Chapter 8

Developing Early Number Concepts and Number Sense

Young children enter school with many ideas about number. These ideas should be built upon as we work with them to develop new relationships. It is sad to see the large number of students in grades 4, 5, and beyond who essentially know little more about number than how to count. It takes time and a variety of experiences for children to develop a full understanding of number that will grow into more advanced number-related concepts. This chapter emphasizes the development of number ideas for numbers up to about 20. These foundational ideas can all be extended to larger numbers, operations, basic facts, and computation.

BIG IDEAS

1. Counting tells how many things are in a set. When counting a set of objects, the last word in the counting sequence names the quantity.
2. Numbers are related through comparisons of quantities including greater than and less than relationships. These comparisons are made through one-to-one correspondence of objects in sets. The number 7, for example, is more than 4, is two less than 9, is composed of 3 and 4, is three away from 10, and can be quickly recognized in several patterns of dots. These ideas extend to composing and decomposing larger numbers such as 17, 57, and 370.
3. Number concepts are intimately tied to operations with numbers based on situations in the world around us. Application of number relationships to problem solving marks the beginning of making sense of the world in a mathematical manner.

Mathematics CONTENT CONNECTIONS

Early number development is related to other mathematics curriculum in two ways: content that enhances the development of number (measurement, data, operations) and content directly affected by how well early number concepts have been developed (basic facts, place value, and computation).

- **Operations** (Chapter 9): As children solve story problems for any of the four operations, they count on, count back, make and count groups, and make comparisons. In the process, they form new relationships and methods of working with numbers.
- **Measurement** (Chapter 19): Selecting an appropriate unit and then determining measures of length, area, size, or weight is an important use of number. Measurement involves the counting and comparing of quantities found in the world in which the child lives.
- **Data** (Chapter 21): Data analysis involves counts and comparisons to both aid in developing number and connecting it to real-world situations. Comparing bar lengths on a bar graph helps young students compare quantities through an organized format.
- **Basic Facts** (Chapter 10): A rich and thorough development of number relationships is a critical foundation for mastering basic facts. Otherwise, facts are rotely memorized and easily forgotten. With knowledge of number relationships, facts that are forgotten can be easily constructed.
- **Place Value and Computation** (Chapters 11,12, and 13): Ideas that contribute to procedural fluency and flexibility in computation are extensions of how numbers are related to 10 and how numbers can be taken apart (decomposed) and recombined (composed).

Promoting Good Beginnings

In 2007, NCTM produced a position statement emphasizing that all children need an early start in learning mathematics. This emphasis on readiness aligns with the recent findings of the National Mathematics Advisory Panel (2008) and the National Research Council (NRC) (2009). This statement suggests several research-based recommendations to help teachers develop high-quality learning activities for children aged 3 to 6:

1. Enhance children's natural interest in mathematics and their instinct to use it to make sense of their world.
2. Build on children's experience and knowledge, taking advantage of familiar contexts.
3. Base mathematics curriculum and teaching practices on a solid understanding of both mathematics and child development.
4. Use formal and informal experiences in the curriculum and teaching practices to strengthen children's problem-solving and reasoning processes.
5. Provide opportunities for children to explain their thinking as they interact with mathematics in deep and sustained ways.
6. Support children's learning by thoughtfully and continually assessing children's mathematical knowledge, skills, and strategies.

PAUSE and REFLECT

Although all of these recommendations are critical, which two are most important for your own professional growth?

In 2009, the NRC established the Committee on Early Childhood Mathematics to examine research on how mathematics is taught and learned in children's early years. Unfortunately, they found a lack of opportunities for learning mathematics in early childhood settings, especially as compared to opportunities in language and literacy development. The studies showed that young children who are starting out behind their peers, such as those growing up in disadvantaged circumstances, do not catch up. They found that what a kindergartner or first-grade child knows about mathematics is a predictor of not only their math achievement (National Mathematics Advisory Panel, 2008) but also their reading achievement.

This NRC committee also identified the foundational mathematics content in number for early learners, grouping it into three core areas: number, relations, and operations. This chapter will begin with the first two core areas; then Chapter 9 will provide an intensive focus on the meaning of the operations. Please note that as you develop students' initial abilities in counting, the conversations about number relationships begin. Therefore, the activities and concepts in this chapter are not sequential but coexist in a rich environment of mathematical experiences where students see connections between and among numbers.

The Number Core: Quantity, Counting, and Knowing How Many

Families help children count their fingers, toys, people at the table, and other small sets of objects. Questions concerning "Who has more?" or "Are there enough?" are part of the daily lives of children as young as 2 or 3 years old. Considerable evidence indicates that these children have beginning understandings of the concepts of number and counting (Baroody, Li, & Lai, 2008; Clements & Sarama 2009; Gelman & Gallistel, 1978). We therefore include abundant activities to support the different experiences that young children and older students with disabilities need to gain a full understanding of number concepts.

Quantity and the Ability to Subitize

Children explore quantity before they can count. They can identify which cup is bigger or which plate of potato chips has more chips. Soon they need to attach an amount to the quantities to explore them in greater depth. When you look at an amount of objects, sometimes you are able to just "see" how many are there, particularly for a small group. For example, when you roll a die and know that it is five without counting the dots, that ability to "just see it" is called *subitizing*. There are times when you are able to do this for even larger amounts, when you break dots in a pattern of ten by seeing five in one row and mentally doubling it to get a total of 10. "Subitizing is a fundamental skill in the development of students' understanding of number" (Baroody, 1987, p. 115). Subitizing is a complex skill that needs to be developed and practiced through experiences with patterned sets.

Many children learn to recognize patterned sets of dots on standard dice due to the many games they have played. Similar instant recognition (subitizing) can be developed for other patterns (see Figure 8.1). Naming these amounts immediately without counting aids in "counting on" (from a known patterned set) or learning combinations of numbers

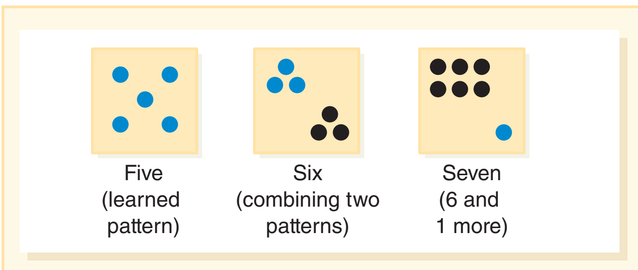

FIGURE 8.1 Recognizing a patterned set.

(seeing a pattern set of two known smaller patterns). Please note that many textbooks present illustrations of small quantities that are less than helpful in encouraging subitizing, so use objects organized in patterns that are symmetric before moving to more challenging images.

Good materials to use in pattern recognition activities include a set of dot plates. These can be made using paper plates and the peel-off dots available in office supply stores. A collection of patterns is shown in Figure 8.2. Note that some patterns are combinations of two smaller patterns or a pattern with one or two additional dots. These should be made in two colors to support early learners. Keep the patterns compact and organized. If the dots are too spread out, the patterns are hard to identify. Explore the activities "Speedy Pictures 1" and "Speedy Pictures 2" on the website www.fi.uu.nl/rekenweb/en, where students can practice subitizing and recognizing patterned sets using flashed images of fingers, dice, beads on a frame, or eggs in a carton holding 10.

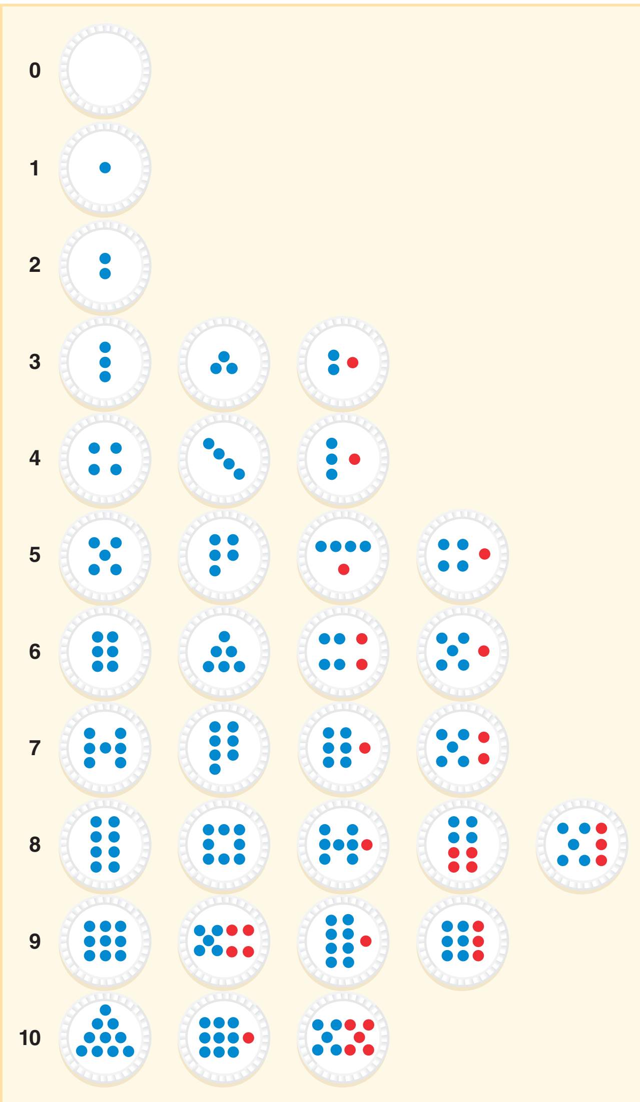

FIGURE 8.2 A collection of dot patterns for "dot plates."

