How Simple Is Your Rational Expression? Examples

1. Rational expressions are algebraic expressions whose numerator and denominator are polynomials. The expression $\dfrac{5x+3}{y}$, $\dfrac{2}{x}$, and $\dfrac{a-2}{a^2+4}$ are examples of rational expressions or algebraic fractions.

2. A fraction indicates division. Zero cannot be used as a denominator because division by zero is undefined.

3. Therefore, any value assigned to a variable that results in a denominator of zero must be excluded from the domain of the variable (it may be necessary to review factoring.)

4. For example, the following would have exclusions:

$\dfrac{5}{x}$, **would exclude x = 0.**

$\dfrac{3x+7}{x+4}$, **would exclude x = -4.**

$\dfrac{y^2-5}{x^2-5x+6}$, **would exclude x = 2 and x = 3.**

> **Emphasize that x ≠ 4 because –4 + 4 = 0.**

> **x^2 – 5x + 6 factored is (x – 2)(x – 3). Apply the Zero Product Property to show that x ≠ 2 or x ≠ 3.**

5. **Example:** State the value(s) of the variable that must be excluded:

$\dfrac{5x}{x+7}$

Exclude the values for which x + 7 = 0.
x + 7 = 0
x = -7

6. **Example:** State the values of the variable that must be excluded:

$$\frac{2a-3}{a^2-a-12}$$

Exclude the values for which a^2 – a – 12 = 0.
a^2 – a – 12 = 0
(a – 4)(a + 3) = 0
a = 4 or a = -3
Therefore, "a "cannot equal 4 or –3.

Factor a^2 – a – 12 = 0.

Use the Zero Product Property.

7. Recall that to simplify an algebraic fraction such as $\dfrac{14a^2bc}{42abc^2}$, first factor the numerator and denominator. Then simplify common factors.

$$\frac{14a^2bc}{42abc^2} \Rightarrow \frac{2\bullet 7\bullet a\bullet a\bullet b\bullet c}{2\bullet 3\bullet 7\bullet a\bullet b\bullet c\bullet c} \Rightarrow \frac{a}{3c}$$

Notice that a ≠ 0, b ≠ 0, and c ≠ 0.

The GCF is 14abc

This same procedure can be used to simplify rational expressions having polynomials in the numerator and denominator.

8. **Example:** Simplify➔ $\dfrac{3a^2 + a - 2}{a^2 + 7a + 6}$. State the excluded values of "a."

$$\dfrac{3a^2 + a - 2}{a^2 + 7a + 6} = \dfrac{(a+1)(3a-2)}{(a+6)(a+1)} = \dfrac{(a+1)(3a-2)}{(a+6)(a+1)}$$

$$= \dfrac{3a - 2}{a + 6}$$

The excluded values of "a" are any values for which a^2 + 7a + 6 = 0➔(a + 6)(a + 1) = 0➔a = -6 or a = -1.
Therefore, a cannot equal –6 or –1.

Stress that factors must be reduce, not terms.

9. **Example:** Simplify➔ $\dfrac{2x - 2y}{y^2 - x^2}$. State the excluded values of "x" and "y."

$$\dfrac{2x - 2y}{y^2 - x^2} = \dfrac{2(x - y)}{(y - x)(y + x)} = \dfrac{2(x - y)}{(y - x)(y + x)}$$

$$= \dfrac{-2}{y + x} \text{ or } -\dfrac{2}{y + x}$$

The excluded values of "x" and "y" are any values for which y^2 – x^2 = 0➔ (y – x)(y + x) = 0 ➔y = x or y = -x

Therefore, y cannot equal x or –x.

Another way to state the excluded values is x cannot equal y or –y.

Name:_____
Date:_____
Class:_____

How Simple Is Your Rational Expression? Worksheet

Simplify each rational expression. State the excluded values of the variables.

1. $\dfrac{y-3}{y^2-9}$

2. $\dfrac{r^3-r^2}{r-1}$

3. $\dfrac{3m^2}{6m^2-3m}$

4. $\dfrac{c-6}{c^2-12c+36}$

5. $\dfrac{m^2-36}{m^2+5m-6}$

6. $\dfrac{4x+8}{x^2+6x+8}$

7. $\dfrac{9-a^2}{a^2-a-6}$

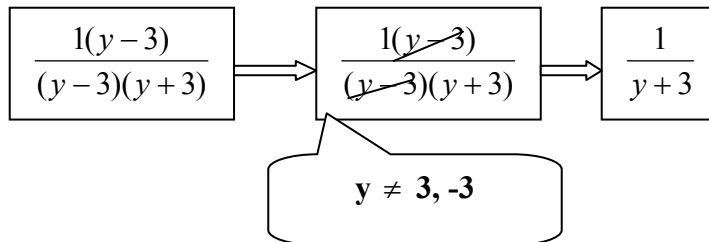8. $\dfrac{g^2+g-2}{g^2-3g+2}$

9. $\dfrac{a^2-9}{a^2+6a-27}$
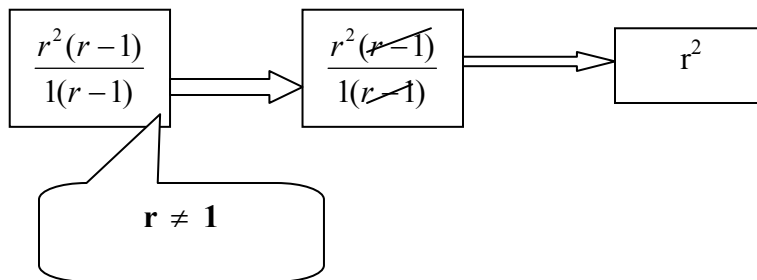
10. $\dfrac{k^2-1}{k^2+2k+1}$

