natural MATHS STRATEGIES

BOOK 3

9-11 year olds

CD-ROM PLANNER inside

By Ann and Johnny Baker

about the AUTHORS

Ann and Johnny Baker are partners in Natural Maths, an educational publishing and consultancy business. The aim of Natural Maths is to bridge the gap between educational research and classroom practice in maths education. Their focus is on the effective implementation of mental strategies with real-life connections in the classroom. They have written more than 40 books together as well as many articles. They also present at Gifted and Talented days of excellence and maths camps, and are currently working together to create exciting learning and teaching environments involving interactive whiteboards.

Ann is passionate about maths and pedagogy. Her mission is to engage students in worthwhile and realistic mathematics as well as to raise the intellectual quality of maths lessons. She believes it is vital that students have strategies for mental computation and develop the disposition to work through problem solving situations and investigations. Her frequent work in classrooms ensures that all of the strategies and activities she presents are tried and tested.

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Ann and Johnny live in the Gold Coast Hinterland with their two sons and a small menagerie of pets and delight in the local flora and fauna that share their home and garden.

Natural Maths Strategies – Book 3
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Maths Planner

The Maths Planner is a software package provided at the front of this book which supports the maintenance of planned units of work to cover the syllabus. The Planner has facilities for:

- incorporating the activities from this book and other schemes into a complete mathematics progression
- maintaining assessment records in conjunction with annotated work samples
- summarising class and individual progress.

Assessment Guide

Each unit contains two examples of the type of assessment activity that will allow students to show their understanding of a particular topic. Examples of student responses to these and to the problematised situations are given in the associated publications *Natural Maths Strategies Assessment Guide for Book 3* with work samples. This book shows how work samples can be annotated to create a portfolio of the student's achievement. The book also gives a summary of key vocabulary, strategies, representations and understandings that students at this level might be expected to demonstrate.

BIG IDEAS IN MATHS

The mathematical content of this series is organised around 12 **big ideas** that are relevant to teaching maths at this level. This chart gives an overview of the big ideas and their links to the 12 units of work.

		Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
		Number Sense	Number and Measurement	Number and Money Strategies	Patterns and Sequences	Chance and Data	Number Situations
N3.1	Number sense	■		■			■
N3.2	Addition and subtraction			■			■
N3.3	Multiplication and division	■		■			■
N3.4	Money			■			
M3.1	Measurement		■				
M3.2	Time	■	■				
S3.1	Shape				■		
S3.2	Position in space				■		
CD3.1	Chance					■	
CD3.2	Data					■	■
PA3.1	Pattern				■		
PA3.2	Equivalence				■		

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BIG IDEAS IN MATHS

Unit 7	Unit 8	Unit 9	Unit 10	Unit 11	Unit 12	
Position in Space	Shape	Decimals and Large Numbers	Measurement and Construction	Chance and Data	Putting it all Together	
		■			■	Representing, comparing and ordering numbers to 9 999 using effective counting strategies.
		■			■	Identifying and solving addition and subtraction problems using a range of strategies and methods of recording.
		■			■	Identifying and solving multiplication and division problems using a range of strategies and methods of recording.
		■			■	Applying numerical operations to transactions involving money.
			■			Using standard units when estimating, measuring and comparing the size of objects.
				■		Reading and recording time in digital and analog formats; interpreting calendars and timetables in everyday contexts.
	■		■			Describing 2-D and 3-D shapes in formal terms; relating 2-D and 3-D shapes through nets.
■						Creating maps and plans to describe location; using the major compass points and alphanumeric grids.
				■		Identifying possible outcomes in a chance situation and determining the likelihood of outcomes.
				■		Collecting and representing data and describing features of the chosen representation.
						Creating and comparing number patterns based on relationships.
						Working with arithmetical expressions that involve all operations.

INTRODUCTION

The activities in this book provide starting points for three-part lessons that focus on the big ideas for teaching maths to 9 to 11-year-olds. It is intended that the activities also be used in conjunction with the Book 3 Maths Planner CD-ROM, which enables the teacher to maintain complete class records of progress. Accordingly, the book is organised into 12 units, with each unit containing mental routines, problematised situations, an investigation/s, games and assessment activities to match the big ideas in maths. The units are intended as starting points for teachers to build on to suit the range of learners that they are working with.

The maths curricula are divided into five strands:

- Number (including Money)
- Space
- Measurement
- Chance and Data
- Patterns and Algebra

Within each of the strands there are a number of big ideas, or concepts, which focus on the syllabus. This book is organised into units of work based on the current research into the developmental sequence in which students generally acquire those concepts. Guiding the choice of activities has been the research that suggests that "children are capable of grasping key mathematical concepts at an earlier age than previously thought". Researchers also "urge teachers to help children think mathematically rather than merely memorise algorithms and hone their computational skills" (see Checkley, K. 1999).

Three-part lessons

Both the Maths Planner and this book are much more than a facility to help teachers meet their syllabus requirements. The activities provide the type of resource needed to implement a three-part lesson process. In outline, a three-part lesson includes:

- a **mental routine** to develop the student's self-confidence and repertoire in mathematical thinking
- a **problematised situation** where the student applies their own thinking to a situation that they can engage with
- a time for **reflection** in which strategies and solutions are shared, compared and formalised, through which:
  1. we begin from where the students are
  2. we build on their understanding through the sharing of ideas
  3. formal mathematical methods are subordinate to methods that the students invent and which work effectively for them
  4. students learn to value each other's ideas, working as a community of learners rather than as individuals.

This approach to the teaching and learning of mathematics has its roots in research findings, and brings these findings to life through activities that have been found to fully engage students in mathematical discovery, discussion and understanding.

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Part 1: Mental Routines	Part 2: Problematised Situations	Part 3: Reflection

Number sense, fluency with mathematical and strategic thinking and estimation skills are the foundational building blocks of all later mathematics. Worksheets and mental arithmetic tests are anathema to risk-taking, reflective thinking and seeking out efficient strategies that will develop automaticity in number facts based on deep understanding.

Five years ago we began to question the relevance of paper and pencil, worksheets and "drill and kill" methods in the development of foundational basic number facts and understandings. We began by testing a few mental activities that involved the whole class simultaneously in fun and relevant activities. As we did so, we observed that when students are engaged in mental activities, certain conditions need to be present for them to obtain maximum benefits. These are:

- Concrete materials need to be provided for students to use as tools.
- Feedback is immediate, and involves sharing and discussing strategies and showing equal respect for all the students.
- Errors are seen as learning opportunities for all.
- Questions provide success for all as well as challenges for some.
- All students need to be engaged at their own level during the process.
- Students see themselves as a community of learners where everyone has a role to play in the development of thinking and learning.

It was with these criteria in mind that we began to explore the potential of "mental routines", as we have chosen to call them.

The purpose of mental routines is to develop useful strategies that will lead to mastery and a solid foundation in basic maths concepts. Mental strategies as far as possible should relate to the methods that students develop intuitively and within their own culture. They should also relate as far as possible to the ways in which those strategies are applied in the real world.

> This means that mathematics instruction must use contexts and pedagogies that allow students to use their own cultural, ethnic, and gender preferences and approaches.
>
> *Ladson-Billings, 1994*

When we refer to the conditions that need to be present for the effective development of mental strategies, we see that this view is clearly reflected.

