Chapter 1A -- Real Numbers

Types of Real Numbers

	Name(s) for the set	Symbol(s) for the set
1, 2, 3, 4, …	Natural Numbers Positive Integers	
…, -3, -2, -1	Negative integers	
0, 1, 2, 3, 4, …	Non-negative Integers Whole Numbers	
…, -3, -2, -1, 0, 1, 2, 3, …	Integers	

Note: ______________________________ is the German word for number.

$\frac{4}{5}$, $\frac{-17}{13}$, $\frac{1}{29}$	Rational Numbers	

Note: This is the first letter of _________________________

1. Circle One:

a) 5 is a rational number TRUE FALSE

b) $\frac{\pi}{4}$ is a rational number TRUE FALSE

c) $\frac{\sqrt{16}}{5}$ is a rational number TRUE FALSE

d) $\frac{\sqrt{17}}{5}$ is a rational number TRUE FALSE

e) 0 is a rational number TRUE FALSE

Definition: If a, b $\in \mathbb{Z}$ and $b \neq 0$, then $\frac{a}{b} \in \mathbb{Q}$.

Translation: ____

Decimal Expansions of Rational Numbers:

Fact: The decimal expansions of rational numbers either

________________________ or ______________________________

Examples $\frac{1}{4}$ = ____________________ (This is a terminating decimal.)

$\frac{4}{9}$ = __________________ (This is a repeating decimal.)

You know how to express a terminating decimal as a fraction. For example $.25 = \frac{25}{100}$, but how do you express a repeating decimal as a fraction?

2. Express these repeating decimals as the quotient of two integers:

a) $.373737\ldots$ or $\overline{.37}$

Let $x = .37373737\ldots$

Then $100x =$

b) $\overline{.123}$

Let $x = .123123123\ldots$

Then

There are a lot of rational numbers, but there are even more irrational numbers. Irrational numbers cannot be expressed as the quotient of two integers. Their decimal expansions never terminate nor do they repeat.

Examples of irrational numbers are ____

The union of the rational numbers and the irrational numbers is called the

________________________________ or simply the _____________________. It is denoted by ______.

Math Symbols:

$\in$

$\forall$

$\exists$

iff

$\Leftrightarrow$ **or** $\leftrightarrow$

Properties of Real Numbers

4. What properties are being used?	Name of Property	
a) $(-40)+8(x+5)=8(x+5)+(-40)$		$a+b=b+a \quad \forall \; a,b\in\mathbb{R}$
b) $8(x+5)+(-40)=8x+40+(-40)$		$a(b+c)=ab+ac \quad \forall \; a,b,c\in\mathbb{R}$
c) $8x+40+(-40)=8x+(40+(-40))$		$(a+b)+c=a+(b+c) \quad \forall \; a,b,c\in\mathbb{R}$
d) $0+5=5+0=5$		$0+a=a+0=a \quad \forall \; a\in\mathbb{R}$
e) $7+(-7)=(-7)+7=0$		For each $a\in\mathbb{R}$, $\exists$ another element of $\mathbb{R}$ denoted by $-a$ such that $a+(-a)=(-a)+a=0$
f) $6\cdot 1=1\cdot 6=6$		$1\cdot a=a\cdot 1=a \quad \forall \; a\in\mathbb{R}$
g) $4\cdot\left(\frac{1}{4}\right)=\left(\frac{1}{4}\right)\cdot 4=1$		For each $a\in\mathbb{R}$, $a\neq 0$ $\exists$ another element of $\mathbb{R}$ denoted by $\frac{1}{a}$ such that $a\cdot\left(\frac{1}{a}\right)=\left(\frac{1}{a}\right)\cdot a=1$

When do we use the commutative property of addition? Consider $7+28+43$.

The rules for order of operation tell us to add left to right, but the commutative property of addition allows us to change the order and make our life easier

$$7+28+43= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

When do we use the associative property of multiplication? Consider 16×25 .

