AP Calculus

CHAPTER 7 WORKSHEET

INVERSE FUNCTIONS

Name________________________________

Seat # ______ Date ____________________

Derivatives of Inverse Functions

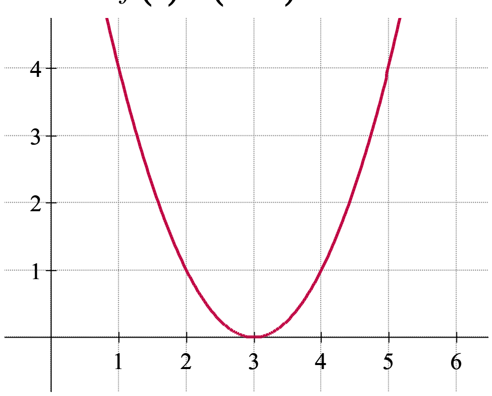
In 1-3, use the derivative to determine whether the function is strictly monotonic on its entire domain and therefore has an inverse.

1. $f(x) = \frac{x^4}{4} - 2x^2$
2. $g(x) = (x+a)^3 + b$
3. $h(x) = 2 - x - x^3$

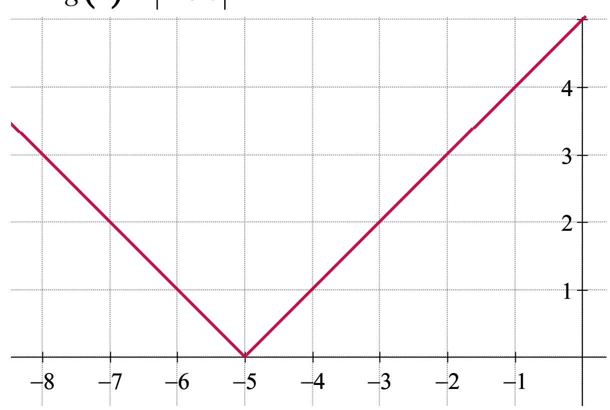
4. **Think About It...** Find the derivative of $y = \tan x$. Notice that the subject derivative has the same sign for all values of x, so $y = \tan x$ is a monotonic function. However, $y = \tan x$ is not a one-to-one function. Why?

In 5-6, (a) "delete" part of the graph of the function shown so that the part that remains is one-to-one. Then, (b) find the inverse of the remaining part and (c) state its domain. (Note: there is more than one correct answer for these questions!)

5. $f(x) = (x-3)^2$



6. $g(x) = |x+5|$



In 7-9, find the derivative of the inverse function at the corresponding value.

7. Given $f(x) = x^3 + 2x - 1$, find $\frac{d}{dx}\left[f^{-1}\right]\Big|_{x=2}$ *(Note: you may need to use guess and check to solve an equation involved in this problem.)*

8. Given $g(x) = 2x^5 + x^3 + 1$, find $\frac{d}{dx}\left[g^{-1}\right]\Big|_{x=1}$

9. Given $h(x) = \sin x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ find $\frac{d}{dx}\left[h^{-1}\right]\Big|_{x=1/2}$

SEE OTHER SIDE

10. Selected values of a strictly monotonic function $g(x)$ and its derivative $g'(x)$ are shown on the table below.

x	-3	-1	1	4
$g(x)$	5	1	0	-3
$g'(x)$	-4	$-\frac{1}{5}$	$-\frac{1}{6}$	-2

a) Find $\left(g^{-1}\right)'(1)$

b) Find $\left(g^{-1}\right)'(-3)$

11. Selected values of a strictly monotonic function $h(x)$ and its derivative $h'(x)$ are shown on the table below.

x	-1	0	2	4
$h(x)$	-5	-1	4	7
$h'(x)$	3	$\frac{1}{2}$	$\frac{1}{6}$	5

Let $f(x)$ be a function such that $f(x)=h^{-1}(x)$.

a) Find $f'(-1)$

b) Find $f'(4)$

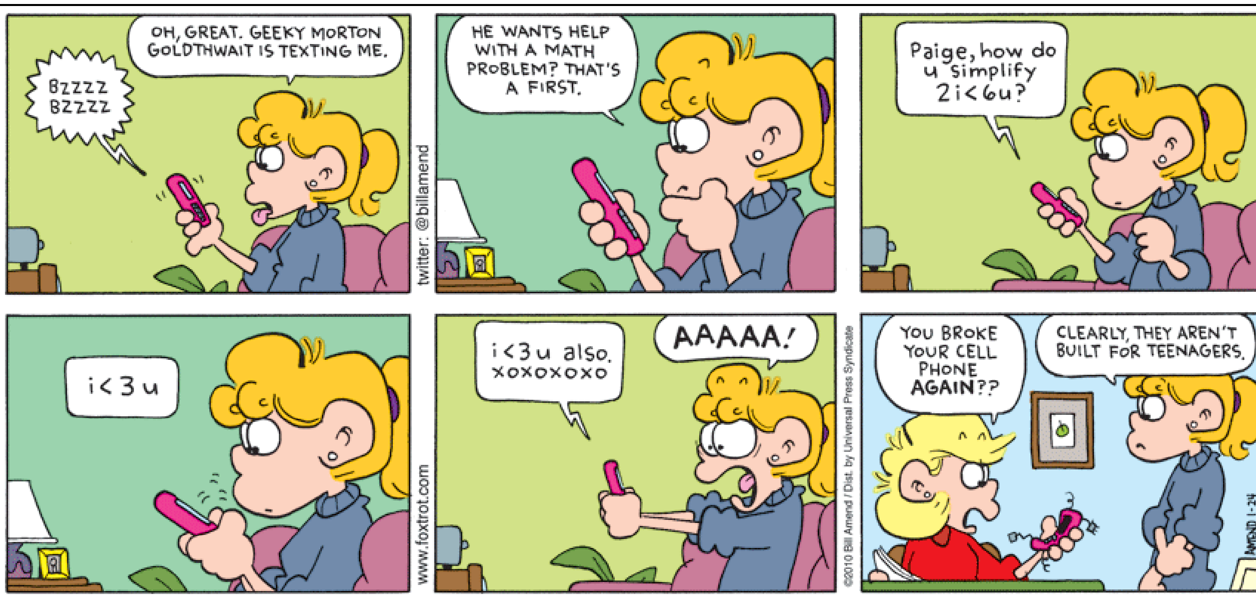
True or False? In 12-15, determine whether the statement is true or false. Justify your answer.

