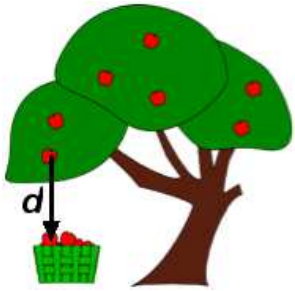
Lesson 23

Negatives in Formulas and Expressions

We've used the formula for the distance between two points on the number line with negative numbers. In this section we explore further formulas using negatives, and substituting negative numbers for variables in expressions. It also provides a chance to practice arithmetic operations with the negatives to help solidify the procedures in your mind.

Falling Objects

The gravity of the earth makes things fall. We can observe that they do not fall at a constant speed, but go faster and faster until they hit the ground. The speed increases as the object falls, and an object dropped from a great height is going faster when it hits the ground than one dropped from lower down. Since the direction of the motion is down, the distance fallen can be expressed as a negative number, which gives both the distance and the direction of the motion.



If ***t*** is the time (in seconds) an object has been fallling, then the distance ***d*** (in feet) that it has fallen is given by the formula

$$d = -16t^2$$

Notice that the formula only applies while the object is falling, before it has hit the ground.

Example: **Find the distance an object has fallen after 1 second, 2 seconds, and 3 seconds.**

When substituting a number for the variable ***t*** , put it in parentheses. Note in the order of operations that it is the number ***t*** that is squared, so be sure to square before multiplying.

$d = -16t^2$	$t = 1$	$d = -16(1)^2 = -16(1) = -16$	**The object fell 16 feet.**
	$t = 2$	$d = -16(2)^2 = -16(4) = -64$	**The object fell 64 feet.**
	$t = 3$	$d = -16(3)^2 = -16(9) = -144$	**The object fell 144 feet.**

It can seem a little foolish to include the negative in the formula. We already know the direction, so if we didn't include it we would still know that the object fell – no one would think that it floated upward! But the expression for the distance the object fell under the influence of gravity can be combined with other expressions to give the height of an object at any time, and then the negative indicating direction is significant.

Note that the new formula in the next problem is not giving the distance the object has traveled. Instead it is telling the height of the object above (or below?) the ground.

Example: **A tennis ball is shot from the ground straight up at a velocity of 80 feet per second. The height of the ball at any time *t* is given by the formula** $h = -16t^2 + 80t$.

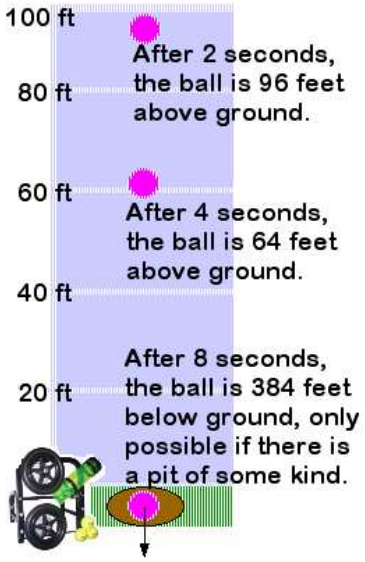
Find the height of the ball after 2 seconds, 4 seconds, and 8 seconds.

Think about the situation and take a look at the formula before beginning. When we shoot a tennis ball into the air, it goes up for awhile, reaches a high point, and begins to fall. The formula has a positive term, **80t**, for the distance traveled up, and a negative term **–16t²**, for the distance traveled down. The height at any time *t* is the result of the two different impulses of the ball, one **up** from the force of the shot, and one **down**, from gravity.

Example: A tennis ball, continued.

$h = -16t^2 + 80t$

$t = 2$	$h = -16(2)^2 + 80(2)$ $= -16(4) + 80(2)$ $= -64 + 160 = 96$	The tennis ball is 96 feet above the ground.
$t = 4$	$h = -16(4)^2 + 80(4)$ $= -16(16) + 80(4)$ $= -256 + 320 = 64$	The tennis ball is 64 feet above the ground.
$t = 8$	$h = -16(8)^2 + 80(8)$ $= -16(64) + 80(8)$ $= -1024 + 640 = -384$	The tennis ball is 384 feet below the ground. ?



Notice that the negative value for height in the example **t = 8** means that the ball is below ground. Once the ball hits the ground, the formula no longer applies, so unless the ball is falling into a hole in the ground, the last value is probably not applicable.

