Diversification in the Internet Economy: The Role of For-Profit Mediators

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We investigate market forces that would lead to the emergence of new classes of players in the sponsored search advertising market. We report a multi-fold diversification triggered by an inherent feature of the sponsored search market, namely, *capacity constraints*, arising from the fact that there is a limit on the number of available advertisement slots, especially for the *popular keywords*. As a result, a significant pool of interested advertisers are left out. We present a comparative study of two scenarios motivated by capacity constraints - one where the additional capacity is provided by for-profit agents (or, mediators), who compete for slots in the original auction, draw traffic, and run their *own sub-auctions*, and the other, where the additional capacity is provided by the auctioneer herself, by essentially acting as a mediator and running a *single combined auction*. The quality of the additional capacity is measured by its *fitness* factor. We observe that the single combined-auction model seems inferior to the mediator-based model and market becomes more *capacity efficient* in the latter. For instance, the revenue of the auctioneer always increases when mediators are involved, unlike the auctioneer based scenario where often there is a tradeoff between the revenue and the capacity. Further, the social value (i.e. efficiency) always increases when mediators are involved. Thus, our analysis indicates that there are *significant opportunities for diversification* in the internet economy and we should expect it to continue to develop richer structure, with room for different types of market entities and mechanisms to coexist.

Categories and Subject Descriptors: H.4.m [**Information Systems**]: Miscellaneous

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A preliminary version of this work was presented in WINE 2007 wherein the mediator-based model was studied [Singh et al. 2007].
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1. INTRODUCTION

Sponsored search advertising(SSA), where advertisers pay to appear alongside the algorithmic/organic search results, is a significant growth market and is largely responsible for the success of Internet Search giants such as Google and Yahoo!. The statistics show that the growth of the overall online advertising market has been around 30% every year, as compared to the 1-2% of the traditional media, and is expected to increase to $35.4 billion in 2012 from around $20 billion in 2007.

In this form of advertising, the Search Engine allocates the advertising space using an auction. Advertisers bid upon specific keywords (i.e. query words). When a user searches for a keyword, the search engine (i.e. the auctioneer) allocates the advertising space to the bidding merchants based on their bid values and *quality scores*, and their ads are listed accordingly. Usually, the sponsored search results appear in a separate section of the page designated as "sponsored links" above/below or to the right of the organic/algorithmic results and have similar display format as the algorithmic results. Each position in such a list of sponsored links is called a *slot*. Whenever a user clicks on an ad, the corresponding advertiser pays an amount specified by the auctioneer; hence, the term *Cost Per Click* (CPC). Generally, users are more likely to click on a higher ranked slot, and therefore, advertisers prefer to be in higher ranked slots and compete for them. The auction currently used by Google and Yahoo! is a generalization of the Vickrey auction [Vickrey 1961], and is referred to as the *GSP* (Generalized Second Price) mechanism. GSP is tailored to the unique requirements of SSA, and has quite different incentive properties than the original Vickrey auction, and has been extensively studied in recent years [Edelman et al. 2007; Varian 2007; Lahaie 2006; Aggarwal et al. 2006; Lahaie and Pennock 2007].

The analysis of the underlying SSA models has so far primarily focused on the scenario where advertisers/bidders interact directly with a primary auctioneer or AdNetwork, e.g., they bid for ad-space at leading search engine and publisher portals. The market, however, has evolved rapidly, and is already witnessing the spontaneous emergence of several categories of companies who are trying to mediate or facilitate the auction process. The main focus of such entities is to generate relevant leads or traffic for the advertisers. For example, a whole genre of companies, collectively referred to as the *online lead generation market*, specialize in aggregating traffic to their sites by bidding for keywords on major portals and search engines. Then, instead of selling services and products themselves, they have advertisers signed up on their sites to capture the funneled traffic. The exact pricing model for the leads sold at these sites varies a great deal, including cost-per-thousand impressions (CPM), CPC and Cost per Action (CPA), where "action" could imply completion of a certain transaction by the lead at the advertiser's site. Examples of such companies include, Oversee.net, LeadClick Media, ad pepper, ValueClick etc. and, according to IDC[1], the lead-generation market is the fastest growing segment of online advertising and in 2007 raked in more than $1.5 billion in revenues.

There are several *unexplored fundamental issues* that come up when one considers the *combined system* comprising both the primary auctioneers and the mediators. For example, in the above mentioned lead-generation scenario, advertisers have the choice to either

[1] http://www.gpbullhound.com/research.php, http://tmginteractive.com/Sector%20Report%20Online%20Lead %20Generation%20March%202007.pdf

directly place their ads on the search portals (i.e., the primary auctioneers), or buy leads from the mediators, or pursue both avenues. Then, are there inherent unmet demands or inefficiencies in the SSA model that enable mediators to fill an economic need and survive, or would in the long term the primary auctioneers simply change mechanisms or improve efficiency and take over the services provided by the mediators? How are the revenues of the primary auctioneers effected by the presence of the mediators? What are some of the mechanisms that the middlemen can use so as to carve out an efficient niche? In other words, we ask whether the primary auctioneers and the for-profit mediators can *coexist in an economic and game theoretic sense*, and if they do coexist, then what ramifications would it have on the overall efficiency and the utility of the advertisers.

In the present work, we adopt a fundamental approach, where we first identify an inherent feature of the sponsored search advertising market, namely *capacity constraints*, which could be key to the emergence of new market entities. This natural constraint in the SSA framework arises from the fact that there is a limit on the number of available advertisement slots (or *slots that receive any clicks from users*), especially for the *popular keywords*, and as a result, a significant pool of advertisers are left out. Consequently, new market mechanisms, as well as, new for-profit agents are likely to emerge to combat or to make profit from the opportunities created by scarcity in ad-space inventory. We show that this unmet need, i.e., the need of a large pool of advertisers to get leads, triggers a 3-fold diversification in the terms of

—the emergence of new market mechanisms
—the emergence of new for-profit agents, and
—the participation of a wider pool of bidders/advertisers.

First, we propose a model where the additional capacity is provided by for-profit agents (or, mediators), who compete for slots in the original auction, draw traffic, and run their own sub-auctions. We show that the revenue of the auctioneer, as well as the social value (i.e. efficiency), always increase when mediators are involved. Next, we ask the question- what if the auctioneer wants to provide the additional capacity herself by essentially acting herself as a mediator and running a *single combined auction*? Do the revenue of the auctioneer and overall efficiency improve or do they degrade in such a model? We show that, unlike the mediator-based model, there is often a tradeoff between the revenue and the capacity, and there is a phase transition from possibly a gain in terms of revenue to a loss as the *fitness* (a measure of the quality of the additional capacity) increases, meaning that there is a critical fitness value beyond which the auctioneer always loses in revenue. However, there exist scenarios where the revenue of the auctioneer could indeed increase by increasing capacity. In the case of efficiency, the result is more in consonance with the mediator-based model, i.e., the efficiency increases as fitness increases. However, unlike the mediator-based model, the efficiency could indeed decrease by increasing capacity.

Our results and analysis indicate that for-profit mediators that can increase capacity and ad inventory space can indeed coexist along with primary auctioneers. In fact, they add significantly to the overall efficiency and the utility of the advertisers. Thus, the SSA market becomes more *capacity efficient* by the involvement of such mediators, and we should expect such entities to proliferate and continue to thrive. The market, however, has had concerns about certain other kinds of mediators, particularly those that abuse inefficiencies present in the market, and both Google and Yahoo! have taken measures to actively discourage and eliminate such entities. As discussed in the following, the mediators that we have analyzed are very different in nature and can survive only if they enhance both user experience, and quality of traffic for the advertisers.

(1) Our model is motivated by a few examples from *Google Adwords*, provided in the *APPENDIX*. For example, the mediator "business.com" bids for the keyword "888 number" and then sells the leads at its own site via a second auction. Similarly, the mediator "personalloans.com" does the same for the keyword "easy loans". Of course, the real world mediators use mechanisms other than secondary auctions to sell the leads that they aggregate. The analysis of such combined systems (i.e., keyword auctions at the primary site and different pricing mechanisms at the mediators' sites) can also be carried out in a manner similar to the approach adopted in this paper.

