Lesson 18

Writing Equations for Word Problems

Math on the Street

New York City math teacher George Nobl had a table in Times Square where he gave out candy bars to passers-by who could solve word problems. He did this for many years to promote the fun of mathematics. Read more here.

The dreaded word problem is the scariest part of algebra for many students. The stylized language, unlikely situations, and tricky translations into mathematical symbols can seem like an impossible challenge. But solving the formal word problems in a math text is a helpful step toward actually using mathematics in real life. It turns out that it's not as difficult as you might think.

Problems With Formulas

We've already solved a lot of word problems by changing the sentences in the problem into formulas. This technique helps organize the problems into types. For example, many problems use the distance-rate-time formula, and we can see their relationship through the formula, even though in some problems we are finding the distance, in others the rate or time.

Formulas So Far

To use a formula, you must know what physical quantity each variable represents and then substitute those values for the variable(s). In a word problem, one quantity represented by a variable is unknown – once you make the substitution(s), solve the equation for the unknown quantity.

Example: Your small business makes greeting cards and sells them online. Each card costs $2 in materials and labor, and you have monthly fixed costs of $125 for advertising. If your total costs last month were $635, how many cards did you make?

Use the formula **Total Cost = Cost per Item • No of Items + Fixed Cost**

$$T = CN + F$$

Read the problem carefully so that you can fill in the equation with the values you know:

"your total costs last month were $635" T = $635;
"Each card costs $2" C = $2/item;
"you have monthly fixed costs of $125" F = $125

$$635 = 2N + 125$$

Notice that there is one variable left in the equation. N stands for the number of items. The question asked in the problem is "How many cards did you make?" If we find N, we can answer the question.

$$2N + 125 = 635$$
$$2N = 510$$
$$N = 255$$

$$2N + 125 - 125 = 635 - 125$$
$$2N / N = 510 / N$$

You made 255 cards.

Formula Quick Summary

Can you explain what each variable in the formula stands for?

Perimeter of a Rectangle	$P = 2(L + W) = 2L + 2W$
Area of Rectangle	$A = LW$
Volume of Box	$V = LWH$
Perimeter of Triangle	$P = a + b + c$
Area of Triangle	$A = \frac{bh}{2}$
Celsius-Fahrenheit Conversion	$F = \frac{9C}{5} + 32$
Distance = Rate • Time	$d = rt$
Total Cost = Cost per Item • No of Items + Fixed Cost	$T = CN + F$

Arithmetic to Algebra

You can make up a formula for any word problem situation and add it to the list, but memorizing a lot of formulas can get really out of hand. If you can figure out the mathematical relationship of the quantities from the problem, you can set up an individual equation for a problem without bothering with a formula. Many students are familiar with doing this for arithmetic problems, but have trouble using a variable (especially because it is not really necessary in these simple problems). This practice with simple one-step problems will help with more complicated word problems later.

Compare the two problems. They are structured exactly the same way, but have different quantities that are missing.

Four friends shared equally in a lottery win of $2 million. How much did each friend get?

$2 million is $2,000,000. Four friends split it, so I should divide:

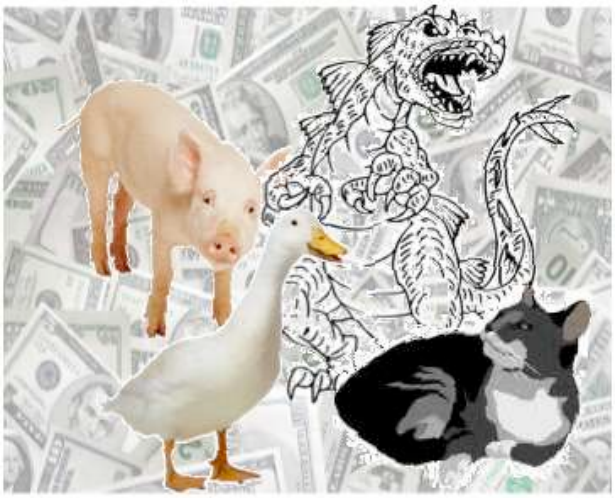
jackpot ÷ no. of friends = each share

2,000,000 ÷ 4 =

= 500,000

They each got $500,000.