Fahrenheit and Celsius

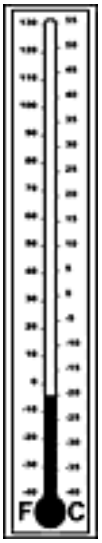
We've used the formula to convert Celsius temperatures to Fahrenheit temperatures for quite a while, but now we can handle those frigid temperatures below zero.

Example: Convert the temperatures 0°C, –10°C and –20°C to their Fahrenheit equivalents, using the formula

$$F = \frac{9C}{5} + 32 .$$

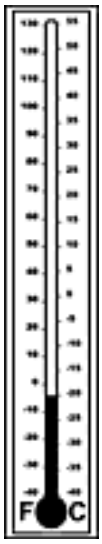
It's usually helpful to put parentheses around a number you're substituting in a formula, but it's really important when substituting negative numbers. Otherwise the negative sign might be mistaken for a subtraction.

$C = 0$	$F = \frac{9(0)}{5} + 32 = \frac{0}{5} + 32$ $= 0 + 32 = 32$	0°C = 32°F
$C = -10$	$F = \frac{9(-\cancel{10}^{\,2})}{\cancel{5}} + 32$ $= -18 + 32 = 14$	–10°C = 14°F
$C = -20$	$F = \frac{9(-\cancel{20}^{\,4})}{\cancel{5}} + 32$ $= -36 + 32 = -4$	–20°C = –4°F



Example: Convert the temperatures 5ºF, –4ºF and –13 ºF to their Celsius equivalents, using the formula

$$C = \frac{5(T-32)}{9}.$$



$F = 5$	$C = \frac{5(5-32)}{9} = \frac{5(-27)}{9} = -15$	5ºF = –15ºC
$F = -4$	$C = \frac{5((-4)-32)}{9} = \frac{5(-36)}{9} = -20$	–4ºF = –20ºC
$F = -13$	$C = \frac{5((-13)-32)}{9} = \frac{5(-45)}{9} = -25$	–13ºF = –25ºC

Did you notice that the formula converted –4ºF to –20ºC? That's just as we would expect from the previous problem, where we converted –20ºC to –4ºF.

Profit and Loss

Businesses sometimes experience losses, expressed with negative numbers. If the money going out, **Cost,** is greater than the money coming in, **Revenue**, the business is losing money. If we write the basic formula

Profit = Revenue – Cost

P = R – C

the "profit" will actually be a loss when the costs exceed the revenue.

Example: A business has costs of $45,900 per month. Find the profit each month with the revenues given.

DEC $R = \$87{,}200$	$P = 87200 - 45900 = 41300$	The business made $41,300 profit in December.
JAN $R = \$47{,}200$	$P = 47200 - 45900 = 1300$	The business made $1,300 profit in January.
FEB $R = \$23{,}700$	$P = 23700 - 45900 = -22200$	The business had losses of $22,200 in February.

A positive result is a profit and a negative result is a loss.

Substitution in Arbitrary Expressions
In these examples we simply practice algebra and arithmetic skills. You must substitute the given number for the correct variable in the equation, use the correct order of operations, and evaluate to find the answer.

Example: Evaluate the expression $x^2 - 25$ for the given values of x.

	$x^2 - 25$
$x = -5$	$(-5)^2 - 25 = 25 - 25 = 0$
$x = -2$	$(-2)^2 - 25 = 4 - 25 = -21$
$x = 0$	$(0)^2 - 25 = 0 - 25 = -25$
$x = 3$	$(3)^2 - 25 = 9 - 25 = -16$
$x = 6$	$(6)^2 - 25 = 36 - 25 = 11$

Example: Evaluate the expression $9 - x$ for the given values of x.

	$9 - x$
$x = 11$	$9 - (11) = 9 + -11 = -2$
$x = 9$	$9 - (9) = 0$
$x = 0$	$9 - (0) = 9$
$x = -3$	$9 - (-3) = 9 + 3 = 12$
$x = -6$	$9 - (-6) = 9 + 6 = 15$

Example: Compare the results when evaluating the two expressions $2x - 3x$ and $-x$.
(Remember that $-x$ is shorthand for $-1x$.)