The mental routines make an excellent lesson starter as they arouse enthusiasm and encourage the students to feel part of a learning community. They need last no more than 10 minutes, but in that time every student has been engaged and challenged to take risks with their current understanding.

INTRODUCTION PART 1

Classroom management

For each **mental routine**, provide a laminated mini-whiteboard of the task resource card for each student and suitable writing materials. We call these "mini-whiteboards", as felt pens can be used to ring or mark ideas on the laminated cards.

The teacher begins by posing simple, closed questions that enable everyone to be successful. Soon, the questions change to a more open type, where more than one answer can be found. This enables students to begin to work at their own level. Finally, the process is flipped, and the students ask the questions, trying to determine a solution to the problem that the teacher has posed.

Card masters that are to be laminated and used as mini-whiteboards.

Target strategies give a focus to the lesson.

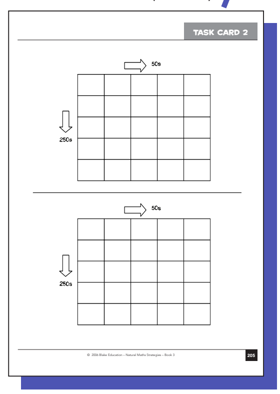
Closed questions enable the teacher to see who has "got it" and which strategies are being used.

Open questions show the students that there is often more than one method and more than one right answer to a question.

Flip questions give the students the opportunity to practise the language of maths.

EXPLORING NUMBERS ON THE EM

Task Card 2 (2 cards)



Target strategies

- Whole numbers to 9 999
- Patterns of numbers to 9 999
- Counting patterns to 9 999
- Addition of friendly numbers to 9 999

Closed questions

Before using these questions, ask the students to mark their grids in 50s from 0 to 1 200, going left to right as in writing. You will also need to make it clear to the students that they start on 0 and count the jumps from there.

If I count by 50s to 200, how many jumps will I make?

How many jumps on the grid if I count by 100s to the end of the last row?

How could I use the grid to count by 150s?

I jumped by 250 each time. What numbers did I land on?

Open questions

I only landed on 100s when I jumped. What number might I have started from and what might I have jumped by?

I made four jumps that took me into the 1 000s. What sized jumps might I have made?

The jumps that I made would never land on the number 1 000. What size jumps might I have made?

I made two different counting patterns on the board. They only met on two squares. What might the counting patterns have been?

Flip questions

You are going to play *Guess My Counting Sequence*. For this game you can ask me questions about:

- number of jumps (Did you make more than five jumps?)
- starting numbers (Did you start at the beginning? . . . on a round number?)
- size of jumps (Were your jumps smaller than 250?)
- finishing numbers (Did you finish on a number that was greater than . . . ?).

Remember though that to win you need to state the starting number, every number jumped on and the finishing number.

Note: A mental routine can be revisited for up to two weeks by changing the level of the question content as the students' vocabulary, skills and strategies improve.

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Part 1: Mental Routines	Part 2: Problematised Situations	Part 3: Reflection

We use the term **problematised situation** to describe the type of activities that will allow students to engage with realistic (to them) situations as described in the research from the Freudenthal Institute. The situations provide the kinds of challenges that encourage students to construct their own ideas, strategies and mathematical understandings as they grapple with them. The students, as described earlier, are developing their own mathematical tools, which can be formalised by the teachers when appropriate.

The problematised situations provided have multiple entry points and many methods of solution. If the numbers are too hard, they can be reduced; if they are too easy, they can be increased. Some students will draw pictures or act out the solution with objects, whereas others may use a more symbolic approach using numbers or tallies. Some will present solutions in an organised fashion whereas others will be more muddled. It is the sharing and reflecting on the range of strategies that will broaden the possibilities for the students and allow them to enter into mathematical thinking from their very first experiences. The focus in the primary classroom is shifting toward an emphasis on mathematical reasoning and problem solving in a true sense. This new focus helps students learn to describe, compare and discuss their multiple approaches to solving real problems. In the classrooms where we have been working, we have noted that all the students engage well with the problems and have shown an increased interest in maths along with a really firm conceptual understanding. It is not difficult to teach the students algorithms and procedures when they are ready for them and have firm foundations in place.

The reflection, as described further on and included in each of the presented problematised situations, is central to this approach. Part of the preparation for the reflection is the process of observing the strategies that the students use and of listening to their explanations. From the information gathered, it is possible to extend, consolidate and formalise learning during the reflection process. When the students are working, it is possible to gather information about what they do know and what they can do. For instance, when a student is working out the total value of a pile of coins, a simple question such as “Would it help to sort the coins before you work out their value?” may act as a prompt from which the students can show that they can make groups of coins with a total value of $1. Annotating the work samples makes it possible to record this information so that decisions about future planning can be made. A range of work samples will eventually give a clear picture of a student’s progress towards understanding the big ideas.

Classroom management

The body of a three-part lesson is often taken up with a **problematised situation** in which the activity is introduced with as little scaffolding as possible. The activity can be structured to enable the students to work independently, in small groups or collaboratively in larger groups, either as they wish or to suit the teacher's assessment purposes.

The problematised situations require the students to work mathematically, to draw on their own experiences and often to invent their own methods of recording and finding a solution.

We give a specification of the problem that can be displayed in a prominent place for all to refer to.

TUCKSHOP MONEY

Every day the tuckshop sorts the loose change into the same denominations before counting.
Today they had
twenty-six 5-cent coins,
thirty-nine 10-cent coins,
forty-three 20-cent coins
twenty-two 50-cent coins.
How much loose change did they have?

The resources list is a suggestion only – it needs to be tailored to your class's needs.

Resources

Toy money, bundling materials.

The activity guide makes suggestions for running the activity.

The activity

This is a multi-step problem and every step within it can be solved in a number of ways; hands-on, pictorially, using additive or multiplicative strategies or formal abstra ods of recording. Remind the students that you want them to show all of their thin methods of finding answers so that they can share them later. You will notice that s dents draw all the coins or representations of them and then count by 5s, 10s, . . . slow method and can be used at the reflection to look for ways of reducing the a drawing needed. For instance if we know that one row of ten 5-cents is fifty cents, w need to draw the next row. We can simply double it. Some students will remember t are twenty 5-cent coins to the dollar and will use that as their kick-off point. Look fo range of strategies for the reflection.

A link to the most relevant big idea helps to provide a focus for observation of the activity.

After the investigation, each situation leads on to the final part of the lesson – the reflection stage.

Reflection

As the students share their methods, ask them to:

- explain how two methods that at first do not look similar are actually representing the same ideas
- identify the strengths and weaknesses of each method
- identify a strategy that they will use next time you give them a similar money problem.

The students can write their own money problems for others to try.

N3.1 Representing, comparing and ordering numbers to 9 999 using effective counting stategies.

As the students are working on the problem, the teacher has opportunities to observe methods of recording, strategies used, problems encountered and fix-up strategies used. This is important preparation for the final reflection stage.