(2) The success of a mediator depends on how well she creates the additional capacity, and how efficiently she sells them. The first aspect is captured by a *fitness* factor in our model, which is essentially a measure of the quality of the additional capacity provided by the mediator. The second aspect is captured by the value she derives by selling the additional capacity. Both these quantities are formally defined later in the paper. Intuitively, it is important that the fitness of the mediator be very good so that she can ensure a better value (i.e. revenue from selling additional capacity) and be competent in bidding for and obtaining a slot in the primary auction (i.e. at the search portal). Consequently, a mediator with poor fitness will not be able to survive in the market. Thus, the kind of mediators we study in this paper are specifically the ones who can efficiently create extra capacity (i.e., increase ad inventory) while enhancing user experience.

(3) In general, there could be several other inefficiencies in the SSA framework, and the market may naturally see the emergence of different kinds of for-profit agents as a result of these inefficiencies. For example, the tail queries or infrequent keywords, can easily comprise 40% or more of the total query volume at any search portal. Individually, each such keyword is difficult to identify, and even if identified, it does not have high enough volume to be attractive enough for advertisers to place bids on. Consequently, a significant fraction of queries are never matched to any advertisement, even though the users may have specific and well-defined commercial intent behind such queries. The existence of such high-volume but poorly monetized query traffic has led to the emergence of a *separate class* of mediators, collectively referred to as the search engine or *click arbitrage* sector. In an ideal world, such mediators could enhance user experience by better capturing *user intention*, i.e., by buying a large number of infrequent keywords with similar intent or from a particular vertical sector (e.g., health, finance, or travel), and then funneling the traffic to a site that shows relevant ads (i.e., on topics related to the users' original queries), but based on high-priced keywords. Since the infrequent keywords are cheaper to buy, the mediators can turn a profit by showing ads for keywords that are more popular, and hence more expensive.
In reality, however, capturing user intention in a large-scale fashion is a very difficult problem, and many such click arbitrageurs end up buying infrequent keywords at a cheap price and selling them at a higher price by taking the users to pages full of ads that are not necessarily related to the original keywords (and may be put by a different auctioneer), without regard to and often compromising user experience. Most of the time, users just click on these pricey but irrelevant ads to make their way out of those pages. Given that the pricing mechanism is PPC (pay-per-click) based, the advertisers

do pay for all such junk clicks, making fortunes for these arbitrageurs. In the long run, however, such an abuse of the inefficiency (i.e., the inability of the primary auctioneer to capture commercial user intention for a large enough fraction of query volume) is automatically eliminated as advertisers figure out the diminishing conversion rates, thereby decreasing their bids and paying much less or nothing for these junk clicks. Also, the auctioneers may take smart actions to ban such arbitrageurs (since the traffic from these arbitrage companies are of poor quality or being funneled to a competitor), and the company Geosign being banned by Google is a prime example of this[2]. *It is important to reiterate* that the mediators we discuss in this paper, however, *do not fall in this category of short term profiteers*.

Now we discuss the formal setup for the standard sponsored search auctions which will be helpful in the presentation of our model for creating additional capacity. Formally, in the current models, there are K slots to be allocated among N ($\geq K$) bidders (i.e. the advertisers). A bidder i has a true valuation v_i (known only to the bidder i) for the specific keyword and she bids b_i. The expected *click through rate* (CTR) of an ad put by bidder i when allocated slot j has the form $CTR_{i,j} = \gamma_j e_i$ i.e. separable in to a position effect and an advertiser effect. γ_j's can be interpreted as the probability that an ad will be noticed when put in slot j and it is assumed that $\gamma_j > \gamma_{j+1}$ for all $1 \leq j \leq K$ and $\gamma_j = 0$ for $j > K$. e_i can be interpreted as the probability that an ad put by bidder i will be clicked on if noticed and is referred to as the *relevance* of bidder i. The payoff/utility of bidder i when given slot j at a price of p per-click is given by $e_i\gamma_j(v_i - p)$ and they are assumed to be rational agents trying to maximize their payoffs.

As of now, Google as well as Yahoo! use schemes closely modeled as RBR(rank by revenue) with GSP(generalized second pricing). The bidders are ranked in the decreasing order of e_ib_i and the slots are allocated as per these ranks. For simplicity of notation, assume that the ith bidder is the one allocated slot i according to this ranking rule, then i is charged an amount equal to $\frac{e_{i+1}b_{i+1}}{e_i}$ per-click. This mechanism has been extensively studied in recent years [Edelman et al. 2007; Varian 2007; Lahaie 2006; Aggarwal et al. 2006; Lahaie and Pennock 2007]. The solution concept that is widely adopted to study this auction game is a refinement of Nash equilibrium independently proposed by Varian [Varian 2007] and Edelman et al [Edelman et al. 2007]. Under this refinement, the bidders have no incentive to change to another positions even at the current price paid by the bidders currently at that position. Edelmen et al [Edelman et al. 2007] calls it *locally envy-free equilibria* and argue that such an equilibrium arises if agents are raising their bids to increase the payments of those above them, a practice which is believed to be common in actual keyword auctions. Varian [Varian 2007] called it *symmetric Nash equilibria(SNE)* and provided some empirical evidence that the Google bid data agrees well with the SNE bid profile. In particular, an **SNE** bid profile b_i's satisfy

$$(\gamma_i - \gamma_{i+1})v_{i+1}e_{i+1} + \gamma_{i+1}e_{i+2}b_{i+2} \leq \gamma_i e_{i+1}b_{i+1} \leq (\gamma_i - \gamma_{i+1})v_ie_i + \gamma_{i+1}e_{i+2}b_{i+2} \tag{1}$$

for all $i = 1, 2, \ldots, N$. Now, recall that in the RBR with GSP mechanism, the bidder i pays an amount $\frac{e_{i+1}b_{i+1}}{e_i}$ per-click, therefore the expected payment i makes per-impression

[2]http://www.techdirt.com/articles/20080319/020719583.shtml, http://www.techcrunch.com/2008/03/18/how-geosign-blew-160-million/

is $\gamma_i e_i \frac{e_{i+1} b_{i+1}}{e_i} = \gamma_i e_{i+1} b_{i+1}$. Thus the best **SNE** bid profile for advertisers (worst for the auctioneer) is minimum bid profile possible according to Equation 1 and is given by

$$\gamma_i e_{i+1} b_{i+1} = \sum_{j=i}^{K} (\gamma_j - \gamma_{j+1}) v_{j+1} e_{j+1} \tag{2}$$

and therefore, the revenue of the auctioneer at this minimum *SNE* is

$$\sum_{i=1}^{K} \gamma_i e_{i+1} b_{i+1} \;=\; \sum_{i=1}^{K} \sum_{j=i}^{K} (\gamma_j - \gamma_{j+1}) v_{j+1} e_{j+1} = \sum_{j=1}^{K} (\gamma_j - \gamma_{j+1}) j v_{j+1} e_{j+1}. \tag{3}$$

For the comparative analysis, in the present work, we assume that the auction used to sell the original slots (i.e., without any additional capacity), the single combined auction in the auctioneer-based-model run by the primary auctioneer to sell the original slots together with the additional slots created by him, as well as, the two auctions in the mediator-based model (one run by the primary auctioneer to sell the original slots and the other run by the mediator to sell the additional slots created by her), are all run via RBR with GSP i.e. the mechanism currently being used by Google and Yahoo!. The solution concept we use is Symmetric Nash Equilibria(SNE)/locally envy-free equilibria [Edelman et al. 2007; Varian 2007]. Nevertheless, as evident from the intuition behind the proofs provided later in the paper, the results hold true for other interesting allocation and pricing mechanisms as well.

2. THE MODEL

We will refer to the scenario where the additional capacity is created by a for-profit mediator as **MDC** (**M**ediator **D**riven additional **C**apacity) and the scenario where the additional capacity is created by the auctioneer as **ADC** (**A**uctioneer **D**riven additional **C**apacity).