The problem is straightforward arithmetic. The key words "shared equally" tell us to divide.



Four friends shared equally in a lottery win. Each friend got $500,000. How much was the jackpot they won?

Four friends split a jackpot, so I should divide:

jackpot ÷ no. of friends = each share

jackpot ÷ 4 = 500,000

Written algebraically, we let the variable J stand for the unknown jackpot amount:

$$\frac{J}{4} = 500{,}000$$

To solve the equation, we multiply both sides by 4.

$$4 \cdot \frac{J}{4} = 500{,}000 \cdot 4$$

$$J = 2{,}000{,}000$$

The jackpot was $2,000,000.

The problems are the same type, because the highlighted mathematical relationship is the same in both problems. Only the unknown quantity is different.

If you have been well-trained in arithmetic and have a good understanding of the relationship between multiplication and division, your immediate instinct in the second problem was probably to multiply 4 • 500,000. You have skipped the equation writing step altogether and done the step needed to solve the problem. This is very good thinking, and will serve you well – congratulations! Right now, though, it will be be temporarily a bit frustrating to have to go through the slower step of writing the algebra equation, but please stick with it. We're not as interested in efficient problem solving as in building translation skills for algebra, and you'll see that you actually understand the structure of the problems in a new way.

Example: **Use the structure sentence from the first problem to write an algebraic equation for the second problem.**

A box containing 200 toothpicks was spilled. 167 toothpicks were on the floor. How many were left in the box? Structure sentence: Number Originally in Box – Number Spilled = Number Left in Box **200 – 167 = 33** **33 toothpicks were left in the box.** 	A box of toothpicks was spilled. 167 toothpicks were on the floor, and 33 were left in the box. How many toothpicks were originally in the box? Number Originally in Box – Number Spilled = Number Left in Box Write an algebraic equation and solve: $N - 167 = 33$ $N - 167 + 167 = 33 + 167$ $N = 200$ There were originally 200 toothpicks.

It's somewhat arbitrary whether we structure these simple problems as arithmetic or algebra. For example, we could have written a different structure sentence:

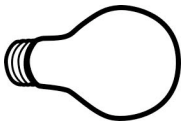
Number Originally in Box = Number Spilled + Number Left in Box

This is just another one of the four related addition/subtraction problems for the situation. Using this new structure sentence would have required an algebra equation for the first problem, and simple arithmetic for the second. So what are we trying to do?

The real goal is to start thinking of the quantities in the problem by name and to structure their relationship mathematically. You may need a variable to write the equation or the problem may only require arithmetic, but the thought process of naming the quantities will deepen your understanding of algebra. Eventually you will feel so at ease with using variables that you'll be able to work with them as if they were numbers.

Example: **Use the structure sentence**

Number of Boxes • Number of Items per Box = Total Number of Items



to write an equation for each of the word problems below.

Four boxes, each containing 24 light bulbs, were smashed. How many light bulbs were smashed? $4 \cdot 24 = 96$	Some boxes, each containing 24 lightbulbs, were smashed. 96 light bulbs were smashed in all. How many boxes was that? $B \cdot 24 = 96$	Four boxes of lightbulbs, containing 96 light bulbs in all, were smashed. How many light bulbs were in each box? $4N = 96$

Using a structure sentence is really just like having a formula for the problem. To really feel confident in solving a word problem you need to be able to write the structure sentence itself. Let's tackle that next.

Analyzing the Mathematical Structure
Word problems call on your practical everyday knowledge of the world, and also on your ability to recognize key words that represent mathematical operations.

Example: **Highlight the three quantities in the problem. Write an equation relating the quantities.**

Sam works 22 hours per week at a grocery store. He makes $15 per hour. What is Sam's weekly wage?