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Part 1: Mental Routines	Part 2: Problematised Situations	Part 3: Reflection

In the busy classroom the end of the lesson approaches all too quickly and as a result the **reflection** is often neglected. Yet the reflection is the most important part of the lesson. It is the time when the students use mathematical language to explain what they have done and see that there are many strategies for solving problems, and that some are more effective than others. It is also the time when the teacher can formalise a particular idea, concept or process and scaffold the students to the next level. In fact there are some who go so far as to say that if you didn't do a reflection then the students will probably retain nothing. The development of a community of learners who share, listen and learn from each other is at the heart of this approach to mathematics. The reflection time sets up the mathematical culture of the classroom with its tight-knit community of learners. It allows for mathematical mind journeys and adds to the excitement of learning mathematics.

The principles of rigorous reflection are:

- the identification of a range of strategies to share and discuss
- the use of one or more errors to show the value of checking results and of developing a fix-up strategy
- celebrating risk-taking, inventiveness, mathematical reasoning and learning from mistakes
- building on, extending and presenting more formal methods of recording as students demonstrate readiness for them
- positive, constructive feedback with a focus on feed forward – what you will do next time.

Through the dialogues and participation of all students in the class, the reflection stage becomes crucial to the development of a community of learners, through which active involvement in learning mathematics is successfully fostered.

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OTHER CONSIDERATIONS

Concrete materials

It may seem like a contradiction to say that hands-on materials could be used as part of mental routines but let's explore this idea a little further. It is our belief that mental maths can involve the use of concrete materials and does not have to be totally abstract. For instance when students are first becoming familiar with doubles and near doubles (doubles plus or minus 1), the use of materials such as Unifix cubes can become a mental routine. The students can be asked closed questions such as, "Show me a near double that makes 7." They may hold up a stack of 3 and a stack of 4 to show double 3 plus 1 or they may hold up two stacks of 4 and then snap off 1 Unifix cube to show double 4 take 1. The beauty of this is that the students can actually see the cubes and show how they match the strategies. This means that language and visual imagery combine the information into a meaningful whole.

> So there are two points to keep in mind as we discuss the uses of tools: First, meaning is not inherent in the tool; students construct meaning for it. Second, meaning developed **for** tools and meaning developed **with** tools both result from actively **using** tools. Teachers don't need to provide long demonstrations before allowing students to use tools; teachers just need to be aware that when students are using tools, they are working on two fronts simultaneously: what the tool means and how it can be used effectively to understand something else.
>
> *Heibert, J. et al. 1997*

This use of invented tools is equally important when the students are working on the problematised situations described below. You will notice that we have provided hands-on resources for each mental routine. These can be photo-enlarged, reduced or copied as appropriate. We laminate ours because we know that they will be used time and time again and we want students to interact with them.

They are used repeatedly and have uses outside those initially presented. The students enjoy using water-soluble felt pens and a tissue to clean them. The use of darker coloured pens means that you can see what the students record and watch their thinking as they find their answers. The students can hold their cards up for everyone to see and this means that they see a broad range of possible answers during the open questions. The resources are also used to develop adaptive reasoning during the flip questioning.

Feedback

Feedback should be immediate and useful, and should create a win–win situation for all rather than the competitive win–lose situation that so many students are familiar with. By this we mean that there is no place for the over-learning of number facts or for the stressful learning and testing practices that often typify mental arithmetic.

The intention is to replace this with a situation where students share their solutions and strategies, where they consider the benefits of different approaches to something as simple as 40 plus 70, and in so doing, receive valuable feedback for making comments such as:

> "I did my rainbow fact to 10 and then added 1 on. Then I added a zero to make it into 10s."
>
> "I took 1 from the 7, and put it with the 4 and knew I had to double 5 plus 1 before I added 0."
>
> "I just knew 4 + 7, so 40 + 70 is easy."
>
> "I knew 30 + 70 is 100, and 10 more is 110."
>
> "I thought it was 100 because I took 10 from 70 and added 60 to 40, but then I forgot the extra 10 that was left over. "

Teacher feedback is barely necessary, is it? The last example shows that the student, while listening to the others, realised that an error had been made and wondered how and where.

INTRODUCTION

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There was no embarrassment about the error, just a willingness to share with others a common error in such methods. The final questions from the teacher enhance this feedback by requiring the students to reflect on what was said as they answer questions such as:

"Which strategy did you think worked best in this example and why?"

"Which strategy would you like to explore a little further?"

"When we do another one, will you stick with the strategy you just used or try a different one?"

Through answering these questions the students have been engaged in the feedback process and have been asked to self-reflect and use that reflection as a planning tool ready for next time.

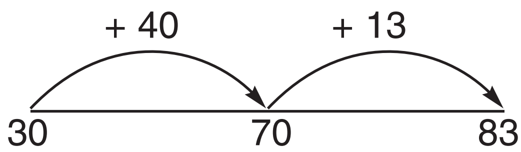
Methods for recording

Throughout the units there is an emphasis on the students inventing their own ways of recording their methods of solving the problems presented. In the past an early emphasis on algorithms has often led to students:

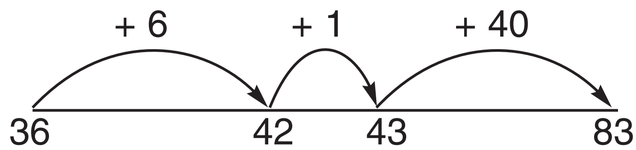
- seeing the pencil and paper algorithm as the only way of working on operations
- using pencil and paper to compute additions as simple as 3 + 4 = 7 that really do not need to be written algorithmically
- experience problems with place value when they incorrectly line up the numbers, don't trade properly and treat every column as a column of 1s
- making silly errors with place value especially where zeros are concerned
- really not developing number sense and an understanding of place value.

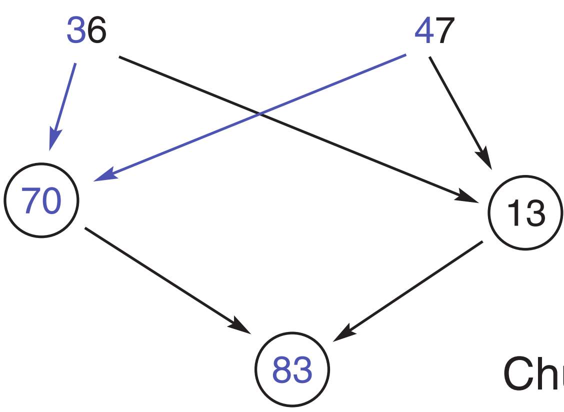
While we are not advocating that students do not use an algorithm, we are advocating that the students know that other and often more efficient methods exist, ones that make number sense and place value visible. By formalising the students' methods onto open number lines, chunking and zigzag methods, we are giving them tools to select from and build on rather than procedures to be applied. When the students are confidently using these methods, it is not difficult for them to understand algorithms with or without regrouping, and to fully understand the principles of base 10 that underpin them. Examples of the open number line, chunking and zigzag methods are included in the units and on the next page where we present two ways of doing the same 2-digit addition using each of the methods.