—**Additional/Secondary Slots:**

—***How are the slots created?*** In **MDC**, the mediator participates in the original auction run by the search engine (called *p-auction*) and competes with advertisers for slots (called *primary slots*). Suppose that in the p-auction, the slot assigned to the mediator is l, then effectively, the additional slots are obtained by forking this *primary slot* in to L additional slots, where $L \leq K$. By forking we mean the following: on the associated landing page the mediator puts some information relevant to the specific keyword associated with the p-auction along with the space for additional slots. Let us call these additional slots as *secondary slots*. In **ADC**, similarly, the additional slots are obtained by forking *one* of the original slots. Here, the auctioneer puts her own ad/link in that slot, and on the associated landing page, she puts some information relevant to the specific keyword along with space for additional slots. We consider the single fork case in **ADC** and single mediator case in **MDC** for the sake of simplicity of presentation and so that the calculations do not get unwieldy, but the results can be extended to the case where the auctioneer forks multiple slots (in **ADC**) and adds additional capacity, or there are more than one mediators involved (in **MDC**).

—***Fitness*** and ***New position based CTRs:*** The quality of the additional/secondary slots is measured by a *fitness factor*. Let the probability associated with the ad put by the auctioneer (in **ADC**) or the mediator (in **MDC**) for creating additional capacity to be clicked, if noticed, be denoted as $\tilde{f}$. In **MDC**, this is actually the relevance score of the mediator in the p-auction. Moreover, the position-based CTRs for the additional slots in the landing page will in general be different than on the main page, and it might actually improve, say by a factor of α. This means that the position based CTR for the jth additional slot on the associated landing page is modeled as $\alpha\gamma_j$. Therefore, we can define a fitness factor f to indicate the effective quality of additional slots being created, which is equal to $\tilde{f}\alpha$. Thus, if the original slot being forked is l, and there are L additional slots being created on the landing page, then the *effective* position based CTRs for the additional slots thus obtained are $\gamma_l f\gamma_1, \gamma_l f\gamma_2, \ldots, \gamma_l f\gamma_L$ respectively. Clearly, $f\gamma_1 < 1$; however, f itself could be greater than 1.

—**A single combined auction vs two uncoupled auctions:** The major difference in the two scenarios **MDC** and **ADC** is that in **MDC** there are two uncoupled auctions- the one run by the auctioneer to sell the primary slots where the mediator also competes for a slot (i.e. p-auction), and the other run by the mediator to sell the secondary slots (called s-auction), however in **ADC** there is a *single* auction run by the auctioneer to sell primary as well as secondary slots. Thus there is no s-auction in **ADC**. For the comparative analysis, we assume that the single combined auction in **ADC** as well as the two auctions in **MDC**, are all run via RBR with GSP (i.e. the mechanism currently being used by Google and Yahoo!) and the solution concept we use is Symmetric Nash Equilibria(SNE)/locally envy-free equilibria [Edelman et al. 2007; Varian 2007].

—***p-auction:*** In **MDC**, the mediator participates in the original auction run by the search engine and compete with advertisers for a primary slot. For the ith agent (an advertiser or a mediator), let v_i^p and b_i^p denote her true valuation and the bid for the p-auction respectively. Further, let us denote $v_i^p e_i^p$ by s_i^p where e_i^p is the relevance score of ith agent for p-auction. There are still K slots for this p-auction, and the position based CTRs are still the same as in the case without additional capacity.
In **ADC**, in the combined auction there are now $\tilde{K} = K + L - 1$ slots and for each slot there will be a probability of being noticed if an advertiser is assigned to that slot i.e. its position based CTR. We rename the slots in the decreasing order of their CTRs. That is, the jth slot is the one having jth maximum of the elements from the set $\{\gamma_1, \gamma_2, \ldots, \gamma_{l-1}, \gamma_{l+1}, \ldots, \gamma_K\} \cup \{\gamma_l f\gamma_1, \gamma_l f\gamma_2, \ldots, \gamma_l f\gamma_L\}$ and its CTR is denoted by $\tilde{\gamma}_j$. For the sake of simplicity, we assume that there are no ties i.e. no two slots have the same position based CTRs. Therefore, like γ_j's we have $\tilde{\gamma}_j > \tilde{\gamma}_{j+1}$ for all $1 \leq j \leq K + L - 1$ and $\tilde{\gamma}_j = 0$ for all $j \geq K + L$. Further note that, $\tilde{\gamma}_j = \gamma_j$ for $j \leq l - 1$, and $\tilde{\gamma}_l < \gamma_l$. Therefore $\tilde{\gamma}_j - \tilde{\gamma}_{j+1} = \gamma_j - \gamma_{j+1}$ for $j < l - 1$, $\tilde{\gamma}_{l-1} - \tilde{\gamma}_l > \gamma_{l-1} - \gamma_l$, and $\tilde{\gamma}_j - \tilde{\gamma}_{j+1}$ could be greater than or less than $\gamma_j - \gamma_{j+1}$ for $l \leq j \leq K$ depending on how the new position based CTRs are distributed among the old ones.

—***s-auction:*** In **ADC**, there is no s-auction. In **MDC**, the mediator runs her individual sub-auction for selling the secondary slots. For an advertiser there is another type of valuations and bids, the ones associated with s-auctions. For the ith agent, let v_i^s and b_i^s denote her true valuation and the bid for the s-auction respectively. In general, the two types of valuations or bids corresponding to p-auction and the s-auctions might differ a lot. We also assume that $v_i^s = 0$ and $b_i^s = 0$ whenever i is a mediator. Further, for the advertisers who do not participate in one auction (p-auction or s-auction), the corresponding true valuation and the bid are assumed to be zero. Also, for notational convenience let us denote $v_i^s e_i^s$ by s_i^s, where e_i^s is the relevance score of ith agent for the s-auction. Further, the s-auction is not coupled to the p-auction, meaning that the corresponding auctions are independent of each other in the sense that for a player who participates in both the auction games- the problem of maximizing the combined payoff from the two auctions is same as the problems of maximizing the payoffs from the individual auctions independently. This is indeed very reasonable in practice because the conversion rates (and consequently the valuations) derived at the two auction sites would be generally different, and further they have the flexibility of reporting different bid for the two auctions.

—***Freedom of participation:*** In **ADC**, since the auctioneer runs a single combined auction to sell original slots together with the additional ones, a bidder is allowed *only* to bid for all slots (original slots plus the additional ones) together and not for the two kind of slots individually. This is unlike in **MDC**, where the mediator runs her own sub-auction. Advertisers are free to bid for primary as well as secondary slots, and in general report two bid values - one to the auctioneer for the p-auction, and the other to the mediator for the s-auction.

—***True valuation of mediator:*** The true valuation of the mediator (for the p-auction) is derived from the expected revenue (total payments from advertisers) she obtains from her corresponding s-auction *ex ante*. This way of deriving the true valuation for the mediator is reasonable because, the mediator can participate in the p-auction several times and run her corresponding s-auction and can estimate the revenue she is deriving from the s-auction.

—**Capacity:** The *capacity* is defined as the sum of position based CTRs. Thus the capacity in the original model without the additional/secondary slots is $\sum_{j=1}^{K} \gamma_j$. In **ADC** or **MDC**, it is $\sum_{i=1, i \neq l}^{K} \gamma_j + \gamma_l f \sum_{i=1}^{L} \gamma_i$. Note that for a fixed L, l, the capacity increases iff f increases, for a fixed l, f, it increases iff L increases.

3. RESULTS

3.1 The MDC Scenario

We first discuss the change in the revenue of the auctioneer due to the involvement of the mediator and our observation as noted in Result 3.1 is that it always increases. Intuitively, when the mediator participates for buying the primary slots, it increases the competition in the p-auction and therefore the revenue of the auctioneer goes up. Thus, as long as the mediator has a good enough fitness that guarantees her a slot in the p-auction, the auctioneer definitely gains in terms of revenue. Further, the better the mediator's valuation is, the better slot the mediator gets allocated, bringing forth more gain in revenue. Besides keeping a good fitness factor f, there is another smart way for the mediator to improve her true valuation. She could actually run many subauctions related to the specific keyword in question. This can be done as follows: besides providing the additional slots on the landing page, the information section of the page could contain links to other pages wherein further additional slots associated with a related keyword could be provided[3].

RESULT 3.1. *Increasing the capacity via mediators improves the revenue of the auctioneer.*

For the formal proof of Result 3.1, we will first need to discuss the incentive properties of the two uncoupled auctions, the p-auction and the s-auction respectively, and in particular

[3] For example, the mediator "personalloans.com".

the bid profiles at their respective SNE's.