$16\times 25=$ ____________________________________

Is subtraction commutative? ________________ Example: _________________________

Is division commutative? ________________ Example: _________________________

Is subtraction associative? ________________ Example: _________________________

Is division associative? ________________ Example: _________________________

Absolute Value

$|4|=$ __________ $|-9|=$ __________

Formal definition for absolute value

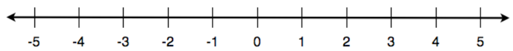
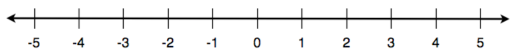
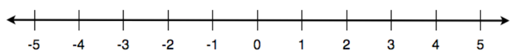
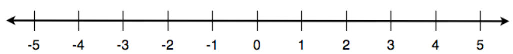
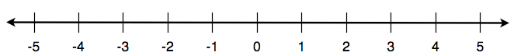
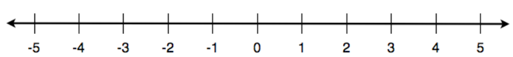
$$|x|=\begin{cases} \\ \\ \end{cases}$$

It will help if you learn to think of absolute value as ____________________________

(Notice that -4 is 4 units away from the origin.)

Distance between two points:

3.) Find the distance between

a) 4 and 1		
b) -1 and 5		
c) 4 and -2		
d) -3 and -4		
e) 1 and x		
f) x and y		

Notice that $|4-1| = |1-4|$ and $|-3-2| = |2-(-3)|$. In general $|x-y| =$ ________

Notice that $|-5+4| =$ _____ whereas $|-5|+|4| =$ _____ So, in general $|x+y| \neq$ _______

Suggested problems:

Text: 1-9, 22, 23

My Previous Exams: Spring 2013 1A: 3, 4, Spring 2014 1A: 13b, c, d, e

Dr. Scarborough's Previous Exams Fall 2013 I: p2:2, Fall 2012 I: p2:1

Dr. Scarborough's Fall 2013 WIR 1: 1, 2, 4, 6, 21

Dr. Kim's Fall 2014 WIR:

Chapter 1B - Exponents and Radicals

Multiplication is shorthand notation for repeated addition $4+4+4+4+4=5\times 4$

Exponents are shorthand notation for repeated multiplication. So

$2\times 2\times 2\times 2\times 2=$ ________ $x \bullet x \bullet x \bullet x=$ ___________

You probably know that $3^{-1}=$ __________ and $x^{-5}=$ _____________ this is because

by definition $x^{-1}=$ ___________ . In general if $m\in \mathrm{N}$, then $x^{-m}=$ ___________

Properties of exponents

Example	In general
$a^4 a^3$	$x^m x^n$
$\frac{b^8}{b^5}$	$\frac{x^m}{x^n}$
$\frac{y^3}{y^3}$	x^0 for $x\neq 0$

Note: $0^0\neq 0$ and $0^0\neq 1$ 0^0 is an ___________________________________

More Properties of Exponents

Example	In general
$(xy)^3$	$(xy)^m$
$\left(\frac{x}{y}\right)^2$	$\left(\frac{x}{y}\right)^m$

1. Simplify

a) $(-8)^2$ b) -8^2 c) 3^0

Dividing is the same as ____

d) $\left(\dfrac{1}{y^{-6}}\right)$

e) $\left(\dfrac{a}{b}\right)^{-1}$

e) $\dfrac{x^{-3}}{x^2}$

f) $\left(\dfrac{y^4}{y^{-5}}\right)^{-1}$

g) $\left(\dfrac{12a^{-3}b^2}{4a^4b^{-1}}\right)^{-2}$

h) $\dfrac{\left(\dfrac{1}{3}\right)^{-1005} - 27^{334}}{9^{502} + 9^{501}}$

Roots or Radicals

TRUE or FALSE: $\sqrt{4} = \pm 2$.

Important distinction: $x^2 = 25$ has ______ solutions: x = _______ and x = _____

$\sqrt{25}$ represents exactly _________ number. $\sqrt{25}$ = ________.

Recall that $2^5 = 32$. Now suppose you have $x^5 = 32$ with the instructions: "Solve for x."

How do you express x in terms of 32? The vocabulary word is __________________

In this case x equals the _______________________________.

The notation is $x =$ ____________________ or $x =$ ____________________

This second notation is called a fractional exponent.

Examples: $\sqrt[4]{81}$ = $(81)^{\left(\frac{1}{4}\right)}$ = ____________ because $3^{\square}$ = _________.