Examples of recording methods for the addition 36 + 47 are given here:

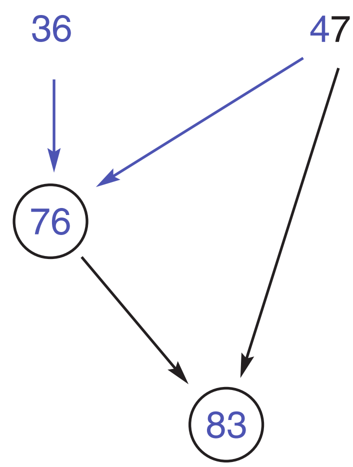


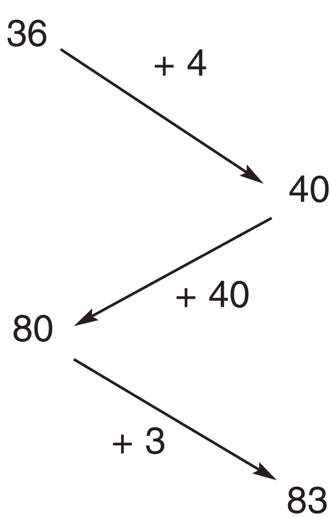
Open number line



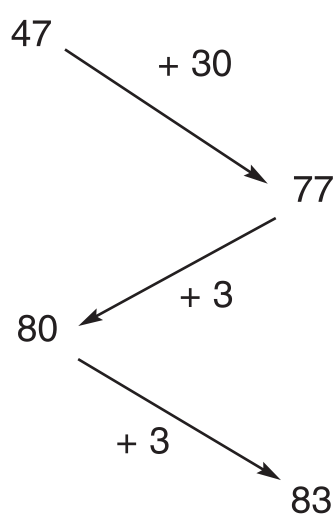


Chunking





Zigzag



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Engagement

The ten or so minutes set aside for mental routines are fast and pacey. They may involve concrete materials, number cards, 100 squares, dice, bottle tops, . . . The students engage with the activities because they are different to the rest of the lesson. When we first began exploring the mental activities that we suggest here, we had no idea how much fun and of course how much learning would flow from them. We soon realised that we didn't need to make up a new mental activity every day because the nature of the tasks and the students' interest in them meant that they could be used and easily adapted over several days, and hence the term "mental routines". We now use the routines for several consecutive days, all the time watching to see the level of engagement and, of course, we switch to a new routine if we think the interest is dwindling.

As we introduce each routine, we use the meta-language of the strategies or process that go with it. At first it was our intention to simply immerse the students in the meta-language but the students were so captivated by words, such as "chunking", "zigzag", "subitise" and "unitise", that they soon wanted to use them too.

Watching the students engage with the activities has been rewarding for us and for them too. When they are having fun and are engaged, they seem to be hungry for more. We have seen even the switched-off learners re-engage through the mental routines.

Learning opportunities

It appears that there is little value in participating in mental activities which are too easy or which are already well-developed. Activities, then, need to be just on the edge of the students' comfort zone, scaffolding them to the next level. If this is the case then obviously errors in computation are going to occur from time to time. We found at first when we worked in this way, students would be derisive and snicker at errors. We also found that some students would not have a go for fear of failure. It was interesting however to see how quickly this mindset turned around. The students seemed to embrace the idea of using errors as learning opportunities and were often heard saying, "Oh good, a learning opportunity." Very often as a student is explaining their strategy, they notice their own errors and are keen to fix them up on the spot. Other times, though, the error is not noticed. For instance, recently a 6-year-old girl while responding to a reading of the *The Very Hungry Caterpillar* showed how she had drawn two rows of foods that the caterpillar had eaten during the week. She counted in 2s but only touched one thing at a time. She was unperturbed by her error; she had counted by 1s earlier and written the number 26 on her paper. One of the other students was very impressed by her ability to count so fast and so far by 2s, but another asked her why she only touched one thing at a time. She stood perplexed looking at her page and asked the other student to come up and show what she meant. She laughed out loud when she saw what she should have done and proceeded to repeat her counting correctly this time.

> We recall when children have wanted to share mistakes; after presenting their solutions, they explain their errors. The children also become very supportive of one another and understand that errors are a natural part of doing mathematics. Errors often can lead to new understandings about the concept.
>
> *Trafton, P. and Thiessen, D. 1999*

Polya's *See, Plan, Do, Check* model

How to Solve It by George Polya was first published in 1945. Over 50 years later, the words he used in the preface to this short but remarkably influential book are as fresh and relevant today as they were then.

> A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on the mind and character for a lifetime.
>
> Thus, a teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations, he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking.
>
> *Polya, G., 1945*

We recommend that you encourage the students to follow Polya's *See, Plan, Do, Check* method of problem solving shown here.

See	What is the important information in the problem? Can you express the problem in your own words? What do you need to find out or do?
Plan	How will you try to solve the problem? What method of solution will you try? (Guess and Check, Draw a Diagram, Act it Out, and so on) Have you done a problem like this one before?
Do	Carry out your plan. Make notes on success and failures. Write up your solution.
Check	How do you know your answer is correct? Have you found all possible solutions? Can you solve the problem a different way?

When students come to communicate their solutions, it is important that they share the approach taken, their method of solution and the checks they made of correctness.

Community of learners

To gain the most from the activities given in this book, the students need to become a community of learners. They need to really listen to the ideas of others, give positive feedback, ask questions, make suggestions and comparisons, and finally to evaluate the strategies presented by others. They need to feel safe to take a risk, present their ideas and to comment on the ideas of others. They need to learn to justify their viewpoints and stick with them. For instance, if after hearing how friendly numbers can be used for an addition a student still prefers a chunking strategy then they should be able to explain why they prefer it. And at the end of the day if the response is, "Because I know it always gives me a correct answer", then that justification has to be seen by all as valid for that student at that time, and as such should be respected.

> Learning to be a member of a mathematical community means taking ownership of the goals and accepting the norms of social interaction. Why is it important that classrooms become mathematical communities and that all students participate? Because such communities provide rich environments for developing deep understandings of mathematics.
>
> *Heibert, J. et al. 1997*

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UNIT 11

Chance and Data

Focus

In everyday conversations people make statements about chance that are poorly informed and often completely untrue. A common expression is "There's nothing on TV tonight, let's get a DVD." Clearly this is an untrue statement and cannot be backed up with data. "It always rains on the weekend" is a similar comment founded on one or two bad experiences rather than on any form of data collection. Students at this stage of their development can be challenged to think about the poorly informed statements that they make. They need to find ways of collecting data to support or refute their statements.

Statistics and information are presented in many different ways depending on the purpose and audience. Simple bar graphs where one cell equals one item (for example, a person or car) are replaced in larger data sets with cells that represent, say, 100 people or 1 000 000 people. The key to the graph is now an important piece of the jigsaw when trying to interpret the sample size and the meaning of each cell. The variety of ways to present data is much broader than the simple bar graphs and tallies that students have met previously. So too is the purpose for which data can be used. Students need opportunities to create data sets and present them in many different formats, and to interpret information from ready-prepared graphical representations. This unit covers multiple forms of data representation and the generation of a variety of different types of data sets all of which can be used to support or refute statements of chance and probability.