Suppose the allocations for the p-auction and s-auction are $\sigma : \{1, 2, \ldots, N\} \longrightarrow \{1, 2, \ldots, N\}$ and $\tau : \{1, 2, \ldots, N\} \longrightarrow \{1, 2, \ldots, N\}$ respectively. Then the payoff of the ith agent from the combined auction (p-auction and s-auction together) is

$$u_i = \gamma_{\sigma^{-1}(i)} \left(s_i^p - r^p_{\sigma^{-1}(i)+1} \right) + \tilde{\gamma}_{\tau^{-1}(i)} \left(s_i^s - r^s_{\tau^{-1}(i)+1} \right)$$

$$\text{where} \quad r_j^p = b^p_{\sigma(j)} e^p_{\sigma(j)}, \quad r_j^s = b^s_{\tau(j)} e^s_{\tau(j)}.$$

From the mathematical structure of payoffs and strategies available to the bidders wherein two different uncorrelated values can be reported as bids in the two types of auctions independently of each other (i.e. since the two auctions are uncoupled), it is clear that the equilibrium of the combined auction game is the one obtained from the equilibria of the p-auction game and the s-auction game each played in isolation. In particular at minimum *SNE* [Edelman et al. 2007; Varian 2007],

$$\gamma_i r^p_{i+1} = \sum_{j=i}^{K} (\gamma_j - \gamma_{j+1}) s^p_{\sigma(j+1)} \text{ for all } i = 1, 2, \ldots, K$$

and

$$\tilde{\gamma}_i r^s_{i+1} = \sum_{j=i}^{L} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) s^s_{\tau(j+1)} \text{ for all } i = 1, 2, \ldots, L$$

which implies that (recall that the *effective* position based CTRs for the secondary slots are $\gamma_l f \gamma_1, \gamma_l f \gamma_2, \ldots, \gamma_l f \gamma_L$ respectively)

$$\gamma_i r^s_{i+1} = \sum_{j=i}^{L-1} (\gamma_j - \gamma_{j+1}) s^s_{\tau(j+1)} + \gamma_L s^s_{\tau(L+1)} \text{ for all } i = 1, 2, \ldots, L \quad \text{where}$$

$$s^p_{\sigma(l)} = s^p_M = f \sum_{j=1}^{L} \gamma_j r^s_{j+1} = f \left(\sum_{j=1}^{L-1} (\gamma_j - \gamma_{j+1}) j s^s_{\tau(j+1)} + \gamma_L L s^s_{\tau(L+1)} \right)$$

is the true valuation of the mediator multiplied by her relevance score as per our definition, which is the expected revenue she derives from her s-auction *ex ante* given a slot in the p-auction.

Proof of Result 3.1: The revenue of the auctioneer with the participation of the mediator is

$$R = \sum_{j=1}^{K} \gamma_j r^p_{j+1} = \sum_{j=1}^{K} (\gamma_j - \gamma_{j+1}) j s^p_{\sigma(j+1)}$$

and similarly, the revenue of the auctioneer without the participation of the mediator is

$$\begin{aligned}
R_0 &= \sum_{j=1}^{K}(\gamma_j - \gamma_{j+1}) j s^p_{\tilde{\sigma}(j+1)} \\
&\quad \text{(where } \tilde{\sigma}(j) = \sigma(j) \text{ for } j < l \text{ and } \tilde{\sigma}(j) = \sigma(j+1) \text{ for } j \geq l) \\
&= \sum_{j=1}^{l-2}(\gamma_j - \gamma_{j+1}) j s^p_{\sigma(j+1)} + \sum_{j=l-1}^{K}(\gamma_j - \gamma_{j+1}) j s^p_{\sigma(j+2)} \\
\therefore R - R_0 &= \sum_{j=max\{1,l-1\}}^{K} (\gamma_j - \gamma_{j+1}) j (s^p_{\sigma(j+1)} - s^p_{\sigma(j+2)}) \geq 0
\end{aligned}$$

wherein the last inequality follows from the observation that
$s^p_{\sigma(i)} \geq s^p_{\sigma(i+1)} \forall i = 1, 2, \ldots, K+1$ at *SNE*. □

Now let us turn our attention to the change in the efficiency and as we will note below in the Result 3.2, the efficiency always improves by the participation of the mediator. The basic intuitions behind an increase in efficiency are that the allocation at *SNE* is an efficient one [Edelman et al. 2007; Varian 2007], and that the mediator brings in more value by accommodating more advertisers with a high *collective* value. Further, better the fitness and higher the values of advertisers in the s-auction, better the efficiency gain will be. Furthermore, it is implicit in our analysis that the *user experience*, measured in terms of total clickability, also improves when the fitness is good[4]. Thus, we can indeed say that the social welfare (i.e. the total welfare of all the parties involved) improves. Moreover, even the payoffs of all the advertisers will increase if the mediator has a high enough fitness (ref. APPENDIX).

RESULT 3.2. *Increasing the capacity via mediators improves the efficiency.*

Proof: Let E and E_0 denote the efficiency with and without the participation of the mediator respectively, then we have

$$\begin{aligned}
E_0 &= \sum_{j=1}^{K} \gamma_j s^p_{\tilde{\sigma}(j)} = \sum_{j=1}^{l-1} \gamma_j s^p_{\sigma(j)} + \sum_{j=l}^{K} \gamma_j s^p_{\sigma(j+1)}, \\
E &= \sum_{j=1}^{l-1} \gamma_j s^p_{\sigma(j)} + \sum_{j=l+1}^{K} \gamma_j s^p_{\sigma(j)} + \gamma_l f \sum_{j=1}^{L} \gamma_j s^s_{\tau(j)} \\
\therefore E - E_0 &= \gamma_l f \sum_{j=1}^{L} \gamma_j s^s_{\tau(j)} - \sum_{l}^{K}(\gamma_j - \gamma_{j+1}) s^p_{\sigma(j+1)} = \gamma_l f \sum_{j=1}^{L} \gamma_j s^s_{\tau(j)} - \gamma_l r^p_{l+1} \geq 0
\end{aligned}$$

wherein the last inequality holds becuase
$\gamma_l f \sum_{j=1}^{L} \gamma_j s^s_{\tau(j)} \geq \gamma_l f \sum_{j=1}^{L} \gamma_j r^s_{j+1} = \gamma_l s^p_{\sigma(l)} \geq \gamma_l r^p_{l+1}$ at *SNE* . □

[4] Athey and Ellison [2007] is an example of work that takes user experience explicitly into account, although not in the setting of the present paper.

3.2 The ADC Scenario

Now, we ask the question- what if the auctioneer wants to provide the additional capacity herself by essentially acting herself as a mediator and running a *single combined auction*? Does the revenue of the auctioneer/efficiency improve or does it degrade in such a model? Our observation is that, unlike the mediator-based model, often there is a tradeoff between the revenue and the capacity, and there is a phase transition from possibly a gain in terms of revenue to a loss as the *fitness* increases, meaning that there is a critical fitness value beyond which the auctioneer always loses in revenue. However, there exist scenarios where the revenue of the auctioneer could indeed increase by increasing capacity. The following results formalize some worst case scenarios. Some further discussion is provided in *APPENDIX*.

Remark: *Since there is no s-auction in the ADC scenario, for notational simplicity, henceforth we will drop the superscripts p. We also assume that the ith bidder is the one allocated slot i when ranked in the decreasing order of $e_i b_i$.*

RESULT 3.3. *Let s_i's satisfy $(j-1)s_j \geq js_{j+1}$ for all $j \geq 2$. Recall that l is the primary slot being forked in to additional slots.*

(*1*) *For $l = 1$, if γ_j's satisfy $(\gamma_1 - \gamma_2) \geq (\gamma_j - \gamma_{j+1})$ for all $1 \leq j \leq K$ then there exists no fitness factor f such that the revenue of auctioneer increases*[5].

