Chapter 1A -- Real Numbers

Math Symbols:

$\in$	$\forall$
$\exists$	$\ni$
iff	$\Leftrightarrow$ **or** $\leftrightarrow$
$\cup$	$\cap$

Example: Let A = {4, 8, 12, 16, 20, ...} and let B = {6, 12, 18, 24, 30, ...}

Then $A \cup B$= ____

and $A \cap B$= ____

Sets of Numbers

	Name(s) for the set	Symbol(s) for the set
1, 2, 3, 4, ...	Natural Numbers Positive Integers	
..., -3, -2, -1	Negative integers	
0, 1, 2, 3, 4, ...	Non-negative Integers Whole Numbers	
..., -3, -2, -1, 0, 1, 2, 3, ...	Integers	

Note: ________________________________ is the German word for number.

$\frac{4}{5}$, $\frac{-17}{13}$, $\frac{1}{29}$	Rational Numbers	

Note: This is the first letter of ______________________________

1. Circle One:

a) 5 is a rational number TRUE FALSE

b) $\frac{\pi}{4}$ is a rational number TRUE FALSE

c) $\frac{\sqrt{16}}{5}$ is a rational number TRUE FALSE

d) $\frac{\sqrt{17}}{5}$ is a rational number TRUE FALSE

e) 0 is a rational number TRUE FALSE

Definition of a rational number: If a, b $\in \mathbb{Z}$ and $b \neq 0$, then $\frac{a}{b} \in \mathbb{Q}$.

Translation: ____

____

Decimal Expansions of Rational Numbers:

Fact: The decimal expansions of rational numbers either

____________________________ or ____________________________

Examples $\frac{1}{4}$ = _________________________ (This is a terminating decimal.)

$\frac{4}{9}$ = ______________________ (This is a repeating decimal.)

You know how to express a terminating decimal as a fraction. For example $.25 = \frac{25}{100}$, but how do you express a repeating decimal as a fraction?

2. Express these repeating decimals as the quotient of two integers:

a) $.373737\ldots$ or $\overline{.37}$

Let $x = .37373737\ldots$

Then $100x =$

b) $\overline{.123}$

Let $x = .123123123\ldots$

Then

There are a lot of rational numbers, but there are even more irrational numbers. Irrational numbers cannot be expressed as the quotient of two integers. Their decimal expansions never terminate nor do they repeat.

Examples of irrational numbers are ____

The union of the rational numbers and the irrational numbers is called the

______________________________ or simply the _____________. It is denoted by _______.

Properties of Real Numbers

4. What properties are being used?	Name of Property	
a) $(-40)+8(x+5)=8(x+5)+(-40)$		$a+b=b+a \quad \forall \; a,b\in\mathbb{R}$
b) $8(x+5)+(-40)=8x+40+(-40)$		$a(b+c)=ab+ac \quad \forall \; a,b,c\in\mathbb{R}$
c) $8x+40+(-40)=8x+(40+(-40))$		$(a+b)+c=a+(b+c) \quad \forall \; a,b,c\in\mathbb{R}$
d) $0+5=5+0=5$		$0+a=a+0=a \quad \forall \; a\in\mathbb{R}$
e) $7+(-7)=(-7)+7=0$		For each $a\in\mathbb{R}$, $\exists$ another element of $\mathbb{R}$ denoted by $-a$ such that $a+(-a)=(-a)+a=0$
f) $6\cdot 1=1\cdot 6=6$		$1\cdot a=a\cdot 1=a \quad \forall \; a\in\mathbb{R}$
g) $4\cdot\left(\frac{1}{4}\right)=\left(\frac{1}{4}\right)\cdot 4=1$		For each $a\in\mathbb{R}$, $a\neq 0$ $\exists$ another element of $\mathbb{R}$ denoted by $\frac{1}{a}$ such that $a\cdot\left(\frac{1}{a}\right)=\left(\frac{1}{a}\right)\cdot a=1$

Notice that the commutative property considers the ________________ of the numbers.

When demonstrating the associative property, the order of the numbers stays the same

but the ____________________ changes.

When do we use the commutative property of addition? Consider $7+28+43$.

The rules for order of operation tell us to add left to right, but the commutative property of addition allows us to change the order to make our life easier:

$7+29+43=$ ____

When do we use the associative property of multiplication? Consider 16×25 .

$16\times 25=$ ____

Is subtraction commutative? ___________ Example: _____________________________

Is division commutative? ___________ Example: _____________________________

Consider $(12-3)-4$ = __________________ while $12-(3-4)=$ ___________________

So is subtraction associative? ___________

Consider $(12\div 3)\div 4$ = __________________ while $12\div (3\div 4)=$ ___________________

So is division associative? ___________

Notice that there are 2 identities:

The **additive identity** is _______________. The **multiplicative identity** is ____________.

Notice there are 2 kinds of inverses.

The additive inverse of 4 is _____________. Note that ________________________________

The additive inverse of -9 is _____________________. Note that ________________________

What is true about the sum of a number and its additive inverse? ________________________

Every real number has an additive inverse.

The multiplicative inverse of 4 is ________. Note that __________________________________

The multiplicative inverse of -9 is _______________ Note that __________________________

What is true about the product of a number and its multiplicative inverse? _________________

Almost all of the real numbers have a multiplicative inverse.

Which real number does NOT have a multiplicative inverse? ___________________________

Absolute Value

$|4| =$ ________________ $|-9| =$ ________________

Formal definition for absolute value

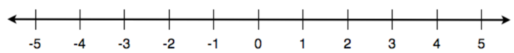
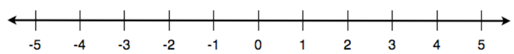
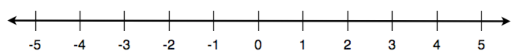
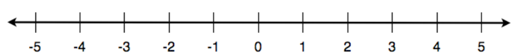
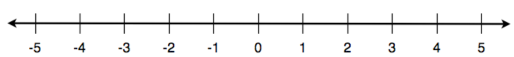
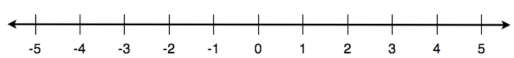
$$|x| = \begin{cases} \\ \\ \end{cases}$$

It will help if you learn to think of absolute value as ____________________________________

(Notice that -4 is 4 units away from the origin.)

Distance between two points:

3.) Find the distance between

a) 4 and 1		
b) -1 and 5		
c) 4 and -2		
d) -3 and -4		
e) 1 and x		
f) x and y		

Notice that $|4-1| = |1-4|$ and $|-3-2| = |2-(-3)|$.

In general $|x-y| =$ _________

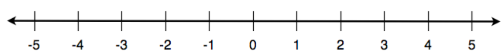
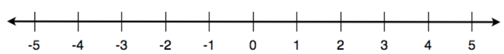
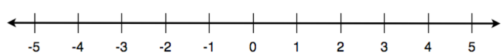
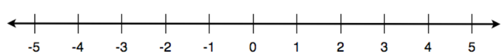
Notice that $|-5+4| =$ ______ whereas $|-5|+|4| =$ ______

So, in general $|x+y| \neq$ ________

A **solution set** is the set of values that make a (mathematical) statement true.

Interval Notation

Determine the solution sets for the following inequalities

	Number Line	Inequality	Interval Notation
$\|x\| \le 3$			
$\|x\| > 3$			
$\|x\| > -3$			
$\|x\| < -3$			

Suggested problems:

Text: 1-9, 22, 23

My Previous Exams: Fall 2014 1A: 11c & 13 a, b, d, e, f, g

Spring 2013 1A: 3, 4, Spring 2014 1A: 13b, c, d, e

Chapter 3C -- Linear Equations in Two Variables

A ________________________ is any set of ordered pairs of real numbers.

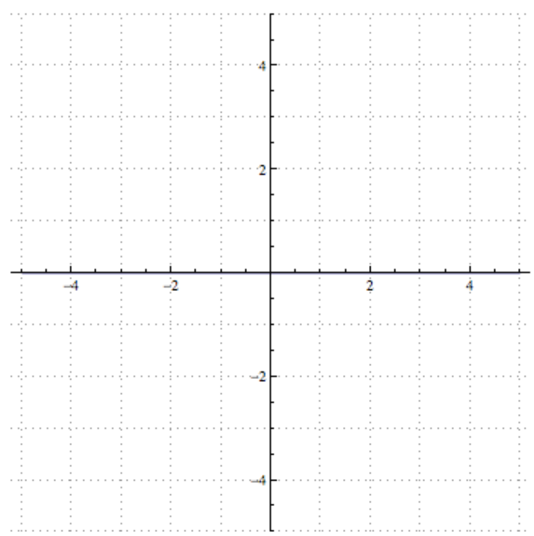
A relation can be finite: {(-3, 1), (-3, -1), (0, 5), (1, -3), (2, 3)}

This is a set of ordered pairs of real numbers, so it is a _________________________.

Each ordered pair of real numbers corresponds to a ______________ on the Cartesian plane.

The set of all points corresponding to a relation is the _____________________ of the relation.

Graph the relation given above.



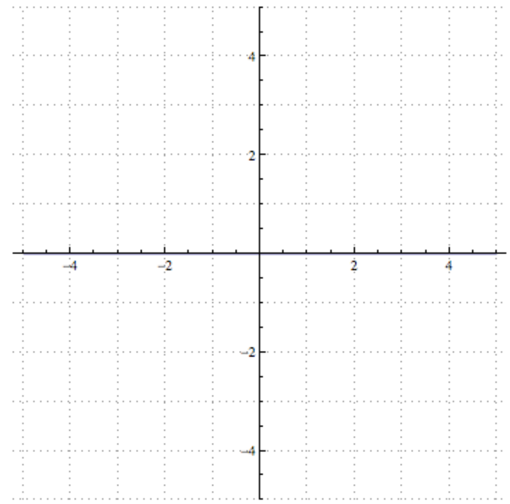
The __________________ of a relation is the set of all first elements of the ordered pairs.

The __________________ of a relation is the set of all second elements of the ordered pairs.

What is the domain of this relation? ____________________________________

What is the range of this relation? ____________________________________

A relation can be infinite:



Consider $\{(x,y) \mid -3 \le x < -2,\ 1 < y \le 4\}$

List 4 different elements of that set.

______________ ______________

______________ ______________

Graph the relation

What is the domain of this relation?