Activity 8.1

STUDENTS *with* SPECIAL NEEDS

Learning Patterns

To introduce patterns, provide each student with 10 counters and a piece of paper or a paper plate as a mat. Hold up a dot plate for about five seconds. "Make the pattern you saw on the plate using the counters on the mat. How many dots did you see? What did the pattern look like?" Spend time discussing the configuration of the pattern and how many dots. Then show the plate so they can self-check. Do this with a few new patterns each day. To modify this activity for students with disabilities, you may need to give the student a small selection of dot plates. Then instead of creating the pattern with counters, they find the matching plate.

Activity 8.2

Dot Plate Flash

Hold up a dot plate for one to three seconds. "How many dots did you see? What did the pattern look like?" Children like to see how quickly they can recognize the pattern and say how many dots. Include easy patterns first and then add more dots as their confidence builds. Students can also flash dot plates to each other as a workstation activity.

STOP

Instant recognition activities with dot plates are exciting and can be done in 5 minutes at any time of day and at any time of year.

Early Counting

Meaningful counting activities begin when children are 3 and 4 years of age, but by the end of kindergarten (CCSSO, 2010), children should be able to count to 100. The counting process cannot be forced, so for children to have an understanding of counting, they must construct this idea. Only the counting sequence of number words is a rote procedure. The *meaning* attached to counting is the key conceptual idea on which all other number concepts are developed.

The Development of Verbal Counting Skills. Counting is a complex task with typical developmental progressions found in a path called a *learning trajectory* (Clements and Sarama, 2009). This trajectory can help you see what the

TABLE 8.1

LEARNING TRAJECTORY FOR COUNTING

Levels of Thinking	Characteristics
Precounter	Here the child has no verbal counting ability. A young child looking at three balls will answer "ball" when asked how many. The child does not associate a number word with a quantity.
Reciter	This child verbally counts using number words, but not always in the right order. Sometimes they say more numbers than they have objects to count, skip objects, or repeat the same number.
Corresponder	A child at this level can make a one-to-one correspondence with numbers and objects, stating one number per object. If asked "How many?" at the end of the count, they may have to recount to answer.
Counter	This student can accurately count objects in an organized display (in a line, for example) and can answer "How many?" accurately by giving the last number counted (this is called *cardinality*). They may be able to write the matching numeral and may be able to say the number just after or before a number by counting up from 1.
Producer	A student at this level can count out objects to a certain number. If asked to give you five blocks, they can show you that amount.
Counter and Producer	A child who combines the two previous levels can count out objects, tell how many are in a group, remember which objects are counted and which are not, and respond to random arrangements. They begin to separate tens and ones, like 23 is 20 and 3 more.
Counter Backwards	A child at this level can count backward by removing objects one by one or just verbally as in a "countdown."
Counter from Any Number	This child can count up starting from numbers other than one. They are also able to immediately state the number before and after a given number.
Skip Counter	Here the child can skip-count with understanding by a group of a given number—tens, fives, twos, etc.

Source: Based on Clements and Sarama (2009).

overarching goals of counting are and how you can help a child move to more sophisticated levels of thinking. Table 8.1 is adapted from their research and is a selection of levels and sublevels identified as benchmarks (pp. 30–41).

As a starting point, verbal counting has at least two separate skills. First, a child must be able to produce the standard list of counting words in order: "One, two, three, four. . . ." Second, a child must be able to connect this sequence in a one-to-one correspondence with the objects in the set being counted. Each object must get one and only one count. As part of these skills, students should recognize that each counting number identifies a quantity that is one more than the previous number and that the new quantity is embedded in the previous quantity (see Figure 8.3). This knowledge will be helpful later in breaking numbers apart.

Experience and guidance are major factors in the development of these counting skills. Many children come to kindergarten able to count sets of 10 or beyond. At the same time, children with weak background knowledge may require additional practice. The size and arrangement of the set are also factors related to success in counting. Obviously, longer number strings require more practice to learn. The first 12 counting words involve no pattern or repetition, and many children do not easily recognize a pattern in the teens. Children learning the skills of counting—that is matching oral number words with objects—should be given sets of blocks or counters that they can move or pictures of sets that are arranged in an organized pattern for easy counting.

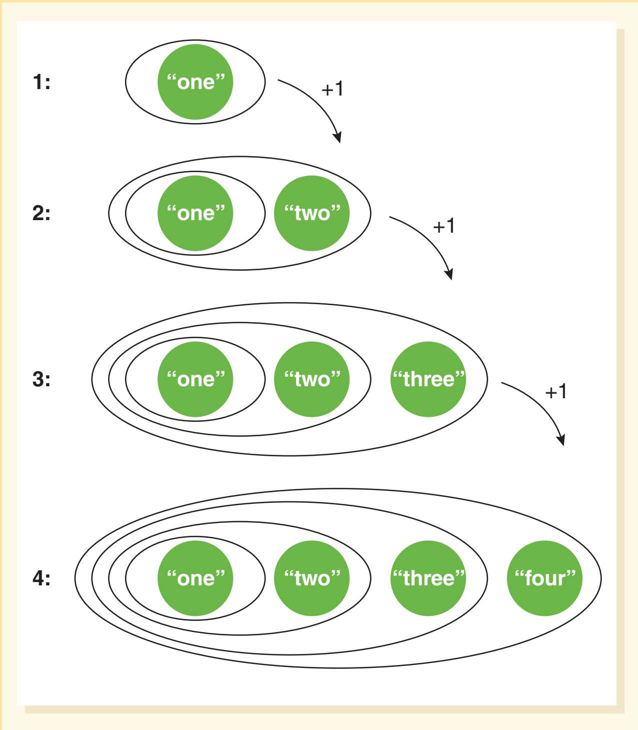

FIGURE 8.3 In counting, each number is one more than the previous number.

Source: National Research Council. (2009). *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity,* p. 27. Reprinted with permission from the National Academy of Sciences, courtesy of the National Academies Press, Washington, DC.

Meaning Attached to the Counting of Objects. Fosnot and Dolk (2001) make it clear that an understanding of cardinality and the connection to counting is not a simple task for 4-year-olds. Children will learn *how* to count (matching counting words with objects) before they understand that the last count word indicates the *amount* of the set or the set's *cardinality* as shown in Figure 8.4. Children who have made this connection are said to have the *cardinality principle*, which is a refinement of their early ideas about quantity. Most, but certainly not all, children by age $4\frac{1}{2}$ have made this connection (Fosnot & Dolk, 2001).

For many students, especially students with disabilities, it is important to have a plan for counting. The children should count objects from left to right, move the objects as they count or point and touch them as they say each number out loud. Consistently ask, "How many do you have in all?" at the end of each count.

FORMATIVE Assessment Notes

Young children who can count orally may not have attached meaning to their counts. Here is a **diagnostic interview** that will help you assess a child's thinking. Show a child a card with five to nine large dots in a row so that they can be easily counted. Ask the child to count the dots. If the count is accurate, ask, "How many dots are on the card?" Early on, many children will need to count again. One indication the child is beginning to grasp the meaning of counting will be that they do not need to recount. Now ask the child, "Please get the same number of counters as there are dots on the card." Here is a sequence of indicators to watch for, listed in order from a child who does not attach meaning to the count to one who is using counting as a tool:

Does the child not count but instead make a similar pattern with the counters?
Will the child recount?
Does the child place the counters in a one-to-one correspondence with the dots?
Or does the child count the dots and retrieve the correct number of counters?
Is the child confident that there is the same number of counters as dots?

As the child shows competence with patterned sets, move to counting random dot patterns. ■

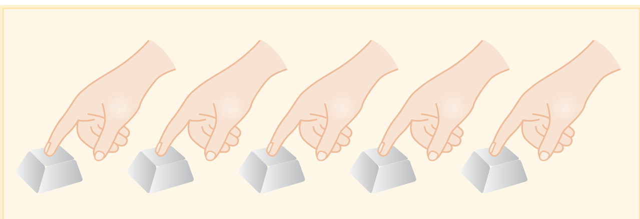

FIGURE 8.4 The student has learned cardinality if, after counting five objects, he or she can answer, "How many do you have in all?" with "Five."

STOP

To develop the full understanding of counting, engage children in games or activities that involve counts and comparisons. The following is a suggestion. ●

Activity 8.3

STUDENTS *with* SPECIAL NEEDS

Fill the Tower

Create a game board with four "towers." Each tower is a column of twelve 1-inch squares with a star at the top. Children take turns rolling a die and collecting the indicated number of counters. They then place these counters on one of the towers. The object is to fill all of the towers with counters. As an option, require that the towers be filled exactly so that a roll of 5 cannot be used to fill four empty squares. A modification for students with disabilities would be to use a die with only 2 or 3 on it. You can increase the number choices on the die when you have evidence that the student is counting accurately.

This game provides opportunities for you to talk with children about number and assess their thinking. Watch how they count the dots on the die. Ask, "How do you know you have the right number of counters?" and "How many counters did you put in the tower? How many more do you need to fill the tower?"

Regular classroom tasks, such as counting how many napkins are needed at snack time, are additional opportunities for children to learn about number and for teachers to listen to students' ideas.

Thinking about Zero. Young children need to discover the number zero (Clements & Sarama, 2009). Surprisingly it is not a concept that is easily grasped without intentionally building understanding. Three- and four-year-olds can begin to use the word *zero* and the numeral 0 to symbolize there are no objects in the set. With the dot plates discussed previously (see Figure 8.2), use the zero plate to formally discuss what it means that there is no dot on the plate. We find that because early counting often involves touching an object, zero is sometimes not included in the count. Zero is one of the most important digits in the base-ten system, and purposeful conversations about it are essential. Activities 8.1, 8.2, and 8.11 are useful in exploring the number zero.