(2) *For any $l \geq 2$, the gain in the revenue of the auctioneer is a decreasing function of f and L.*

Proof of Result 3.3(1): Let R and R_0 denote the revenue of the auctioneer with and without the additional capacity respectively. Let us define

$$i_0 = \max_{1\leq i\leq K} \{i : \gamma_1 f \gamma_1 < \gamma_i\}$$

then $\tilde{\gamma}_j = \gamma_{j+1}$ for all $1 \leq j \leq i_0 - 1$, $\tilde{\gamma}_{i_0} = \gamma_1 f \gamma_1$, and $\tilde{\gamma}_j \geq \gamma_j$ for all $j \geq i_0 + 1$. Clearly, $i_0 \geq 1$. Now,

$$\begin{aligned}
R_0 &= \sum_{j=1}^{K} (\gamma_j - \gamma_{j+1}) j s_{j+1} = \gamma_1 s_2 - \sum_{j=2}^{K} \gamma_j \left[(j-1)s_j - js_{j+1}\right] \\
R &= \tilde{\gamma}_1 s_2 - \sum_{j=2}^{K+L-1} \tilde{\gamma}_j \left[(j-1)s_j - js_{j+1}\right] \\
\therefore R - R_0 &= (\tilde{\gamma}_1 - \gamma_1)s_2 - \sum_{j=2}^{K} (\tilde{\gamma}_j - \gamma_j) \left[(j-1)s_j - js_{j+1}\right] \\
&\quad - \sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j \left[(j-1)s_j - js_{j+1}\right].
\end{aligned}$$

[5] Note that the conditions on γ_j's hold when they are geometrically decreasing (i.e. when $\gamma_j = r^{j-1}, 1 \leq j \leq K$ for some $r < 1$ and 0 otherwise), which is a very good approximation in practice [Abrams and Ghosh 2007; Feng et al. 2006].

Therefore, when $i_0 \geq 2$, we have

$$\begin{aligned}
R - R_0 \;=\; & -(\gamma_1 - \gamma_2)s_2 + \sum_{j=2}^{i_0-1}(\gamma_j - \gamma_{j+1})\left[(j-1)s_j - js_{j+1}\right] \\
& +(\gamma_{i_0} - \gamma_1 f\gamma_1)\left[(i_0-1)s_{i_0} - i_0 s_{i_0+1}\right] \\
& - \sum_{j=i_0+1}^{K}(\tilde{\gamma}_j - \gamma_j)\left[(j-1)s_j - js_{j+1}\right] - \sum_{j=K+1}^{K+L-1}\tilde{\gamma}_j\left[(j-1)s_j - js_{j+1}\right] \\
\leq\; & -(\gamma_1 - \gamma_2)s_2 + (\gamma_1 - \gamma_2)\left[s_2 - i_0 s_{i_0+1}\right] \\
& \text{(recall that } (\gamma_1 - \gamma_2) \geq (\gamma_j - \gamma_{j+1}) \text{ and } (j-1)s_j - js_{j+1} \geq 0 \;\; \forall\, j \geq 2\text{)} \\
=\; & -(\gamma_1 - \gamma_2)i_0 s_{i_0+1} < 0.
\end{aligned}$$

When $i_0 = 1$, we have

$$\begin{aligned}
R - R_0 \;=\; & -(\gamma_1 - \gamma_1 f\gamma_1)s_2 - \sum_{j=2}^{K}(\tilde{\gamma}_j - \gamma_j)\left[(j-1)s_j - js_{j+1}\right] \\
& - \sum_{j=K+1}^{K+L-1}\tilde{\gamma}_j\left[(j-1)s_j - js_{j+1}\right] \\
<\; & 0. \quad \square
\end{aligned}$$

Proof of Result 3.3(2): Let

$$i_0 = \max_{1 \leq i \leq K}\{i : \gamma_l f\gamma_1 < \gamma_i\}$$

then $\tilde{\gamma}_j = \gamma_j$ for all $1 \leq j \leq l-1$, $\tilde{\gamma}_j = \gamma_{j+1}$ for all $l \leq j \leq i_0 - 1$, $\tilde{\gamma}_{i_0} = \gamma_l f\gamma_1$, and $\tilde{\gamma}_j \geq \gamma_j$ for all $j \geq i_0 + 1$. Clearly, $i_0 \geq l \geq 2$.

$$\begin{aligned}
\therefore R \;=\; & \tilde{\gamma}_1 s_2 - \sum_{j=2}^{K+L-1}\tilde{\gamma}_j\left[(j-1)s_j - js_{j+1}\right] \\
=\; & \gamma_1 s_2 - \sum_{j=2}^{l-1}\gamma_j\left[(j-1)s_j - js_{j+1}\right] - \sum_{j=l}^{i_0-1}\gamma_{j+1}\left[(j-1)s_j - js_{j+1}\right] \\
& -\gamma_l f\gamma_1\left[(i_0-1)s_{i_0} - i_0 s_{i_0+1}\right] - \sum_{j=i_0+1}^{K+L-1}\tilde{\gamma}_j\left[(j-1)s_j - js_{j+1}\right]
\end{aligned}$$

Now let us increase f to $f^{'}$ and denote the new position based CTRs as $\tilde{\gamma}_j{}^{'}$'s and the new revenue of the auctioneer as $R^{'}$ then two cases arise - one where i_0 does not change and other where it changes to $i_0 - 1$.

Case 1: when i_0 does not change by increasing f to $f^{'}$. Clearly, $\tilde{\gamma}_j{}^{'} \geq \tilde{\gamma}_j$ for all $j \geq i_0 + 1$ as we will be choosing elements from a set with larger values. Also recall that s_i's satisfy $(j-1)s_j - js_{j+1} \geq 0$ for all $j \geq 2$. Therefore, the second last term in the expression of R strictly decrease and the last term also decreases and we get $R^{'} < R$.

Case 2: When i_0 changes to $i_0 - 1$ by increasing f to $f^{'}$. In this case, $\tilde{\gamma}_j^{'} = \gamma_j$ for all $1 \leq j \leq l-1$, $\tilde{\gamma}_j^{'} = \gamma_{j+1}$ for all $l \leq j \leq i_0 - 2$, $\tilde{\gamma}_{i_0-1}^{'} = \gamma_l f^{'} \gamma_1$, and $\tilde{\gamma}_j^{'} \geq \tilde{\gamma}_j$ for all $j \geq i_0$. Therefore,

$$\begin{aligned} R^{'} - R &= \gamma_{i_0} \left[(i_0 - 2) s_{i_0-1} - (i_0 - 1) s_{i_0} \right] - \gamma_l f^{'} \gamma_1 \left[(i_0 - 2) s_{i_0-1} - (i_0 - 1) s_{i_0} \right] \\ &\quad - \sum_{j=i_0}^{K+L-1} (\tilde{\gamma}_j^{'} - \tilde{\gamma}_j) \left[(j-1) s_j - j s_{j+1} \right] \\ &\leq (\gamma_{i_0} - \gamma_l f^{'} \gamma_1) \left[(i_0 - 2) s_{i_0-1} - (i_0 - 1) s_{i_0} \right] < 0. \quad \square \end{aligned}$$

Recall that the bidders are characterized by their true valuations v_i^p's and their relevance scores e_i's and $s_i = e_i v_i$. Thus, there is a wide pool of bidders satisfying the conditions in the above result, and therefore indicating a significant tradeoff between revenue of the auctioneer and the capacity. Intuitively, the conditions $(j-1)s_j \geq j s_{j+1}$ state that the s_i's are well separated, and therefore the payments that the bidders make at **SNE** are also well separated. Increasing capacity (via increasing f or L) means essentially selling a fraction of the clicks at a lower price. When s_i's are well separated, the extra revenue coming from the newly accommodated bidders still fall short of that lost due to lower payments from the other bidders. Further, the Result 3.3 suggests that there is a phase transition from possibly positive gain in the revenue to negative as f increases, and there is a critical f beyond which the auctioneer always loses.

Now let us look at the change in efficiency due to added capacity in **ADC**. In this case, the result is more in consonance with **MDC**, i.e., the efficiency increases as fitness increases. However, unlike in **MDC**, the efficiency could indeed decrease by increasing capacity.