	$2x - 3x$	$-x$
$x = 5$	$2(5) - 3(5) = 10 - 15 = -5$	$-(5) = -5$
$x = 3$	$2(3) - 3(3) = 6 - 9 = -3$	$-(3) = -3$
$x = 0$	$2(0) - 3(0) = 0 - 0 = 0$	$-(0) = -1(0) = 0$
$x = -2$	$2(-2) - 3(-2) = -4 - (-6) = -4 + 6 = 2$	$-(-2) = -1(-2) = 2$
$x = -7$	$2(-7) - 3(-7) = -14 - (-21) = -14 + 21 = 7$	$-(-7) = -1(-7) = 7$

The two expressions are equal, and the results are equal no matter what values are substituted for x.

Example: Compare the results when evaluating the two expressions

$$\frac{x}{x^2} \text{ and } \frac{1}{x}.$$

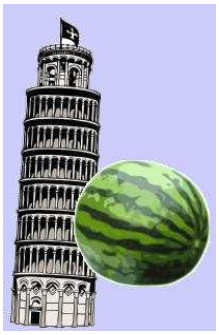
	$\frac{x}{x^2}$	$\frac{1}{x}$
$x = 5$	$\frac{(5)}{(5)^2} = \frac{\cancel{5}}{\cancel{5} \cdot 5} = \frac{1}{5}$	$\frac{1}{5} = \frac{1}{5}$
$x = 3$	$\frac{(3)}{(3)^2} = \frac{\cancel{3}}{\cancel{3} \cdot 3} = \frac{1}{3}$	$\frac{1}{3} = \frac{1}{3}$
$x = 0$	$\frac{(0)}{(0)^2} = \frac{0}{0 \cdot 0}$ undefined	$\frac{1}{0}$ undefined
$x = -2$	$\frac{(-2)}{(-2)^2} = \frac{\cancel{-2}}{\cancel{-2} \cdot -2} = \frac{1}{-2} = -\frac{1}{2}$	$\frac{1}{-2} = -\frac{1}{2}$
$x = -7$	$\frac{(-7)}{(-7)^2} = \frac{\cancel{-7}}{\cancel{-7} \cdot -7} = \frac{1}{-7} = -\frac{1}{7}$	$\frac{1}{-7} = -\frac{1}{7}$

The expressions are equal, and the results of evaluating with any value of x are the same for both expressions.

Lesson 23: Negatives in Formulas and Expressions

Worksheet

Name _____



1. A person on top of a 200 foot building decides to drop a watermelon over the side. The formula for the height of the watermelon t seconds after dropping is

$$h = -16t^2 + 200$$

What is the height of the watermelon after 1 second?

What is the height of the watermelon after 3 seconds?

What is the height of the watermelon after 4 seconds?

What does it mean for the watermelon to have a negative height?

If there is no pit for it to fall into and the watermelon hit the ground, about how long did that take?

2. From ground level a bullet is fired straight up into the air at the rate of 2100 feet per second. If we ignore air resistance, the formula for the height of the bullet t seconds after firing is

$$h = -16t^2 + 2100t$$

What is the height of the bullet 30 seconds after firing?

What is the height of the bullet 60 seconds after firing?

What is the height of the bullet 120 seconds after firing?

Does the bullet seem to be on the way up or on the way down at 30 seconds?

Does the bullet seem to be on the way up or on the way down at 120 seconds?

3. Use the formula to convert the Celsius temperatures to their Fahrenheit equivalents.

$$F = \frac{9C}{5} + 32$$

–15ºC

–100ºC

4. Use the formula to convert the Fahrenheit formulas to their Celsius equivalents.

$$C = \frac{5(F - 32)}{9}$$

–22ºF

–58ºF

5. Use the table of revenue and costs to find the profit each month, using the formula ***P = R – C.***

MONTH	REVENUES	COSTS	PROFIT
JAN	$4,902	$3,677	
FEB	$3,560	$3,648	
MAR	$5,007	$5,298	

If you consider the 3-month period as a whole, has the business made a profit or experienced a loss?