Numeral Writing and Recognition

Kindergartners are expected to write numbers up to 20 (CCSSO, 2010). Helping children read and write the 10 single-digit numerals is similar to teaching them to read and write letters of the alphabet. Neither has anything to do with number concepts. Numeral writing does not have to be repetitious practice, but it can be engaging. For example, ask children to trace over pages of numerals, make numerals from clay, trace them in shaving cream on their desks, write them on the board or in the air, and so on.

The calculator is a good instructional tool for numeral recognition. Early calculator activities can also help develop familiarity with other symbols on the keypad so that more complex activities are possible. While these numerals may be familiar to students from other cultures, the naming of the numerals is not. Activities that move between objects, numerals, and number names are important for all learners, particularly English language learners.

Activity 8.4

Number Tubs

Give each child four to six closed margarine tubs, each containing a different number of pennies or counters. (Foam counters work well.) The tubs are then mixed up. The teacher asks the child to find the tub with a particular number of counters. After the child looks inside and counts to find the correct tub, a new twist can be added. You can allow them to mark the tubs with sticky notes to show what is inside. At first, children may make four dots to show four counters, but eventually, with your encouragement, they will write the numeral. Then the students recognize the value of writing the numbers in a form that all can understand and that doesn't require recounting.

Activity 8.5

STUDENTS *with* SPECIAL NEEDS

Find and Press

Give each child a calculator and ask them to press the clear key. Say a number, and have children press that number on the calculator. If you have a digital projector, you can show the children the correct key so that they can confirm their responses, or you can write the number on the board for children to self-check. Begin with single numbers. Later, progress to two or three numbers called out in succession. For example, call, "Three, seven, one." Children press the complete string of numbers as called. Some children with disabilities may need calculators with large keys spaced apart so that they can enter a number. For students with limited mobility, there is a nice online four function calculator at www.online-calculator.com/full-screen-calculator.

Perhaps the most common preschool and kindergarten exercises have children match sets with numerals. Children are given pictured sets (e.g., frogs) and asked to write or match the number that tells how many. Alternatively, they may be given a number and told to make a set with that many objects. When children are successful with these matching-numeral-to-sets activities, it is time to move on to more advanced concepts, like counting on and counting back.

Counting On and Counting Back

Although the forward sequence of numbers is relatively familiar to most young children, counting on from a particular number and counting back are often difficult skills. In particular, for English language learners counting back is more difficult (try counting back in a second language you have learned). Frequent short practice sessions are recommended.

Activity 8.6

Up and Back Counting

Counting up to and back from a target number in a rhythmic fashion is an important counting exercise. For example, line up five children and five chairs in front of the room. As the whole class counts from 1 to 5, the children sit down one at a time. When the target number, 5, is reached, it is repeated; the child who sat on 5 now stands, and the count goes back to 1. As the count goes back, the children stand up one at a time, and so on, "1, 2, 3, 4, 5, 5, 4, 3, 2, 1, 1, 2, . . ." Children find exercises such as this both fun and challenging. Any rhythmic movement (clapping, turning around) can be used as the count goes up and back.

This last activity is designed to help students become fluent with the number-word sequence in both forward and reverse order and to begin counts with numbers other than 1. Although not easy for young students, these activities do not yet address the meaning of counting on or counting back. Children will later realize that counting on is adding and counting backward is subtracting. Fosnot and Dolk (2001) describe the ability to count on as a "landmark" on the path to number sense.

Activity 8.7

Counting On with Counters

Give each child a collection of 10 or 12 counters that the children line up left to right. Tell them to count four counters and push them under their left hands or place them in a cup (see Figure 8.5). Then say, "Point to your hand. How many are there?" (Four.) "So let's count like this: fooour . . . (slowly, pointing to their hand), five, six. . . ." Repeat with other numbers under the hand.

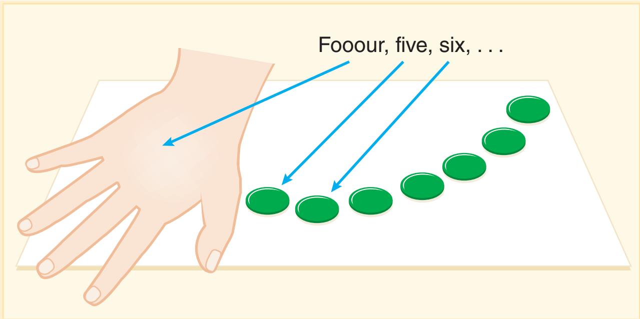

FIGURE 8.5 Counting on: "Hide four. Count, starting from the number of counters hidden."

Activity 8.8

Real Counting On

STUDENTS *with* SPECIAL NEEDS

This game for two requires a deck of cards (numbers 1 to 7), a die, a paper cup, and counters. The first player turns over the top number card and places the indicated number of counters in the cup. The card is placed next to the cup as a reminder of how many are inside. The second player rolls the die and places that many counters next to the cup (see Figure 8.6). Together they decide how many counters in all. A record sheet with columns for "In the Cup," "On the Side," and "In All" will support students' organization. Increase the highest number in the card deck when the children have mastered the smaller numbers. For students with disabilities, you may want to just use a single number in the cup (such as 5) and have them just count on from the number in the cup until they are fluent with that number.

Observe how children determine the total amounts in Activity 8.8. Children who are not yet counting on may empty the counters from the cup or will count up from one without emptying the counters. As children continue to play, they will eventually count on as that strategy becomes meaningful and useful.

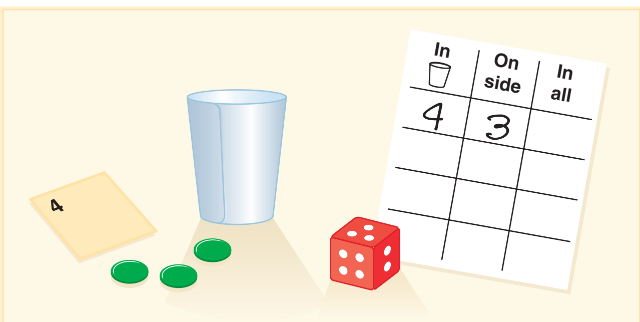

FIGURE 8.6 How many in all? How do children count to tell the total? Empty the counters from the cup? Count up from 1 without emptying the counters? Count on?

The Relations Core: More Than, Less Than, and Equal To

The concepts of "more," "less," and "same" are basic relationships contributing to the overall concept of number. Almost any child entering kindergarten can choose the set that is *more* if presented with two sets that are quite obviously different in number. In fact, Baroody (1987) states, "A child unable to use 'more' in this intuitive manner is at considerable educational risk" (p. 29). Classroom activities should help children build on and refine this basic notion that links to their ability to count.

Though the concept of less is logically related to the concept of more (selecting the set with more is the same as *not* selecting the set with less), the word *less* proves to be more difficult for children than *more*. A possible explanation is that children have many opportunities to use the word *more* but have limited exposure to the word *less*. To help children with the concept of less, frequently pair it with *more* and make a conscious effort to ask "Which is less?" questions as well as "Which is more?" questions. For example, suppose that your class correctly selected the set that has more from the two sets given. Immediately follow with "Which is less?" In this way, the concept can be connected with the better-known idea and the term *less* can become familiar.

For all three concepts (more/greater than, less/less than, and same/equal to), children should construct sets using counters as well as make comparisons or choices (Which is less?) between two given sets. The following activities should be conducted in a spirit of inquiry accompanied with requests for explanations. "Can you show me how you know this group has less?"

Activity 8.9

Make Sets of More/Less/Same

STUDENTS *with* SPECIAL NEEDS

At a workstation, provide about eight cards with pictures of sets of 4 to 12 objects (or use large dot cards); a set of counters; word cards labeled *More, Less,* and *Same;* and paper plates or low boxes to support students with disabilities. Next to each card, have students make three collections of counters: a set that is more than the amount on the card, one that is less, and one that is the same (see Figure 8.7). Start students with disabilities with matching the set that is the same.

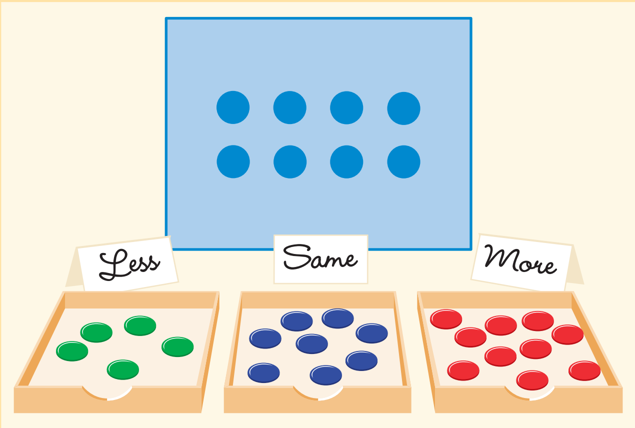

FIGURE 8.7 Making sets that are more, less, and the same.

Activity 8.10

STUDENTS *with* SPECIAL NEEDS

Find the Same Amount

Give children a collection of cards with sets on them. Dot cards are one option (see Blackline Masters 3–8). Have the children pick any card in the collection and then find another card with the same amount to form a pair. Continue finding other pairs. This activity can be altered to have children find dot cards that are "less" or "more." Some students with disabilities may need a set of counters with a blank ten-frame to help them "make" a pair instead of finding a pair.

Observe children as they do these tasks. (This is also a good opportunity for diagnostic interviews.) Note that some children make comparisons of more or less without assigning numerical values. Children whose number ideas are tied to counting and nothing more will select cards at random and count each dot looking for the same amount. Others will estimate and begin by selecting a card that appears to be the same number of dots. This demonstrates a significantly higher level of understanding. Also observe how the dots are counted. Are the counts made accurately? Is each dot counted only once? Does the child touch the dot? A significant milestone occurs when children recognize small patterned sets without counting.