1 The first step is to recognize what quantities are involved in the problem. Look for numbers and their units, and look for the unknown quantity in the question.

Sam works 22 hours per week at a grocery store. He makes $15 per hour. What is Sam's weekly wage?

2 To relate the quantities in a mathematical equation, you use your practical knowledge about wages. Multiply your hourly wage by your hours to find your paycheck:

$15 per hour • 22 hours per week = weekly wage

If you write out the units, you can see that they work out perfectly:

$$\frac{\$15}{1\ \text{hour}} \cdot \frac{22\ \text{hours}}{1\ \text{week}} = \frac{\$330}{1\ \text{week}}$$

Example: **Highlight the three quantities in the problem. Write an equation relating the quantities.**

A massive recall of eggs involved 22,200,000 eggs. How many dozens was this?

1 Highlight the quantities involved. Be sure you understand what the problem is asking.

A massive recall of eggs involved 22,200,000 eggs. How many dozens was this?
"How many dozens?" means "How many cartons holding 12 eggs each were there?"

2 Use your practical knowledge of the mathematical relationships:

Number of cartons • 12 eggs in each carton = total number of eggs (which is 22,200,000)

$$N \cdot 12 = 22{,}0000{,}000$$
$$12N = 22{,}0000{,}000 \qquad 12N / 12 = 22{,}0000{,}000 / 12$$
$$N = 1{,}850{,}000$$

1,850,000 dozens of eggs were recalled.

Example: **Write an equation and solve the problem.**

Shara spent $105 on school supplies for her kids, consisting of 3 backpacks and 15 notebooks. Notebooks cost $2 each. How much did each backpack cost?

1 Highlight the quantities involved. Be sure you understand what the problem is asking.

Shara spent $105 on school supplies for her kids, consisting of 3 backpacks and 15 notebooks. Notebooks cost $2 each. How much did each backpack cost?

The quantities are:

- The total amount spent on school supplies, **$105**.
- The number of backpacks: **3 backpacks**
- The number of notebooks: **15 notebooks**
- The cost per notebook: **$2 per notebook**
- The cost per backpack: UNKNOWN (Let's call it ***b* dollars per backpack.)**

2 Use your practical knowledge of the mathematical relationships:

15 notebooks @ $2 per notebook = **$30** for notebooks

How much does a backpack cost? ***b* dollars per backpack**

3 backpacks @ $*b* per backpack = **$3*b*** for backpacks

Cost of notebooks + Cost of backpacks = Cost of school supplies

30 + 3*b* = 105

$$3b + 30 = 105 \qquad 3b + 30 - 30 = 105 - 30$$

$$3b = 75 \qquad 3b / 3 = 75 / 3$$

$$b = 25$$

Each backpack cost $25.

Lesson 18: Word Problems

Worksheet

Name _____

1. Use the structure sentence Number of boxes • Number of Items per Box = Total Number of Items to write an equation for each problem. Use a variable where necessary. You don't need to solve the equations.

a. "I accidentally ordered two thousand boxes each containing fifty plastic hula dancers." "Oh no. How many do we have now?"	b. "I accidentally ordered too many boxes each containing fifty plastic hula dancers. Now we have 100,000 plastic hula dancers…" "How many boxes did you order?"	c. "I accidentally ordered two thousand boxes of plastic hula dancers. Now we have 100,000 plastic hula dancers." "You're kidding, how many were in each box?

2. At your wedding reception, you'll have:

- a number of Tables;
- a number of Guests per Table;
- a total number of Guests.

Write a structure sentence relating the quantities above.

Use the same structure sentence to write equations for all the problems. You don't need to solve the equations.

a. We'll set up 30 tables, each seating 6 guests. How many guests?	b. We'll set up 30 tables to accommodate the 180 guests. How many guests per table?	c. We'll seat the 180 guests in tables of 6. How many tables?