6. Evaluate each expression when $x = -5$.

a. $x + 1$

b. $x^2 + 1$

c. $(x + 1)^2$

d. $x - 1$

e. $1 - x$

f. $1 - x^2$

g. x^3

h. $(-x)^3$

Lesson 23: Negatives in Formulas and Expressions

Homework 23A

Name _____

1. Simplify.

$-6+1$	$-6-1$	$(-6)(-1)$	$\frac{-6}{-1}$
$8-(-2)$	$-2-8$	$\frac{8}{-2}$	$(-2)(8)$
$3-(5)(-6)$	$\frac{\lvert 9-12 \rvert}{3}$	$(5)(-2)(-3)$	$2\lvert 3-6 \rvert$
$(-3)^2-(-3)^3$		$\frac{(-2)(3)-(-6)}{4-5}$	

3. Translate the words to mathematics.

a. The 10th floor basement is lower than the 5th floor basement.

b. It is better to have $20 than to owe $20.

c. The sum of 5 and –2 is 3.

d. 18 members of the negative team met 22 members of the positive team in battle, with the result that the positive team won by 4.

e. The distance between the 14th floor and the 3rd floor basement is 17 floors.

4. Simplify the algebraic expressions.

a. $-7x-8x-9x$

b. $6x-4y-7x+5y$

c. $-2(5x-7)$

d. $2(-3x+1)$

e. $-4(x-2)+2(2x+2)$

4. Solve the equations.

a. $5a-9=11$	b. $3b+b=18+6$	c. $\frac{4c}{5}=6$

6. Solve the problems using the given formulas.

a. A person standing on a balcony 80 feet above ground tosses a water balloon straight up at the rate of 10 feet per second. The height of the ballon ***t*** seconds after it is tossed is given by the formula

$$h=-16t^2+10t+80$$

What is the height of the water balloon after 2 seconds?

b. One branch store is located 18 miles north of the main office, and the other is located 13 miles south of the main office. Find the distance between the stores using the formula

$$d=|a-b|$$

c. Convert –40ºF to Celsius using the formula

$$C=\frac{5(F-32)}{9}$$

7. Evaluate the expressions for the given values of the variables.

a. **7x – 15**

	$7x-15$
$x=7$	
$x=1$	
$x=0$	
$x=-1$	
$x=-3$	

b. **9 – x²**

	$9-x^2$
$x=7$	
$x=1$	
$x=0$	
$x=-1$	
$x=-3$	

Lesson 23: Negatives in Formulas and Expressions

Homework 23A Answers

1. Simplify.

$-6+1 = -5$	$-6-1 = -6 + -1 = -7$	$(-6)(-1) = 6$	$\frac{-6}{-1} = 6$
$8-(-2) = 8 + 2 = 10$	$-2-8 = -2 + -8 = -10$	$\frac{8}{-2} = -4$	$(-2)(8) = -16$
$3-(5)(-6)$ $= 3 - (-30) = 3 + 30$ $= 33$	$\frac{\lvert 9-12 \rvert}{3} = \lvert -3 \rvert / 3$ $= 3/3 = 1$	$(5)(-2)(-3)$ $= 5(6) = 30$	$2\lvert 3-6 \rvert = 2\lvert -3 \rvert$ $= 2 \cdot 3 = 6$
$(-3)^2 - (-3)^3$ $= (-3)(-3) - (-3)(-3)(-3) = 9 - (-27)$ $= 9 + 27 = 36$		$\frac{(-2)(3)-(-6)}{4-5} = \frac{-6-(-6)}{4+-5} = \frac{0}{-1} = 0$	

3. Translate the words to mathematics.

a. The 10th floor basement is lower than the 5th floor basement.

$$-10 < -5$$

b. It is better to have \$20 than to owe \$20.

$$20 > -20$$

c. The sum of 5 and –2 is 3.

$$5 + -2 = 3$$

d. 18 members of the negative team met 22 members of the positive team in battle, with the result that the positive team won by 4.

$$-18 + 22 = 4$$

e. The distance between the 14th floor and the 3rd floor basement is 17 floors.