Context

The context for this unit is *There's not much on TV*. This context has been chosen because the majority of students watch TV every day. They are interested in what's on and often organise their out-of-school activities around favourite shows. TV guides are easy to come by and provide plenty of information that can be quantified and represented in many ways to answer questions about or persuade others of a particular point of view. TV guides offer opportunities to collect information about favourite shows, types of shows represented, whether shows are gender- or age-based and whether programs are on at the best times. The questions provided are suggestions only. The students will have their own ideas and questions that they want to research. These may be topical and tied into current events such as the World Cup, the Olympics, the election or whatever is topical at the time.

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Developmental sequence

Students at this stage of their development are beginning to:

1. classify ideas and information in a variety of ways, recognising that the same idea or information can belong simultaneously in more than one category
2. use keys or legends to interpret information
3. see the relationships between different representations of the same data sets
4. develop ideas about sample sets and the effect that sample size has on probability
5. use language such as “There is a 1 in 6 chance of throwing a 6 on a die but only a 1 in 36 chance of throwing double 6 with two dice”.

THE TV GUIDE

Resources

Make photocopies of a current TV Guide for one day of the week, showing only the programs within times that the students might normally be allowed to watch television.

Target strategies

- Reading a TV guide timetable
- Using data from a timetable to compare TV channels
- Classifying TV shows in different of ways

Closed questions

What time is . . . on?

What channel is . . . on?

How many cartoons are on between 3 o'clock and 4:30?

Which channel has the most kids' shows between 3 o'clock and 6 o'clock?

How much time is there for the news over all the channels? How does this compare with the amount of time given to comedy shows?

Open questions

If I wanted to watch the news between 3 o'clock and 6:30, what time and which channel would you suggest for me?

I want to watch a sit com, a quiz show and the news. How might I be able to plan that?

What would you plan for your afternoon viewing?

If I wanted to watch two complete shows of thirty minutes or less, what might I watch?

Flip questions

I have planned to watch TV for one hour continuously. You can ask me time, channel and program type questions to find out what I am going to watch and when.

I will only answer Yes or No to your questions. You could ask questions like:

> "Will you begin watching before 4 o'clock?"
>
> "Will you finish watching after 5:30?"
>
> "Will you watch more than one channel?"
>
> "Are you going to include the news?"

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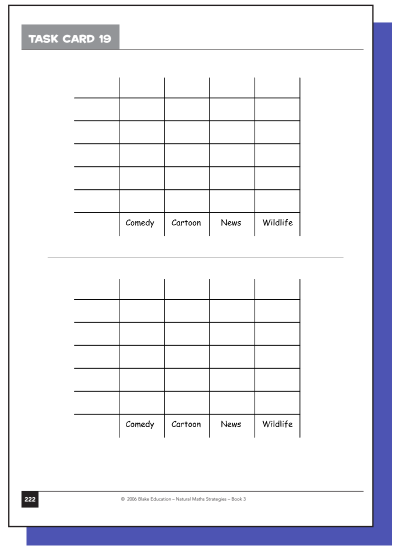
TOP VIEWING

Target strategies

- Representing data on a graph
- Identifying the value of a cell on a graph
- Using/making a key to show the value of a cell
- Inventing symbols to represent values for a cell

Task Card 19 (2 cards)



Closed questions

If each cell on the graph represents one person, how would you show that 6 people watched the news?

If 12 people watched the news and the value for each cell was 4 people, how would you show that on the graph and on the key?

Ten people watched TV last night. The news was as popular as the cartoons, comedies were watched by 5 people, and only 1 person watched a wildlife show. What should the graph of last night's viewing look like?

Open questions

Twenty people watched the news last night. How could you show that on your graph?

If 100 people watched TV last night and more than half of them watched the news, what might the graph look like and what would the key look like?

One thousand people watched the TV last night. Half of them watched the news, and 100 of them watched a cartoon. What might the graph and the key look like?

Last night more people watched cartoons than any other show. The same number of people watched all the other shows. What might the graph look like and what sample size might have been used?

Flip questions

My sample size for last night's viewing was 100 people. You can ask me questions that will help you to work out what my graph looks like. Questions could include:

"Is the value of a cell on your graph more than ten people?"

"Was the news the favourite show?"

"Did any two shows have the same number of people watching them?"

"Did fewer than 10 people watch the cartoons?"

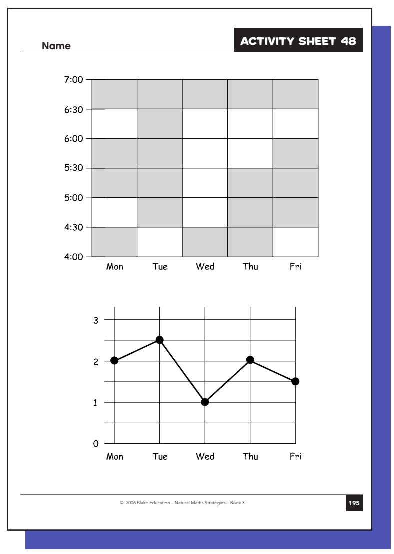
WHAT'S IN A GRAPH?

The grid shows the TV viewing of a family over a few days and so does the line graph.

Write some questions for each method of showing the data.

Compare the two methods of showing the information.

Activity Sheet 48



Resources

Grid or graph paper.

The activity

Ensure the students can interpret each representation and then explain to them that they need to list some questions that they could ask about the family's TV viewing. Model one or two questions if necessary, for instance:

"At what time in the evening do the family watch the least TV? Why might that be the case?"

"At what time do the family watch the most TV? Why might that be the case?"

"Do the family watch the same amount of TV on any of the days?"

When the students use the data to answer their questions, they need to think about which method of representation made it easier to find the answer and why.

The students might like to try other methods of showing the data.

Reflection

CD3.2 Collecting and representing data and describing features of the chosen representation.

Ask the students to share some of their questions and explain which method of representing the data works best for each question. Use the actual data representations to try to answer each question, and look at the strengths and weaknesses. Some students may have attempted to show the data in a different form, so use any alternative representations in the same way.

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KID'S TIME

TV between 3:30 p.m. and 6 p.m. should be kids' time.
There is no set time slot just for kids' TV?
True or false?

Resources

TV guides for one week, graph or grid paper.

The activity

Allow time for the students to decide whether they think there is a real kids' time slot. Ask them to make statements such as "agree" or "disagree" or "strongly disagree" that there is a real kids' TV time slot.

Explain to the students that they are going to investigate the kids' TV timeslot and that they need to find a way of presenting their data on graphs or tables, or some other form that makes it quite clear what is on offer during that time. You might like to suggest that the students work in groups so that they can share the workload. Students might decide to investigate one channel each or one time slot each. There will be a variety of ways of tackling this problem as well as ways of representing the data collected, so select a range for the reflection.

Reflection

CD3.2 Collecting and representing data and describing features of the chosen representation.

Share some of the methods of organising the data collection as well as the methods of representing it. Ask the students to discuss how effectively a particular method shows the information as well as any surprises that they had as they charted the information.

Review the original question and the students' responses to it and ask them to comment on whether the data has changed their perspective at all and if so, why or how? You could help the students to write an example of a letter to the TV channels containing the results of the data so that they can see what the students think.

PROBLEMATISED SITUATIONS

THE "KIDS' PROGRAM OF THE WEEK" AWARD

Did you know that the programs that are the most popular with some people can be the most unpopular with others?

You either love or hate some programs! There is not much in between.