3. You take your total monthly income, subtract your bills, and what's left is for discretionary spending.

a. Write a structure sentence relating the quantities:

b. Use the structure sentence to write an equation for each problem below. You don't need to solve the equations.

a. Sheila was making $3112 per month, and her bills were $2877 per month. How much was left for discretionary spending?	a. Pete was making $2036 per month, and he had only $18 left over for discretionary spending. How much were his bills?	b. After paying his bills of $1662 per month, Joaquin still had $1233 left over for discretionary spending. How much was his monthly income?

4. The fan cost index for baseball comprises the price of four average tickets, two small beers, four small sodas, four hot dogs, parking for one car, two game programs, and two adult-sized caps. (Source)

Fan Cost Index = Cost of: (4 tickets + 2 beers + 4 sodas + 4 hot dogs + 1 parking + 2 programs + 2 caps)

- tickets cost $22 each
- beer costs $5 each
- soda costs $3 each
- hot dogs cost $6 each
- parking costs $30
- programs cost $5 each
- caps cost $ UNKNOWN

San Francisco's fan cost index was $212 in 2008. Write an equation and find the price of one cap.

Lesson 18: Word Problems

Homework 18A

Name _____

1. A rectangle has length 45 inches. The width is 10 inches less than the length.

a. Find the width of the rectangle.

b. Find the perimeter of the rectangle.

c. Find the area of the rectangle.

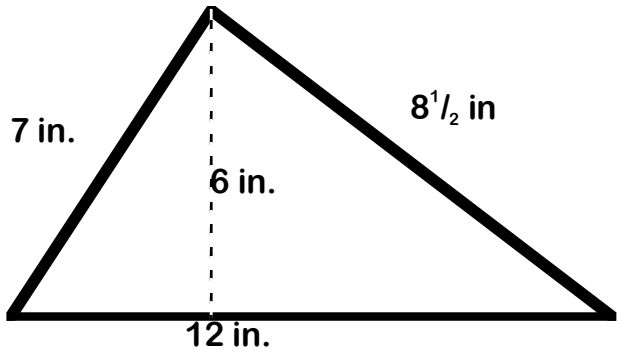
2. A box has length 18 inches. The width is 2 inches more than the length. The height is half the length.

a. Find the width of the box.

b. Find the height of the box.

c. Find the volume of the box.

3. A triangle has the measurements shown.



a. Find the perimeter.

b. Find the area.

4. a. The formula to convert Celsius to Fahrenheit temperature is $F = \frac{9C}{5} + 32$. Find the Fahrenheit temperature equivalent to 110ºC.

b. The formula to convert Fahrenheit to Celsius temperature is $C = \frac{5(F - 32)}{9}$. Find the Celsius temperature equivalent to 113ºF.

5. Is 323 a prime number? If not, find the prime factorization.

6. Write an algebraic expression showing the sum of two terms. The first term has coefficient 3 and variable *x*. The second term is the constant 5.

7. Use the distance-rate-time formula to write an equation and solve the problem.

a. A car drove 498 miles in 6 hours. What was the car's average rate?

b. Another car drove 252 miles at an average speed of 72 mph. How long did it take?

8. Use the structure sentence

No. Boxes • No. Items per Box = No. Items

to write equations for the problems. Then solve.

a. "For our fundraiser we got 200 cases of candy bars. That's 5000 candy bars."
How many candy bars in one case?

b. "Well, for our fundraiser we got cases that have 40 per case. But we also bought 5000 candy bars!"
How many cases did they buy?

9. Write a structure sentence relating the quantities

- Original Number of Homework Problems
- Number of Homework Problems Done
- Number of Homework Problems Left to Do

Use the sentence to write equations and solve.

a. You had 30 homework problems, but you diligently did 27 of them. How many do you have left to do?

b. You did 18 problems, but you still have 22 more to do. How many did you have to start with?

10. Write an equation and solve:

The amusement park charges $55 for an adult and $40 for a child. The group took 24 children, and their cost for the day at the park was $1180. How many adults went along?

Lesson 18: Word Problems

Homework 18A Answers

1. A rectangle has length 45 inches. The width is 10 inches less than the length.

a. Find the width of the rectangle.

The width is 10 in. less than the length, 45 in.

So, the width is 45 - 10 = 35 inches.

b. Find the perimeter of the rectangle.

L = 45 in., W = 35 in.; P = 2L + 2W

P = 2(45) + 2(35) = 160 inches

c. Find the area of the rectangle.

L = 45 in., W = 35 in.; A = LW

A =(45 in)(35 in) = 1575 in^2

2. A box has length 18 inches. The width is 2 inches more than the length. The height is half the length.

a. Find the width of the box.

The width is 2 more than the length, 18 in.

So, the width is 18 + 2 = 20 inches.

b. Find the height of the box.

The height is half the length, 18 in.

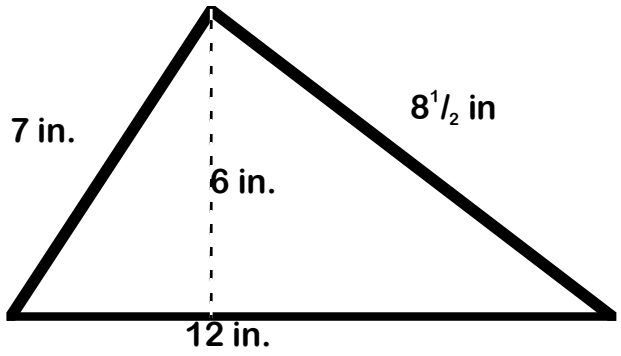
So, the height is 18/2 = 9 inches.

c. Find the volume of the box.

L = 18 in; W = 20 in; H = 9 in; V = LWH

V = (18 in)(20 in)(9 in) = 3240 in^3

3. A triangle has the measurements shown.



a. Find the perimeter.

P = 7 in + 8^1/$_2$ in + 12 in = 27^1/$_2$ in.

b. Find the area.

A = bh/2 = (12 in)(6 in)/2 = 36 in^2

4. a. The formula to convert Celsius to Fahrenheit temperature is $F = \frac{9C}{5} + 32$. Find the Fahrenheit temperature equivalent to 110ºC.

$$F = \frac{9(110)}{5} + 32 = 198 + 32 = 230$$

110°C = 230°F

b. The formula to convert Fahrenheit to Celsius temperature is $C = \frac{5(F - 32)}{9}$. Find the Celsius temperature equivalent to 113ºF.

$$C = \frac{5(113 - 32)}{9} = \frac{5(81)}{9} = 45$$

113°F = 45°C

5. Is 323 a prime number? If not, find the prime factorization.

323 is not divisible by 2, 3, 5, 7, 11, or 13, (the first six primes), but it is divisible by 17, so it is NOT PRIME.

323 = 17 • 19

6. Write an algebraic expression showing the sum of two terms. The first term has coefficient 3 and variable *x*. The second term is the constant 5.

$3x + 5$

7. Use the distance-rate-time formula to write an equation and solve the problem.

a. A car drove 498 miles in 6 hours. What was the car's average rate?

$d = rt$ $498 = r \cdot 6$

$6r = 498$ $6r / 6 = 498 / 6$

$r = 83$ The car's rate was 83 mph.

b. Another car drove 252 miles at an average speed of 72 mph. How long did it take?

$d = rt$ $252 = 72t$

$72t = 252$ $72r / 72 = 252 / 72$

$t = \frac{7}{2}$ The time was $3\frac{1}{2}$ hours.

8. Use the structure sentence

No. Boxes • No. Items per Box = No. Items

to write equations for the problems. Then solve.

a. "For our fundraiser we got 200 cases of candy bars. That's 5000 candy bars."
How many candy bars in one case?

$200N = 5000$ $200N / 200 = 5000 / 200$

$N = 25$

There are 25 candy bars per case.

b. "Well, for our fundraiser we got cases that have 40 per case. But we also bought 5000 candy bars!"
How many cases did they buy?

$B \cdot 40 = 5000$ $40B / 40 = 5000 / 40$

$B = 125$

They bought 125 cases.

