 = 3+5i$ and $z_2 = 1+2i$
The distance between z_1 and z_2 is



Just for fun, calculate $z_1 - z_2$ = ____

And $|z_1 - z_2|$ = ____

What we have seen is that for this particular z_1 and z_2, the distance between these points is equal to ___________________. But this is actually true for all complex numbers.

The distance between two general points z_1 and z_2 is ________________

5. Solve $0 = x^2 + 4$

6. Solve $3x^2 + 1 = 2x$

7. Solve $8x^4 + 18x^2 - 5 = 0$

Suggested Problems:

Text: 1-12

My Previous Exams: Fall 2014: 11, a, b, 12 Fall 2013 1A: 5,
Spring 2013 1A: 7, Fall 2012 1A: 5

Chapter 9 -- Vectors

Remember that $\mathbb{R}$ is the set of real numbers, often represented by the number line,

$\mathbb{R}^2$ is the notation for the 2-dimensional plane. (Think about the Cartesian plane which is drawn by intersecting 2 numbers lines. $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$)

$\mathbb{R}^3$ is the notation for 3-dimensional space. (Imagine another number line passing through the origin, perpendicular to the plane.)

Vectors are used to represent quantities such as force and velocity which have both

________________________ and ________________________.

The magnitude of a vector corresponds to its ________________.

Vectors are directed line segments. (Like the cross between a segment and a ray.)

Each of these directed line segments has an initial (starting) point and a terminal (ending) point.

Naming Vectors:

Vectors in $\mathbb{R}^2$ are defined by their terminal point on the Cartesian plane, assuming their initial point is at the origin.

For example, the vector $\langle 3,4 \rangle$ has its initial point at the origin and its terminal point at $(3,\ 4)$.

Notice that distinction between the vector $\langle 3,4 \rangle$ and its terminal point $(3,\ 4)$.

$\langle 3,4 \rangle$ is a vector. It is a directed line segment. It has magnitude and direction.

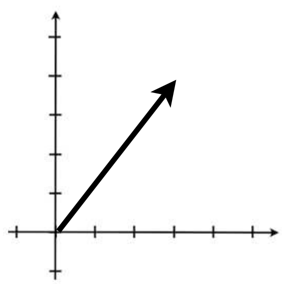
$(3,\ 4)$ is simply an ordered pair of real numbers.

Definition: A vector $\vec{v}$ in the Cartesian plane is *defined by* an ordered pair of real numbers in the form $\langle v_x, v_y \rangle$. We write $\vec{v} = \langle v_x, v_y \rangle$ and call v_x and v_y the ______________________

of vector $\vec{v}$. Specifically

v_x is the ______________________ and v_y is the ______________________ of the vector.

The graphical representation of $\langle 3,4 \rangle$ is given in Figure 1.



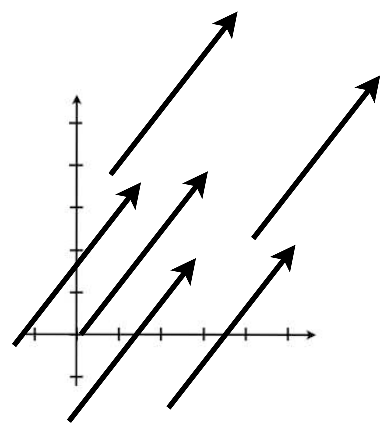


Figure 2

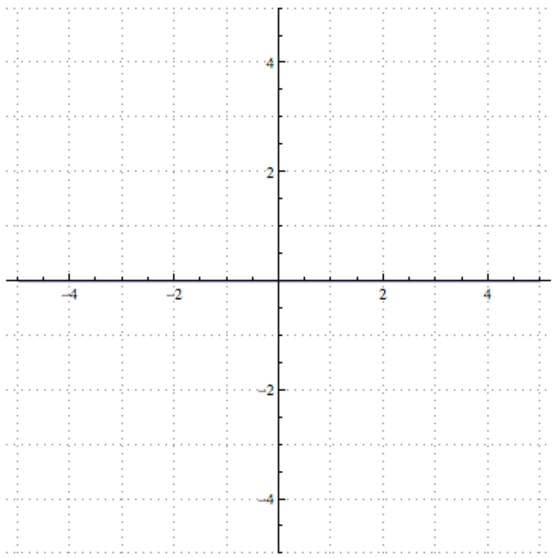
Note: Although the vectors in Figure 2 have different initial points and different terminal points, they are all equivalent to $\langle 3,4 \rangle$ because they have the same magnitude and direction. Usually what is important about the vector $\vec{v}$ is not where it is, but how long it is and which way it points.