RESULT 3.4. *The efficiency is an increasing function of fitness f.*

Proof: Let E denote the efficiency when additional capacity is added. Let us define

$$i_0 = \max_{1 \leq i \leq K} \{ i : \gamma_l f \gamma_1 < \gamma_i \}$$

then $\tilde{\gamma}_j = \gamma_j$ for all $1 \leq j \leq l-1$, $\tilde{\gamma}_j = \gamma_{j+1}$ for all $l \leq j \leq i_0 - 1$, $\tilde{\gamma}_{i_0} = \gamma_l f \gamma_1$, and $\tilde{\gamma}_j \geq \gamma_j$ for all $j \geq i_0 + 1$. Clearly, $i_0 \geq l$. Then,

$$E = \sum_{j=1}^{K+L-1} \tilde{\gamma}_j s_j = \sum_{j=1}^{l-1} \gamma_j s_j + \sum_{j=l}^{i_0-1} \gamma_{j+1} s_j + \gamma_l f \gamma_1 s_{i_0} + \sum_{j=i_0+1}^{K+L-1} \tilde{\gamma}_j s_j .$$

Now let us increase f to $f^{'}$ and denote the new position based CTRs as $\tilde{\gamma_j}^{'}$'s and the new efficiency as $E^{'}$ then two cases arise - one where i_0 does not change and other where it changes to $i_0 - 1$.

Case 1: when i_0 does not change by increasing f to $f^{'}$. Clearly, $\tilde{\gamma_j}^{'} \geq \tilde{\gamma}_j$ for all $j \geq i_0 + 1$ as we will be choosing elements from a set with larger values. Therefore, the second last term in the expression of E strictly increase and the last term also increases and we get $E^{'} > E$.

Case 2: When i_0 changes to $i_0 - 1$ by increasing f to $f^{'}$. In this case, $\tilde{\gamma}_j^{'} = \gamma_j$ for all $1 \leq j \leq l-1$, $\tilde{\gamma}_j^{'} = \gamma_{j+1}$ for all $l \leq j \leq i_0 - 2$, $\tilde{\gamma}_{i_0-1}^{'} = \gamma_l f^{'} \gamma_1$, and $\tilde{\gamma}_j^{'} \geq \tilde{\gamma}_j$ for all

$j \geq i_0$. Therefore,

$$\begin{aligned} E^{'} &= \sum_{j=1}^{l-1} \gamma_j s_j + \sum_{j=l}^{i_0-2} \gamma_{j+1} s_j + \gamma_l f^{'} \gamma_1 s_{i_0-1} + \sum_{j=i_0}^{K+L-1} \tilde{\gamma}_j s_j. \\ &\geq E + (\gamma_l f^{'} \gamma_1 - \gamma_{i_0}) s_{i_0-1} > E. \quad \square \end{aligned}$$

4. DISCUSSIONS

Having established some results on how improving the capacity via mediator driven model and auctioneer driven model effect the revenue and efficiency, we would now like to compare the two models in terms of these two parameters, which are considered two fundamental bench-marking metrics in mechanism design theory [Krishna 2002].

First, recall that the auctioneer's revenue always increases in **MDC** (Result 3.1) and in fact the revenue of the auctioneer increases as the *fitness* of the mediator increases, thus there is no conflict between the revenue and the capacity in that scenario. However, as we saw that in the **ADC** often there is a tradeoff between the revenue and the capacity (Result 3.3). Therefore, in terms of revenue, **MDC** is superior to **ADC**. A typical tradeoff curve is shown in the Figure 1. Further, recall that the efficiency always increases in **MDC** and in fact the efficiency increases as the *fitness* of the mediator increases, thus there is no conflict between the efficiency and the capacity in that scenario. However, in **ADC**, although the efficiency increases when fitness increases, it could go well below the efficiency in the scenario without any additional capacity. A typical tradeoff curve is shown in the Figure 1. Therefore, even in terms of efficiency the **MDC** is superior to **ADC**. Hence, we can conclude that the **ADC** is indeed inferior to the **MDC** and the market becomes more capacity efficient by the participation of mediators.

It is instructive to note that, in the **MDC** scenario, as long as the p-auction and the s-auction are not coupled, and the traffic for the s-auction site is drawn from the p-auction site, it does not matter who adds capacity and runs the s-auction, a mediator or the primary auctioneer. But in order to add the necessary capacity and run the secondary auctions effectively, the mediators have to specialize in the particular sector (e.g., loans, finance, business logistics etc.) that they are adding capacity to. Thus, if the primary auctioneer (e.g., a search engine portal) wants to also play the role of the mediator then it will also have to develop the necessary sales force and business infrastructure. For example, if Google wanted to take on the role of business.com and personalloans.com (see APPENDIX) then it will have to develop a support and sales force that will reach out to small-businesses and to loan companies, which might detract it from its core business. Thus, we expect that separate mediator entities, specializing in different sectors, will continue to coexist with the giant search portals, who specialize on being the primary auctioneers and the source of primary traffic or leads. Our results are further confirmed by a recent empirical study [Gunawardana et al. 2008].

APPENDIX

A. MDC: ADVERTISERS' PAYOFFS

Clearly, for the newly accommodated advertisers, that is the ones who lost in the p-auction but win a slot in s-auction, the payoffs increase from zero to a positive number. Now let us see where do these improvements in the revenue of the auctioneer (Result 1), in payoffs of

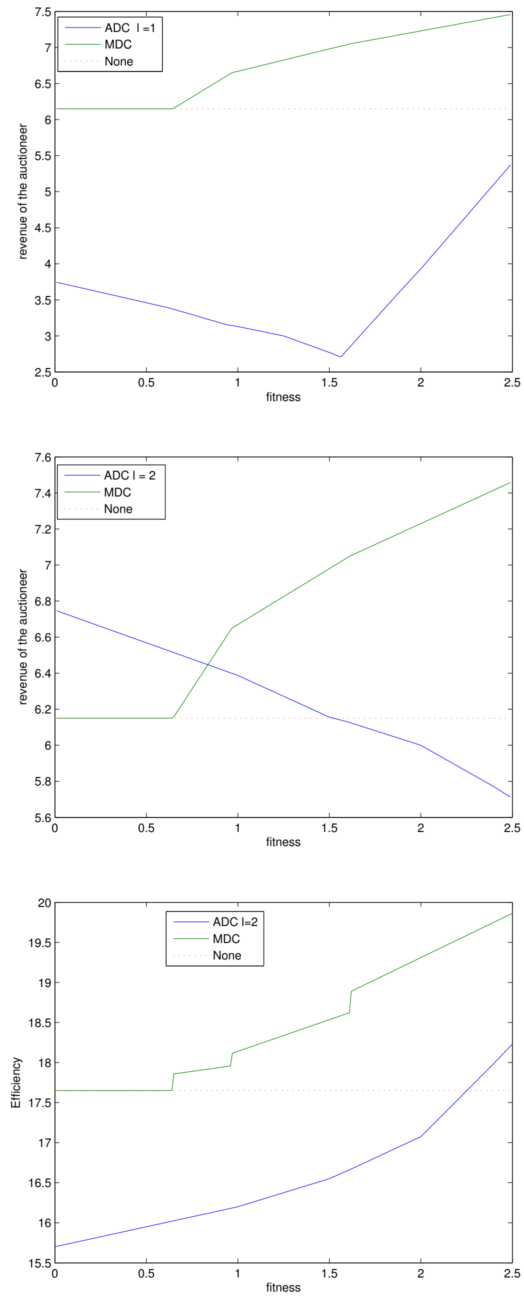

Fig. 1. Tradeoff curves for Auctioneer's Revenue/Efficiency: MDC vs ADC. The data used is the following: $N = 8, K = 5, L = 3, \gamma = [0.4 \quad 0.25 \quad 0.2 \quad 0.15 \quad 0.10]$,
$s^p = [25 \quad 20 \quad 8 \quad 5 \quad 3 \quad 2 \quad 1.5 \quad 1], s^s = [0 \quad 0 \quad 10 \quad 4 \quad 3 \quad 6 \quad 0 \quad 0]$.