$$\lvert 14 - (-3) \rvert = 17$$

4. Simplify the algebraic expressions.

a. $-7x - 8x - 9x$

$$= -7x + -8x + -9x$$
$$= -15x + -9x = -24x$$

b. $6x - 4y - 7x + 5y$

$$= 6x + -7x + -4y + 5y$$
$$= -1x + 1y = -x + y$$

c. $-2(5x - 7)$

$$= -10x + 14$$

d. $2(-3x + 1)$

$$= -6x + 2$$

e. $-4(x - 2) + 2(2x + 2)$

$$= -4x + 8 + 4x + 4$$
$$= 0x + 12 = 12$$

4. Solve the equations.

a. $5a-9=11$

$5a-9=11$ $\quad 5a-9+9=11+9$

$5a=20$ $\quad 5a/5=20/5$

$a=4$

b. $3b+b=18+6$

$4b=24$ $\quad 4b/4=24/4$

$b=6$

c. $\frac{4c}{5}=6$

$\frac{4c}{5}=6$ $\quad \frac{4c}{5}\cdot 5=6\cdot 5$

$4c=30$ $\quad 4c/4=30/4$

$c=\frac{30}{4}=\frac{15}{2}$

6. Solve the problems using the given formulas.

a. A person standing on a balcony 80 feet above ground tosses a water balloon straight up at the rate of 10 feet per second. The height of the ballon ***t*** seconds after it is tossed is given by the formula

$$h=-16t^2+10t+80$$

What is the height of the water balloon after 2 seconds?

$h=-16(2)^2+10(2)+80$

$=-16(4)+20+80$

$=-64+100=36$ **feet above ground**

b. One branch store is located 18 miles north of the main office, and the other is located 13 miles south of the main office. Find the distance between the stores using the formula

$$d=|a-b|$$

$d=|18-(-13)|=|18+13|$

$=|31|=31$ **miles between stores**

c. Convert –40°F to Celsius using the formula

$$C=\frac{5(F-32)}{9}$$

$C=\frac{5((-40)-32)}{9}=\frac{5(-72)}{9}=-40$

$-40°F=-40°C$

7. Evaluate the expressions for the given values of the variables.

a. **7x – 15**

	$7x-15$
$x=7$	$7(7)-15=49-15=34$
$x=1$	$7(1)-15=7+-15=-8$
$x=0$	$7(0)-15=0+-15=-15$
$x=-1$	$7(-1)-15=-7+-15=-22$
$x=-3$	$7(-3)-15=-21+-15=-36$

b. **9 – x²**

	$9-x^2$
$x=7$	$9-(7)^2=9-49=-40$
$x=1$	$9-(1)^2=9-1=8$
$x=0$	$9-(0)^2=9-0=9$
$x=-1$	$9-(-1)^2=9-1=8$
$x=-3$	$9-(-3)^2=9-9=0$

Lesson 23: Negatives in Formulas and Expressions

Homework 23B

Name _____

1. Simplify.

$3-8$	$-3-8$	$-3-(-8)$	$(-3)(-8)$
$-7+-6$	$-7-(-6)$	$(7)(-6)$	$\frac{7}{-1}$
$1-(8)(-5)$	$(-2)^3$	$(-1)(-5)(-4)$	$17-(5+2)$
$\frac{\lvert -9-(-1)\rvert}{-4}$		$\frac{-8}{3}\cdot\frac{9}{-4}$	

3. Translate the words to mathematics.

a. A temperature of 0º is warmer than a temperature of –4º.

b. It is worse to owe $100 than it is to owe $90.

c. The product of 5 and –2 is –10.

d. If you have $40 and you owe $60, your net worth is –$20.

e. The distance between the points –12 and –3 on the number line is 9.

4. Simplify the algebraic expressions.

a. $-2x-8x+5x$

b. $3x-9y-2x+7y$

c. $-5(-2x-7)$

d. $-1(6x+7)$

e. $-4(x+2)+2(x+4)$

4. Solve the equations.

a. $2a - 7 = 11$	b. $b + 9b = 70$	c. $\frac{4c}{3} = 6$

6. Solve the problems using the given formulas.

a. A person standing on a balcony 124 feet above ground tosses a water balloon straight up at the rate of 12 feet per second. The height of the ballon ***t*** seconds after it is tossed is given by the formula

$$h = -16t^2 + 12t + 120$$

What is the height of the water balloon after 3 seconds?

b. One branch store is located 10 miles north of the main office, and the other is located 13 miles north of the main office. Find the distance between the stores using the formula

$$d = |a - b|$$

c. Convert –49ºF to Celsius using the formula

$$C = \frac{5(F - 32)}{9}$$

7. Evaluate the expressions for the given values of the variables.

a. **–3x + 15**

	$-3x + 15$
$x = 7$	
$x = 1$	
$x = 0$	
$x = -1$	
$x = -3$	

b. **$8 - x^3$**

	$8 - x^3$
$x = 7$	
$x = 1$	
$x = 0$	
$x = -1$	
$x = -3$	