$(125)^{\left(\frac{1}{3}\right)}$ = ____________ because $5^{\square}$ = _________.

In general if $a^n = b$ then ____________________

and if ____

So in shorthand __________________________________

Properties of Roots (just like properties of exponents!)

Example	In general
$\sqrt[3]{27 \cdot 8}$ $\sqrt[3]{27}\sqrt[3]{8}$	$\sqrt[m]{xy}$
$\sqrt{\frac{36}{4}}$ $\left(\frac{\sqrt{36}}{\sqrt{4}}\right)$	$\frac{\sqrt[m]{x}}{\sqrt[m]{y}}$

Now consider $8^{\frac{2}{3}}$. Notice that $\left(8^2\right)^{\frac{1}{3}} =$ ______________________________

Also $\left(8^{\left(\frac{1}{3}\right)}\right)^2 =$ ______________________________

So it would seem that $8^{\frac{2}{3}} =$ ______________________________

In fact, more generally, it is true that $x^{\left(\frac{m}{n}\right)} =$ ______________________________

Similarly, consider $\sqrt[3]{\sqrt{64}} = \left(64^{\left(\frac{1}{2}\right)}\right)^{\frac{1}{3}} =$ ______________________________

Whereas, $\sqrt{\sqrt[3]{64}} = \left(64^{\left(\frac{1}{3}\right)}\right)^{\frac{1}{2}} =$ ______________________________

So it would seem that $\sqrt[3]{\sqrt{64}}$ ______________________________

In general $\sqrt[m]{\sqrt[n]{x}}$ ______________________________

Simplify $\sqrt[n]{a^n}$ **for all** $n \in \mathbb{N}$**.**

In recitation you graphed $y = \sqrt[3]{x^3}$. Hopefully you found that its graph looked just like the graph of $y = x$. In other words, you saw that $\sqrt[3]{x^3} =$ ____________

You also graphed $y = \sqrt[2]{x^2}$. Hopefully you found that its graph looked just like the graph of $y = |x|$. In other words, you saw that $\sqrt[2]{x^2} =$ ____________

Most people believe that $\sqrt[5]{2^5} =$ _______ and agree easily that $\sqrt[n]{2^n} =$ _______ $\forall\ n$

But $\sqrt[4]{(-2)^4}$ = ________________________ while $\sqrt[5]{(-2)^5} =$ ________________________

So hopefully you see believe $\sqrt[n]{(-2)^n} = 2$ whenever n is ___________

and $\sqrt[n]{(-2)^n} = -2$ whenever n is ___________

So to simplify $\sqrt[n]{a^n}$ for any $a \in \mathbb{R}$ and any $n \in \mathbb{N}$ we have

$$\sqrt[n]{a^n} = \begin{cases} \\ \\ \end{cases}$$

Simplifying Radicals

A radical expression is simplified when the following conditions hold:

1. All possible factors (“perfect roots”) have been removed from the radical.
2. The index of the radical is as small as possible.
3. No radicals appear in the denominator.

2. Simplify

a) $\sqrt[5]{-32}$

b) $(-216)^{\left(\frac{1}{3}\right)}$

c) $-9^{\left(\frac{1}{2}\right)}$

d) $\sqrt{-64}$

e) $\sqrt[6]{(-13)^6}$

f) $\sqrt[4]{16x^4}$

g) $\sqrt[3]{x^5}$

h) $\sqrt[3]{648x^4y^6}$

i) $\sqrt[8]{16x^4y^{12}}$

j) $\left(\frac{-125}{64}\right)^{\frac{-2}{3}}$

Rationalizing the Denominator

"Rationalizing the denominator" is the term given to the techniques used for eliminating radicals from the denominator of an expression without changing the value of the expression. It involves multiplying the expression by a 1 in a "helpful" form.

3. Simplify

a) $\frac{1}{\sqrt{2}}$

b) $\frac{1}{\sqrt[3]{2}}$

c) $\frac{1}{\sqrt[3]{4x}}$

d) $\sqrt{\frac{(9x)^3 y^{-4}}{50x^8 y^{-5}}}$

Notice: $\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$ = ____.