Activity 8.11

More, Less, or the Same

This activity is for partners or a small group. Use Blackline Master 1 (make four to five of each card) to make a deck of more-or-less cards as shown in Figure 8.8. You will also need a set of number cards (Blackline Master 2) with the numbers 3 to 10 (two each). One child draws a number card and places it face up. That number of counters is put into a cup. Next, another child draws one of the more-or-less cards and places it next to the number card. For the More cards, counters are added to the cup. For the Less cards, counters are removed from the cup. For Zero cards, no change is made. Once the cup has been adjusted, children predict how many counters are now in the cup. The counters are emptied and counted and a new number card is drawn.

"More, Less, or the Same" can also be played with the whole class. The words *more* and *less* can be paired or substituted with *add* and *subtract* to connect these ideas with the arithmetic operations, even if they have not yet been formally introduced.

The calculator can be used to practice relationships of one more than, two more than, one less than, and two less than. Also use it to show the pattern of adding or subtracting zero.

Early Number Sense

Howden (1989) described *number sense* as a "good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (p. 11). In *Principles and Standards for School Mathematics*, the term *number sense* is used freely throughout the Number and Operations standard.

FIGURE 8.8 Materials to play "More or Less" (see Blackline Master 1).

"As students work with numbers, they gradually develop flexibility in thinking about numbers, which is a hallmark of number sense. . . . Number sense develops as students understand the size of numbers, develop multiple ways of thinking about and representing numbers, use numbers as referents, and develop accurate perceptions about the effects of operations on numbers" (NCTM, 2000, p. 80).

PAUSE and REFLECT

You have begun to see some of the early foundational ideas about number. Stop now and make a list of all of the important ideas that you think children should *know about* the number 8 by the time they finish first grade. (The list could be about any number from, say, 6 to 12.) Put your list aside, and we will revisit your ideas later.

The discussion of number sense begins as we look at the relationships and connections children should be making about smaller numbers up to 20. But "good intuition about numbers" does not end with these smaller whole numbers. Children continue to develop number sense as they use numbers in operations, build an understanding of place value, and devise flexible methods of computing and making estimates involving large numbers, fractions, decimals, and percents.

The ideas of early numeracy discussed to this point are the rudimentary aspects of number. Unfortunately, many textbooks move directly from these beginning ideas to addition and subtraction, leaving students with a very limited collection of ideas about number and number relationships to bring to these new topics. The result is that children often continue to count by ones to solve simple story problems and have difficulty mastering basic facts. Early number sense requires significant attention in pre-K–2 programs.

Relationships Between Numbers 1 Through 10

Once children acquire a concept of cardinality and can meaningfully use their counting skills, little more is to be gained from the kinds of counting activities described so far. Also, more relationships beyond the general "more or less" decision must be created for children to develop number sense, a flexible concept of number not completely tied to counting. Figure 8.9 illustrates three types of number relationships that children can and should develop:

- *One and two more, one and two less.* The two-more-than and two-less-than relationships involve more than just the ability to count on two or count back two. Children should know that 7, for example, is 1 more than 6 and also 2 less than 9.
- *Anchors or "benchmarks" of 5 and 10.* Because 10 plays such a large role in our numeration system and because two fives make up 10, it is very useful to develop relationships for the numbers 1 to 10 connected to the anchors of 5 and 10.
- *Part-part-whole relationships.* To conceptualize a number as being made up of two or more parts is the most important relationship that can be developed about numbers. For example, 7 can be thought of as a set of 3 and a set of 4 or a set of 2 and a set of 5.

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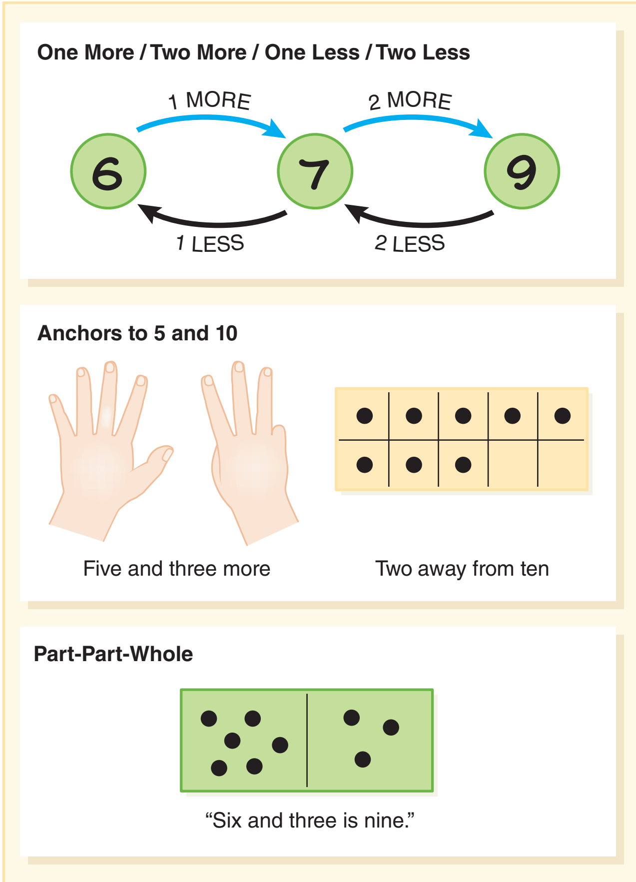


FIGURE 8.9 Three number relationships for children to develop.

The principal tool that children will use as they construct these relationships is the one number tool they possess: counting. Initially, then, you will notice a lot of counting, and you may wonder if you are making progress. Have patience! As children construct new relationships and begin to use more powerful ideas, counting will become less and less necessary.

One and Two More, One and Two Less

When children count, they don't often reflect on the way one number is related to another. Their goal is only to match number words with objects until they reach the end of the count. To learn that 6 and 8 are related by the corresponding relationships of "two more than" and "two less than" requires reflection on these ideas. Counting on (or back) one or two counts is a useful tool in constructing these ideas.

Note that the relationship of "two more than" is significantly different from "comes two counts after." This latter relationship is applied to the string of number words, not to the quantities they represent. A comes-two-after relationship can even be applied to letters of the alphabet as the letter *H* comes two after the letter *F.* However, there is no numeric or quantitative difference between *F* and *H.* The quantity 8 would still be two more than 6 even if there were no number string to count these quantities. It is the numeric relationship you want to develop.

The following activity focuses on the two-more-than relationship and is a good place to begin.

Activity 8.12

Make a Two-More-Than Set

Provide students with six dot cards (Blackline Masters 3–8). Their task is to construct a set of counters that is two more than the set shown on the card. Similarly, spread out eight to ten dot cards, and ask students to find another card for each that is two less than the card shown. (Omit the 1 and 2 cards for two less than, and so on.)

In activities such as 8.12 in which children find a set or make a set, they can also add numeral cards (see Blackline Master 2) to all of the sets involved. Then they can be encouraged to take turns reading the associated number sentence to their partner. If, for example, a set has been made that is two more than a set of four, the child can say the number sentence, "Two more than four is six" or "Six is two more than four." The next activity combines the relationships.

Activity 8.13

A Calculator Two-More-Than Machine

Teach children how to make a two-more-than machine. Press 0 [+] 2 [=]. This makes the calculator a two-more-than machine. Now press any number—for example, 5. Children hold their finger over the [=] key and predict the number that is two more than 5. Then they press [=] to confirm. If they do not press any of the operation keys (+, −, ×, ÷), the "machine" will continue to perform in this way.

What is really happening in the two-more-than machine is that the calculator "remembers" or stores the last operation, in this case "+2," and adds that to whatever number is in the window when the [=] key is pressed. If the child continues to press [=], the calculator will continue to count by twos. At any time, a new number can be pressed followed by the equal key. To make a two-less-than machine, press 2 [−] 2 [=]. (The first press of 2 is to avoid a negative number.) In the beginning, students may accidentally press operation keys, which change what their calculator is doing. Soon, however, they get the hang of using the calculator as a function machine.

The "two-more-than" calculator will give the number that is two more than any number pressed, including those with two or more digits. The two-more-than relationship should be extended to two-digit numbers as soon as students are exposed to them. One way to do this is to ask for the number that is two more than 7. After getting the correct answer, ask, "What is two more than 37?" and similarly for other numbers that end in 7. When you try this for 8 or 9, expect difficulties and creative responses such as two more than 28 is "twenty-ten." In the first grade, this struggle can generate a "teachable moment." The "More or Less" activity can be extended to larger numbers if no actual counters are used.

Anchoring Numbers to 5 and 10

We want to help children relate a given number to other numbers, specifically 5 and 10. These relationships are especially useful in thinking about various combinations of numbers. For example, in each of the following, consider how the knowledge of 8 as "5 and 3 more" and as "2 away from 10" can play a role: $5 + 3$, $8 + 6$, $8 - 2$, $8 - 3$, $8 - 4$, $13 - 8$. For example, $8 + 6$ may be thought of as $8 + 2 + 4$ ("Up Over 10" strategy). Later similar relationships can be used in the development of mental computation skills on larger numbers such as $68 + 7$.

The most common and perhaps most important model for exploring this relationship is the ten-frame. The ten-frame is simply a 2×5 array in which counters or dots are placed to illustrate numbers (see Figure 8.10). Ten-frames can be drawn on a sheet of paper (see Blackline Master 10). Nothing fancy is required, and each child can have one. There is a nice virtual manipulative of the ten-frame with four associated games that develop counting and addition skills at http://illuminations.nctm.org/activitydetail.aspx?id=75.

For children in pre-K, kindergarten, or early first grade who have not yet explored a ten-frame, it is a good idea to begin with a five-frame. (See a virtual five-frame at NCTM's Illuminations website: http://illuminations.nctm.org/ActivityDetail.aspx?ID=74.) This row of five sections can be drawn on a sheet of paper (or Blackline Master 9 can be used). Provide children with about 10 counters that will fit in the five-frame sections and conduct the following activity.