9. Write a structure sentence relating the quantities

- Original Number of Homework Problems
- Number of Homework Problems Done
- Number of Homework Problems Left to Do

Problems – Problems Done = Problems Left

Use the sentence to write equations and solve.

a. You had 30 homework problems, but you diligently did 27 of them. How many do you have left to do?

$30 - 27$

$= 3$ problems left to do

b. You did 18 problems, but you still have 22 more to do. How many did you have to start with?

$P - 18 = 22$ $P - 18 + 18 = 22 + 18$

$P = 40$ You had 40 problems to start with.

10. Write an equation and solve:

The amusement park charges $55 for an adult and $40 for a child. The group took 24 children, and their cost for the day at the park was $1180. How many adults went along?

Cost for group
= cost for adults + cost for children

Cost for children: 24 children @ $40 per child is $24 \cdot 40 = \$960$

Cost for adults: UNKNOWN number of adults (use the variable A) @ $55 per adult is $55A

Equation: $1180 = 55A + 960$

$55A + 960 = 1180$

$55A + 960 - 960 = 1180 - 960$

$55A = 220$ $55A / 55 = 220 / 55$

$A = 4$ Four adults went along.

Lesson 18: Word Problems

Homework 18B

Name _____

1. A rectangle has length 84 cm. The width is 12 cm more than the length.

a. Find the width of the rectangle.

b. Find the perimeter of the rectangle.

c. Find the area of the rectangle.

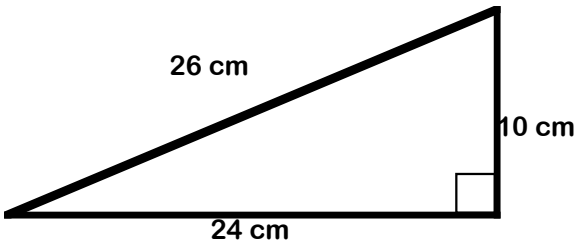
2. A box has length 25 cm. The width is 5 cm less than the length. The height is 5 cm more than the length.

a. Find the width of the box.

b. Find the height of the box.

c. Find the volume of the box.

3. A triangle has the measurements shown.



Because the triangle has a square corner, the side measuring 10 cm is also the height of the triangle.

a. Find the perimeter.

b. Find the area.

4. a. The formula to convert Celsius to Fahrenheit temperature is $F = \frac{9C}{5} + 32$. Find the Fahrenheit temperature equivalent to 60ºC.

b. The formula to convert Fahrenheit to Celsius temperature is $C = \frac{5(F - 32)}{9}$. Find the Celsius temperature equivalent to 59ºF.

5. Is 319 a prime number? If not, find the prime factorization.

6. Write an algebraic expression showing the sum of two terms. The first term has coefficient 9 and variable *y*. The second term is the constant 1.

7. Use the distance-rate-time formula to write an equation and solve the problem.

a. The factory workers assembled 203 products in 7 hours. What was their production rate?

b. At the other plant, the workers assembled 132 products at a rate of 33 products per hour. How long did that take?

8. Use the structure sentence

No. Boxes • No. Items per Box = No. Items

to write equations for the problems. Then solve.

a. "I got this great deal on toilet paper. Now we just need to find room in the garage. There are 40 cartons! That's 3840 rolls of toilet paper! We'll never be out of toilet paper again!" *How many rolls per carton?*

b. "You know, my grandma did that, but she lived in New Orleans. She lost 5040 rolls in the flood! Each carton of 48 rolls was a soggy mess." *How many cartons?*

9. Write a structure sentence relating the quantities

- Original Length of Board
- Length of Piece Cut Off
- Length of Piece Remaining

Use the sentence to write equations and solve.

a. The carpenter cut a 6 foot length from the 8 foot board.

b. The carpenter cut 3 feet from the board, leaving 15 feet remaining.

10. Write an equation and solve:

The amusement park charges $80 for an adult and $50 for a child. A group of six adults took some children, and their cost for the day at the park was $1680. How many children went along?