To simplify $\frac{1}{\sqrt{2}-\sqrt{5}}$ we multiply it by 1 in the form of ______________________.

So $\dfrac{1}{\sqrt{2}-\sqrt{5}} =$ and $\dfrac{41}{2+3\sqrt{5}} =$

Adding and Subtracting Radical Expressions

Terms must be alike to combine them with addition or subtraction. Radical terms are alike if they have the same index and the same radicand. (The radicand is the expression under the radical sign.)

4. Simplify

a) $\sqrt{5}+2\sqrt{7}-3\sqrt{5}-\sqrt{7}$

b) $\sqrt[3]{16x^4}-3x\sqrt{18x}-x\sqrt[3]{250x}$

c) $\dfrac{1}{\sqrt{3}}-\dfrac{1}{\sqrt{27}}$

Please notice $\sqrt{9}+\sqrt{16}$ = ______________________ Whereas $\sqrt{25}$ = ___________

In other words _____________________________

In general ___________________________

Suggested Problems:

Text: 1-32

My Previous Exams: Spring 2014 2A: 11, 14, Fall 2013 1A: 1, 2 ,
Spring 2013 1A: 1, 2, Fall 2012 1A: 1 a, b, d,

Dr. Scarborough's Previous Exams Fall 2013 I: p2:2, Fall 2012 I: p2:1

Dr. Scarborough's Fall 2013 WIR 1: 3, 5, 7-10, 12-13, 15-16, 19-20,
WIR 2: 2,
WIR 3: 12,13, 21, 22, 24

Dr. Kim's Fall 2014 WIR:

Chapter 1C - Polynomials

Which of these are examples of polynomials?

	Polynomial	Not a polynomial	Polynomials should not have
x^2+2x+1			
$x^7+\pi$			
$\left(\sqrt{2}\right)x^2+2x+1$			
$4\sqrt{x}-1$			
$\frac{1}{x}+3x^2$			
$\left(5x^3+3x^2-2\right)^{\frac{1}{4}}$			
$\left(5x^3+3x^2-2\right)^2$			
$x^2+\sin x$			
5			

Polynomials are expressions of the form $a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$ where $a_i\in\mathbb{R}$ and $n\in\mathbb{N}$

n is called ______________________________

a_n is called ______________________________

a_0 is called ______________________________

If *n* = 2, the polynomial is called a ________________________

If *n* = 3, the polynomial is called a ________________________

A polynomial with 2 terms is called a ________________________

A polynomial with 3 terms is called a ________________________

Factoring polynomials will help us in later chapters when we need to find the roots of polynomials.

Techniques for Factoring Polynomials

Common Factors: $12x^4 + 4x^2 + 2x =$ ____________________

Factor by Grouping: $3x^3 + x^2 - 12x - 4$ (Factoring by grouping is your best bet if you have a long cubic!)

Factor using special patterns:

Look what happens when you multiply and simplify:

$(x-3)(x+3) =$ ____________________

$(4-y)(4+y) =$ ____________________

Difference of Squares	$a^2 - b^2 =$

Look what happens when you expand

$(x+5)^2 =$ ____________________

$(x-2)^2 =$ ____________________

Square of a Binomial	$a^2 + 2ab + b^2 =$
	$a^2 - 2ab + b^2 =$

a) $x^2 - 144$

b) $16x^4 - y^6$

c) $a^2 + 10a + 25$

d) $x^4 + 8x^2 + 16$

Look what happens when you multiply

$(a+b)(a^2 - ab + b^2)$

Sum of Cubes	$a^3 + b^3$
Difference of Cubes	$a^3 - b^3$

1. Factor

a) $27 - y^3$

b) $x^6 + 64$

c) $27x^6 + 64$

d) $x^5 - 4x^3 - x^2 + 4$

e) $3x^3 + 6x^2 - 12x - 24$

f) $16x^4 - 25x^2$

Factoring quadratics, lead coefficient 1:

2. Factor : $x^2 + 10x + 16$ = ____

Find 2 numbers whose product is _______ and whose sum is ___________

3. Factor: $x^2 - 2x - 48$ = ____

Find 2 numbers whose product is _______ and whose sum is ___________

Factoring quadratics with lead coefficient is not 1

Use the Blankety-Blank Method!