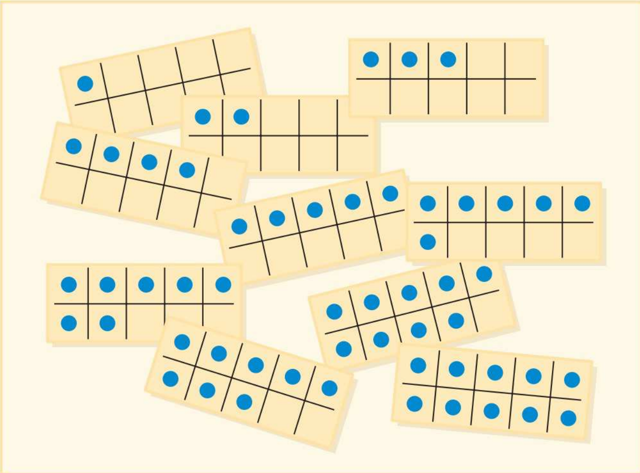


FIGURE 8.10 Ten-frames.

Activity 8.14

Five-Frame Tell-About

Explain that only one counter is permitted in each section of the five-frame. No other counters are allowed on the five-frame mat. Have children show 3 on their five-frame, as seen in Figure 8.11(a). "What can you tell us about 3 from looking at your mat?" After hearing from several children, try other numbers from 0 to 5. Children may place their counters on the five-frame in any manner. For example, with four counters, a child with two on each end may say, "It has a space in the middle" or "It's two and two." Accept all correct answers. Focus attention on how many more counters are needed to make 5 or how far away from 5 a number is. Next try numbers between 5 and 10. As shown in Figure 8.11(b), numbers greater than 5 are shown with a full five-frame and additional counters on the mat but not in the frame. In discussion, focus attention on these larger numbers as 5 and some more: "Seven is five and two more."

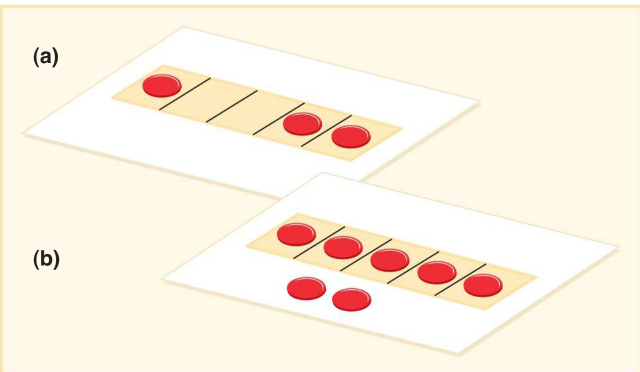


FIGURE 8.11 A five-frame focuses on the 5 anchor. Counters are placed one to a section, and students describe how they see their number in the frame.

Notice that the five-frame focuses on the relationship to 5 as an anchor for numbers and does not anchor numbers to 10. When five-frames have been used for a week or so, introduce ten-frames. Play a ten-frame version of a "Five-Frame Tell-About," but soon introduce the following rule for showing numbers on the ten-frame: *Always fill the top row first, starting on the left, the same way you read. When the top row is full, counters can be placed in the bottom row, also from the left.* This will produce the "standard" way to show numbers on the ten-frame as in Figure 8.10.

For a while, many children will continue to count every counter on their ten-frame. Some will take all counters off and begin each new number from a blank frame. Others will soon learn to adjust numbers by adding on or taking off only what is required, often capitalizing on a row of five without counting. Do not pressure students. With continued practice, all students will grow. How they are using the ten-frame provides you with insights into students' current number concept development; therefore, ten-frame questions can be used as diagnostic interviews.

Activity 8.15

ENGLISH LANGUAGE LEARNERS

Crazy Mixed-Up Numbers

This activity is adapted from the classic resource *Mathematics Their Way* (Baratta-Lorton, 1976). All children make their ten-frame show the same number. Then the teacher calls out random numbers between 0 and 10. After each number, the children change their ten-frames to show the new number. If working with ELLs, consider saying the number in their native language or writing the number. Children can play this game independently by preparing lists of about 15 "crazy mixed-up numbers." One child plays "teacher" and the rest use the ten-frames.

"Crazy Mixed-Up Numbers" is much more of a problem-solving situation than it may first appear. How do you decide how to change your ten-frame? Some children clear off the entire frame and start over with each new number. Others have learned what each number looks like. To add another dimension, have the children tell, *before changing their ten-frames*, how many more counters need to be added or removed. If, for example, the frames showed 6, and the teacher called out "4," the children would respond, "Subtract!" and then change their ten-frames accordingly. A discussion of how they know what to do is valuable.

Ten-frame flash cards are an important variation of ten-frames and can be made from cardstock (see Blackline Masters 15–16). A set of 20 cards consists of a 0 card, a 10 card, and two each of the numbers 1 to 9. The cards allow for simple practice that reinforces the 5 and 10 anchors as in the following activity.

Activity 8.16

ENGLISH LANGUAGE LEARNERS

Ten-Frame Flash

Flash ten-frame cards to the class or a small group and see how quickly the children can tell how many dots are shown. This activity is fast-paced, takes only a few minutes, can be done at any time, and is a lot of fun. For ELLs, coming up with the English word for the number may take more time, so either pair students with similar language skills, or encourage students to use their preferred language in playing the game.

Important variations of "Ten-Frame Flash" include:

- Saying the number of spaces on the card instead of the number of dots
- Saying one more than the number of dots (or two more, one less, or two less)
- Saying the "10 fact"—for example, "Six and four make ten"
- Adding the flashed card to a card they have at their desk (for challenging advanced learners)

Ten-frame tasks are surprisingly challenging for some students, as there is a lot to keep in their working memory. Students must reflect on the two rows of five, the spaces remaining, and how a particular number is more or less than 5 and how far away from 10. How well students can respond to "Ten-Frame Flash" is a good quick diagnostic assessment of their current number concept level. Consider interviews that include the variations of the activity listed above. Because the distance to 10 is so important, another assessment is to point to a numeral less than 10 and ask, "If this many dots were on a ten-frame, how many blank spaces would there be?" Or you can also simply ask, "If I have seven, how many more do I need to make ten?"

Part-Part-Whole Relationships

PAUSE and REFLECT

Before reading on, gather eight counters. Count out the set of counters in front of you as if you were a kindergartner.

Any child who has learned how to count meaningfully can count out eight objects as you just did. What is significant about the experience is what it did *not* cause you to think about. Nothing in counting a set of eight objects will cause a child to focus on the fact that it could be made of two parts. For example, separate the counters you just set out into two piles and reflect on the combination. It might be 2 and 6, 7 and 1, or 4 and 4. Make a change in your two piles of counters and say the new combination to yourself. Focusing on a quantity in terms of its parts has important implications for developing number sense. A noted researcher in children's number concepts, Resnick (1983), stated that a major conceptual achievement of young learners is the interpretation of numbers in terms of part and whole relationships.

Basic Ingredients of Part-Part-Whole Activities. Most part-part-whole activities focus on a single number for the entire activity. For example, a pair of children might work on breaking apart or building the number 7 throughout the activity. They can either build (compose) the designated quantity in two or more parts, or else they start with the full amount and separate it into two or more parts (decompose). Kindergarten children will usually begin these activities working on the number 4 or 5. As concepts develop, children can extend to numbers 6 to 12. A wide variety of materials and formats for these activities can help maintain student interest.

When children do these activities, have them say or "read" the parts aloud or write them down on some form of recording sheet (or do both). Reading or writing the combinations serves as a means of encouraging reflective thought focused on the part–whole relationship. Writing can be in the form of drawings, numbers written in blanks (a group of _____ cubes and a group of _____ cubes), or addition equations if these have been introduced ($3 + 5 = 8$ or $8 = 2 + 6$). There is a clear connection between part-part-whole concepts and addition and subtraction ideas.

Part-Part-Whole Activities. The following activity and its variations may be considered the "basic" part-part-whole task.

Activity 8.17

Build It in Parts

Provide children with one type of material, such as connecting cubes or squares of colored paper. The task is to see how many different combinations for a particular number they can make using two parts. (If you wish, you can allow for more than two parts.) Use a context that will be familiar to your students, or consider a piece of children's literature. For example, ask how many different combinations of six hats the peddler in the book *Caps for Sale* (Slobodkina, 1938) can wear, limiting the color choices to two to start. (Note that the book is also available in Spanish for some ELLs.) Each different combination can be displayed on a small mat. Here are just a few ideas, each of which is illustrated in Figure 8.12.

- **Use two colors of counters such as lima beans spray painted on one side (also available in plastic).**
- **Make bars of connecting cubes of two different colors. Keep the colors together.**
- **Make combinations using two dot strips—strips of cardstock about 1 inch wide with stick-on dots.**
- **Make combinations of two Cuisenaire rods connected as a train to match a given amount.**

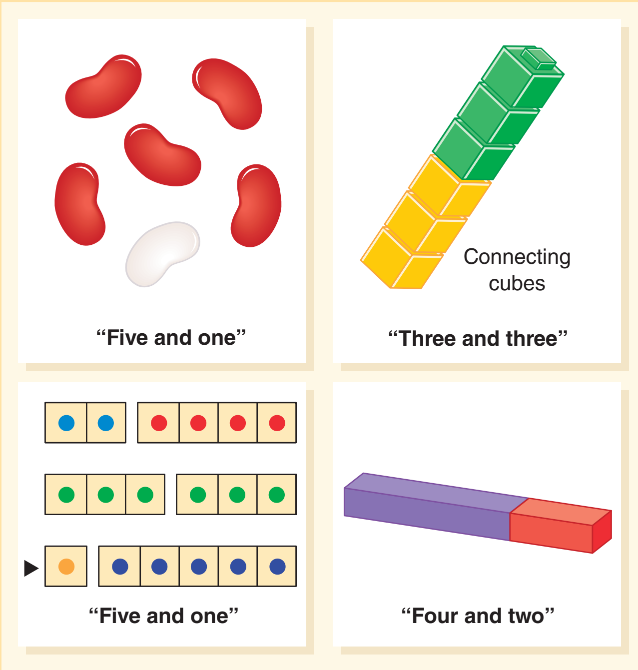


FIGURE 8.12 Assorted materials for building parts of 6.