Your job is to find the program that most people are prepared to watch and to give it an award.

The activity

Usually when students conduct a survey, they collect data about favourites, not about least favourites. In the real world this can equate to something as awkward as finding that most people like meat-lover's pizza, ordering meat-lover's pizzas and then finding out someone is a vegetarian who will not eat it. To find the most popular show in this case means to find a show that *everyone* watches sometimes, not one that most people watch all the time and some people never watch. Make this clear to the students and then allow time for them to decide on their sample population and plan their survey. It is always important to try out a survey before doing a final draft of the question in case the researcher ends up collecting too much data. When the data collection is complete, ask the students to present their findings as a graph to the rest of the class.

Reflection

CD3.2 Collecting and representing data and describing features of the chosen representation.

When the students present their findings, focus the discussion and comparisons on:

- age group of sample populations and sample size
- survey questions
- data collection methods
- data presentation methods
- results and differences in them (for example, an older population would have a different set of preferences to a younger population)
- comments about whether the most popular TV show was actually the most popular with everyone.

As with the earlier activity, you could help the students to write an example of a letter to a TV channel giving their results and comments.

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THE NEWS IS ON!

In any hour of the day there is a one-in-four chance that the news will be on TV.
Agree or disagree?
What are the chances that there will be news on at any time between 6 a.m. and 9 p.m.?

Resources

TV guide for one day of the week.

The activity

You may need to clarify parts of the problem with the students. For instance "in any hour" can be taken to mean that between any o'clock and the next, there will either be a news headlines or a "news" show. Ask the students to say to what extent they agree and to tender alternative chances, for instance, to change the probability to one in eight across a two-hour time period.

Explain to the students that they can use a TV guide to investigate the truth in the statement. Ask them to clearly present their data and their methods for interpreting it ready for the reflection.

Reflection

CD3.1 Identifying possible outcomes in a chance situation and determining the likelihood of outcomes.

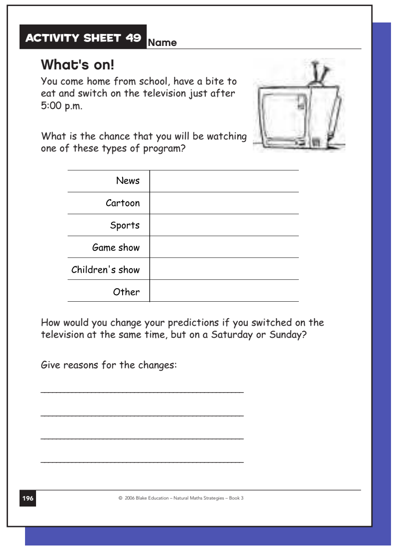
Allow time for the students to present their data collection methods and their results. Look for similarities and differences in the methods of sorting out the information and in presenting it. Ask the students to present their findings in terms of probability and to consider how the probability changes at certain times of the day or night. Ask the students to comment on whether or not the probability matched their expectations. Some students might feel that the news is on every few minutes and will be surprised at the actual frequency, so discuss the need for data in the light of some everyday ideas of never, always, often, . . .

WHAT'S ON?

Resources

TV guides for 2 or 3 weeks.

Activity Sheet 49


ACTIVITY SHEET 49 Name

What's on!

You come home from school, have a bite to eat and switch on the television just after 5:00 p.m.

What is the chance that you will be watching one of these types of program?

News	
Cartoon	
Sports	
Game show	
Children's show	
Other	

How would you change your predictions if you switched on the television at the same time, but on a Saturday or Sunday?

Give reasons for the changes:

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The investigation

The students need to work in teams on this investigation as it relies on collecting data from two or three weeks of TV guides. The information collected can be entered on charts such as the one on the activity sheet, and there needs to be one chart for weekdays and one for weekends. Encourage the students to make predictions for each weekday and to give reasons for their predictions, before they compare their predictions with actual data that they have collected. Encourage them to identify the types of factors that cause changes to the patterns in programming.

At each stage, encourage them to use appropriate language of chance and data. For instance they might say that there is a 1 in 5 chance of finding a game show because there are 5 channels, or that there is a 1 in 25 chance because there are 5 channels and 5 days in the working week. When they do make such statements, ask them to check their accuracy by using the TV guides.

The students can then investigate the days of the weekend using the same procedures.

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A TIMETABLE FOR YOUNG PEOPLE

Have a close look at the way a TV timetable is organised.
Do you agree that this is not a good way to organise the information?
How could the information be organised so that a 6-year-old would find it easy to see what program is showing at a given time?

Resources

A large display card, scissors, glue, a current TV guide.

The investigation

The students can work in teams on this investigation but if you can arrange to survey some young students in another class, this would give the whole investigation a sense of reality.

The teams should notice that the starting times on a TV guide do not line up, nor is the length of a program indicated by the vertical space that it occupies on the timetable. Encourage the students to make preliminary designs that can be submitted for audience approval. Differences can be made by such things as:

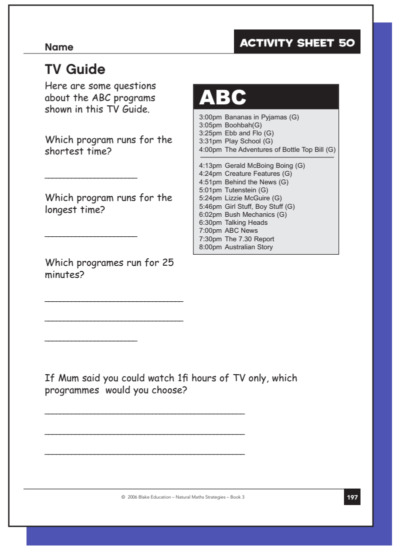
- the use of colour to mark the passage of time
- an indication as to what kind of show it is, using team-invented symbols for the different types of programs
- using one time line and positioning the programs on it.

TV GUIDE

Resources

Use a current TV schedule as an alternative to the activity sheet.

Activity Sheet 50


Name

ACTIVITY SHEET 50

TV Guide

Here are some questions about the ABC programs shown in this TV Guide.

Which program runs for the shortest time?

Which program runs for the longest time?

Which programes run for 25 minutes?

If Mum said you could watch 1fi hours of TV only, which programmes would you choose?

ABC

3:00pm Bananas in Pyjamas (G)
3:05pm Boohbah(G)
3:25pm Ebb and Flo (G)
3:31pm Play School (G)
4:00pm The Adventures of Bottle Top Bill (G)
4:13pm Gerald McBoing Boing (G)
4:24pm Creature Features (G)
4:51pm Behind the News (G)
5:01pm Tutenstein (G)
5:24pm Lizzie McGuire (G)
5:46pm Girl Stuff, Boy Stuff (G)
6:02pm Bush Mechanics (G)
6:30pm Talking Heads
7:00pm ABC News
7:30pm The 7.30 Report
8:00pm Australian Story

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Prior experiences

The students will be ready for this activity if they have had experiences with:

- reading simple timetables
- reading times in digital format
- working out durations of timespans

Observer's guide

The problem presented on the activity sheet asks the students to read the timetable, program lengths and plan a one and a half hour TV viewing session. As the students work on each of the questions, observe the ease with which they follow the timetable and the strategies used to work out the time durations. Some students may use 100 as their base for computing, forgetting that there are 60 minutes in an hour. Some students will count on in intervals to find the length of each show whereas others might count back from the finishing time.