____

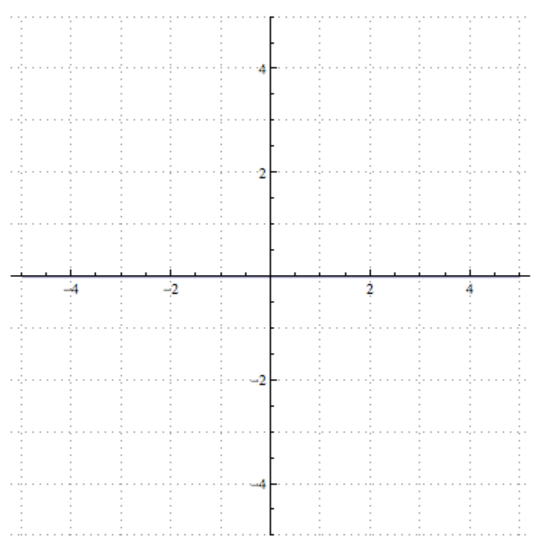
What is the range of this relation? _________________________________

Equations can be used to define relations

2. Consider the set $\{(x,y) \mid x + y = 3\}$

List several elements of that set. ____

It is important to note that the elements of this relation are the solutions to the equation.



Graph the relation

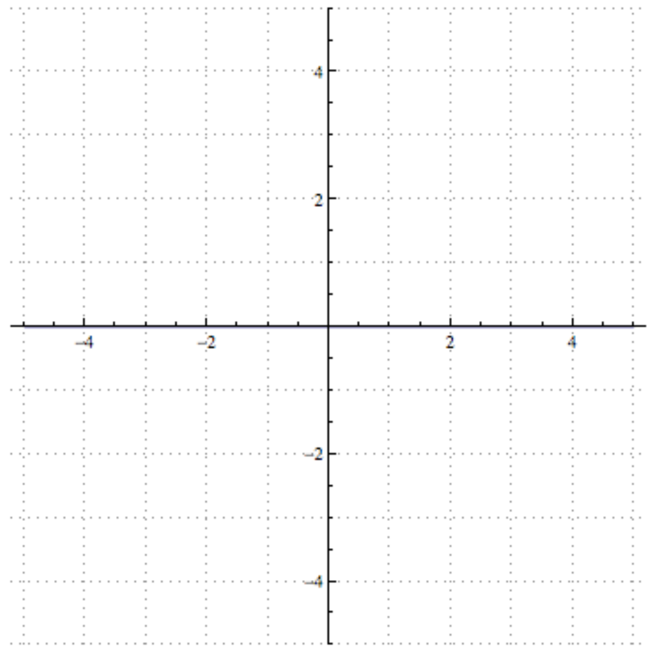
What is the domain of this relation?

____

What is the range of this relation?

____

3. Consider the set $\{(x,y) \mid x = y^2\}$



List several elements of that set.

____

____

It is important to note that the elements of this relation are the solutions to the equation.

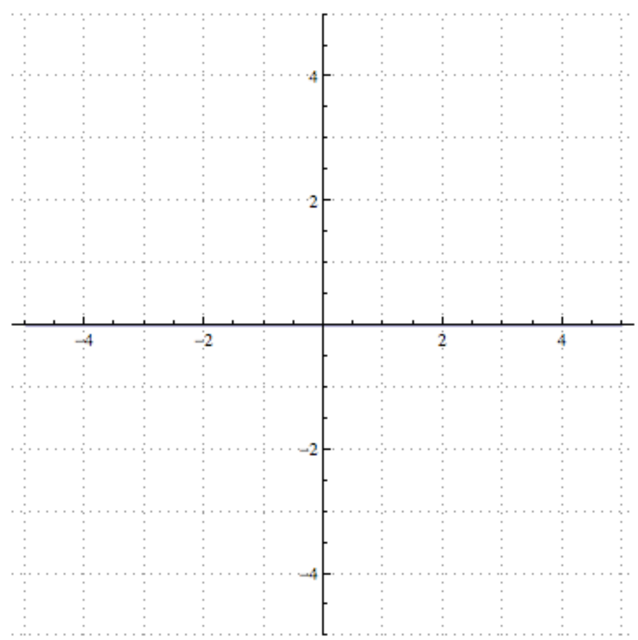
What is the domain of this relation?

____

What is the range of this relation?

____

4. Consider the set $\{(x,y) \mid y = x^2\}$



List several elements of that set.

____

____

It is important to note that the elements of this relation are the solutions to the equation.

What is the domain of this relation?

____

What is the range of this relation?

____

Functions

True or False:

________ A function is a relation in which no two different ordered pairs have the same first element.

_________ A function is a relation where there is only one output (called the y-value), for each input (called the x-value).

_______ A function is a rule that assigns exactly one element in a set B (called the range) to each element in a set A (called the domain).

Which of the following represent a function?

{(1, 2), (1, 3), (2, 3)} This set of ordered pairs is ____________________

{(1, 3), (2, 3), (3, 3), (4, 3)} This set of ordered pairs is ____________________

Functions as a Rule:

1. If $f(x) = 3x + 4$ then $f(2) =$ ____________________

2. If $f(x) = 3x^2 + 4x + 1$ then $f(-1) =$ ____________________

Note: Order of operations dictates that $-3^2 =$ ________ while $(-3)^2 =$ ________

These "rules" create sets of ordered pairs of real numbers $(x, f(x))$

In these cases the domain is the set of numbers for which $f(x)$ is defined.

The range is the set of values that $f(x)$ attains.

The set of all points corresponding to a function is the ____________________ of the function.

A curve in the $xy-$plane is the graph of a function iff no ____________ line

_____________________ the curve _________________________

So if there is a ______________ line that ______________ the curve ____________

then the curve is _________ the graph of a ______________

This is called ____

Looking back at examples 3 and 4 on page 11,

$\left\{(x,y) \middle| \, x = y^2\right\}$ Is a relation, but __________________________________

It ______________________ the vertical line test.

$\left\{(x,y) \middle| \, y = x^2\right\}$ Is a relation and a _________________________________

It ______________________ the vertical line test.

So what we are seeing is that there are 2 common ways to define a function.

A function can be defined with an equation such as $\left\{(x,y) \middle| \, y = x^2\right\}$

or a function can be defined with a rule such as $\left\{(x,\ f(x)) \middle| \, f(x) = x^2\right\}$

But having seen all of this, the truth is that we seldom use the set notation, just the equation or the $f(x)$ notation.

Linear Equations

Which of the following is a linear equation in 2 variables?

$3x+4y=5$ $\quad$ $\pi x+4y=\sqrt{5}$ $\quad$ $x+y+z=0$ $\quad$ $y=x-9$

$3x+xy+4y=5$ $\quad$ $\sqrt{x}+4y=2$ $\quad$ $x^2+4y=2$ $\quad$ $y=\dfrac{2}{x}$

$v=32t+16$ $\quad$ $y=2$ $\quad$ $x=\sqrt{5}$ $\quad$ $x=y$

Any equation that can be written in the form ______________________________ where A

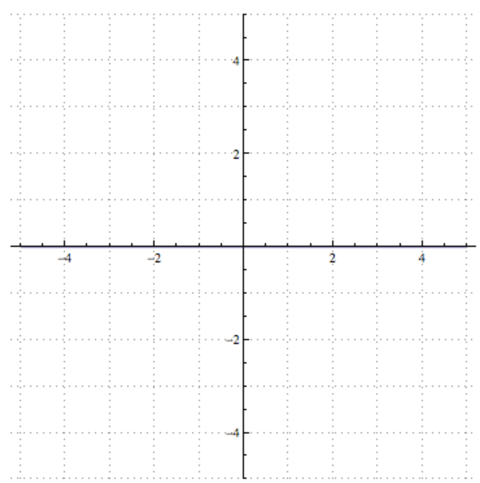
and B are not both ____________________ is called a ________________________________

You already know a lot about lines, but let's review quickly:

Consider $x+y=4$

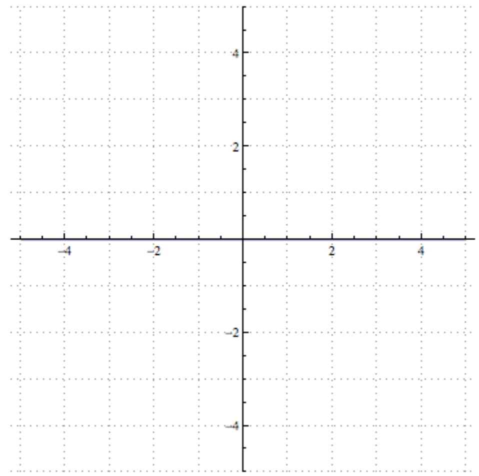
.

x	y



When you create a table to list points to create the graph, you are finding the solutions to the equation. Solutions to linear equations in two variables are ordered pairs of real numbers that when substituted into the equation create an identity. The coordinates of all of the points on the line are solutions to the equation. How many solutions exist for this equation?

Graphing lines with both intercepts



Consider $x+3y=-3$

When $x=0$, $y=$ __________ and when

$y=0$, $x=$___________

Intercepts (crossings)

The y-intercept is where the graph crosses the

____

It occurs when ________________________

The x-intercept is where the graph crosses the ___________________________

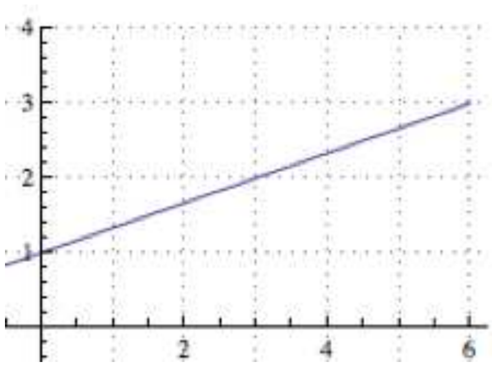
It occurs when ________________________

Consider $2x+y=4$. Solve this equation for y . _________________________________

What is the y-intercept? When $x=0$, $y=$ ______

What is the slope of this line?

Slope Calculate the slope m of the line on the graph below.



$m=\dfrac{\Delta y}{\Delta x}=$

Note Δ is the Greek letter ___________.