As you observe students working on the "Build It in Parts" activity, ask them to "read" a number sentence to go with each of their combinations. Encourage students to read their number sentences to each other. Two or three students working together may have quite a large number of combinations, including repeats. Remember, the students are focusing on the combinations.

The following activity is a step toward a more abstract understanding of combinations that make 5 (or other totals). Students can do these mentally or use counters. Allowing options is both a good instructional strategy and a way to see how ready students are for addition.

Activity 8.18

Two Out of Three

Make lists of three numbers, two of which total the whole that students are focusing on. Here is an example list for the number 5:

2–3–4

5–0–2

1–3–2

3–1–4

2–2–3

4–3–1

With the list on the board, students can take turns selecting the two numbers that make the whole. As with all problem-solving activities, students should be challenged to justify their answers.

Missing-Part Activities. An important variation of part-part-whole activities is referred to as *missing-part* activities. In a missing-part activity, students know the whole amount and use their already developed knowledge of the parts of that whole to tell what the covered or hidden part is. If they are unsure, they simply uncover the unknown part and say the full combination. Missing-part activities provide maximum reflection on the combinations for a number. They also serve as the forerunner to subtraction concepts. With a whole amount of 8 but with only 3 showing, the student can later learn to write "8 − 3 = 5."

Missing-part activities require some way for a part to be hidden or unknown. Usually this is done with two students working together or else in a teacher-guided whole-class lesson using a single designated quantity as the whole. The next three activities illustrate variations of this important idea. For any of these activities, you can use a context from familiar classroom events or from a children's book, such as animals hiding in the barn in *Hide and Seek* (Stoeke, 1999).

Activity 8.19

Covered Parts

A set of counters equal to the target amount is counted out, and the rest are put aside. One student places the counters under a margarine tub or piece of cardstock. The student then pulls some out into view. (This amount could be none, all, or any amount in between.) For example, if 6 is the whole and 4 are showing, the other student says, "Four and *two* is six." If there is hesitation or if the hidden part is unknown, the hidden part is immediately shown (see Figure 8.13).

Activity 8.20

Missing-Part Cards

For each number from 4 to 10, make missing-part cards on strips of 3-by-9-inch cardstock. Each card has a numeral for the whole and two dot sets with one set covered by a flap. For the number 8, you need nine cards with the visible part ranging from zero to eight dots. Students use the cards as in "Covered Parts," saying, "Four and two is six" for a card showing four dots and hiding two (see Figure 8.13).

Activity 8.21

STUDENTS *with* SPECIAL NEEDS

I Wish I Had

Hold out a bar of connecting cubes, a dot strip, or a dot plate showing 6 or less. Say, "I wish I had six." The children respond with the part that is needed to make 6. Counting on can be used to check. The game can

focus on a single number (especially as a starting point for students with disabilities), or the "I wish I had" number can change each time (see Figure 8.13). Consider adding a familiar context, like "I wish I had six books to read."

TECHNOLOGY

There are lots of ways you can use computer software to create part-part-whole activities. All that is needed is a program that permits students to create sets of objects on the screen. Scott Foresman's *eTools* (published by Pearson Education) is available free at www.kyrene.org/mathtools. Choose "Counters" and under "workspaces" on the bottom left, select the bucket icon. Then select the bathtub and add boat, duck, or goldfish counters. As shown in Figure 8.14, children can stamp different types of bathtub toys either in the tub (unseen) or outside the tub. The numeral on the tub shows how many are in the tub, or it can show a question mark (?) for missing-part thinking. The total is shown at the bottom. By clicking on the lightbulb, the contents of the tub can be seen, as shown in Figure 8.14(b). This program offers a great deal of diversity and challenge for both part-part-whole and missing-part activities.

(a)

(b)

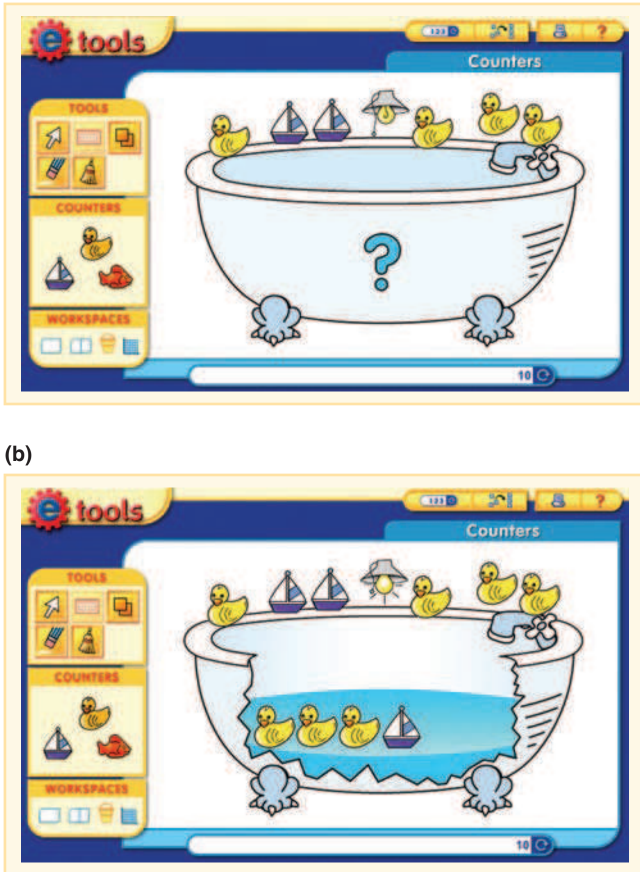


FIGURE 8.14 Scott Foresman's *eTools* software is useful for exploring part-part-whole and missing-part ideas.

Source: Scott Foresman Addison-Wesley Math Electronic-Tools CD-ROM Grade K Through 6.

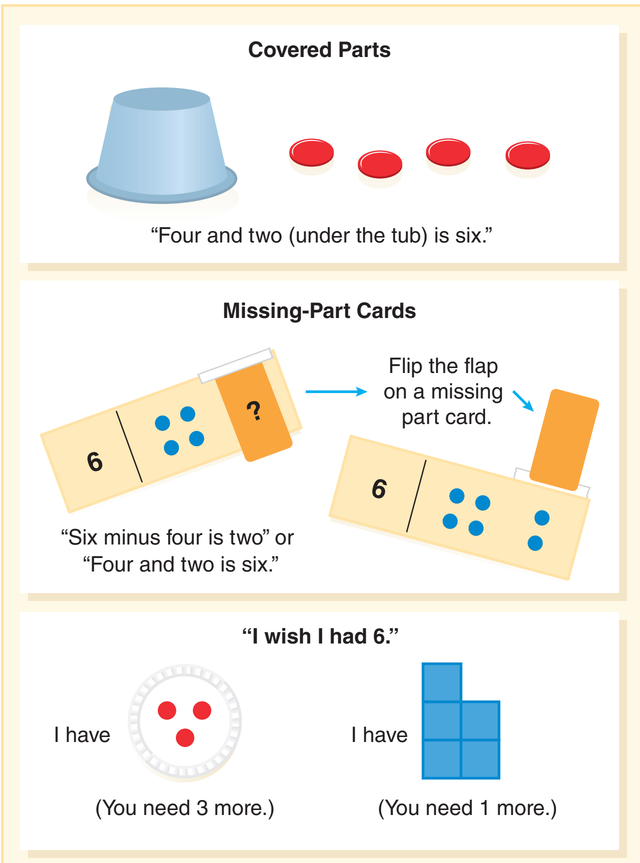


FIGURE 8.13 Missing-part activities.

PAUSE and REFLECT

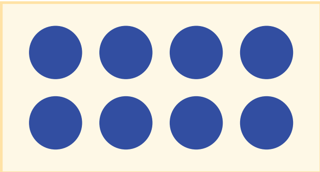
Remember the list you made earlier in the chapter about what students should know about the number 8? Let's refer to it and see if you would add to it or revise it based on what you have read to this point. Do this before reading on.

Here is a possible list of the kinds of things that students should be learning about the number 8 (or any number up to about 12) while they are in pre-K (NCTM, 2006).

- Count to 8 (know the number words and their order)
- Count 8 objects and know that the last number word tells how many
- Write the numeral 8
- Recognize and read the numeral 8

The preceding list represents the minimal skills of number. In the following list are the relationships students should have that contribute to number sense:

- More and less by 1 and 2—8 is one more than 7, one less than 9, two more than 6, and two less than 10
- Patterned sets for 8 such as



- Anchors to 5 and 10: 8 is 3 more than 5 and 2 away from 10
- Part–whole relationships: 8 is 5 and 3, 2 and 6, 7 and 1, and so on (This includes knowing the missing part of 8 when some are hidden.)
- Doubles: double 4 is 8
- Relationships to the real world: my brother is 8 years old; my reading book is 8 inches wide

Dot Cards as a Model for Teaching Number Relationships

We have already seen how dot cards are valuable in developing the Number Core and early explorations in the Relations Core. Here we combine more than one of the relationships discussed so far into several number development activities by using the complete set of cards. As students learn about ten-frames, patterned sets, and other relationships, the dot cards in Blackline Masters 3–8 provide a wealth of activities (see Figure 8.15). The full set of cards contains dot patterns, patterns that require counting, combinations of two and three simple patterns, and ten-frames with "standard" as well as unusual dot placements. When children use these cards for any activity that involves number concepts, the cards help them think flexibly about numbers. The dot cards add another dimension to many of the activities already described and can be used effectively in the following activities.