Two or more vectors with the same magnitude and direction are ___________________

When a vector's initial point is NOT at the origin, we must figure out how to name the vector so that it has the same magnitude and direction as the equivalent vector with initial point at the origin.

Note that you can think of the vector $\langle 3,4 \rangle$ as set of instructions to get from the initial point to the terminal point: "Go to the right 3 and up 4."

Consider a vector with initial point at (3, -4) and terminal point at (-2, 0).



To *name* this vector think about how you would instruct someone to move from the initial point to the terminal point. Those instructions would be to move

right left ________________

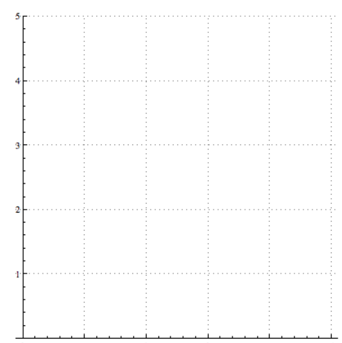
up down ________________

So the name of the vector with initial point at (3, -4)

and terminal point at (-2, 0) is ___________

What is the name of the vector with initial point at (x_0, y_0) and terminal point at (x_T, y_T) ?

What is the length of $\langle 3,4 \rangle$?



Length of a vector:

The magnitude or length of a vector $\langle v_x, v_y \rangle$ is denoted $\left\| \langle v_x, \mathrm{v}_y \rangle \right\|$

$$\left\| \langle v_x, \mathrm{v}_y \rangle \right\| =$$

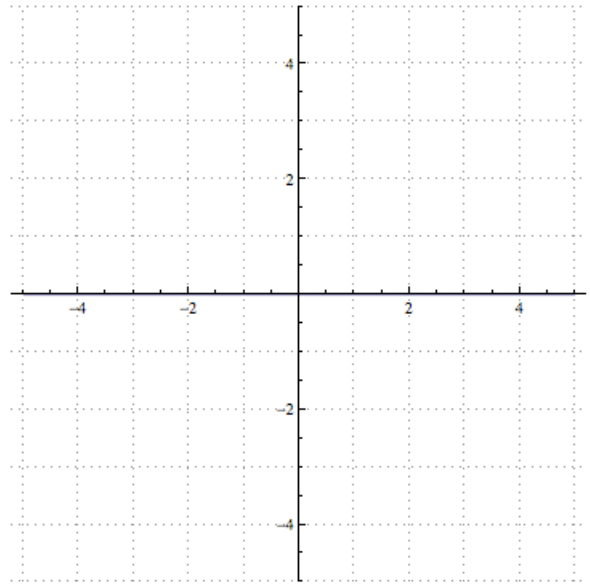
In $\mathbb{R}^3$ the length of a vector $\langle v_x, v_y, v_z \rangle$ is denoted $\left\| \langle v_x, v_y, v_z \rangle \right\|$

$$\left\| \langle v_x, v_y, v_z \rangle \right\| =$$

Adding vectors

If $\vec{u} = \langle 3,\ -5\rangle$ and $\vec{v} = \langle -4,\ 1\rangle$,

then $\vec{u} + \vec{v}$ =



Geometrically, imagine “picking up” vector $\vec{v}$ and putting its initial point at the terminal point of $\vec{u}$. Then the new terminal point of $\vec{v}$ will be at the terminal point of $\vec{u} + \vec{v}$.