4. Factor: $10x^2+11x+3$ = ____

Find 2 numbers whose product is _______ and whose sum is ___________

5. Factor: $3x^2-14x-5$ = ____

Find 2 numbers whose product is _______ and whose sum is ___________

Important Fact: a^2+b^2 does *not* factor over the real numbers.

So x^2+64 cannot be factored nor can y^2+100

Dividing Polynomials (This is a lot like long division of integers!)

6. $y=\dfrac{4x^3-x^2-5x+2}{x-1}$ = ____

7. $y=\dfrac{x^5-3x^3+x^2+9}{x^2+2x}$

Suggested Problems:

Text: 1, 2, 16, 17, 18, 21-38

My Previous Exams: Spring 2014: 8, 10, Fall 2013 1A: 3, 4, 12
Spring 2013 1A: 5, Fall 2012 1A: 1c, 2

Dr. Scarborough's Previous Exams Fall 2013 I: p2:2, Fall 2012 I: p2:1

Dr. Scarborough's Fall 2013 WIR 1: 11, 17, 18 , WIR 3: 17, 28

Dr. Kim's Fall 2014 WIR

Chapter 1D - Rational Expressions

A rational expression is the quotient of two ________________________________

Division by _______________ is not defined for real numbers, so sometimes restrictions must be placed on the variables of rational expressions.

Simplifying Rational Expressions:

1. Simplify the rational expressions and list any restrictions that exist for x.

$$\frac{x^7 - x^4}{x^5 - x^3} =$$

Restrictions on x : ________________________

2. $\frac{x+2}{x^2+4} =$

Restrictions on x : ________________________

Addition and Subtraction of Rational Expressions

3. $\frac{3}{x+2} + \frac{2}{x-3} =$

Restrictions on x : ________________________

4. $\dfrac{x-4}{x^2-2x+4}-\dfrac{x^2-8x+16}{x^3+8}=$

Restrictions on x : ________________________

Multiplication and Division of Rational Expressions

5. $\left(\dfrac{8}{x+2}\right)\bullet\left(\dfrac{x^2-x-2}{x^2-1}\right) =$

Restrictions on x : ________________________

6. $\dfrac{x^2-4x-21}{x+4}\div(x^2+7x+12)=$

Restrictions on x : ________________________

7. $\dfrac{\dfrac{x^3-1}{x+1}}{\dfrac{x-1}{x^2+2x+1}} =$

Restrictions on x : ______________________

Compound Fractions

A compound (or complex) fraction is an expression containing fractions within the numerator and/or the denominator. To simplify a compound fraction, first simplify the numerator, then simplify the denominator, and then perform the necessary division.

8. $\dfrac{\dfrac{3}{x}+\dfrac{1}{2x-7}}{\dfrac{5}{x}+1}$

Restrictions on x : ______________________

7. $\dfrac{3(x+h)^{-2}-3x^{-2}}{h}=$

Restrictions on x : ________________________

Restrictions on h : ________________________

Suggested Problems:

Text: 1-25

My Previous Exams: Fall 2012 1A: 3, 4

Dr. Scarborough's Previous Exams: Fall 2013 I: p8:10, Fall 2012 I: p2:4, p8:10

Dr. Scarborough's Fall 2013 WIR 2: 3, 5 WIR 3: 6, 16, 25

Dr. Kim's Fall 2014 WIR:

Chapter 1E - Complex Numbers

$\sqrt{-16}$ exists!

So far the largest (most inclusive) number set we have discussed and the one we have the most experience with has been named the real numbers.