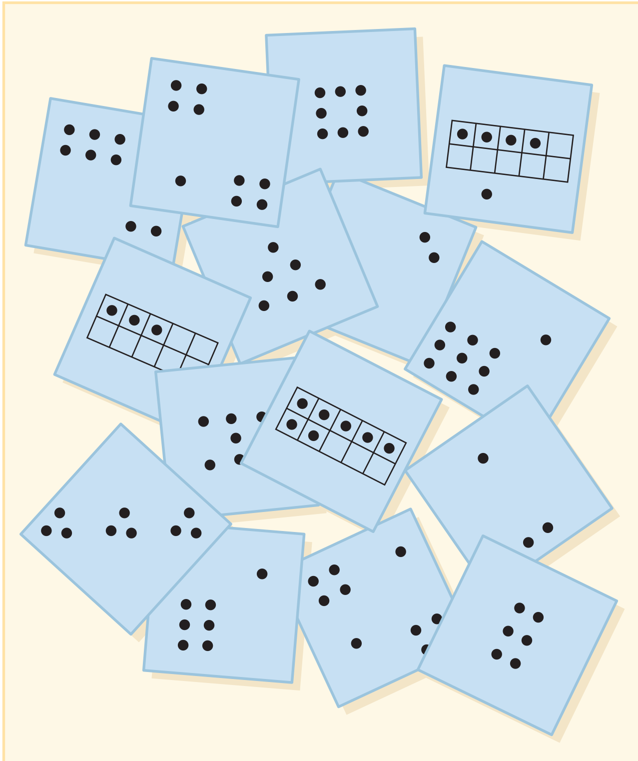


FIGURE 8.15 Dot cards can be made using Blackline Masters 3–8.

Activity 8.22

STUDENTS *with* SPECIAL NEEDS

Double War

The game of "Double War" (Kamii, 1985) is played like the War card game, but on each play, both players turn up two dot cards instead of one. The winner is the player with the larger total number. Students playing the game can and should use many different number relationships to determine the winner without actually finding the total number of dots. A modification of this activity for students with disabilities would have the teacher (or another student) do a "think-aloud" and describe her thinking about the dots using relationships as she figures who wins the round. This modeling is critical for students who struggle.

Activity 8.23

Difference War

Deal out the dot cards to the two players as in regular War, and prepare a pile of about 40 counters. On each play, the players turn over one card from the top of the stack. The player with the greater number of dots wins as many counters from the pile as the difference between the two cards. Used cards are put aside. The game is over when the cards or counters run out. The player with the most counters wins the game. This game can also be played so the person with "less" wins the number of counters in the difference.

Activity 8.24

Number Sandwiches

Select a number between 5 and 12, and have students find combinations of two cards that total that number. They place the two cards back to back with the dot side out. When they have found at least 10 pairs, the next challenge is for the partner to name the number on the other side. The cards are flipped over to confirm. The same pairs can then be used again to name the other hidden part.

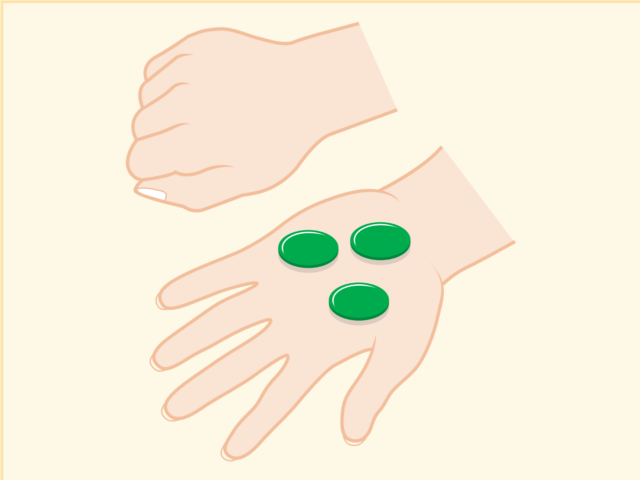


FIGURE 8.16 A missing-part number assessment. "There are eight in all. How many are hidden?"

FORMATIVE Assessment Notes

To assess the important part–whole relationships, use a missing-part **diagnostic interview** (similar to Activity 8.19). Begin with a number you believe the student has "mastered," say, 5. Have the student count out that many counters into your open hand. Close your hand around the counters and confirm that the student knows how many are hidden there. Then remove some and show them in the open palm of your other hand (see Figure 8.16). Ask the student, "How many are hidden?" "How do you know?" Repeat with different amounts removed, trying three or four missing parts for each number. If the student responds quickly and correctly and is clearly not counting in any way, call that a "mastered number" and check it off on your student's assessment record. Then repeat the entire process with the next higher number. Continue until the student begins to struggle. In early kindergarten, you will find a range of mastered numbers from 4 to 8. By the end of kindergarten, students should master numbers through 10 (CCSSO, 2010). ■

Relationships for Numbers 10 Through 20

Even though pre-K, kindergarten, and first-grade students experience numbers up to 20 and beyond daily, it should not be assumed that they will automatically extend the set of relationships they developed on smaller numbers to numbers beyond 10. And yet these numbers play a big part in many simple counting activities, in basic facts, and in much of what we do with mental computation. Relationships with these numbers are just as important as relationships involving the numbers through 10.

Pre-Place-Value Concepts

A set of ten should play a major role in students' early understanding of numbers between 10 and 20. When children see a set of six together with a set of ten, they should know without counting that the total is 16. However, the numbers between 10 and 20 are not an appropriate place to discuss place-value concepts. That is, prior to a much more complete development of place value, students should not be expected to explain the 1 in 16 as representing "one ten." Yet, this work with composing and decomposing numbers from 11 through 19 in kindergarten is seen as an essential foundation for place value (CCSSO, 2010).

PAUSE and REFLECT

Say to yourself, "One ten." Now think about that from the perspective of a child just learning to count to 20! What could "one ten" possibly mean when ten tells me how many fingers I have and is the number that comes after nine? How can it be one of something? ●

Initially, children do not see a numeric pattern in the numbers between 10 and 20. Rather, these number names are simply ten additional words in the number sequence. In some languages, the teens are actually stated as 10 and 1, 10 and 2, 10 and 3. But since this is not the case in English, for many students, the teens provide a significant challenge.

The concept of a unit of ten is challenging for a kindergarten or early first-grade child to grasp. Although some researchers feel it is developmentally challenging (Kamii, 1985), the *Common Core State Standards* suggests that first graders should know that "10 can be thought of as a bundle of ten ones—called a 'ten'" (p. 15). The difficulty in students discussing "one ten and six ones" (what's a one?) does not mean that a set of ten should not figure prominently in the discussion of the teen numbers. The following activity illustrates this idea.

Activity 8.25

Ten and Some More

Use a simple two-part mat and a story that links to whatever counters you are using. You may want to use coffee stirrers as "sticks" with the story *Not a Stick* (Portis, 2007). Then have students count out ten sticks onto the left side of the mat. Next, have them put five sticks on the other side. Together, count all of the sticks by ones. Chorus the combination: "Ten and five is fifteen." Turn the mat around: "Five and ten is fifteen." Repeat with other numbers (9 or less) in a random order, but always keep 10 on the left side. After playing the game for a while, bundle the 10 sticks with a rubber band.

Activity 8.25 is designed to teach the often challenging number names in the "teens" and thus requires teacher modeling. Following this activity, explore numbers through

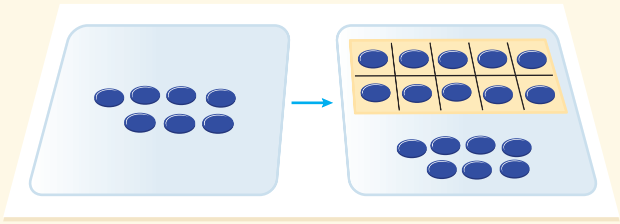


FIGURE 8.17 Extending relationships to the teens.

20 in a more open-ended manner. Provide each child with two ten-frames drawn one under the other on cardstock (see Blackline Master 11). In random order, have students show numbers up to 20 on the frames. Have students discuss how counters can be arranged on the mat so that it is easy to see how many are there. Not every student will use a full set of 10, but as this idea becomes more popular, they will develop the notion that 10 and some more is a teen amount. Then challenge students to find ways to show 26 counters or even more.

Extending More Than and Less Than Relationships

The relationships of one more than, two more than, one less than, and two less than are important for all numbers. However, these ideas are built on or connected to the same concepts for numbers less than 10. The fact that 17 is one less than 18 is connected to the idea that 7 is one less than 8. Students may need help in making this connection.

Activity 8.26

More and Less Extended

On the board, show seven counters and ask what is two more, or one less, and so on. Now add a filled ten-frame to the display (or 10 in any pattern) and repeat the questions. Pair up questions by covering and uncovering the ten-frame as in Figure 8.17.

Number Sense in Their World

Here we examine ways to broaden early knowledge of numbers. Relationships of numbers to real-world quantities and measures and the use of numbers in simple estimations can help students develop flexible, intuitive ideas about numbers. Here are some activities that can help students connect numbers to real situations.