Notice that algebraically and geometrically, $\vec{u} + \vec{v}$.= $\vec{v} + \vec{u}$

In other words, vector addition is ________________________________

Definition: If $\vec{u} = \langle u_x,\ u_y\rangle$ and $\vec{v} = \langle v_x,\ v_y\rangle$, then $\vec{u} + \vec{v}$ = ________________________

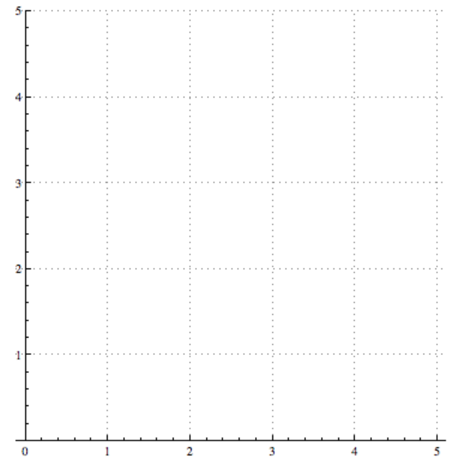
If $\vec{v} = \langle 1,\ 3\rangle$, find $\vec{v} + \vec{v} + \vec{v} + \vec{v}$

This suggests the reasonableness of the following definition:

Scalar Multiplication: (When working with vectors, the term scalar refers to a real number.)

If $\vec{u} = \langle u_x,\ u_y\rangle$ and c is a real number, then the scalar multiple $c\,\vec{u}$ =

If $\vec{v} = \langle 1, 2 \rangle$, then



$2\vec{v}$ = ____________________

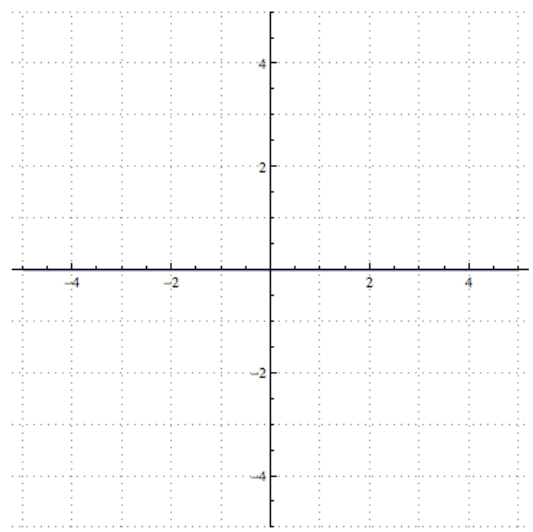
$3\vec{v}$ = ____________________

Notice how $\vec{v}$, $2\vec{v}$ and $3\vec{v}$ all have the

same ____________________

So scalar multiplication is simply the multiplying a vector by a number. The result is another vector.

A scalar of particular interest is -1.



If $\vec{u} = \langle u_x, u_y \rangle$, then

$(-1)\vec{u}$ = $-\vec{u}$ = ____________________

If $\vec{v} = \langle 1, 2 \rangle$, then $-\vec{v}$ = ____________________

Notice how $\vec{v}$ and $-\vec{v}$ lie

____

Direction of a vector:

We say that two nonzero vectors $\vec{u}$ and $\vec{v}$ have the

same direction if ____

opposite directions if ____

Give two examples of vectors that are not equivalent to $\langle 3, -5 \rangle$, but have the same direction.

Give two examples of vectors that have the opposite direction of $\langle 3, -5 \rangle$.

Do $\langle 6, 10 \rangle$ and $\langle 9, 15 \rangle$ have the same direction? If $\langle 6, 10 \rangle$ and $\langle 9, 15 \rangle$ have the same direction then

Do $\langle 4, 9 \rangle$ and $\langle 2, 3 \rangle$ have the same direction? If $\langle 4, 9 \rangle$ and $\langle 2, 3 \rangle$ have the same direction then

Unit Vectors

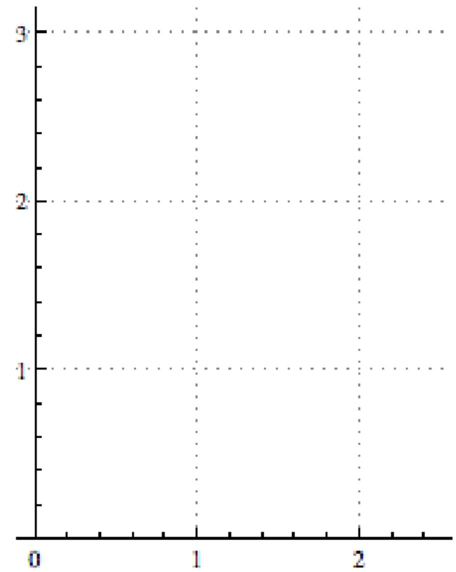
A unit vector is a vector with length ________.