And $\forall\ x \in \mathbb{R}\,,\quad x^2 \geq 0$

But there exists a number (that is not an element of $\mathbb{R}$) named i and $i^2 =$______

Since $i^2 =$______, $\sqrt{-1} =$____________ , so $\sqrt{-16}$ = ______________________________

i is not used in ordinary life, and humankind existed for 1000’s of years without considering i . i is, however, a legitimate number. i , products of i , and numbers like $\sqrt{2}+i$ are solutions to many problems in engineering. So it is unfortunate that it was termed “imaginary!”

i and numbers like $4i$ and $-3i$ are called ____________________________________

Numbers like $\sqrt{2}+i$ and $\frac{17}{3}-5i$ are called ___________________________________

More formally $\mathbb{C}$ is the set of all numbers ______________________________________

When a complex number has been simplified into this ___________ form, it is called

____

1. Put the following complex numbers into **standard form**:

a) $\left(16+\sqrt{-9}\right)+\left(-13-\sqrt{-100}\right)$ = ____

b) $(a+bi)+(c+di)$ = ____

c) $(2+3i)(4+5i)$ = ____

d) $(5+4i)(3-2i)$ = ____

e) $i^3 =$ ____________ f) $i^4 =$ ____________ g) $i^5 =$ ____________

Graphing Complex Numbers

When we graph elements of $\mathbb{R}$, we use ______________________________

When we graph elements of $\mathbb{C}$, we use the complex plane which will seem a lot like the Cartesian plane.

In the Cartesian plane, a point represents ________________________________

In the complex plane, a point represents _________________________________

The horizontal axis is the ____

The vertical axis is the ____

2. Graph and label the following points on the complex plane:

A $3+4i$

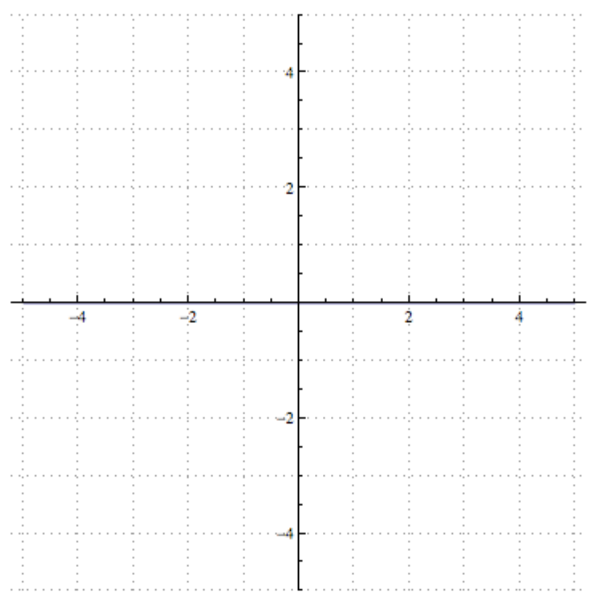
B $-2+i$

C $\frac{3}{2}-3i$

D $3i$

E -4

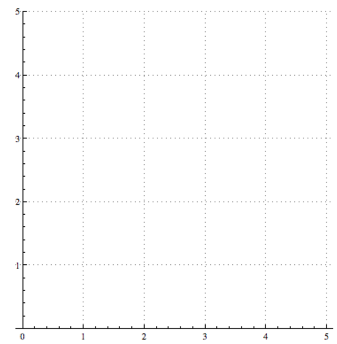
F $-1-2i$



Absolute Value of a Complex Number:

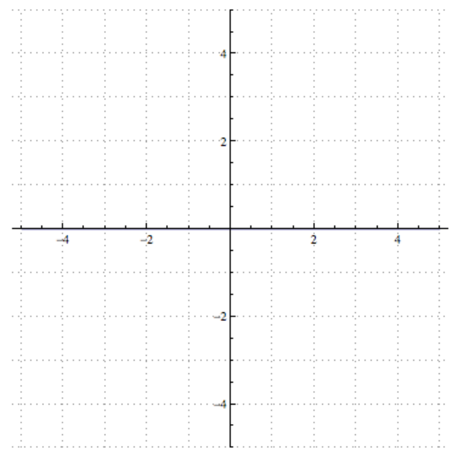
If $z \in \mathbb{C}$ then $|z|$ is defined as its ______________________________________

Calculate $|3+4i|$ (How far is $3+4i$ from the origin?)



Consider $a+bi$, an arbitrary element of $\mathbb{C}$.

What is $|a+bi|$? (How far is $a+bi$ from the origin?)



