

# Pre-Calculus 11 Chapter 6 Rational Expressions and Equations.

Date: \_\_\_\_\_ Block: \_\_\_\_\_ Name: key

## Lesson Notes 6.2: Multiplying and Dividing Rational Expressions

### Objectives:

- comparing operations on rational expressions to the same operations on rational numbers
- identifying non-permissible values when performing operations on rational expressions
- determining the product or quotient of rational expressions in simplest form

### Multiplying Rational Expressions

When you multiply rational expressions, you follow procedures **similar** to those for multiplying rational numbers.

$$\left(\frac{5}{8}\right)\left(\frac{4}{15}\right) = \frac{(5)(4)}{(8)(15)} = \frac{(5)(4)}{(2)(4)(3)(5)}$$

$$= \frac{\overset{1}{\cancel{5}}\overset{1}{\cancel{4}}}{2\underset{1}{\cancel{4}}(3)\underset{1}{\cancel{5}}} = \frac{1}{6}$$

$$\left(\frac{4x^2}{3xy}\right)\left(\frac{y^2}{8x}\right) = \frac{(4x^2)(y^2)}{(3xy)(8x)}$$

$$= \frac{\overset{1}{\cancel{4}}x^{\overset{1}{\cancel{2}}}\overset{y}{y^{\overset{1}{\cancel{2}}}}}{\underset{6}{\cancel{3}}x^{\overset{1}{\cancel{1}}}\underset{1}{\cancel{y}}}} = \frac{y}{6}, x \neq 0, y \neq 0$$

Values for the variables that result in any denominator of zero are nonpermissible. Division by zero is not defined in the real-number system.

Example 1) Multiply. Write your answer in simplest form. Identify all non-permissible values.

a)  $\frac{a^2 - a - 12}{a^2 - 9} \times \frac{a^2 - 4a + 3}{a^2 - 4a}$

$$= \frac{\cancel{(a+3)}\cancel{(a-4)}}{\cancel{(a+3)}\cancel{(a-3)}} \cdot \frac{\cancel{(a-1)}\cancel{(a-3)}}{a\cancel{(a-4)}} = \boxed{\frac{a-1}{a}}$$

non-perm.  
 $a \neq -3$   
 $a \neq 3$   
 $a \neq 0$   
 $a \neq 4$

b)  $\frac{d}{2\pi r} \times \frac{2\pi rh}{(d-2)}$

$$= \frac{d \cdot \cancel{2\pi} \cancel{h}}{\cancel{2\pi} \cancel{r} \cdot (d-2)}$$

$$= \frac{dh}{(d-2)}$$

non-perm.  
 $r \neq 0$   
 $d \neq 2$

c)  $\frac{y^2 - 9}{r^3 - r} \times \frac{r^2 - r}{y + 3} = \frac{\cancel{(y+3)}\cancel{(y-3)}}{\cancel{r}\cancel{(r-1)}(r+1)} \cdot \frac{\cancel{r}\cancel{(r-1)}}{\cancel{(y+3)}}$

$$= \frac{y-3}{r+1}$$

non-perm.  
 $r \neq 0, r \neq 1, r \neq -1, y \neq -3$

## Dividing Rational Expressions

Dividing rational expressions follows similar procedures to those for dividing rational numbers.

### Multiply by the Reciprocal

$$\frac{5}{3} \div \frac{1}{6} = \frac{5}{3} \times \frac{6}{1} = 10$$

$$\frac{3x^2}{y^2} \div \frac{x}{y} = \frac{3x^2}{y^2} \times \frac{y}{x} = \frac{3x}{y}, x \neq 0, y \neq 0$$

Example 2) Determine the quotient in simplest form. Identify all non-permissible values.

a)  $\frac{x^2 - 4}{x^2 - 4x} \div \frac{x^2 + x - 6}{x^2 + x - 20}$

$$= \frac{(x+2)(x-2)}{x(x-4)} \div \frac{(x+3)(x-2)}{(x+5)(x-4)}$$

$$= \frac{(x+2)\cancel{(x-2)}}{x\cancel{(x-4)}} \cdot \frac{(x+5)\cancel{(x-4)}}{(x+3)\cancel{(x-2)}}$$

$$= \frac{(x+2)(x+5)}{x(x+3)}$$

non permissible:  
 $x \neq 0$   
 $x \neq 4$   
 $x \neq -5$   
 $x \neq -3$   
 $x \neq 2$

b)  $\frac{c^2 - 6c - 7}{c^2 - 49} \div \frac{c^2 + 8c + 7}{c^2 + 7c}$

$$= \frac{(c+1)(c-7)}{(c+7)(c-7)} \div \frac{(c+7)(c+1)}{c(c+7)}$$

$$= \frac{\cancel{(c+1)}\cancel{(c-7)}}{(c+7)\cancel{(c-7)}} \cdot \frac{c\cancel{(c+7)}}{\cancel{(c+7)}\cancel{(c+1)}}$$

$$= \frac{c}{c+7}$$

non permissible:  
 $c \neq -7$   
 $c \neq 7$   
 $c \neq 0$   
 $c \neq -1$

Example 3) Simplify. What are the non-permissible values?