Is $\vec{v} = \langle 1, 2 \rangle$ a unit vector? ______

What is the length of $\langle 1, 2 \rangle$? $\|\langle 1, 2 \rangle\|$ = ________

What if you wanted a unit vector that is in the same direction as $\langle 1, 2 \rangle$?

You would want a scalar c, so that $\|c\langle 1, 2 \rangle\| = 1$



What if you wanted a unit vector that is in the same direction as $\langle v_x, v_y \rangle$?

You would want a scalar c, so that $\|c\langle v_x, v_y \rangle\| = 1$

If $\vec{v} = \langle v_x, v_y \rangle$, then ________________________ is a unit vector in the same direction as $\vec{v}$.

If $\vec{v} = \langle -1, 3 \rangle$, what is the unit vector that is in the opposite direction as $\vec{v}$?

There are two really important unit vectors: $\vec{i}$ = ________ and $\vec{j}$ = ________

In $\mathbb{R}^3$ $\vec{i}$ = ____________ $\vec{j}$ = ____________ and $\vec{k}$ = ____________

Sometimes these are named $\overrightarrow{e_1}$ = ____________ and $\overrightarrow{e_2}$ = ____________

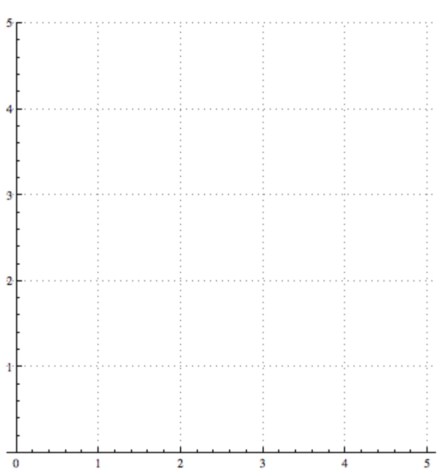
In $\mathbb{R}^3$ $\overrightarrow{e_1}$ = ____________ $\overrightarrow{e_2}$ = ____________ and $\overrightarrow{e_3}$ = ____________

Notice that if $\vec{v} = \langle 3,-5 \rangle$, then

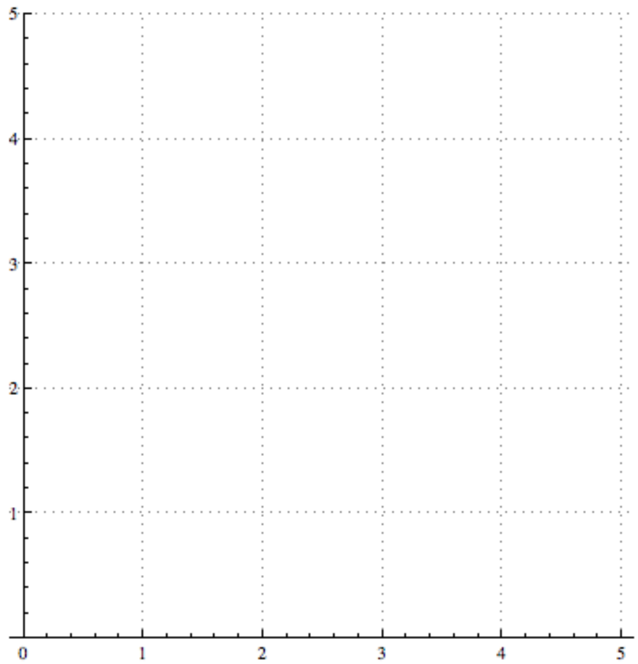
If $\vec{v} = \langle v_x, v_y \rangle$, then

In other words every vector is a ________________________________ of $\vec{i}$ and $\vec{j}$.