12. If $f(x)$ is an even function, then $f^{-1}(x)$ exists.

13. If the inverse of f exists, then the y-intercept of f is an x-intercept of f^{-1}.

14. If $f(x)=x^n$ where n is odd, then $f^{-1}(x)$ exists.

15. There exists no function f such that $f=f^{-1}$.



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ANSWER KEY

Derivatives of Inverse Functions

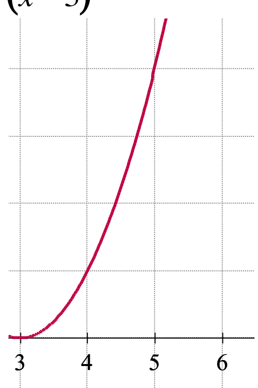
1. $f'(x) = x^3 - 4x = x(x^2 - 1)$
Performing a sign analysis, $f'(x) < 0$ if $x < 0$, but $f'(x) > 0$ if $x > 0$ (except at $x = 1$), so this is not a strictly monotonic function and it does not have an inverse function.

2. $g'(x) = 3(x + a)^2$
Performing a sign analysis, $g'(x) > 0$ for all values of x, except at $x = -a$. So this is a strictly monotonic function and it has an inverse function.

3. $h'(x) = -1 - 3x^2$
Performing a sign analysis, $h'(x) < 0$ for all values of x. So this is a strictly monotonic function and it has an inverse function.

4. $y' = \sec^2 x$. We have $y' = \sec^2 x > 0$, for all values of x included in the domain of $\sec x$. Therefore $y = \tan x$ is always increasing. But the graph of $y = \tan x$ has vertical tangent lines and it does not pass the horizontal line test: $y = \tan x$ is not a one-to-one function.

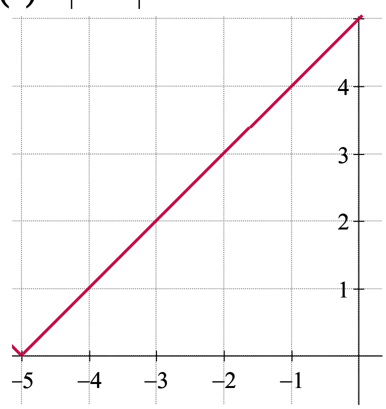
5. $f(x) = (x - 3)^2$



$f^{-1}(x) = \sqrt{x} + 3$

Domain of $f^{-1}(x)$ is $[0, +\infty)$

6. $g(x) = |x + 5|$



$g^{-1}(x) = x - 5$

Domain of $g^{-1}(x)$ is $[0, +\infty)$

7. Given $f(x) = x^3 + 2x - 1$, $\left.\dfrac{d}{dx}\left[f^{-1}\right]\right|_{x=2} = \dfrac{1}{f'(1)} = \dfrac{1}{5}$

8. Given $g(x) = 2x^5 + x^3 + 1$, $\left.\dfrac{d}{dx}\left[g^{-1}\right]\right|_{x=1} = \dfrac{1}{g'(0)} =$ undefined

9. Given $h(x)=\sin x$ on the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, $\frac{d}{dx}\left[h^{-1}\right]\Big|_{x=1/2}=\dfrac{1}{h'\left(\frac{\pi}{6}\right)}=\dfrac{2}{\sqrt{3}}=\dfrac{2\sqrt{3}}{3}$

10.

x	-3	-1	1	4
$g(x)$	5	1	0	-3
$g'(x)$	-4	$-\frac{1}{5}$	$-\frac{1}{6}$	-2

a) $\left(g^{-1}\right)'(1)=\dfrac{1}{g'(-1)}=-5$

b) $\left(g^{-1}\right)'(-3)=\dfrac{1}{g'(4)}=-\dfrac{1}{2}$

11.

x	-1	0	2	4
$h(x)$	-5	-1	4	7
$h'(x)$	3	$\frac{1}{2}$	$\frac{1}{6}$	5

a) $f'(-1)=\dfrac{1}{h'(0)}=2$

b) $f'(4)=\dfrac{1}{h'(2)}=6$

12. If $f(x)$ is an even function, then $f^{-1}(x)$ exists.
FALSE. An even function has symmetry with respect to the y-axis and, therefore, cannot be one-to-one.

13. If the inverse of f exists, then the y-intercept of f is an x-intercept of f^{-1}.
TRUE. Switching x and y coordinates will result in switching x and y intercepts.

14. If $f(x)=x^n$ where n is odd, then $f^{-1}(x)$ exists.
TRUE. If $f(x)=x^n$ where n is odd, its derivative is $f'(x)=nx^{n-1}$ where $n-1$ is even. So $f'(x)>0$ for all values of x except $x=0$. Therefore $f(x)=x^n$ is strictly monotonic.

15. There exists no function f such that $f=f^{-1}$.
FALSE. There are many such functions! Some examples: $y=x$, $y=-x$, $y=-x+a$,…