newly accommodated advertisers, and in the efficiency (Result 2) come from? Only thing left to look at is the change in the payoffs for the advertisers who originally won in the p-auction, that is the winners when there was no mediator. The new payoff for jth ranked advertiser in p-auction is

$$u_{\sigma(j)} = \gamma_j s^p_{\sigma(j)} - \sum_{i=j}^{K} (\gamma_i - \gamma_{i+1}) s^p_{\sigma(i+1)} + u^s_{\sigma(j)}$$

where

$$u^s_{\sigma(j)} = \gamma_l f \gamma_{\tau^{-1}(\sigma(j))} \left(s^s_{\sigma(j)} - r^s_{\tau^{-1}(\sigma(j))+1} \right)$$

is her payoff from the s-auction. Also, for $j \leq l-1$, her payoff when there was no mediator is

$$\begin{aligned} u^0_{\sigma(j)} &= \gamma_j s^p_{\sigma(j)} - \sum_{i=j}^{K} (\gamma_i - \gamma_{i+1}) s^p_{\tilde{\sigma}(i+1)} \\ &= \gamma_j s^p_{\sigma(j)} - \sum_{i=j}^{l-2} (\gamma_i - \gamma_{i+1}) s^p_{\sigma(i+1)} - \sum_{i=l-1}^{K} (\gamma_i - \gamma_{i+1}) s^p_{\sigma(i+2)}. \\ \therefore u_{\sigma(j)} - u^0_{\sigma(j)} &= u^s_{\sigma(j)} - \sum_{i=l-1}^{K} (\gamma_i - \gamma_{i+1})(s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)}) \end{aligned}$$

Similarly, for $j \geq l+1$, her payoff when there was no mediator is

$$\begin{aligned} u^0_{\sigma(j)} &= \gamma_{j-1} s^p_{\sigma(j)} - \sum_{i=j-1}^{K} (\gamma_i - \gamma_{i+1}) s^p_{\sigma(i+2)} \\ \therefore u_{\sigma(j)} - u^0_{\sigma(j)} &= u^s_{\sigma(j)} - \sum_{i=j-1}^{K} (\gamma_i - \gamma_{i+1})(s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)}) \end{aligned}$$

Therefore, in general we have,

$$u_{\sigma(j)} - u^0_{\sigma(j)} = u^s_{\sigma(j)} - \sum_{i=max\{l-1,j-1\}}^{K} (\gamma_i - \gamma_{i+1})(s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)}).$$

Thus, for the jth ranked winning advertiser from the auction without mediation, the revenue from the p-auction decreases by $\sum_{i=max\{l-1,j-1\}}^{K} (\gamma_i - \gamma_{i+1})(s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)})$ and she faces a loss unless compensated for by her payoffs in s-auction. Further, this payoff loss will be visible only to the advertisers who joined the auction game before the mediator and they are likely to participate in the s-auction so as to make up for this loss. Thus, via the mediator, a part of the payoffs of the originally winning advertisers essentially gets distributed among the newly accommodated advertisers. However, when the mediator's fitness factor f is very good, it might be a win-win situation for everyone. Depending on how good the fitness factor f is, sometimes the payoff from the s-auction might be enough to compensate for any loss by accommodating new advertisers. Let us consider an extreme situation when $L = K$ and $\tau = \tilde{\sigma}$. The *gain* in payoff for the advertiser $\sigma(j)$ is

$$\gamma_l f \sum_{i=j}^{K} (\gamma_i - \gamma_{i+1})(s^s_{\sigma(j)} - s^s_{\sigma(i+1)}) - \sum_{i=max\{l-1,j-1\}}^{K} (\gamma_i - \gamma_{i+1})(s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)})$$

Therefore as long as

$$f \geq \frac{\sum_{i=max\{l-1,j-1\}}^{K} (\gamma_i - \gamma_{i+1})(s^p_{\sigma(i+1)} - s^p_{\sigma(i+2)})}{\gamma_l \sum_{i=j}^{K} (\gamma_i - \gamma_{i+1})(s^s_{\sigma(j)} - s^s_{\sigma(i+1)})}$$

the advertiser $\sigma(j)$ faces no net loss in payoff and might actually gain.

B. MDC: EXAMPLES OF FOR-PROFIT MEDIATORS

Please refer to the Figures 2, 3, 4, 5.

C. ADC

Remark: **Since there is no s-auction in the ADC scenario, for notational simplicity, henceforth we will drop the superscripts p. We also assume that the ith bidder is the one allocated slot i when ranked in the decreasing order of $e_i b_i$.**

Value of capacity:

Definition C.1. Let R_0 be the original revenue of the auctioneer without added capacity and R be the new revenue of the auctioneer after adding capacity at their corresponding minimum *SNE* [Edelman et al. 2007; Varian 2007], then the "value of capacity" is defined as $\frac{R-R_0}{R_0}$ i.e. the relative gain in the revenue of auctioneer per impression.

OBSERVATION C.2. *For a given L, if* $\exists l \leq K$ *such that*

$$\eta > 1 - \left(\frac{(\gamma_l - \tilde{\gamma}_l)(l-1)s_l + \sum_{j=K+1}^{K+L-1} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) j s_{j+1}}{\sum_{j=l}^{K} (\gamma_j - \gamma_{j+1}) j s_{j+1}} \right) \quad \textit{where}$$
$$\eta = \min_{K \geq j \geq l} \frac{\tilde{\gamma}_j - \tilde{\gamma}_{j+1}}{\gamma_j - \gamma_{j+1}},$$

then the value of capacity is positive, i.e., revenue of the auctioneer increases by adding capacity.

PROOF. Let $\eta = \min_{l \leq j \leq K} \frac{\tilde{\gamma}_j - \tilde{\gamma}_{j+1}}{\gamma_j - \gamma_{j+1}}$ then we have $\tilde{\gamma}_j - \tilde{\gamma}_{j+1} \geq \eta(\gamma_j - \gamma_{j+1})$ for $l \leq j \leq K$. At their corresponding minimum *SNE* [Edelman et al. 2007; Varian 2007], the original revenue of the auctioneer without added capacity and the new revenue of the auctioneer after adding capacity are $R_0 = \sum_{j=1}^{K} (\gamma_j - \gamma_{j+1}) j s_{j+1}$ and $R = \sum_{j=1}^{\tilde{K}} (\tilde{\gamma}_j -$

Fig. 2. For-Profit Mediator: Shaded links are the ads (the primary slots), and the doubly shaded link is the ad of the mediator personalloans.com

Fig. 3. Secondary slots at personalloans.com

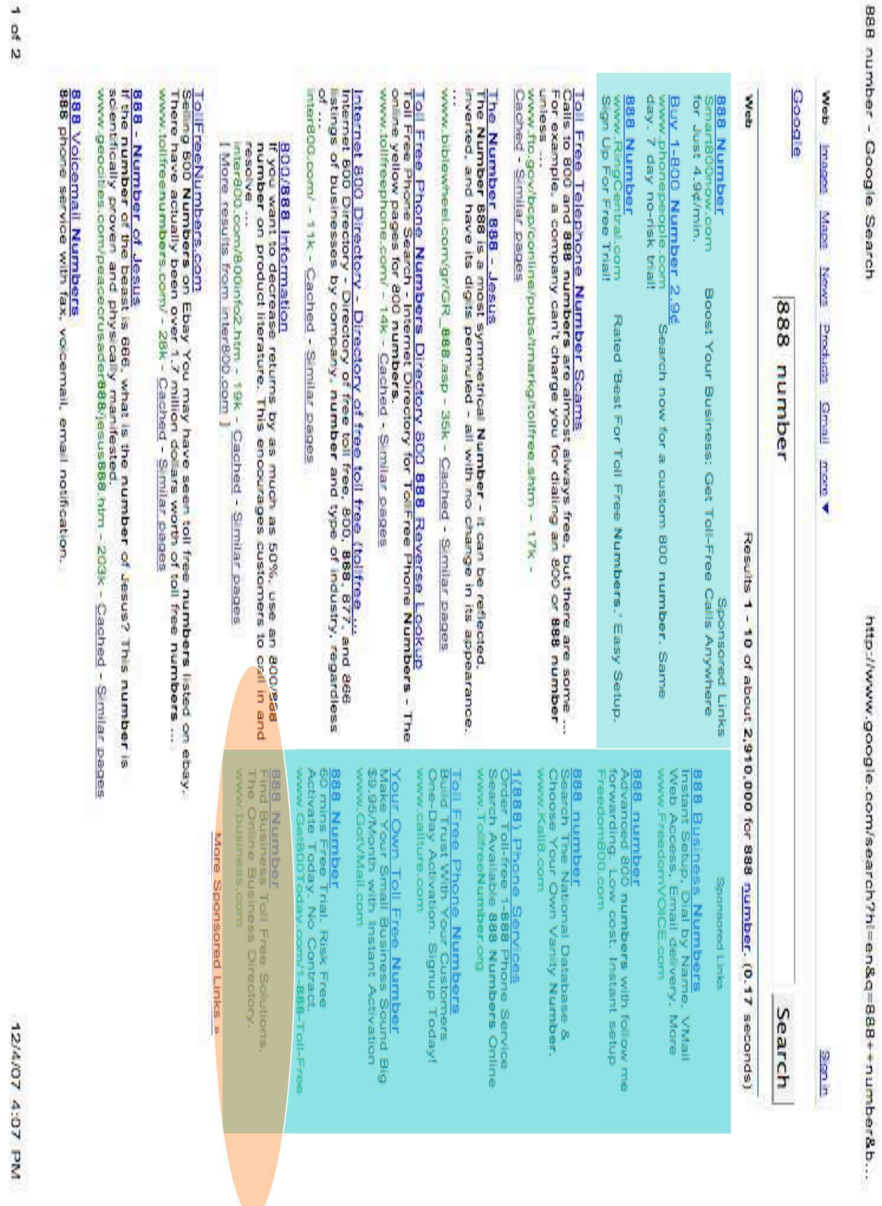

Fig. 4. For-Profit Mediator: business.com

Fig. 5. Secondary slots at business.com

$\tilde{\gamma}_{j+1})js_{j+1}$ respectively.