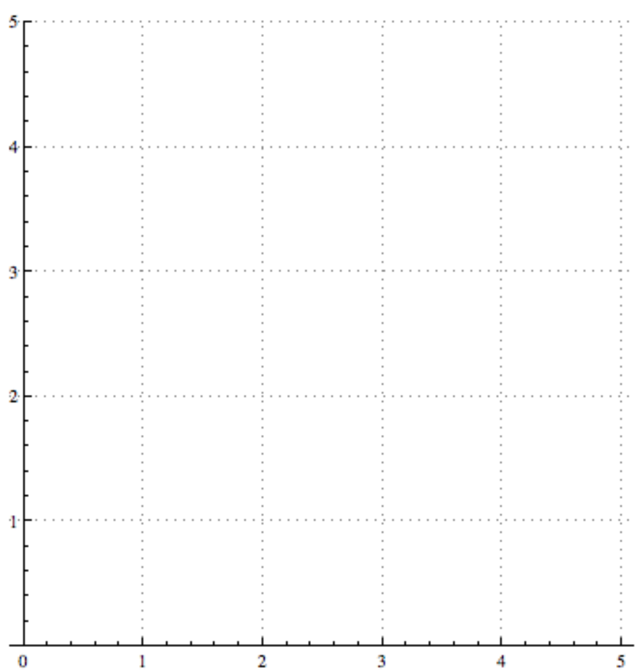
Direction angles: If $\vec{v}$ makes an angle θ with the positive x-axis, then



If $\vec{u} = \langle 5, 2 \rangle$ and $\vec{v} = \langle 1, 5 \rangle$, find the angle θ, between $\vec{u}$ and $\vec{v}$.



If $\vec{u} = \langle u_x, u_y \rangle$ and $\vec{v} = \langle v_x, v_y \rangle$, find the angle θ, between $\vec{u}$ and $\vec{v}$.



Vector Dot Product

If $\vec{u} = \langle u_x, u_y \rangle$ and $\vec{v} = \langle v_x, v_y \rangle$, then the dot product

$$\vec{u} \bullet \vec{v} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Notice that the dot product of two vectors is a ___________________.

The vector dot product is sometimes called the _________________________________

It is important to notice the distinction between the scalar product of two vectors (an operation on two vectors which yields a scalar), and scalar multiplication of a vector (an operation between a scalar and a vector that yields a vector).

Notice also that $\vec{u} \bullet \vec{v}$

Find $\langle 3, 4\rangle \bullet \langle -2, 5\rangle$

Find the angle between the vectors $\langle 3, 4\rangle$ and $\langle -2, 5\rangle$

$\vec{i} \bullet \vec{j} =$

What is the angle between $\vec{i}$ and $\vec{j}$?

$\cos 90^\circ =$

Notice that since $\vec{u} \bullet \vec{v}$ = ________________________________

if $\vec{u}$ and $\vec{v}$ are $\perp$ then $\vec{u} \bullet \vec{v}$ = ____________

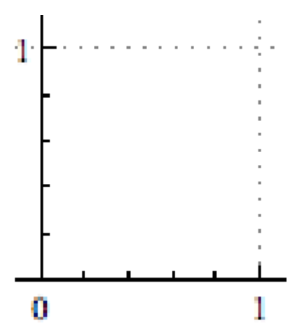
In fact $\vec{u} \bullet \vec{v} = 0$ iff ____

Synonyms for perpendicular include ____

In $\mathbb{R}^3$ If $\vec{u} = \langle u_x, u_y, u_z \rangle$ and $\vec{v} = \langle v_x, v_y, v_z \rangle$,

then the dot product $\vec{u} \bullet \vec{v}$

Find a vector perpendicular to $\langle 1, 1 \rangle$



Find all of the unit vectors perpendicular to $\langle 1, 1 \rangle$.

Find a vector perpendicular to $\langle 3, -4 \rangle$

Find all the unit vectors perpendicular to $\langle 3, -4 \rangle$

An airplane is flying at 300 miles per hour, heading 30 degrees North of East. What are the magnitudes of the North and East components of the velocity?

A wind from due North starts blowing at 40 miles per hour. What is the new velocity of the plane? (magnitude and direction)

A river flows at 3 mph and a rower rows at 6 mph. What heading should the rower take to go straight across the river?

What if the river flowed at 6 mph and the rower rowed at 3 mph?