Δ or D is for _____________________

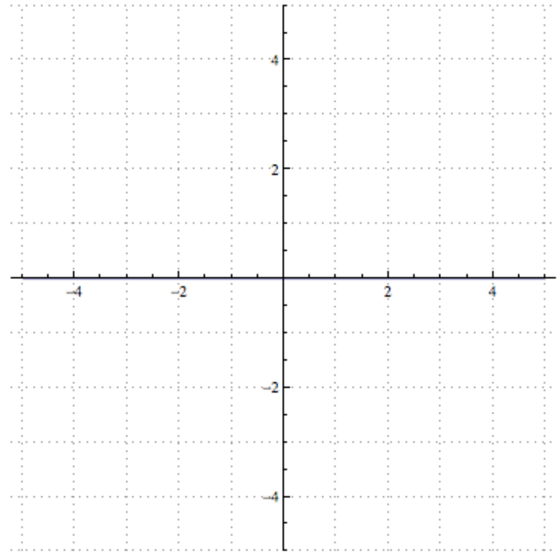
Some people like the phrase

slope = ___________________________

What is the equation of this line? ____________________________

This $y=mx+b$ form of a linear equation is known as the ___________________________ form.

The slope of a horizontal line is ________________



The slope of a vertical line is __________________

Parallel lines do not ________________________.

Parallel lines have the same __________________.

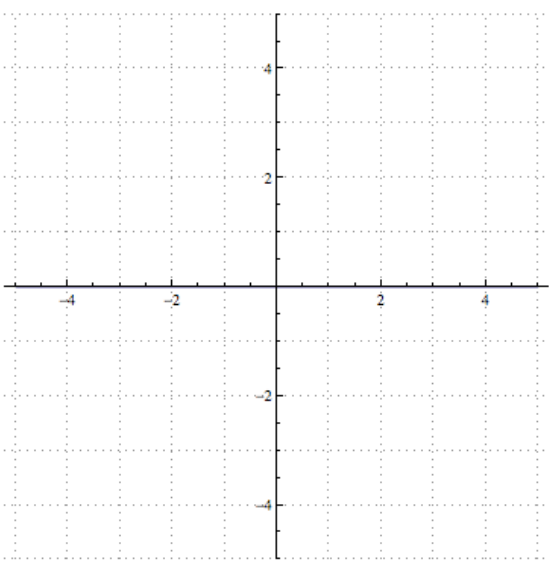
When 2 lines are perpendicular their slopes are

_____________________________.

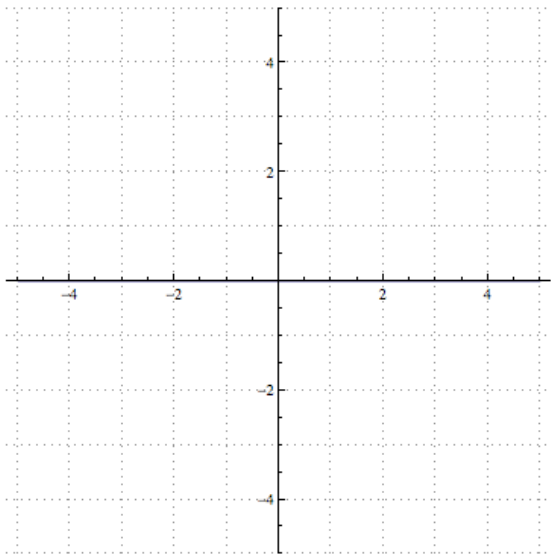
The set of ordered pairs of real numbers $\{(x,y) \mid x=3\}$ is a relation, but it is _________ a

____

Determine the equation of the line through (-1, -2) that is perpendicular to $y=-3x+4$.

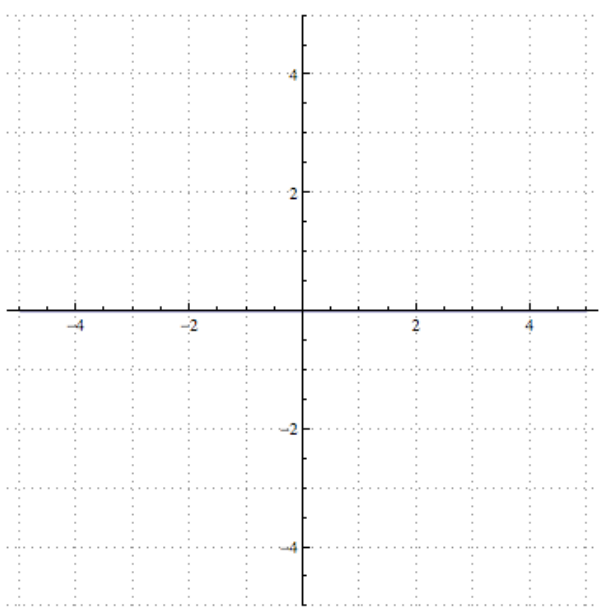


Determine the equation of the line through (1, 2) and (4, 3).

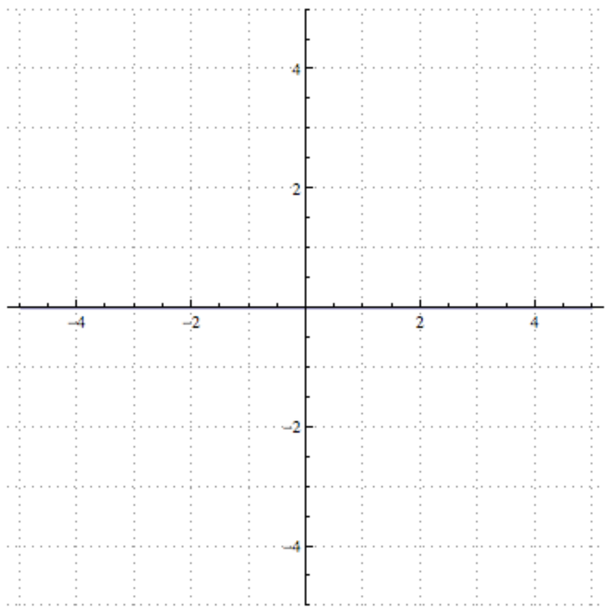


The Midpoint of a Line Segment.

Consider the points A (-3, 0) and B (5, -4). Find the midpoint of the segment connecting these 2 points.



Remember: The number half between *a* and *b* is the average of *a* and *b*



Given a line segment with endpoints (x_1, y_1) and (x_2, y_2), the midpoint of that segment is

Suggested Problems:

Text: 17, 21a, 22 g, h, i , 23d, 27-29

My Previous Exams: Fall 2014 2A: 17

Spring 2014 1A: 1, 2, 13f

Spring 2013 2A: 10

Fall 2012 2A: 5

Chapter 7A Systems of Linear Equations

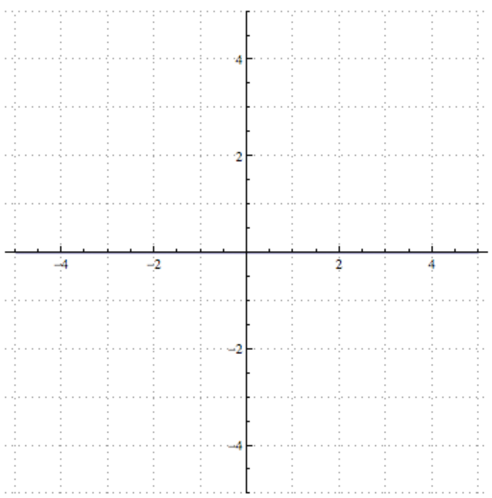
The set $\{(x,y) | \; x+y=3\}$ is a set of ordered pairs of real numbers. It is a function, and all of the elements of this set are solutions to the equation $x+y=3$

Definition: A **solution** to an equation in 2 variables is an ordered pair of real numbers (x, y) that, when substituted into the equation, make the equation an identity.

1. a) List 3 examples of solutions for the equation $x+y=3$.

__________________ __________________ __________________

b) How many solutions exist for the equation $x+y=3$? ____________________



c) Graph all of the solutions of $x+y=3$

2. a) List 3 examples of solutions for the equation

$x-y=1$.______________________________

b) How many solutions exist for the equation

$x-y=1$?______________________________

c) Graph all of the solutions of $x-y=1$

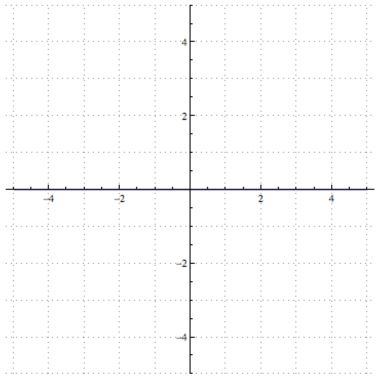
Do these two solution sets have any common elements? ____________________

A solution to a *system* of equations in 2 variables is an ordered pair of real numbers (x, y) that, when substituted into all of the equations in the system, make all of the equations identities.

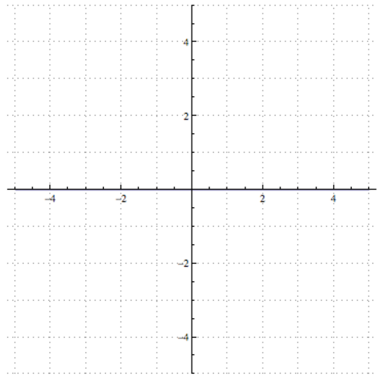
__________________ is the solution to the system $\begin{matrix} x+y=3 \\ x-y=1 \end{matrix}$

because ______________ and ______________

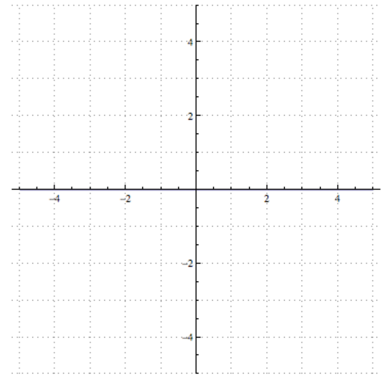
2. How many solutions exist for the following system of equations ?



$$\begin{aligned} x+y&=1 \\ x+y&=2 \end{aligned}$$



$$\begin{aligned} x+y&=1 \\ -x+y&=1 \end{aligned}$$



$$\begin{aligned} x+y&=1 \\ 3x+3y&=3 \end{aligned}$$

Geometrically there are 3 possibilities for the graphs of 2 lines.

Finding solutions to systems of linear equations (when they exist!)

By Graphing Solutions to systems of equations correspond to the intersections of the graphs of the equations.

By Substitution

3. Find all of the solutions, if any exist, to the system $\begin{aligned} x+3y&=7 \\ 2x-y&=7 \end{aligned}$

By Elimination Our goal will be to add or subtract the equations in a way that eliminates one of the variables. Part of the process will remind you of finding a common denominator.