Activity 8.27

Add a Unit to Your Number

Write a number on the board. Now suggest some units to go with it and ask the students what they can think of that fits. For example, suppose the number is 9. "What do you think of when I say nine *dollars*? Nine *hours*? Nine *cars*? Nine *kids*? Nine *meters*? Nine *o'clock*? Nine *hand spans*? Nine *gallons*?" Spend time discussing and exploring each. Let students suggest other appropriate units. Students from different cultures may bring different ideas to this activity, and including these ideas is a way to bring their culture into their school experience.

Activity 8.28

Is It Reasonable?

Select a number and a unit—for example, 15 feet. Could the teacher be 15 feet tall? Could a house be 15 feet wide? Can a man jump 15 feet high? Could three children stretch their arms 15 feet? Pick any number, large or small, and a unit with which students are familiar. Then make up a series of these questions. Also ask, "How can we find out if it is reasonable or not? Who has an idea about what we can do?" Then have the students select the number and unit.

These activities are problem based in the truest sense. Not only are there no clear answers, but students can easily begin to pose their own questions and explore the numbers and units most interesting to them.

Calendar Activities

The National Research Council (2009) has stated that "using the calendar does not emphasize foundational mathematics" (p. 241). They go on to remind early childhood teachers that although the calendar may be helpful in developing a sense of time, it does not align with the need to develop mathematical relationships related to the number 10 because the calendar is based on groups of seven. Although 90 percent of the classrooms surveyed reported using calendar-related activities (Hamre, Downer, Kilday, & McGuire, 2008), there are significant issues with this work being considered the kind of mathematics instruction that will support young learners in reaching mathematical literacy. They conclude, "Doing the calendar is not a substitute for teaching foundational mathematics" (p. 241). The key message is that doing calendar math should be thought of as an "add on" and not take time away from essential pre-K–2 math concepts. If you wish to keep track of the days of school, post and fill ten-frames.

Estimation and Measurement

One of the best ways for students to think of real quantities is to associate numbers with measures of things. Measures of length, weight, and time are good places to begin. Just measuring and recording results will not be very effective unless there is a reason for students to be interested in or think about the result. To help students think about what number might tell how long the desk is or how heavy the book is, it important if they could first write down or tell you an estimate. To produce an estimate is, however, a very difficult task for young children. They do not easily grasp the concept of "estimate" or "about." For example, suppose that you have cut out a set of very large footprints, each about 18 inches long. You would ask, "About how many footprints will it take to measure across the rug in our reading corner?" The key word here is *about*, and it is one that you will need to spend a lot of time helping your students understand. To this end, the request of an estimate can be made in ways that help with the concept of "about."

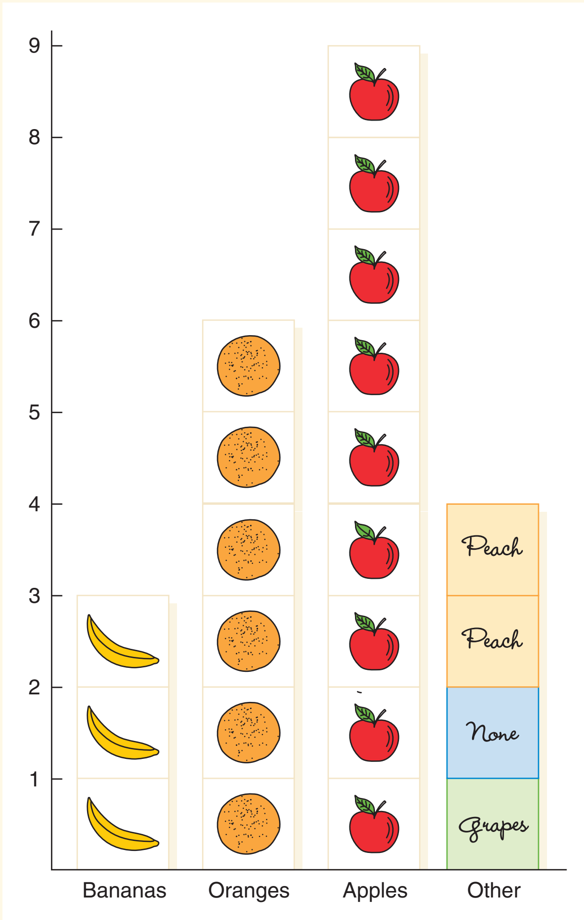
The following questions can be used with early estimation activities:

- *More or less than* _____? Will it be more or less than 10 footprints? Will the apple weigh more or less than 20 blocks? Are there more or less than 15 connecting cubes in this long bar?
- *Closer to* _____ *or to* _____? Will it be closer to 5 footprints or closer to 20 footprints? Will the apple weigh closer to 10 blocks or closer to 30 blocks? Does this bar have closer to 10 cubes or closer to 50 cubes?
- *About* ________? About how many footprints? About how many blocks will the apple weigh? About how many cubes are in this bar?

Asking for estimates using these formats helps students learn what you mean by "about." Every student can make a close estimate with some supportive questions and examples. However, rewarding students for the closest estimate in a competitive fashion will often result in their trying to seek precision and not actually estimate. Instead, discuss all answers that fall into a reasonable range. One of the best approaches is to give students ranges as their possible answers: Does your estimate fall between 10 and 30? Between 50 and 70? Or 100 and 130? Of course, you can make the choices more divergent until they grasp the idea.

Data Collection and Analysis

Graphing activities are good ways to connect students' worlds with number and relationships. Graphs can be quickly made from any student data, such as favorites (ice cream, sports team, pet), number of sisters and brothers, and transportation to school. Graphs can be connected to science, such as an investigation of objects that float or sink. Students can generate ideas for what data to gather.



Class graph showing fruit brought for snack. Paper cutouts for bananas, oranges, apples, and cards for "others."

- Which snack (or refer to what the graph represents) is most, least?
- Which are more (less) than 7 (or some other number)?
- Which is one less (more) than this snack (or use fruit name)?
- How much more is ______ than ______ ? (Follow this question immediately by reversing the order and asking how much less.)
- How much less is ______ than ______ ? (Reverse this question after receiving an answer.)
- How much difference is there between ______ and ______ ?
- Which two bars together are the same as ______ ?

FIGURE 8.18 Relationships and number sense in a bar graph.

Once a simple bar graph is made, it is very important to take time to ask questions (e.g., "What do you notice about our class and our ice cream choices?"). In the early stages of number development, the use of graphs is primarily for developing number relationships and for connecting numbers to real quantities in the students' environment. The graphs focus attention on tallies and counts of realistic things. Equally important, bar graphs clearly exhibit comparisons between numbers that are rarely made when only one number or quantity is considered at a time. See Figure 8.18 (p. 145) for an example of a graph and corresponding questions. At first, students may have trouble with questions involving differences, but these comparison concepts add considerably to students' understanding of number.

RESOURCES *for Chapter 8*

LITERATURE CONNECTIONS

Children's literature abounds with wonderful counting books and visually stimulating number-related books. Have children talk about the mathematics in the story. Begin by talking about the book's birthday (copyright date) and how old the book is. Here are a few ideas for making literature connections to concepts of number.

Ten Little Hot Dogs *Himmelman, 2010*

Here is an example of one of many predictable-progression counting books; this one highlights 10 dachshund puppies climbing on a chair. Children can create their own stories using a mat illustrated with a chair and move counters representing the puppies on or off. Two students can compare the numbers of dogs on their chairs. Who has more puppies? How many more? What combinations for each number are there?

Two Ways to Count to Ten *Dee, 1988*

This folktale is about King Leopard's search for the best animal to marry his daughter. The task devised involves throwing a spear and counting to 10 before the spear lands. Many animals try and fail as counting by ones proves too lengthy. Finally, the antelope succeeds by counting "2, 4, 6, 8, 10." The story is a perfect lead-in to skip counting. Can you count to 10 by threes? How else can you count to 10? How many ways can you count to 48? What numbers can you reach if you count by fives? A hundreds board or counters are useful in helping with these problems.

RECOMMENDED READINGS

Articles

Griffin, S. (2003). Laying the foundation for computational fluency in early childhood. *Teaching Children Mathematics*, *9*(6), 306–309.

This useful article for assessment lays out five stages of number development with a simple addition story problem task followed by activities to develop number for each stage.

Losq, C. (2005). Number concepts and special needs students: The power of ten-frame tiles. *Teaching Children Mathematics*, *11*(6), 310–315.

This article supports struggling learners in the use of a countable and visual model—the ten-frame tile. The ten-frames are positioned vertically to enhance subitizing and provide tools for formative assessment.

Moomaw, S., Carr, V., Boat, M., & Barnett, D. (2010). Preschoolers' number sense. *Teaching Children Mathematics*, *16*(6), 333–340.

How can you best assess young learners? This article offers curriculum-based assessments that can capture number sense concepts through game-like activities.

Books

Dougherty, B., Flores, A., Louis, E., & Sophian, C. (2010). *Developing essential understanding of number and numeration for teaching mathematics in prekindergarten–grade 2.* Reston, VA: NCTM.

This book describes what big mathematical ideas a teacher needs to know about number, how number connects to other mathematical ideas, and how to teach and assess this pivotal topic.

Richardson, K. (2003). *Assessing math concepts: The hiding assessment.* Bellingham, WA: Mathematical Perspectives.

This is one of a series of nine assessment books with diagnostic interviews covering number topics from counting through two-digit numbers. Extensive explanations and examples are provided.

ONLINE RESOURCES

Let's Count to 5 (Grades K–2)
http://illuminations.nctm.org/LessonDetail.aspx?id=U57

Here are seven lessons where children make sets of zero through five objects and connect number words or numerals to the sets. Songs, rhymes, and activities that appeal to visual, auditory, and kinesthetic learners are included. See other Illuminations sites for counting to 10 and 20.

Toy Shop Numbers (Grades K–2)
http://illuminations.nctm.org/LessonDetail.aspx?id=L216

Using the setting of a toy shop, these activities focus on finding numbers in the real world.