$$
\begin{aligned}
\therefore \quad & R - R_0 \\
&= \sum_{j=1}^{K} \left[(\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) - (\gamma_j - \gamma_{j+1})\right] js_{j+1} + \sum_{j=K+1}^{\tilde{K}} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) js_{j+1} \\
&= \sum_{j=l-1}^{K} \left[(\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) - (\gamma_j - \gamma_{j+1})\right] js_{j+1} + \sum_{j=K+1}^{\tilde{K}} (\tilde{\gamma}_j - \widetilde{\gamma_{j+1}})) js_{j+1} \\
&\geq (\gamma_l - \tilde{\gamma}_l)(l-1)s_l + \sum_{j=l}^{K} (\eta - 1)(\gamma_j - \gamma_{j+1}) js_{j+1} + \sum_{j=K+1}^{\tilde{K}} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) js_{j+1} \\
&= (\gamma_l - \tilde{\gamma}_l)(l-1)s_l + \sum_{j=K+1}^{\tilde{K}} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) js_{j+1} - (1-\eta) \sum_{j=l}^{K} (\gamma_j - \gamma_{j+1}) js_{j+1}
\end{aligned}
$$

and hence follows the observation.

We now provide an example to confirm that the above observation does not give a vacuous sufficient condition and the *value of capacity* can indeed be positive.

Example C.3. Let $l = K$ and geometrically decreasing γ_j's [Abrams and Ghosh 2007; Feng et al. 2006] i.e. $\gamma_j = r^{j-1}, 1 \leq j \leq K$ for some $r < 1$ and 0 otherwise. Then $\tilde{\gamma}_j = r^{j-1}$ for $1 \leq j \leq K-1$ and $\tilde{\gamma}_j = fr^{j-1}$ for $K \leq j \leq K+L-1$ and 0 otherwise. Also, let $js_{j+1} \geq (j-1)s_j$ for all $K+1 \leq j \leq K+L-1$ and $(K-1)s_K > Ks_{K+1}$. Then, the condition for Observation C.2 is satisfied. Detailed calculations are provided below.

We have

$$\eta = \min_{K \geq j \geq l} \frac{\tilde{\gamma}_j - \tilde{\gamma}_{j+1}}{\gamma_j - \gamma_{j+1}} = \frac{fr^{K-1}(1-r)}{r^{K-1}} = f(1-r).$$

Now,

$$
\begin{aligned}
& (\gamma_l - \tilde{\gamma}_l)(l-1)s_l + \sum_{j=K+1}^{K+L-1} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) js_{j+1} \\
& \quad \geq (\gamma_K - \tilde{\gamma}_K)(K-1)s_K + Ks_{K+1}(\tilde{\gamma}_{K+1} - \tilde{\gamma}_{K+L}) \\
& \quad = r^{K-1}(1-f)(K-1)s_K + fr^K Ks_{K+1}.
\end{aligned}
$$

Also

$$\sum_{j=l}^{K} (\gamma_j - \gamma_{j+1}) js_{j+1} = r^{K-1} Ks_{K+1} \quad (\text{as } l = K)$$

$$\begin{aligned}
\therefore \quad & 1 - \left(\frac{(\gamma_l - \tilde{\gamma}_l)(l-1)s_l + \sum_{j=K+1}^{K+L-1} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) j s_{j+1}}{\sum_{j=l}^{K} (\gamma_j - \gamma_{j+1}) j s_{j+1}} \right) \\
& \leq 1 - \frac{r^{K-1}(1-f)(K-1)s_K + f r^K K s_{K+1}}{r^{K-1} K s_{K+1}} \\
& = 1 - \left(\frac{(K-1)s_K}{K s_{K+1}} (1-f) + fr \right) \\
& = f - fr + (1-f) - \frac{(K-1)s_K}{K s_{K+1}} (1-f) \\
& = f(1-r) + (1-f) \left(1 - \frac{(K-1)s_K}{K s_{K+1}} \right) \\
& < f(1-r) = \eta.
\end{aligned}$$

OBSERVATION C.4. *For a given L, if* $\exists l \leq K$ *such that*

$$\beta > 1 - \frac{\sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j s_j}{\sum_{j=l}^{K} \gamma_j s_j} \quad \textit{where } \beta = \min_{K \geq j \geq l} \frac{\tilde{\gamma}_j}{\gamma_j}, \tag{4}$$

then the efficiency improves.

Proof: We have

$$\begin{aligned}
E_0 &= \sum_{j=1}^{K} \gamma_j s_j, \quad E = \sum_{j=1}^{K+L-1} \tilde{\gamma}_j s_j = \sum_{j=1}^{l-1} \gamma_j s_j + \sum_{j=l}^{K+L-1} \tilde{\gamma}_j s_j. \\
\therefore E - E_0 &= \sum_{j=l}^{K} (\tilde{\gamma}_j - \gamma_j) s_j + \sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j s_j.
\end{aligned}$$

Let

$$\beta = \min_{K \geq j \geq l} \frac{\tilde{\gamma}_j}{\gamma_j}$$

then

$$E - E_0 \geq \sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j s_j - (1-\beta) \sum_{j=l}^{K} \gamma_j s_j.$$

and hence follows the observation. □

We now provide an example to confirm that the above observation does not give a vacuous sufficient condition and the efficiency can indeed improve.

Example C.5. Let $l = K$ and geometrically decreasing γ_j's as in Example C.3 and let s_i's satisfy $s_{K+j} = \alpha^j s_K$ for some $\alpha < 1$. Then, the condition for Observation C.4 is satisfied when $f > \left(\frac{1-\alpha r}{1-\alpha^L r^L} \right)$. Detailed calculations are provided below.

$$\beta = \frac{\tilde{\gamma}_K}{\gamma_K} = \frac{f r^{K-1}}{r^{K-1}} = f.$$

Also,

$$
\begin{aligned}
\sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j s_j &= \sum_{j=K+1}^{K+L-1} f r^{j-1} s_j \\
&= f r^K s_K \sum_{j=1}^{L-1} r^{j-1}\alpha^j = \alpha f r^K s_K \left(\frac{1 - r^{L-1}\alpha^{L-1}}{1 - r\alpha}\right) \\
\therefore 1 - \frac{\sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j s_j}{\sum_{j=l}^{K} \gamma_j s_j} &= 1 - \frac{\alpha f r^K s_K \left(\frac{1 - r^{L-1}\alpha^{L-1}}{1 - r\alpha}\right)}{r^{K-1} s_K} \\
&= 1 - \alpha f r \left(\frac{1 - r^{L-1}\alpha^{L-1}}{1 - r\alpha}\right) = \frac{1 - r\alpha - \alpha r f + f r^L \alpha^L}{1 - r\alpha} \\
&= \frac{f(1 - r\alpha) + 1 - f - \alpha r + f r^L \alpha^L}{1 - r\alpha} = f + \left(1 - f\frac{1 - r^L\alpha^L}{1 - \alpha r}\right) \\
&< f \text{ if } f > \frac{1 - \alpha r}{1 - r^L \alpha^L}. \quad \square
\end{aligned}
$$

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