Why is elimination a legitimate technique?

$$2x+y=1$$
$$x-y=5$$

Start with the first equation $2x+y=1$

Rewrite the equation adding 5 to both sides

Substitute $x-y$ for the 5 on the left hand side.

In essence we have added the 2 equations to one another.

4. Solve $\begin{aligned} 2x+3y&=4 \\ 5x+6y&=7 \end{aligned}$

5. Solve $\begin{aligned} -6x+7y&=18 \\ 4x+3y&=-12 \end{aligned}$

Applications

6. An airplane makes the 2400 miles trip from Washington D.C. to San Francisco in 7.5 hours and makes the return trip in 6 hours. Assuming that the plane travels at a constant airspeed and that the wind blows at a constant rate from west to east, find the plane's airspeed and the wind rate.

7. To raise funds, the hiking club wants to make and sell trail mix. Their plan is to mix dried fruit worth $1.60 per pound with nuts worth $2.45 per pound to make 17 pounds of a mixture worth $2 per pound. How many pounds of dried fruit and how many pounds of nuts should they use?

8. How many gallons of each of a 60% acid solution and an 80% acid solution must be mixed to produce 50 gallons of a 74% acid solution?

Suggested Problems: **Text: 7 - 17**

My Previous Exams: **Fall 2014 3A: 9**
Spring 2014 1A: 3 **Fall 2013 3A: 11**
Spring 2013 3A: 8 **Fall 2012 3A: 14**

Chapter 8A Angles and Circles

From now on angles will be drawn with their vertex at the ____________________

The angle's initial ray will be along the positive ____________________. Think of the angle's terminal ray as starting along the positive x-axis, and then swinging into its position.

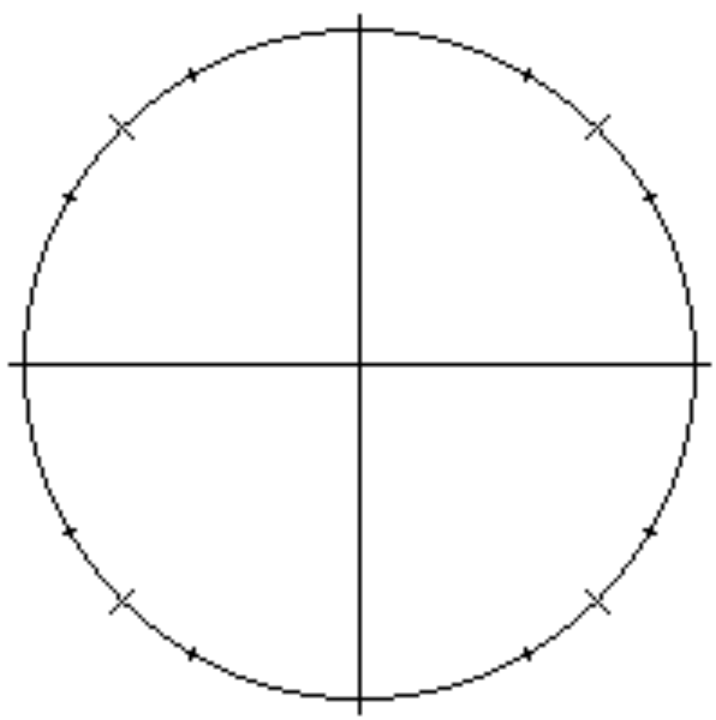
If the terminal ray swung away from the x-axis in a counterclockwise direction, then the angle has ____________________ measure. If the terminal ray swung away from the x-axis in a clockwise direction, then the angle has _________________________ measure.

The circle below has a radius of 1 unit. It is called the _____________________________.

The circumference of a unit circle is _________________

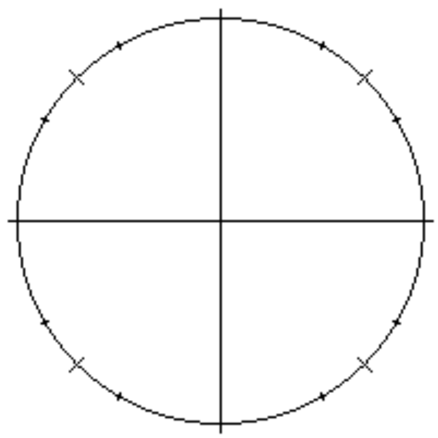
If a terminal ray swings through an entire rotation, you would say it has a measure of _____

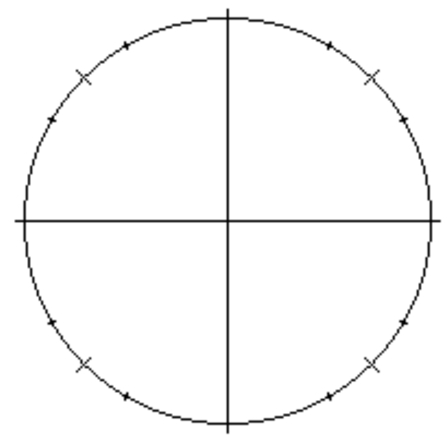
You could also say that it has a measure of _____________________________________.



1. Sketch the following angles on the unit circle below

a) $\frac{\pi}{4}$ b) $\frac{2\pi}{3}$ c) $\frac{7\pi}{6}$ d) $\frac{3\pi}{2}$ e) $\frac{7\pi}{4}$





2. Sketch the following angles on the unit circle above

a) $-\frac{\pi}{4}$ b) $-\frac{2\pi}{3}$ c) $\frac{13\pi}{6}$ d) 3π e) $-\pi$

3π and $-\pi$ are called ___________________________

because they share the same _____________________________

To find an angle that is coterminal to θ, just add or subtract ___________________________

Another way to say this:
To find an angle that is coterminal to θ, just add or subtract ___________________________

3. List 2 other angles that are coterminal angles with $\frac{\pi}{2}$ _________________________

4. List 2 other angles that are coterminal angles with $\frac{2\pi}{3}$ _________________________

An angle is called acute if its measure is between ________________________

An angle is called obtuse if its measure is between ________________________

Two angles are called complementary if the sum of their measures is _______________

An example of complementary angles: $\theta_1 =$ _____________ and $\theta_2 =$ _____________

Two angles are called supplementary if the sum of their measures is _______________

An example of supplementary angles: $\theta_1 =$ _____________ and $\theta_2 =$ ____________

A line which intersects the circle twice is called a ____________________________

A line which intersects the circle at exactly one point is called a _______________

The region inside of a circle is called a ______________________________

Any piece of the circle between two points on the circle is called an _______________

Any line segment between 2 points on the circle is called a_______________

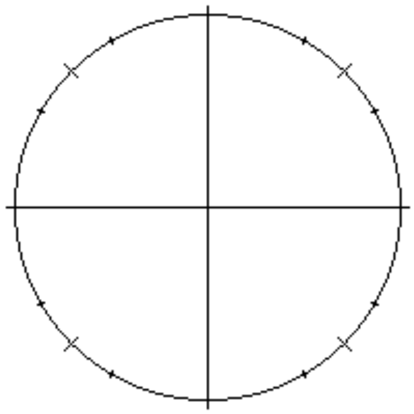
Any piece of the disk between 2 radial lines is called a _______________

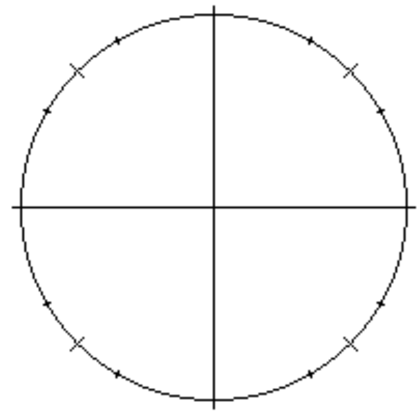
An angle whose vertex is at the center of a circle is called _______________

Three ways to measure angles: **Revolutions -- Degrees -- Radians**

Revolutions	Degrees	Radians
$\frac{3}{2}$		
$3\frac{1}{3}$		
	75	
	480	
		5π
		$\frac{3\pi}{7}$

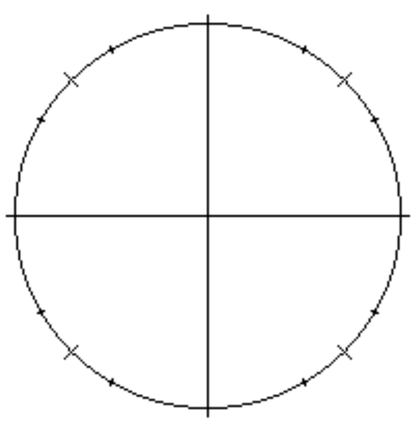
Arc Length: (Think about the fraction of the circumference.)

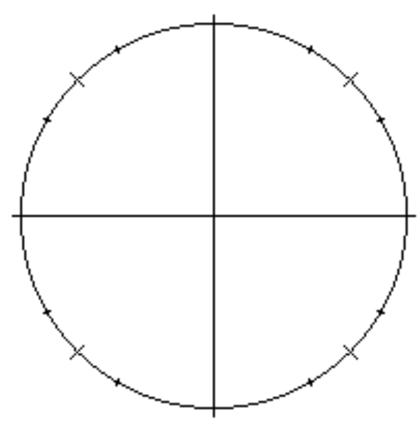




$\theta = 120^\circ$	$\theta = \frac{\pi}{4}$

Area of a sector: (Think about the fraction of the area of the circle.)





$\theta = 30^\circ$	$\theta = \frac{7\pi}{6}$

When using degrees to measure the central angle

Area of a sector =

Length of arc =

When using radians to measure the central angle

Area of a sector =

Length of arc =

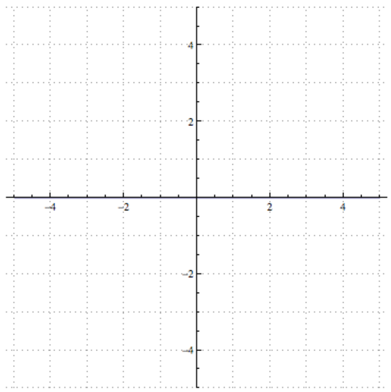
Suggested Problems: **Text: 1 - 12**

My Previous Exams: **Fall 2014 3A: 16** **Spring 2014 1A: 5,**

Spring 2013 3A: 11 **Fall 2012 3A: 9, 11**

4B Graphs of Functions

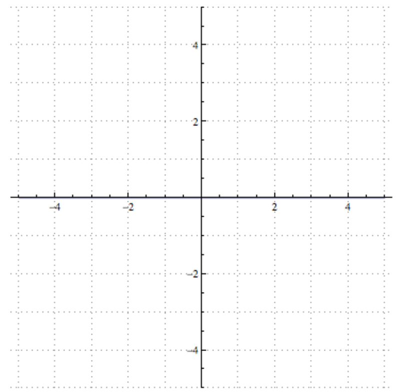
Graph the following library of basic functions.
It is important to be able to recognize and sketch these graphs with ease!