As the students work out their plan for viewing, note the decisions that they take and the methods used for computing. You may want to remind the students that it does not say *continuous viewing* so they could watch a show, switch off and come back later if they want to. Some students may use a time line or an algorithm or other representation for their addition of time depending on the duration of the shows that they choose to watch. Ask them to explain why they chose the method that they did. Some students will convert minutes to hours as they go and some will convert at the end. Ask questions to find out why they chose their methods and to see if they could use a different method as well.

M3.2 Reading and recording time in digital and analog formats; interpreting calendars and timetables in everyday contexts.

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ASSESSMENT ACTIVITIES

I DON'T LIKE THIS PIZZA

Prior experiences

The students will be ready for this activity if they have had experiences with:

- tallying information
- interpreting information presented in table format
- using data to make a decision or answer a question

Activity Sheet 51

ACTIVITY SHEET 51 Name

I don't like this pizza

I had to arrange a pizza party on my birthday so that we could play basket ball and then have some pizza.

So I asked my friends to tell me one pizza they liked and one they didn't like … but now I can't decide what to do with the information I have collected.

Pizza	Like	Dislike
Hawaiian	𝍸	𝍸
Meat Lovers	𝍸 II	𝍸 IIII
Pepperoni	III	𝍸
Cheese and Tomato	𝍸 III	𝍸 I
Supreme	𝍸 I	III
Vegetarian	III	IIII

Everyone voted on one like and one dislike.
How many friends are coming to my basket ball party?

____

Mum said I have to choose 2 types of pizza. Which ones should I choose and why?

____

____

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Observer's guide

The problem presented on the activity sheet asks the students to interpret the information about liked and disliked pizzas listed on the page so that they can make an informed decision about which pizzas to order for a pizza party.

The first question asks them to work out how many people are coming to the pizza party. While this is not a hard question mathematically, the students need to focus on the fact that everyone had two votes. A common error will be to count all the votes, so you may need to remind some students that everyone had two votes *not* one.

The second question requires the use of the information listed and justification for the answer, so tell the students that they cannot just choose any pizzas. They must choose pizzas that will provide something that everyone will eat. As the students make their decision, ask questions to find out what their thinking was. If a well-reasoned decision has not been made, ask questions to encourage a more reasoned decision, for example:

"If there was a vegetarian present, would there be a pizza they would eat?"

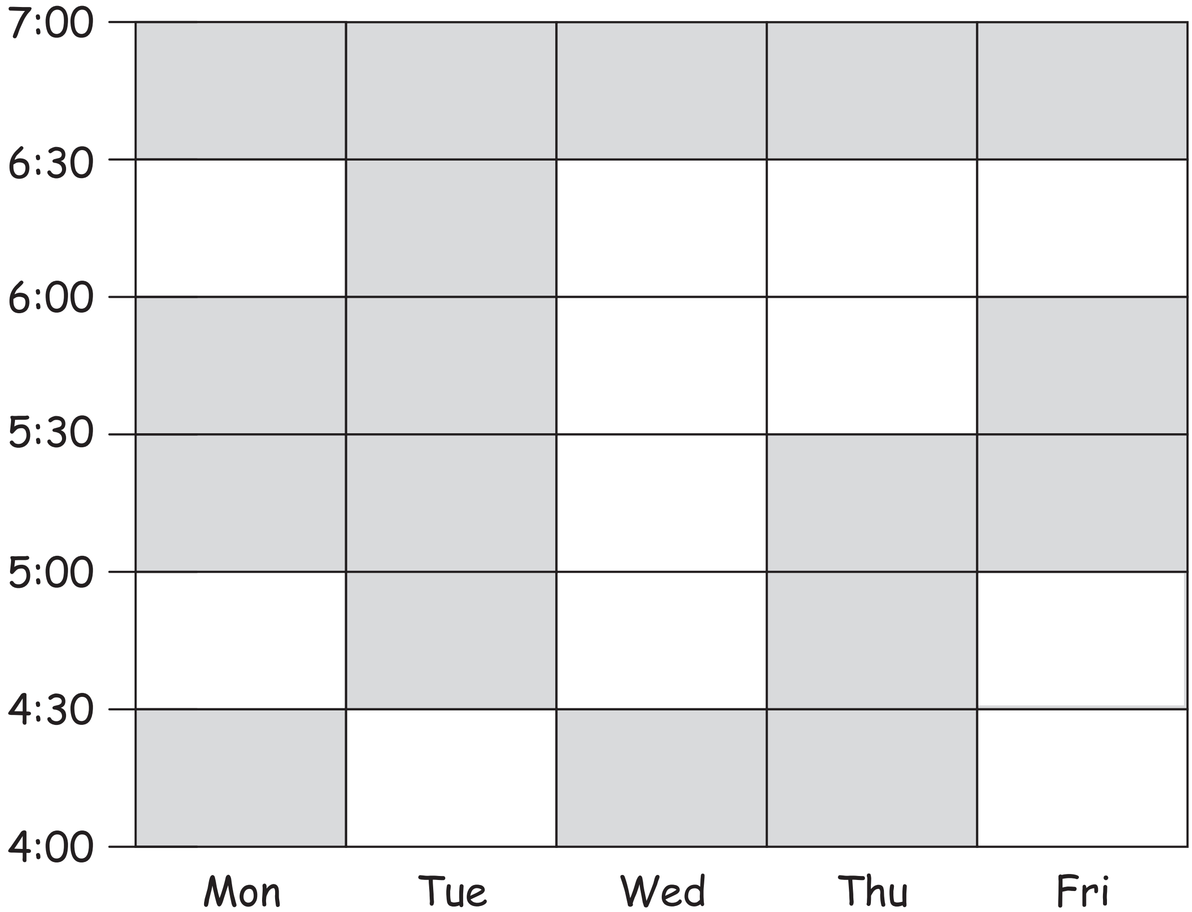
"Which pizza did most people dislike?"