$$\begin{aligned}
 \text{a) } \frac{2m^2 - 7m - 15}{2m^2 - 10m} \div \frac{4m^2 - 9}{6} \times (3 - 2m) &= \frac{(2m+3)(m-5)}{2m(m-5)} \div \frac{(2m+3)(2m-3)}{6} \cdot \underline{(3-2m)} \\
 &= \frac{\cancel{(2m+3)}(m-5)}{2m\cancel{(m-5)}} \cdot \frac{6}{\cancel{(2m+3)}\cancel{(2m-3)}} \cdot \cancel{(3-2m)}(-1)\cancel{(2m-3)} \\
 &= \frac{-6}{2m} = \boxed{\frac{-3}{m}}
 \end{aligned}$$

non-permi.  
 $m \neq 0$   
 $m \neq 5$   
 $m \neq \frac{-3}{2}$   
 $m \neq \frac{3}{2}$

$$\begin{aligned}
 \text{b) } \frac{3x+12}{3x^2-5x-12} \div \frac{12}{3x+4} \times \frac{2x-6}{x+4} \\
 &= \frac{3(x+4)}{(3x+4)(x-3)} \div \frac{12}{(3x+4)} \cdot \frac{2(x-3)}{x+4} \\
 &= \frac{3\cancel{(x+4)}}{\cancel{(3x+4)}(x-3)} \cdot \frac{\cancel{(3x+4)}}{12} \cdot \frac{2\cancel{(x-3)}}{\cancel{(x+4)}} \\
 &= \frac{6}{12} = \boxed{\frac{1}{2}}
 \end{aligned}$$

non-perm.  
 $x \neq \frac{-4}{3}$   
 $x \neq 3$   
 $x \neq -4$

Example 4) Two points on a coordinate grid are represented by M ( $p-1, 2p+3$ ) and N ( $2p-5, p+1$ ).

- a) What is a simplified rational expression for the slope of the line passing through M and N?  
 b) Write a rational expression for the slope of any line that is perpendicular to MN.

$$\begin{aligned}
 \text{a) slope } MN &= \frac{\Delta Y}{\Delta X} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(2p+3) - (p+1)}{(p-1) - (2p-5)} \\
 &= \frac{2p+3-p-1}{p-1-2p+5} = \boxed{\frac{p+2}{-p+4}} = -\boxed{\frac{p+2}{p-4}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \left. \begin{array}{l} l_1 = m_1 \\ l_2 = m_2 \end{array} \right\} l_1 \perp l_2 &= * \boxed{m_1 \cdot m_2 = -1} \\
 &= \boxed{m_1 = \frac{-1}{m_2}} \text{ or } \boxed{m_2 = \frac{-1}{m_1}}
 \end{aligned}$$

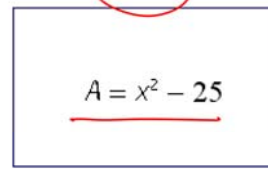
$$m_{MN} = \frac{p+2}{-(p-4)}$$

$$m_{\perp} = \ominus \left[ \frac{\ominus(p-4)}{p+2} \right] = \boxed{\frac{p-4}{p+2}}$$

Example 5) Write an expression to represent the length of the rectangle. Simplify your answer.

$$A = L \cdot W \Rightarrow \frac{A}{W} = L$$

$$L = \frac{x^2 - 25}{x - 4} \div \frac{x^2 + x - 20}{x - 4}$$



$$W = \frac{x^2 + x - 20}{x - 4}$$

$$= \frac{(x-5)(x+5)}{x-4} \div \frac{(x-4)(x+5)}{x-4} = \frac{(x-5)(x+5)}{1} \cdot \frac{(x-4)}{(x-4)(x+5)}$$

$$L = \boxed{x-5}$$

Example 6) What is an expression for the area of  $\triangle PQR$ ? Give your answer in simplest form.

$$\text{Area } \triangle = \frac{1}{2} \cdot b \cdot h$$

$$= \frac{1}{2} \cdot \frac{x^2 - 2x - 24}{x^2 - 9} \cdot \frac{x+3}{x-6}$$

$$= \frac{1}{2} \cdot \frac{(x-6)(x+4)}{(x-3)(x+3)} \cdot \frac{(x+3)}{(x-6)}$$

$$= \frac{(x+4)}{2(x-3)}$$