How Simple Is Your Rational Expression?
Worksheet Key

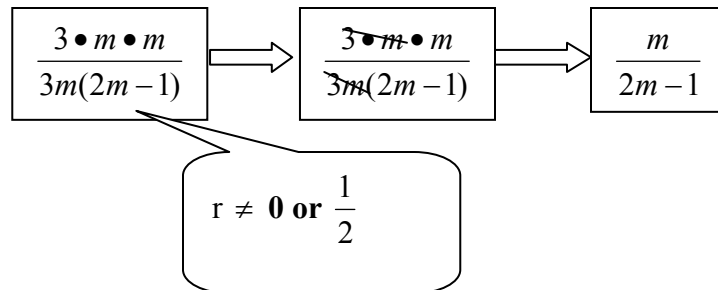Simplify each rational expression. State the excluded values of the variables.
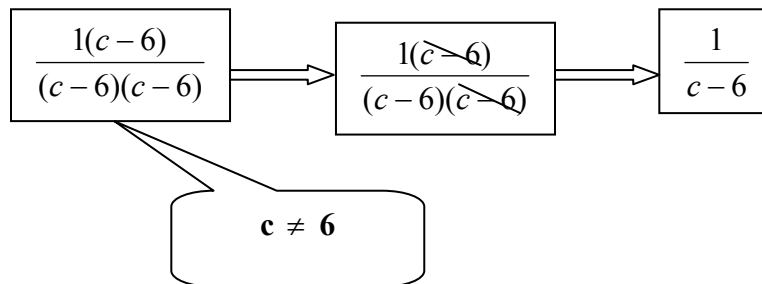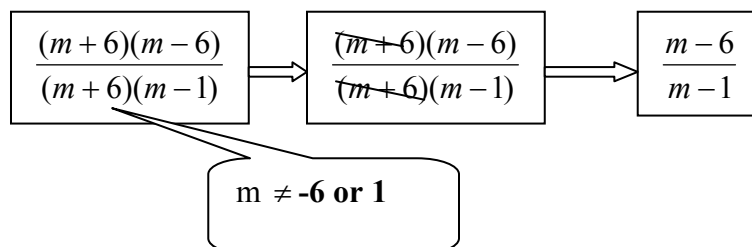
1. $\dfrac{y-3}{y^2-9}$ $\boxed{\dfrac{1(y-3)}{(y-3)(y+3)}}$ → $\boxed{\dfrac{1(\cancel{y-3})}{(\cancel{y-3})(y+3)}}$ → $\boxed{\dfrac{1}{y+3}}$

$y \neq 3, -3$

2. $\dfrac{r^3-r^2}{r-1}$ $\boxed{\dfrac{r^2(r-1)}{1(r-1)}}$ → $\boxed{\dfrac{r^2(\cancel{r-1})}{1(\cancel{r-1})}}$ → $\boxed{r^2}$

$r \neq 1$

3. $\dfrac{3m^2}{6m^2-3m}$ $\boxed{\dfrac{3 \bullet m \bullet m}{3m(2m-1)}}$ → $\boxed{\dfrac{\cancel{3 \bullet m} \bullet m}{\cancel{3m}(2m-1)}}$ → $\boxed{\dfrac{m}{2m-1}}$

$r \neq 0$ or $\dfrac{1}{2}$

4. $\dfrac{c-6}{c^2-12c+36}$ $\boxed{\dfrac{1(c-6)}{(c-6)(c-6)}}$ → $\boxed{\dfrac{1(\cancel{c-6})}{(c-6)(\cancel{c-6})}}$ → $\boxed{\dfrac{1}{c-6}}$

$c \neq 6$

5. $\dfrac{m^2-36}{m^2+5m-6}$ $\boxed{\dfrac{(m+6)(m-6)}{(m+6)(m-1)}}$ → $\boxed{\dfrac{(\cancel{m+6})(m-6)}{(\cancel{m+6})(m-1)}}$ → $\boxed{\dfrac{m-6}{m-1}}$

$m \neq -6$ or 1

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6. $\dfrac{4x+8}{x^2+6x+8}$ → $\dfrac{4(x+2)}{(x+2)(x+4)}$ → $\dfrac{4(\cancel{x+2})}{(\cancel{x+2})(x+4)}$ → $\dfrac{4}{x+4}$

x ≠ -2 or -4

7. $\dfrac{9-a^2}{a^2-a-6}$ → $\dfrac{(3-a)(3+a)}{(a-3)(a+2)}$ → $\dfrac{-(\cancel{a-3})(a+3)}{(\cancel{a-3})(a+2)}$ → $-\dfrac{a+3}{a+2}$

a ≠ 3, -2

8. $\dfrac{g^2+g-2}{g^2-3g+2}$ → $\dfrac{(g+2)(g-1)}{(g-2)(g-1)}$ → $\dfrac{(g+2)(\cancel{g-1})}{(g-2)(\cancel{g-1})}$ → $\dfrac{g+2}{g-2}$

g ≠ **2 or 1**

9. $\dfrac{a^2-9}{a^2+6a-27}$ → $\dfrac{(a-3)(a+3)}{(a-3)(a+9)}$ → $\dfrac{(\cancel{a-3})(a+3)}{(\cancel{a-3})(a+9)}$ → $\dfrac{a+3}{a+9}$

a ≠ **3 or -9**

10. $\dfrac{k^2-1}{k^2+2k+1}$ → $\dfrac{(k-1)(k+1)}{(k+1)(k+1)}$ → $\dfrac{(k-1)(\cancel{k+1})}{(k+1)(\cancel{k+1})}$ → $\dfrac{k-1}{k+1}$

k ≠ **-1**

Student Name: _____

Date: _____

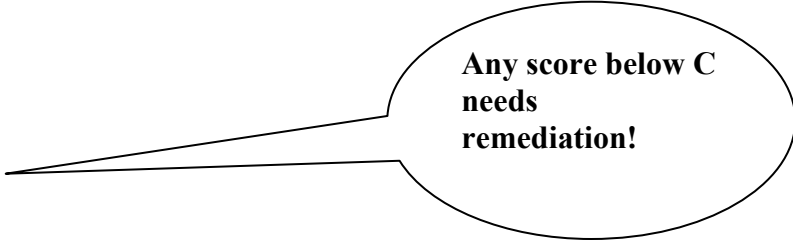## How Simple Is Your Rational Expression? Checklist

1. On question 1, did the student simplify the rational expression completely and state the excluded values?

    a. Yes (20 points)
    b. Simplified completely but did not state excluded values (15 points)
    c. Did not simplify completely but did state excluded values (10 points)
    d. Did not simplify completely and did not state excluded values (5 points)

2. On question 2, did the student simplify the rational expression completely and state the excluded values?

    a. Yes (20 points)
    b. Simplified completely but did not state excluded values (15 points)
    c. Did not simplify completely but did state excluded values (10 points)
    d. Did not simplify completely and did not state excluded values (5 points)

3. On question 3, did the student simplify the rational expression completely and state the excluded values?

    a. Yes (20 points)
    b. Simplified completely but did not state excluded values (15 points)
    c. Did not simplify completely but did state excluded values (10 points)
    d. Did not simplify completely and did not state excluded values (5 points)

4. On question 4, did the student simplify the rational expression completely and state the excluded values?

    a. Yes (20 points)
    b. Simplified completely but did not state excluded values (15 points)
    c. Did not simplify completely but did state excluded values (10 points)
    d. Did not simplify completely and did not state excluded values (5 points)

5. On question 5, did the student simplify the rational expression completely and state the excluded values?

    a. Yes (20 points)
    b. Simplified completely but did not state excluded values (15 points)
    c. Did not simplify completely but did state excluded values (10 points)
    d. Did not simplify completely and did not state excluded values (5 points)

6. On question 6, did the student simplify the rational expression completely and state the excluded values?

   a. Yes (20 points)
   b. Simplified completely but did not state excluded values (15 points)
   c. Did not simplify completely but did state excluded values (10 points)
   d. Did not simplify completely and did not state excluded values (5 points)

7. On question 7, did the student simplify the rational expression completely and state the excluded values?

   a. Yes (20 points)
   b. Simplified completely but did not state excluded values (15 points)
   c. Did not simplify completely but did state excluded values (10 points)
   d. Did not simplify completely and did not state excluded values (5 points)

8. On question 8, did the student simplify the rational expression completely and state the excluded values?

   a. Yes (20 points)
   b. Simplified completely but did not state excluded values (15 points)
   c. Did not simplify completely but did state excluded values (10 points)
   d. Did not simplify completely and did not state excluded values (5 points)

9. On question 9, did the student simplify the rational expression completely and state the excluded values?

   a. Yes (20 points)
   b. Simplified completely but did not state excluded values (15 points)
   c. Did not simplify completely but did state excluded values (10 points)
   d. Did not simplify completely and did not state excluded values (5 points)

10. On question 10, did the student simplify the rational expression completely and state the excluded values?

   a. Yes (20 points)
   b. Simplified completely but did not state excluded values (15 points)
   c. Did not simplify completely but did state excluded values (10 points)
   d. Did not simplify completely and did not state excluded values (5 points)

   e. No, student identified a cluster of data (10 points)

Total Number of Points _____

A   180 points and above

B   160 points and above

C   140 points and above

D   120 points and above

F   119 points and below

Any score below C needs remediation!