The constant function $f(x)=c$
c is a real number

Domain ____________________________

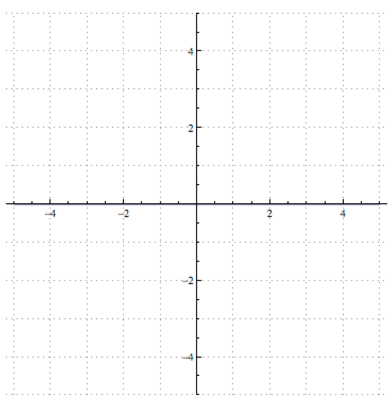
Range ____________________________



Identity function $f(x)=x$

Domain ____________________________

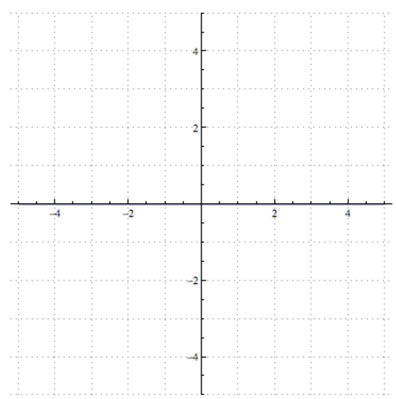
Range ____________________________



Squaring Function
$f(x)=x^2$

Domain ____________________________

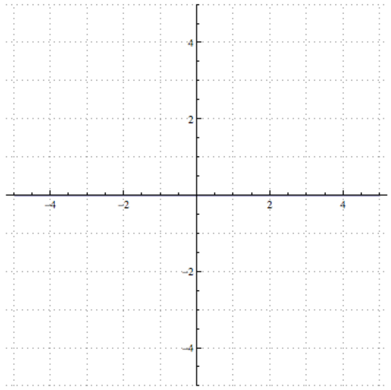
Range ____________________________



Cubing Function
$f(x)=x^3$

Domain ____________________________

Range ____________________________

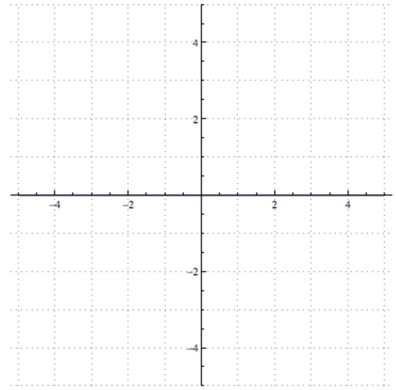


Square Root Function

$$f(x) = \sqrt{x}$$

Domain ______________________

Range ______________________

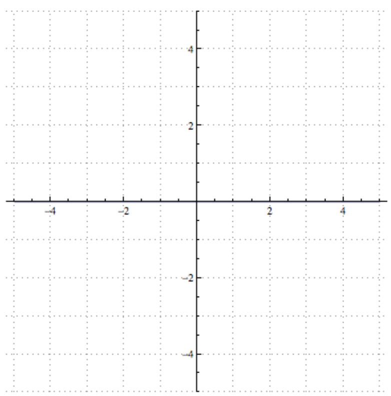


Absolute Value Function

$$f(x) = |x|$$

Domain ______________________

Range ______________________



Reciprocal Function

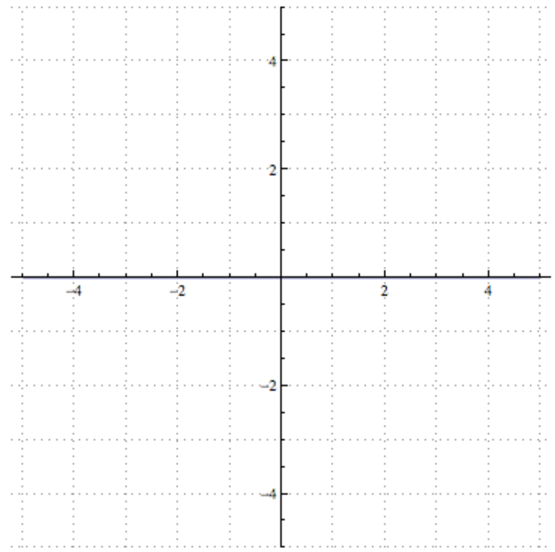
$$f(x) = \frac{1}{x}$$

Domain ______________________

Range ______________________

Piecewise Functions

1 . Let $f(x)=\begin{cases}-x, & x<0\\ 3, & x=0\\ \sqrt{x}\ , & x>0\end{cases}$



a) $f(-2)=$ ____________

b) $f(4)=$ ____________

c) Graph $f(x)$

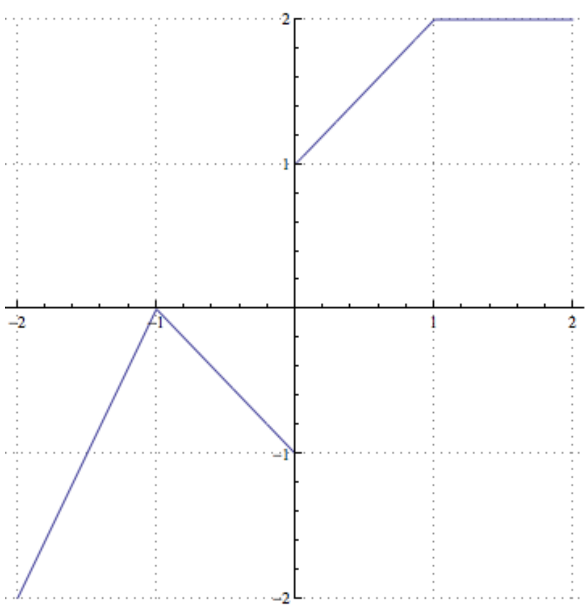
d) $x-$ intercept ______________

e) y -intercept _________________

f) domain _________________

g) range _________________

Assume that the graph below is that of the function $g(x)$.



Evaluate $g(-1)$

$g\left(-\frac{3}{2}\right)$

$g(\sqrt{3})$

Write a definition for $g(x)$

Domain: ________________________________ Range: __________________________________

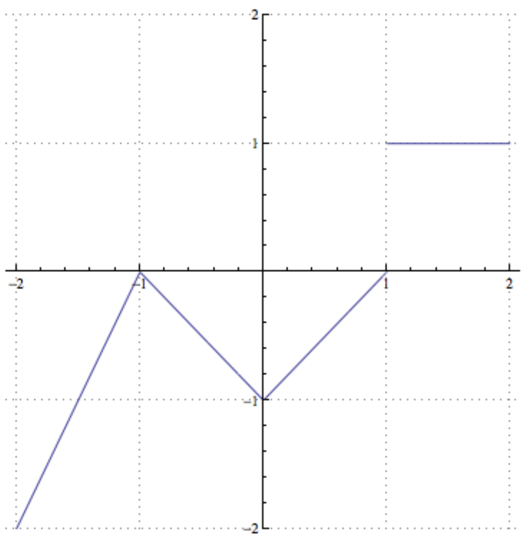
Increasing, Decreasing, and Constant

A function $f(x)$ is increasing on an interval I iff $x_1 < x_2 \rightarrow$ ____________________ $\forall\ x_1, x_2 \in I$

A function $f(x)$ is decreasing on an interval I iff $x_1 < x_2 \rightarrow$ ____________________ $\forall\ x_1, x_2 \in I$

A function $f(x)$ is constant on an interval I iff $x_1 < x_2 \rightarrow$ ____________________ $\forall\ x_1, x_2 \in I$

The graph below is associated with a function $f(x)$



Domain ____________________

Range ____________________

$x-$ intercept ______________

y -intercept ________________

$f(x)$ is increasing on ____________________

$f(x)$ is decreasing on ____________________

$f(x)$ is constant on ____________________

Suggested Problems: **Text: 2-4, 6, 8**

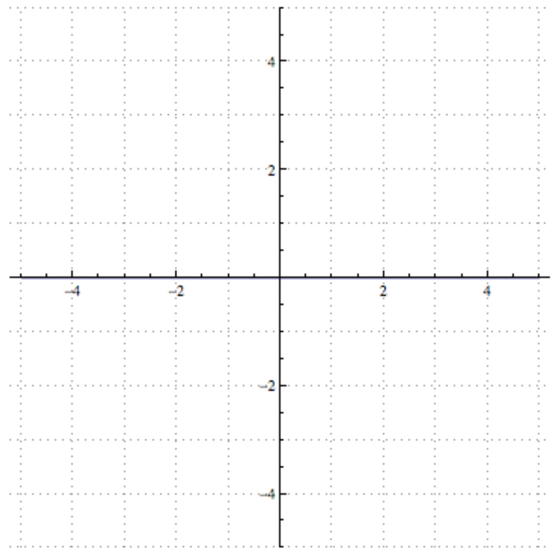
My Previous Exams: **Fall 2014 2A: 15**

Spring 2014 2A: 16, Fall 2012 2A: 10

Chapter 4C -- Transformations of Functions

Vertical Shifts

x	$f(x)=\|x\|$	$g(x)=\|x\|+2$	$h(x)=\|x\|-1$
-3	3		
-2	2		
-1	1		
0	0		
1	1		
2	2		
3	3		



Domain of $g(x)$ ________________ Range of $g(x)$ ______________________

Domain of $h(x)$ ________________ Range of $h(x)$ ______________________

$g(x)$ is increasing on __________________ $g(x)$ is decreasing on ______________

$x-$intercept for $f(x)$ __________________ y-intercept for $f(x)$ ________________

$x-$intercept for $g(x)$ __________________ y-intercept for $g(x)$ ________________

$x-$intercept for $h(x)$ __________________ y-intercept for $h(x)$ ________________

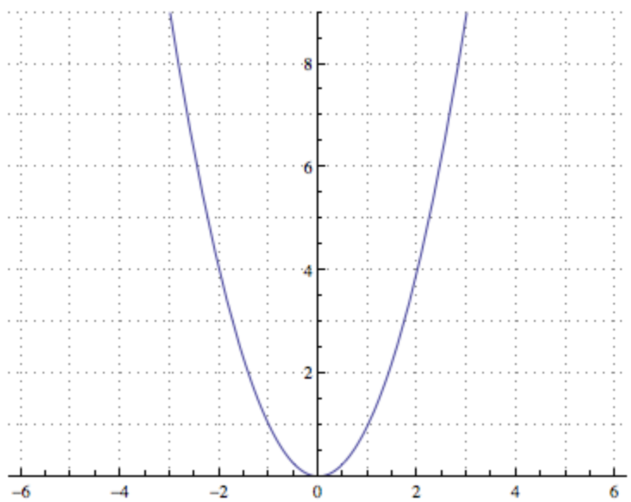
In general, adding a constant to a function shifts the graph ____________________

1. Write a function $g(x)$ that shifts the graph of $f(x)=x^3$ 4 units down. _______________

2. Write a function $h(x)$ that shifts the graph of $f(x)=\sqrt{x}$ 3 units up. _________________

Horizontal Shifts

x	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x)=x^2$			9	4	1	0	1	4	9			
$g(x)=(x-3)^2$												
$h(x)=(x+2)^2$												



vertex of $g(x)$ ________________

vertex of $h(x)$ ________________

$x-$intercept for $f(x)$ ________________

$x-$intercept for $g(x)$ ________________

$x-$intercept for $h(x)$ ________________

y-intercept for $f(x)$ ________________

y-intercept for $g(x)$ ________________

y-intercept for $h(x)$ ________________

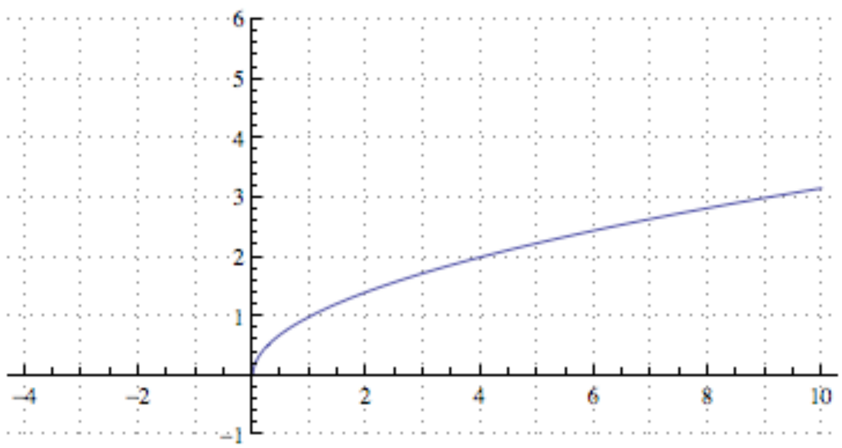
$g(x)$ is increasing on ________________

$g(x)$ is decreasing on ________________

$h(x)$ is increasing on ________________

$h(x)$ is decreasing on ________________

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
$f(x)=\sqrt{x}$					0	1			2					3	
$g(x)=\sqrt{x+4}$															
$h(x)=\sqrt{x-1}$															



For square root functions, there is a value for x which makes the radicand equal 0. I call this the

____

The anchor point for $f(x)$ ____________

The anchor point for $g(x)$ ____________

The anchor point for $h(x)$ ____________

Domain of $f(x)$ __________________

Domain of $g(x)$ ______________

Domain of $h(x)$ ______________

$x-$intercept for $f(x)$ ________________

y-intercept for $f(x)$ ______________

$x-$intercept for $g(x)$ ________________

y-intercept for $g(x)$ ______________

$x-$intercept for $h(x)$ ________________

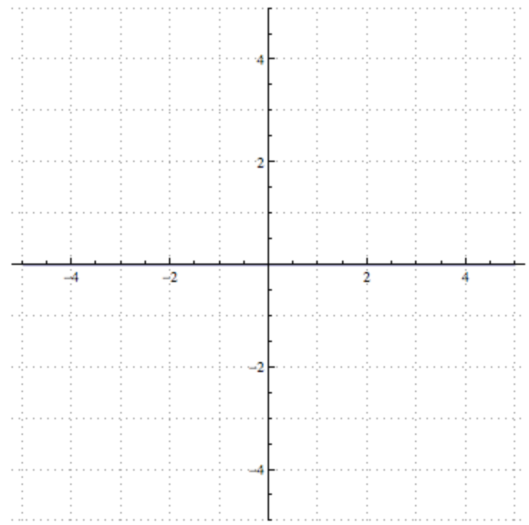
y-intercept for $h(x)$ ______________

$g(x)$ is increasing on ________________

$g(x)$ is decreasing on ____________

In general adding a constant to x before applying the function results in a ______________ shift of the graph.

3. $f(x)=\sqrt{x}$ Write the function $g(x)$ that shifts the graph of $f(x)$ four units to the right.

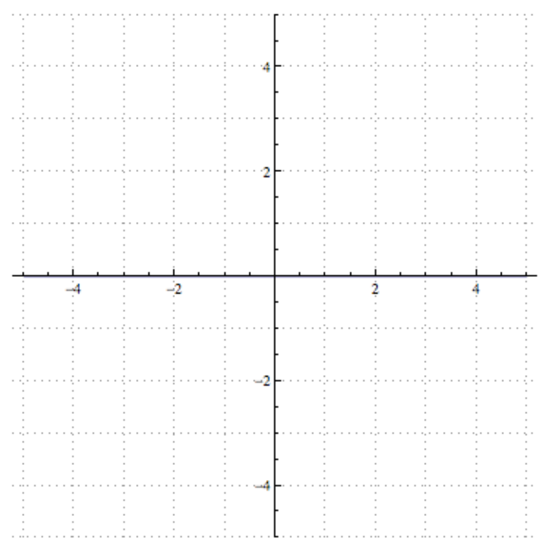


$g(x) =$ ____________________________

Domain of $g(x)$ _____________________

The anchor point for $g(x)$ _____________

4. Write the function $h(x)$ that shifts the graph of $f(x)=|x|$ two units to the left.



$h(x)$ = __________________________

vertex of $h(x) =$ _____________________

$x-$intercept for $h(x)$ __________________

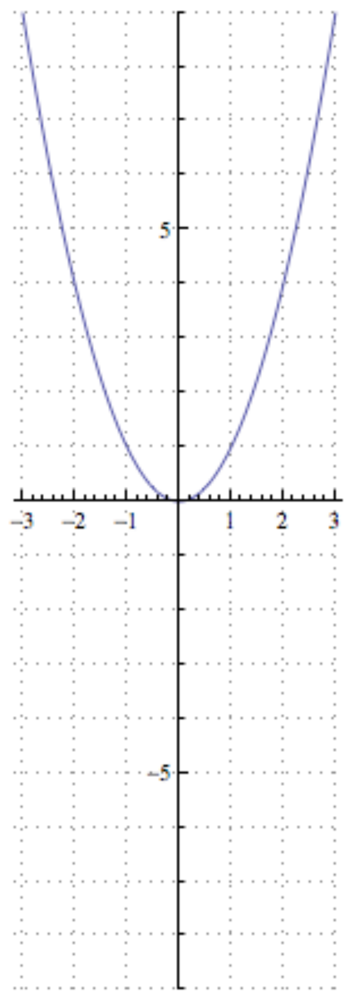
y-intercept for $h(x)$ ________________

Reflections

Let $f(x) = x^2$ Let $g(x) = -f(x) = -x^2$

Range of $f(x)$ ____________________ Range of $g(x)$ ____________________

x	$f(x) = x^2$	$g(x) = -x^2$
-3	9	
-2	4	
-1	1	
0	0	
1	1	
2	4	
3	9	



We can see $-f(x)$ **is a** ________________________ **of** $f(x)$ **through the** ___________

Evaluating $f(-x)$

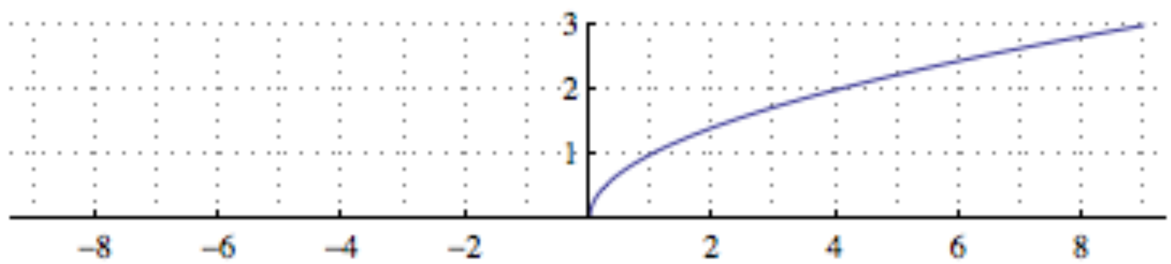
If $f(x) = x^2$ then $f(-x) =$ ____________ If $g(x) = \frac{1}{x}$ then $g(-x) =$ ____________

If $h(x) = x^3$ then $h(-x) =$ ____________

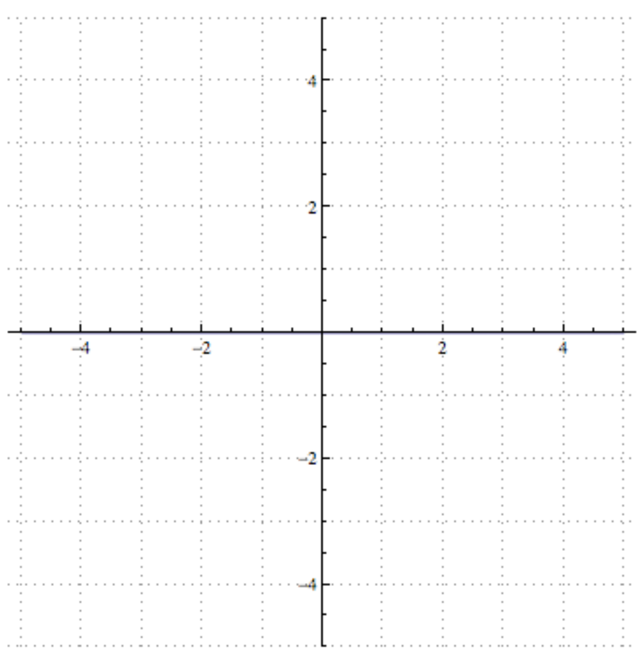
If $r(x) = \sqrt{x}$ then $r(-x) =$ ___________

Let $p(x) = r(-x) =$ ________________ What is the domain of $p(x)$? ____________

Graph $r(x) = \sqrt{x}$ and $p(x) = \sqrt{-x}$



Consider $f(x) = (x-3)^2$ Let $g(x) = f(-x) =$ __________________



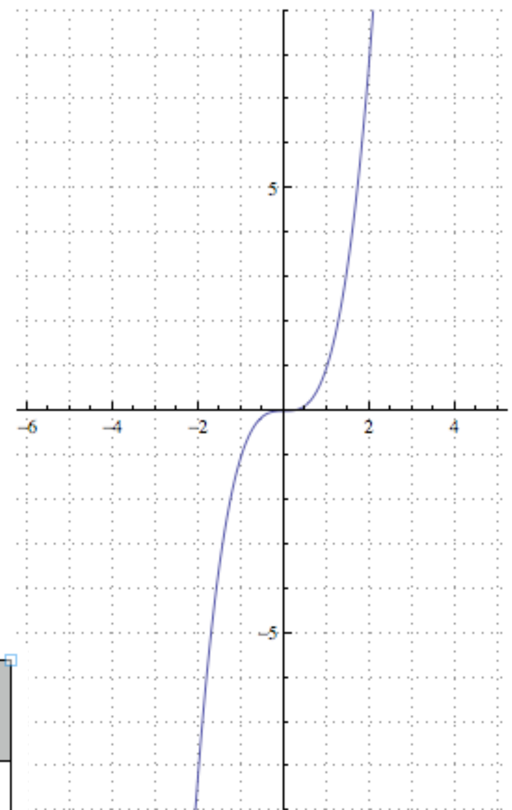
Graph $f(x)$ and $g(x)$.

Here we see that $f(-x)$ is a _____________

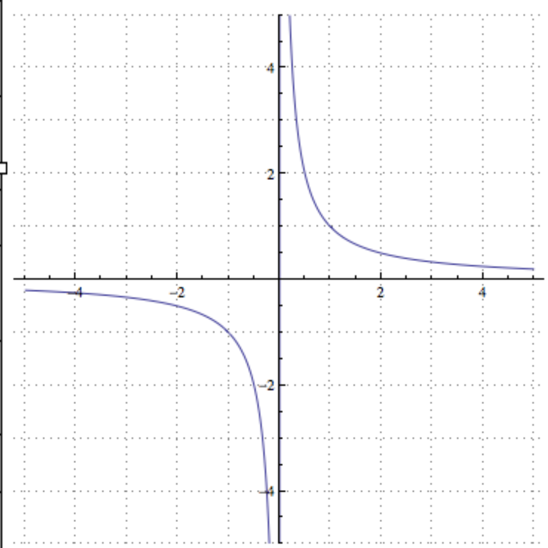
of $f(x)$ through the ___________________

Stretching and Shrinking

x	$f(x)=x^3$	$g(x)=\left(\frac{1}{2}\right)x^3$	$h(x)=\left(\frac{1}{4}\right)x^3$
-2	-8		
-1	-1		
0	0		
1	1		
2	8		



x	$f(x)=\frac{1}{x}$	$g(x)=2\left(\frac{1}{x}\right)$	$h(x)=4\left(\frac{1}{x}\right)$
-4	$-\frac{1}{4}$		
-2	$-\frac{1}{2}$		
-1	-1		
$\frac{1}{2}$	-2		
$\frac{1}{4}$	-4		
0	0		
$\frac{1}{4}$	4		
$\frac{1}{2}$	2		
1	1		
2	$\frac{1}{2}$		
4	$\frac{1}{4}$		



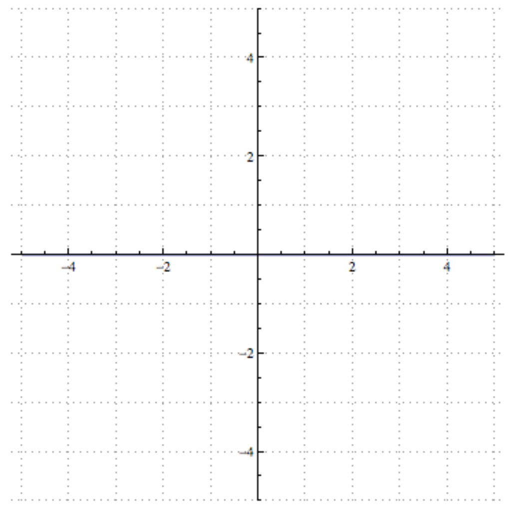
Stretching and Shrinking

If $c>1$, then the transformation $y=cf(x)$ ___________________ $f(x)$ by a factor of c.

Whereas if $0<c<1$, then the transformation $y=cf(x)$ __________ $f(x)$ by a factor of ________.

Combinations of Transformations:

5. $f(x)=2\sqrt{x+1}-6$



a) function ______________________________

b) transformations _______________________

____

____

c) domain ______________________________

d) anchor point ______________________

e) range ________________

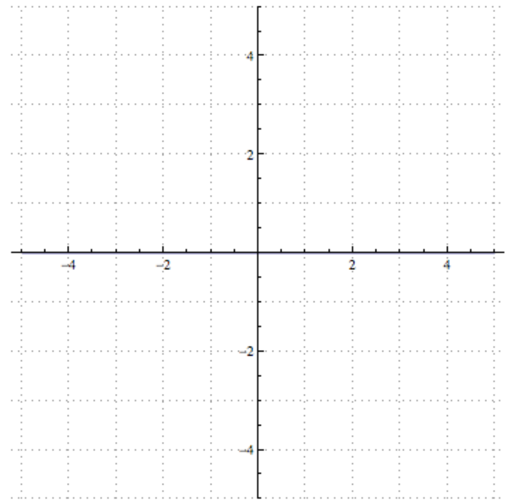
f) $x-$intercept for $f(x)$ _______________

g) y-intercept for $f(x)$ ______________

h) increasing on _________________________

i) decreasing on ___________________

6. $g(x)=\dfrac{(x-2)^2}{-4}+1$



a) function ______________________________

b) transformations _______________________

____

____

c) domain _______________________________

d) vertex ________________________ e) range _________________________

f) $x-$intercept for $f(x)$ _______________ g) y-intercept for $f(x)$ _____________

h) increasing on _________________________ i) decreasing on ____________________

Even Functions:

Notice if $f(x)=x^2$, then $f(-x)=$ ____________________ In other words $f(-x)=$ ________

A function is ___________ **iff** _____________________ **for every** x **in the domain of** $f(x)$

The graph of an even function has symmetry about the _________________________

Look back at the catalog of basic functions. Which of the functions do you think are even? Test the function to determine if it is even.

Odd Functions:

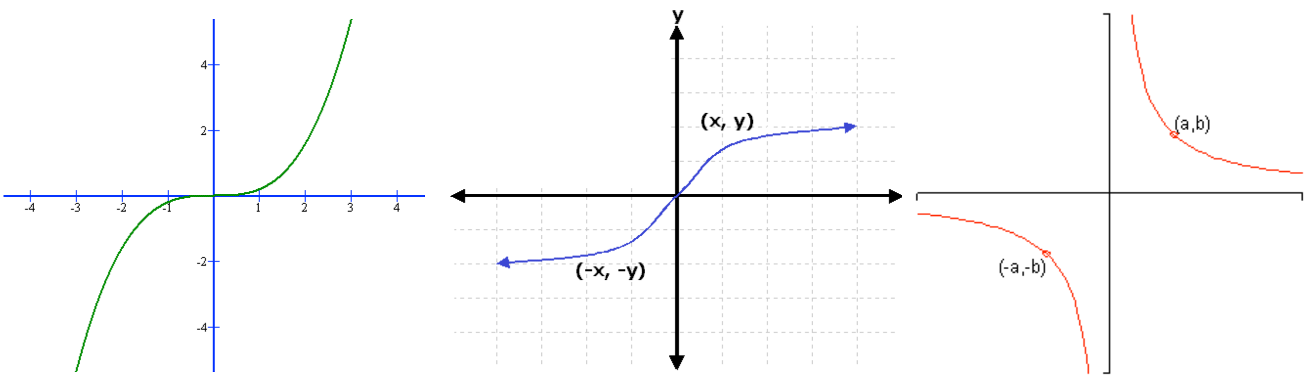
Notice if $f(x)=x^3$, then $f(-x)=$__________________ In other words $f(-x)=$_______

A function is ____________ iff _________________ for every x in the domain of $f(x)$.

The graph of an odd function has symmetry about the ______________

What does it mean for a function to be symmetric about the origin?

All three of these graphs are symmetric about the origin:



Informally, a graph is symmetric about the origin if when it is rotated $180°$ about the origin, the graph looks the same.

By definition, a graph is symmetric about the origin iff for each (x, y) on the graph, _________ is also on the graph.

Interestingly, if a graph is symmetric about the origin, then for each point on the graph, there is a corresponding point on the graph such that the line segment connecting these two points has the origin as its midpoint.

To determine if a function $f(x)$ is even or odd or neither, simplify $f(-x)$.

If $f(-x) = f(x)$, then $f(x)$ is an _____________ function.

If $f(-x) = -f(x)$, then $f(x)$ is an ____________ function.

7. Determine if the following functions are even, odd, or neither

a) $f(x) = 3x^4 - x^2 + 1$

b) $g(x) = x^5 + 1$

c) $h(x) = -x^3 + x^2$

d) $q(x) = \frac{x^2 + 1}{x^3}$

Suggested Problems: Text: 1 -12

My Previous Exams:

Spring14 1A: 11, 12, 13 a&g, 14-16,

Spring 2013 2A: 5abc,

Fall 2014 2A: 1 f, 6, 10,

Fall 2013 2A: 5, 10,

Fall 2012 2A: 2, 7, 10

Chapter 1B - Exponents and Radicals

Multiplication is shorthand notation for repeated addition $4+4+4+4+4=5\times 4$

Exponents are shorthand notation for repeated multiplication. So

$2\times 2\times 2\times 2\times 2=$ ________ $x \bullet x \bullet x \bullet x =$ ___________

You probably know that $3^{-1}=$ __________ and $x^{-5}=$ _____________ this is because

by definition $x^{-1}=$ ___________ . In general if $m\in \mathrm{N}$, then $x^{-m}=$ ___________

Properties of exponents

Example	In general
a^4a^3	x^mx^n
$\frac{b^8}{b^5}$	$\frac{x^m}{x^n}$
$\frac{y^3}{y^3}$	x^0 for $x\neq 0$

Note: $0^0\neq 0$ and $0^0\neq 1$ 0^0 is an ____________________________________

More Properties of Exponents

Example	In general
$(xy)^3$	$(xy)^m$
$\left(\frac{x}{y}\right)^2$	$\left(\frac{x}{y}\right)^m$

1. Simplify

a) $(-8)^2$ b) -8^2 c) 3^0

d) $\left(\dfrac{1}{y^{-6}}\right)$

e) $\left(\dfrac{a}{b}\right)^{-1}$

e) $\dfrac{x^{-3}}{x^{2}}$

f) $\left(\dfrac{y^{4}}{y^{-5}}\right)^{-1}$

g) $\left(\dfrac{12a^{-3}b^{2}}{4a^{4}b^{-1}}\right)^{-2}$

h) $\dfrac{\left(\dfrac{1}{3}\right)^{-1005}-27^{334}}{9^{502}+9^{501}}$

Roots or Radicals

TRUE or FALSE: $\sqrt{4} = \pm 2$.

Important distinction: $x^2 = 25$ has ______ solutions: x = _______ and x = _____

$\sqrt{25}$ represents exactly _________ number. $\sqrt{25}$ = ________.

Recall that $2^5 = 32$. Now suppose you have $x^5 = 32$ with the instructions: "Solve for x."

How do you express x in terms of 32? The vocabulary word is __________________

In this case x equals the ______________________________.

The notation is $x =$ ____________________ or $x =$ ____________________

This second notation is called a fractional exponent.

Examples: $\sqrt[4]{81}$ = $(81)^{\left(\frac{1}{4}\right)}$ = ____________ because $3^{\square}$ = _________.

$(125)^{\left(\frac{1}{3}\right)}$ = ____________ because $5^{\square}$ = _________.

Properties of Roots (just like properties of exponents!)

Example	In general
$\sqrt[3]{27 \cdot 8}$ $\sqrt[3]{27}\sqrt[3]{8}$	$\sqrt[m]{xy}$

Example	In general
$\sqrt{\frac{36}{4}}$ $\left(\frac{\sqrt{36}}{\sqrt{4}}\right)$	$\frac{\sqrt[m]{x}}{\sqrt[m]{y}}$

Now consider $8^{\frac{2}{3}}$. Notice that $\left(8^{2}\right)^{\frac{1}{3}} =$ ______________________________

Also $\left(8^{\left(\frac{1}{3}\right)}\right)^{2} =$ ______________________________

So it would seem that $8^{\frac{2}{3}} =$ ______________________________

In fact, more generally, it is true that $x^{\left(\frac{m}{n}\right)} =$ ______________________________

Similarly, consider $\sqrt[3]{\sqrt{64}} = \left(64^{\left(\frac{1}{2}\right)}\right)^{\frac{1}{3}} =$ ______________________________

Whereas, $\sqrt{\sqrt[3]{64}} = \left(64^{\left(\frac{1}{3}\right)}\right)^{\frac{1}{2}} =$ ______________________________

So it would seem that $\sqrt[3]{\sqrt{64}}$ ______________________________

In general $\sqrt[m]{\sqrt[n]{x}}$ ______________________________

Simplify $\sqrt[n]{a^n}$ for all $n \in \mathbb{N}$.

In recitation you graphed $y = \sqrt[3]{x^3}$. Hopefully you found that its graph looked just like the graph of $y = x$. In other words, you saw that $\sqrt[3]{x^3} =$ ____________

You also graphed $y = \sqrt[2]{x^2}$. Hopefully you found that its graph looked just like the graph of $y = |x|$. In other words, you saw that $\sqrt[2]{x^2} =$ ____________

Most people believe that $\sqrt[5]{2^5} =$ ________ and agree easily that $\sqrt[n]{2^n} =$ ________ $\forall\ n$

But $\sqrt[4]{(-2)^4}$ = ________________________ while $\sqrt[5]{(-2)^5} =$ ________________________

So hopefully you see believe $\sqrt[n]{(-2)^n} = 2$ whenever n is ___________

and $\sqrt[n]{(-2)^n} = -2$ whenever n is ___________

So to simplify $\sqrt[n]{a^n}$ for any $a \in \mathbb{R}$ and any $n \in \mathbb{N}$ we have

$$\sqrt[n]{a^n} = \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$$

Simplifying Radicals

A radical expression is simplified when the following conditions hold:

1. All possible factors (“perfect roots”) have been removed from the radical.
2. The index of the radical is as small as possible.
3. No radicals appear in the denominator.

2. Simplify

a) $\sqrt[5]{-32}$

b) $(-216)^{\left(\frac{1}{3}\right)}$

c) $-9^{\left(\frac{1}{2}\right)}$

d) $\sqrt{-64}$

e) $\sqrt[6]{(-13)^6}$

f) $\sqrt[4]{16x^4}$

g) $\sqrt[3]{x^5}$

h) $\sqrt[3]{648x^4y^6}$

i) $\sqrt[8]{16}$

j) $\left(\frac{-125}{64}\right)^{\frac{-2}{3}}$

Rationalizing the Denominator

"Rationalizing the denominator" is the term given to the techniques used for eliminating radicals from the denominator of an expression without changing the value of the expression. It involves multiplying the expression by a 1 in a "helpful" form.

3. Simplify

a) $\frac{1}{\sqrt{2}}$

b) $\frac{1}{\sqrt[3]{2}}$

c) $\frac{1}{\sqrt[3]{4x}}$

d) $\sqrt{\frac{(9x)^3 y^{-4}}{50x^8 y^{-5}}}$

Notice: $\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)=$ ____.

To simplify $\frac{1}{\sqrt{2}-\sqrt{5}}$ we multiply it by 1 in the form of

So $\frac{1}{\sqrt{2}-\sqrt{5}}$ =

and $\frac{41}{2+3\sqrt{5}}$ =

Adding and Subtracting Radical Expressions

Terms must be alike to combine them with addition or subtraction. Radical terms are alike if they have the same index and the same radicand. (The radicand is the expression under the radical sign.)

4. Simplify

a) $\sqrt{5}+2\sqrt{7}-3\sqrt{5}-\sqrt{7}$

b) $\sqrt[3]{16x^4}-3x\sqrt{18x}-x\sqrt[3]{250x}$

Please notice $\sqrt{9}+\sqrt{16}$ = ________________________ Whereas $\sqrt{25}$ = ____________

In other words ______________________________

In general ____________________________

Suggested Problems: Text: 1-32

My Previous Exams:
Fall 2014 1A: 1
Spring 2014 2A: 11, 14,
Fall 2013 1A: 1, 2 ,
Spring 2013 1A: 1, 2,
Fall 2012 1A: 1 a, b, d,

Chapter 4E - Combinations of Functions

1. Let $f(x)=\sqrt{3-x}$ and $g(x)=\sqrt{3+x}$

a) What is the domain of $f(x)$?

b) What is the domain of $g(x)$?

c) $(f+g)(x)$ =

d) What is the domain of $(f+g)(x)$?

e) $(f-g)(x)$ =

f) What is the domain of $(f-g)(x)$?

g) $(fg)(x)=$

h) What is the domain of $(fg)(x)$?

i) $\left(\frac{f}{g}\right)(x)=$

j) What is the domain of $\left(\frac{f}{g}\right)(x)$?

Practicing with function notation:

2. Let $f(x) = -x^2 + x$ Determine

a) $f(2)$

b) $f(-3)$

c) $f(*)$

d) $f(x^2)$

e) $f(4x)$

f) $f(x+h)$

Function Composition

Definition For functions $f(x)$ and $g(x)$, the composition of functions

$(f \circ g)(x)$ is defined as ___________________________________

3. Let $f(x) = x^2 - 9$ and $g(x) = \dfrac{1}{x+5}$

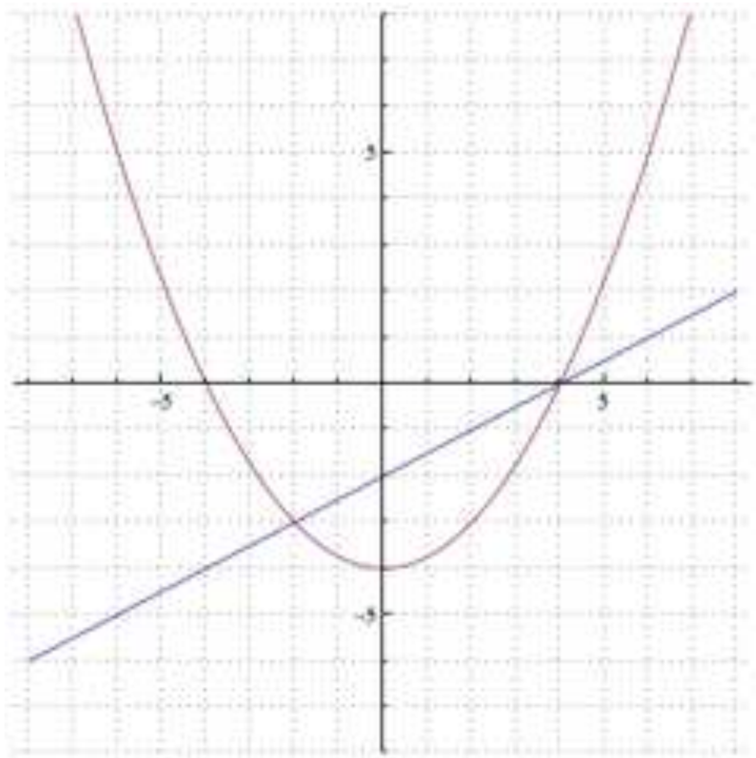
a) $(f \circ g)(-3) =$

b) $(g \circ f)(-3) =$

c) $(f \circ g)(x) =$

d) $(g \circ f)(x) =$

4. Here are the graphs of two functions. If the line is the graph of $f(x)$ and the parabola is the graph of $g(x)$, sketch $(f+g)(x)$



5. Using the same graph and functions, determine

a) $(f \circ g)(4)$

b) $(g \circ f)(4)$

c) $(f \circ g)(0)$

d) $(g \circ f)(0)$

Suggested Problems: **Text:** 3, 8-11

My Previous Exams: Fall 2014 2A: 19 Fall 2013 2A: 3, 9

Spring 2013 2A: 4, 11 Fall 2012 2A: 8