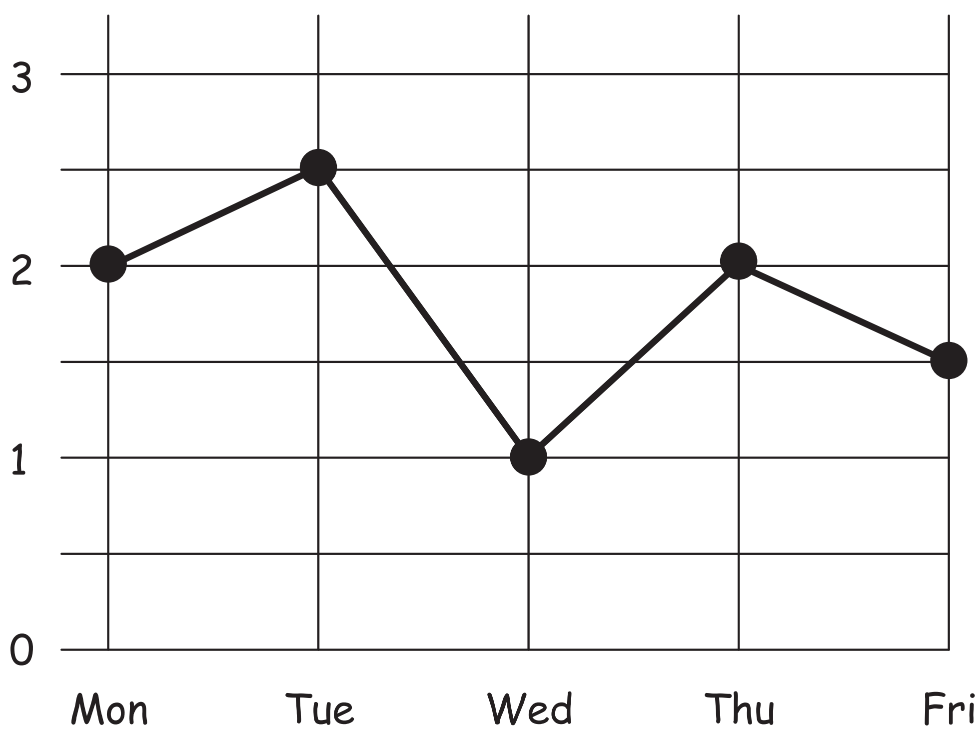
"Is Meat Lovers a good choice?"

CD3.2 Collecting and representing data and describing features of the chosen representation.

Name

What's in a graph?





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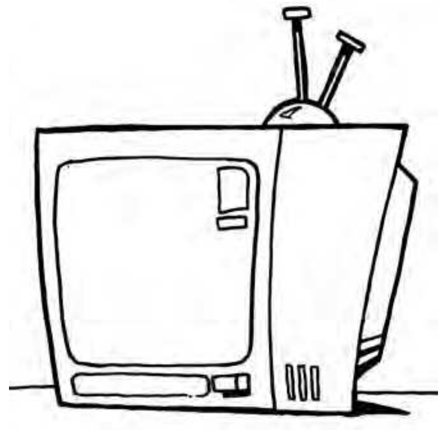
 Name

What's on?

You come home from school, have a bite to eat and switch on the television just after 5:00 p.m.



What is the chance that you will be watching one of these types of program?

News	
Cartoon	
Sports	
Game show	
Children's show	
Other	

How would you change your predictions if you switched on the television at the same time, but on a Saturday or Sunday?

Give reasons for the changes.

____

____

____

____

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TV guide

Here are some questions about the ABC programs shown in this TV guide.

ABC

3:00pm	Bananas in Pyjamas (G)
3:05pm	Boohbah(G)
3:25pm	Ebb and Flo (G)
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4:13pm	Gerald McBoing Boing (G)
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4:51pm	Behind the News (G)
5:01pm	Tutenstein (G)
5:24pm	Lizzie McGuire (G)
5:46pm	Girl Stuff, Boy Stuff (G)
6:02pm	Bush Mechanics (G)
6:30pm	Talking Heads
7:00pm	ABC News
7:30pm	The 7.30 Report
8:00pm	Australian Story

Which program runs for the shortest time?

____

Which program runs for the longest time?

____

Which programs run for 25 minutes?

____

____

____

If Mum said you could watch 1 ½ hours of TV only, which programs would you choose?

____

____

____

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 Name

I don't like this pizza

I had to arrange a pizza party on my birthday so that we could play basketball and then have some pizzas.

So I asked my friends to tell me one pizza they liked and one they didn't like . . . but now I can't decide what to do with the information I have collected.

Pizza	Like	Dislike
Hawaiian	~~IIII~~	~~IIII~~
Meat Lovers	~~IIII~~ II	~~IIII~~ IIII
Pepperoni	III	~~IIII~~
Cheese and Tomato	~~IIII~~ III	~~IIII~~ I
Supreme	~~IIII~~ I	III
Vegetarian	III	IIII

Everyone voted on one like and one dislike.

How many friends are coming to my basketball party?

____

Mum said I have to choose 2 types of pizza. Which ones should I choose and why?

____

____

____

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	Comedy	Cartoon	News	Wildlife

	Comedy	Cartoon	News	Wildlife

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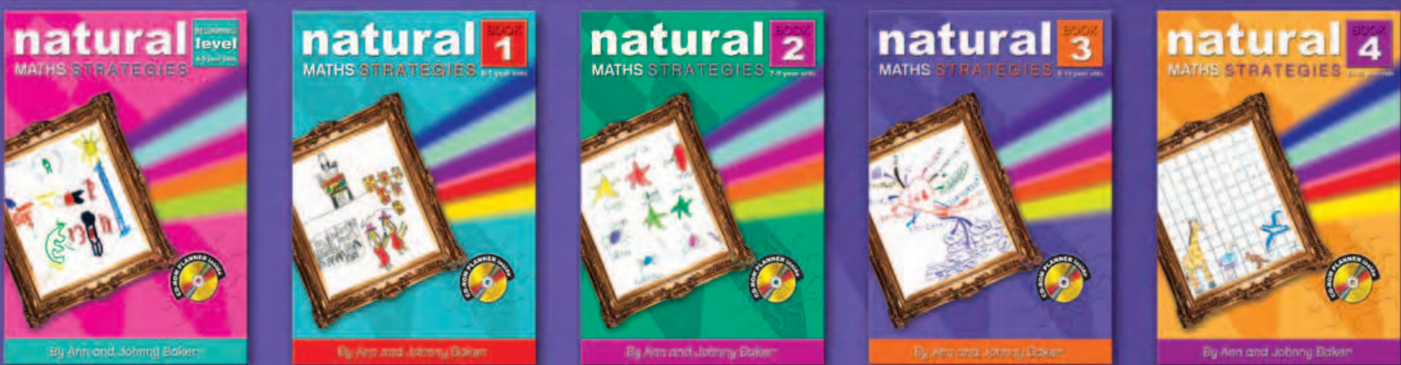

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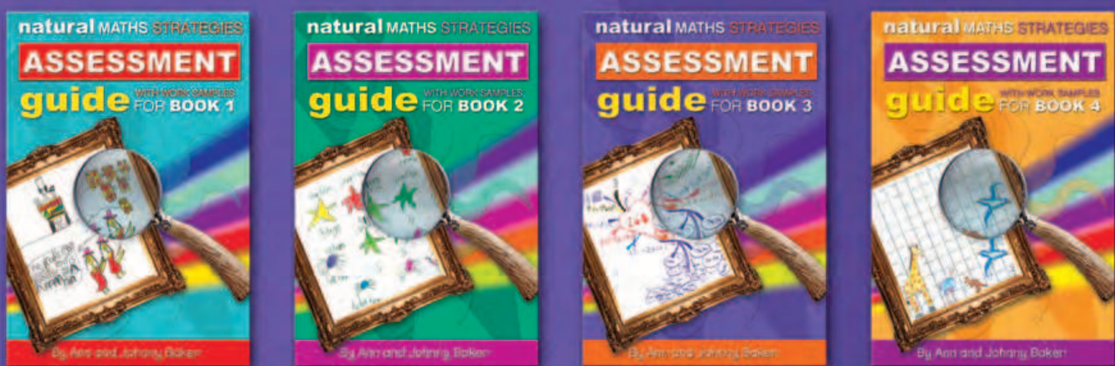

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