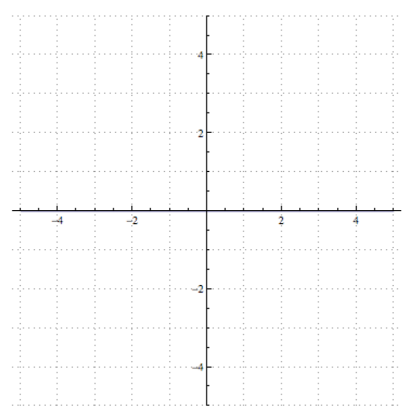
In general $|z|$ = ____________________________

Notice that $|z|$ is a real number.

It is z's distance from the origin.

We did not use i to calculate $|z|$.

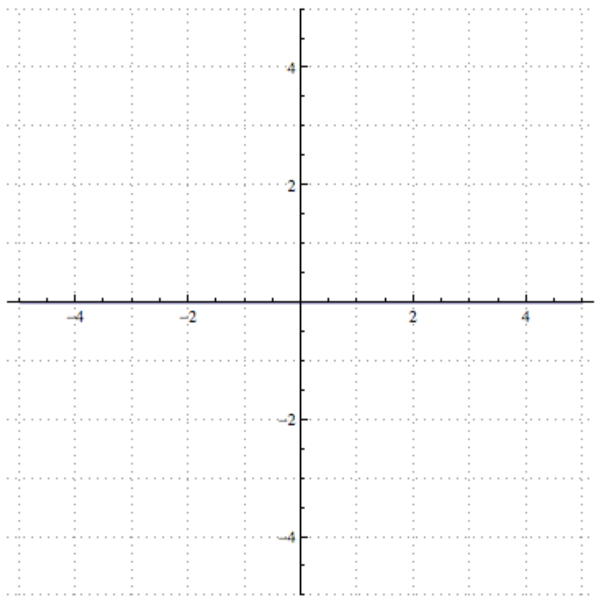
3. Calculate $|-5+i|$. (How far is $-5+i$ from the origin?)



$|-5+i|$ = ___________________

Complex Conjugate

If $z = 3 + 4i$, then $\bar{z}$ = ________________ . If $z = -1 - 2i$, then $\bar{z}$ = __________ .



Graphically the complex conjugate is the

________________________ of the number

through the ________________________

More generally if $z = a + bi$,

then $\bar{z}$ = ______________________

4. Put the following complex numbers into standard form.

a) $\overline{2-3i}$ = ________________ b) $\overline{4i}$ = ___________________

c) $\bar{5}$ = ____________________ d) $\overline{3i-2}$ = ___________________

Finally notice that $(3+4i)(3-4i)$ = ______________________________________

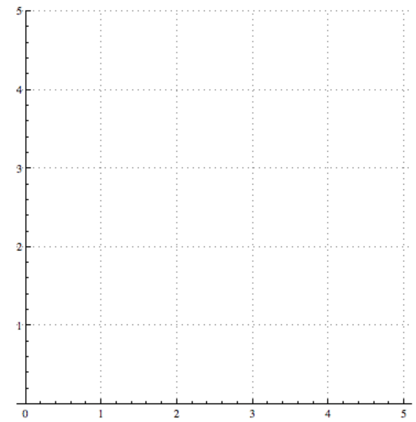
More generally $(a+bi)(a-bi)$ = ____

in other words $z \cdot \bar{z}$ = ____

Distance Between Two Complex Numbers

Plot and label two points in the complex plane $z_1 = 3+5i$ and $z_2 = 1+2i$

The distance between z_1 and z_2 is



Just for fun, calculate $z_1 - z_2$ = ____

And $|z_1 - z_2|$ = ____

What we have seen is that for this particular z_1 and z_2 , the distance between these points is

equal to ___________________. But this is actually true for all complex numbers.

The distance between two general points z_1 and z_2 is ________________

Suggested Problems:

Text: 1-12

My Previous Exams: Fall 2013 1A: 5, Spring 2013 1A: 7, Fall 2012 1A: 5

Dr. Scarborough's Previous Exams: Fall 2013 I: p6:3, Fall 2012 I: p3:7, p6:2

Dr. Scarborough's Fall 2013 WIR 2: 4, 6, 7, 8 WIR 3: 8, 19

Dr. Kim's Fall 2014 WIR: