EFFECT OF ROTATION IN A GENERALIZED THERMOELASTIC MEDIUM WITH TWO TEMPERATURE UNDER THE INFLUENCE OF GRAVITY

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ABSTRACT

In the present problem we study the deformation of a rotating generalized thermoelastic medium with two temperature under the influence of gravity subjected to different type of sources. The components of displacement, force stress, conductive temperature and temperature distribution are obtained in Laplace and Fourier domain by applying integral transforms for Green-Lindsay (G-L) theory of thermoelasticity. These components are then obtained in the physical domain by applying a numerical inversion method. Some particular cases are also discussed in context of the problem. The results are also presented graphically to show the effect of rotation and gravity.

Keywords: Rotation, gravity, generalized thermoelasticity, laplace and fourier transforms, conductive temperature, temperature distribution.

NOMENCLATURE

λ, μ	Lame's constants
ρ	Density
$\vec{u}$	Displacement vector
t_{ij}	Stress tensor
$\tau_0,\ \tau_1$	Thermal relaxation times
$\upsilon = (3\lambda + 2\mu)\alpha_t$	Linear thermal expansion
g	Acceleration due to gravity
$K^{\bullet}$	Coefficient of thermal conductivity
C_E	Specific heat
ϕ	Conductive temperature

η	A constant
a	Two temperature parameter
T_0	Reference temperature

1 INTRODUCTION

Generalized thermoelasticity theories have been developed with the objective of removing the paradox of infinite speed of heat propagation inherent in the conventional coupled dynamical theory of thermoelasticity in which the parabolic type heat conduction equation is based on Fourier's law of heat conduction. This newly emerged theory which admits finite speed of heat propagation is now referred to as the hyperbolic thermoelasticity theory, (Chandrasekharaiah 1998), since the heat equation for rigid conductor is hyperbolic-type differential equation.

There are two important generalized theories of thermoelasticity. The first is due to Lord and Shulman (Lord and Shulman 1967). The second generalization to the coupled theory of thermoelasticity what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. Muller (Muller 1971), in a review of the thermodynamics of thermoelastic solid, proposed an entropy production inequality, with the help of which he consider restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws (Green and Laws 1972). Green and Lindsay (G-L) obtained another version of the constitutive equations (1972). These equations were also obtained independently and more explicitly by Suhubi (Suhubi 1975). This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, not only the heat equations. The classical Fourier law violated if the medium under consideration has a centre of symmetry.

Barber and Martin-Moran (Barber and Martin-Moran 1982) discussed Green's functions for transient thermoelastic contact problems for the half-plane. Barber (Barber 1984) studied thermoelastic displacements and stresses due to a heat source moving over the surface of a half plane. Sherief (Sherief 1986) obtained components of stress and temperature distributions in a thermoelastic medium due to a continuous source. Dhaliwal *et al.* (Dhaliwal *et al.* 1997) investigated thermoelastic interactions caused by a continuous line heat source in a homogeneous isotropic unbounded solid. Chandrasekharaiah and Srinath (Chandrasekharaiah and Srinath 1998) studied thermoelastic interactions due to a continuous point heat source in a homogeneous and isotropic unbounded body. Sharma *et al.* (Sharma *et al.* 2000) investigated the disturbance due to a time-harmonic normal point load in a homogeneous isotropic thermoelastic half-space. Sharma and Chauhan (Sharma and Chauhan 2001) discussed mechanical and thermal sources in a generalised thermoelastic half-space. Sharma *et al.* (Sharma *et al.* 2004), investigated the steady-state response of an applied load moving with constant speed for infinite long time over the top surface of a homogeneous thermoelastic layer lying over an infinite half-space. Recently Deswal and Choudhary (Deswal and Choudhary 2008) studied a two-dimensional problem due to moving load in generalized thermoelastic solid with diffusion.

Chen and Gurtin (Chen and Gurtin 1968) and Chen *et al.* (Chen *et al.* 1968; Chen *et al.* 1969) formulated a theory of heat conduction in deformable bodies, which depends upon two distinct temperatures, the conductive temperature ϕ and the thermodynamic temperature T.

For time-dependent situations, the difference between these two temperatures is proportional to the heat supply, and in the absence of any heat supply, the two temperatures are identical Chen *et al.* (Chen *et al.* 1968). For time-dependent problems, however, and for wave propagation problems in particular, the two temperatures are in general different regardless of the presence of a heat supply.

Some researchers in past have investigated different problem of rotating media. Chand *et al.* (Chand *et al.* 1990) presented an investigation on the distribution of deformation, stresses and magnetic field in a uniformly rotating homogeneous isotropic, thermally and electrically conducting elastic half space. Many authors (Schoenberg and Censor 1973, Clarke and Burdness 1994, Destrade 2004) studied the effect of rotation on elastic waves. Roychoudhuri and Mukhopadhyay (Roychoudhuri and Mukhopadhyay 2000) studied the effect of rotation and relaxation times on plane waves in generalized thermo-visco-elasticity. Ting (Ting 2004) investigated the interfacial waves in a rotating anisotropic elastic half space. (Sharma and his co-workers 2006, 2007a, 2007b, 2008) discussed effect of rotation on different type of waves propagating in a thermoelastic medium. Othman and Song (Othman and Song 2008) presented the effect of rotation in magneto thermoelastic medium. Recently Ailawalia and Narah (Ailawalia and Narah 2008) discussed the effect of rotation due to moving load at the interface of elastic half space and generalized thermoelastic half space.

In classical theory of elasticity the gravity effect is generally neglected. The effect of gravity in the problem of propagation of waves in solids, in particular on an elastic globe, was first studied by Bromwich (Bromwich 1898). Subsequently, investigation of the effect of gravity was considered by (Love 1911) who showed that the velocity of Rayleigh waves is increased to a significant extent by the gravitational field when wavelengths are large. (De and Sengupta 1973; De and Sengupta 1974; De and Sengupta 1976) studied the effect of gravity on surface waves, on the propagation of waves in an elastic layer and Lamb's problem on a plane. Sengupta and Acharya (Sengupta and Acharya 1979) studied the influence of gravity on the propagation of waves in a thermoelastic layer. Das, Acharya and Sengupta (Das, Acharya and Sengupta 1992) investigated surface waves under the influence of gravity in a non-homogeneous elastic solid medium. Abd-Alla and Ahmed (Abd-Alla and Ahmed 1996) investigated Rayleigh waves in an orthotropic thermoelastic medium under gravity field and initial stress. Abd-Alla and Ahmed (Abd-Alla and Ahmed 2003) discussed wave propagation in a non-homogeneous orthotropic elastic medium under the influence of gravity.

In the present investigation we have obtained the expressions for displacement, force stress, conductive temperature and temperature distribution in a rotating generalized thermoelastic medium with two temperature under the influence of gravity by applying Laplace and Fourier transforms subjected to concentrated force, distributed force and moving force. Such types of problems in the rotating medium are very important in many dynamical systems. No attempt has been made so far to study the effect of rotation in generalized thermoelastic medium with two temperature under the influence of gravity.

2 FORMULATION OF THE PROBLEM

We consider a homogeneous generalized thermoelastic half-space with two temperature rotating uniformly with angular velocity $\vec{\Omega} = \Omega\hat{n}$, where $\hat{n}$ is a unit vector representing the direction of the axis of rotation. All quantities considered are functions of the time variable t

and of the coordinates x and z. The displacement equation of motion in the rotating frame has two additional terms Schoenberg and Censor (Schoenberg and Censor 1973): centripetal acceleration, $\Omega\times(\vec{\Omega}\times\vec{u})$ due to time varying motion only and $2\vec{\Omega}\times\vec{u}$ where $\vec{u}=(u_1,0,u_3)$ is the dynamic displacement vector and angular velocity is $\vec{\Omega}=(0,\Omega,0)$. These terms do not appear in non-rotating media.

We consider a normal source acting at the plane surface of generalized thermoelastic half space with two temperature under the influence of gravity. A rectangular coordinate system (x,y,z) having origin on the surface $z=0$ and $z-$ axis pointing vertically into the medium is considered.

3 BASIC EQUATIONS AND THEIR SOLUTIONS

The field equations and constitutive relations in generalized thermoelastic body with two temperature are given by Youssef (Youssef 2006-c)

$$(\lambda+2\mu)\nabla(\nabla.\vec{u})-\mu(\nabla\times\nabla\times\vec{u})-\upsilon\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T=\rho\frac{\partial^2\vec{u}}{\partial t^2}, \tag{1}$$

$$K^*\nabla^2\phi=\rho C_E\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\upsilon T_0\left(\frac{\partial}{\partial t}+\eta\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla.\vec{u}), \tag{2}$$

$$t_{ij}=\lambda u_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i})-\upsilon\left(1+\tau_1\frac{\partial}{\partial t}\right)T\,\delta_{ij}, \tag{3}$$

where

$$T=(1-a\nabla^2)\phi \tag{4}$$

For a two dimensional problem ($xz-$plane) all quantities depends only on space coordinates x,z and time t. The field equations and constitutive relations in a rotating generalized linear thermoelasticity with two temperature under the influence of gravity and without body forces and heat sources are given by

$$\frac{\partial t_{11}}{\partial x}+\frac{\partial t_{13}}{\partial z}+\rho g\frac{\partial u_3}{\partial x}=\rho\left[\frac{\partial^2 u_1}{\partial t^2}-\Omega^2 u_1+2\Omega\frac{\partial u_3}{\partial t}\right], \tag{5}$$

$$\frac{\partial t_{31}}{\partial x}+\frac{\partial t_{33}}{\partial z}-\rho g\frac{\partial u_1}{\partial x}=\rho\left[\frac{\partial^2 u_3}{\partial t^2}-\Omega^2 u_3-2\Omega\frac{\partial u_1}{\partial t}\right], \tag{6}$$

Using equation (3) in equations (5) to (6) we obtain,

$$(\lambda+2\mu)\frac{\partial^2 u_1}{\partial x^2}+\mu\frac{\partial^2 u_1}{\partial z^2}+(\lambda+\mu)\frac{\partial^2 u_3}{\partial x\partial z}+\rho g\frac{\partial u_3}{\partial x}-\upsilon\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}$$
$$=\rho\left[\frac{\partial^2 u_1}{\partial t^2}-\Omega^2 u_1+2\Omega\frac{\partial u_3}{\partial t}\right], \tag{7}$$

$$\mu\frac{\partial^2 u_3}{\partial x^2}+(\lambda+2\mu)\frac{\partial^2 u_3}{\partial z^2}+(\lambda+\mu)\frac{\partial^2 u_1}{\partial x\partial z}-\rho g\frac{\partial u_1}{\partial x}-\upsilon\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}$$
$$=\rho\left[\frac{\partial^2 u_3}{\partial t^2}-\Omega^2 u_3-2\Omega\frac{\partial u_1}{\partial t}\right]. \tag{8}$$

Introducing dimensionless variables defined by,

$$x_i'=\frac{\omega^*}{c_0}x_i,\, u_i'=\frac{\rho c_0\omega^*}{\upsilon T_0}u_i,\, t'=\omega^* t,\, \tau_0'=\omega^*\tau_0,\, \tau_1'=\omega^*\tau_1,\, T'=\frac{T}{T_0},$$
$$\phi'=\frac{\phi}{T_0},\; t_{ij}'=\frac{t_{ij}}{\upsilon T_0},\; \Omega'=\frac{\Omega}{\omega^*}, \tag{9}$$

where $\omega^*=\rho C_E c_0^2/K^*$, $\rho c_0^2=\lambda+2\mu$.

in equations (2), (7), and (8), we obtain the equations of motion in dimensionless form.

We define displacement potentials q and ψ which are related to displacement components u_1 and u_3 as,

$$u_1=\frac{\partial q}{\partial x}-\frac{\partial\psi}{\partial z},\qquad u_3=\frac{\partial q}{\partial z}+\frac{\partial\psi}{\partial x}, \tag{10}$$

in the resulting dimensionless equations, and then apply the Laplace and Fourier transform defined by,

$$\bar f(x,z,p)=\int_0^\infty f(x,z,t)e^{-pt}dt, \tag{11}$$
$$\tilde f(\xi,z,p)=\int_{-\infty}^{\infty}\bar f(x,z,p)e^{i\xi x}dx, \tag{12}$$

we get, (after suppressing the primes),

$$\left[\frac{d^2}{dz^2}-b_1\right]\tilde q-b_2\tilde\psi-b_3\tilde T=0, \tag{13}$$

$$\left[\frac{d^2}{dz^2}-b_4\right]\tilde\psi+b_2\tilde q=0, \tag{14}$$

$$\left[(1+b_6)\frac{d^2}{dz^2}-(b_5+\xi^2)\right]\tilde{\phi}-b_7\left[\frac{d}{dz^2}-\xi^2\right]\tilde{q}=0, \tag{15}$$

where

$$b_1=\xi^2+p^2-\Omega^2,\ b_2=\frac{i\xi g}{\omega^* c_0}+2\Omega p,\ \ b_3=(1+\tau_1 p),$$

$$b_4=\xi^2+\frac{\rho c_0{}^2}{\mu}(p^2-\Omega^2),\ b_5=\xi^2+p(1+\tau_0 p),\ \ b_6=ap(1+\tau_0 p),$$

$$b_7=\varepsilon(p+\eta\tau_0 p^2), \tag{16}$$

Eliminating $\tilde{\phi}$ *and* $\tilde{\psi}$, from equations (13) to (15) we obtain,

$$[\nabla^6+A\nabla^4+B\nabla^2+C]\tilde{q}=0, \tag{17}$$

where

$$f_1=1+b_6+ab_3b_7,\ \ f_2=b_5+\xi^2+b_1(1+b_6)+b_3b_7(1+a\xi^2)+ab_3b_7\xi^2,$$

$$f_3=b_1(b_5+\xi^2)+b_3b_7\xi^2(1+a\xi^2),\ \ f_4=b_2(1+b_6),$$

$$f_5=b_2(b_5+\xi^2),\nabla=\frac{d}{dz},\in=\frac{\upsilon^2 T_0}{\rho K^*\omega^*},$$

$$A=\frac{-1}{f_1}(f_2+f_1b_4),\ \ B=\frac{1}{f_1}(f_3+f_2b_4+f_4b_2),$$

$$C=\frac{-1}{f_1}(f_3b_4-f_5b_2). \tag{18}$$

The solutions of equation (17) satisfying the radiation conditions that $\tilde{q},\tilde{\psi},\tilde{\phi}\to 0$ as $z\to\infty$ are,

$$\tilde{q}=D_1e^{-q_1z}+D_2e^{-q_2z}+D_3e^{-q_3z}, \tag{19}$$

$$\tilde{\psi}=a_1^*D_1e^{-q_1z}+a_2^*D_2e^{-q_2z}+a_3^*D_3e^{-q_3z}, \tag{20}$$

$$\tilde{\phi}=b_1^*D_1e^{-q_1z}+b_2^*D_2e^{-q_2z}+b_3^*D_3e^{-q_3z}, \tag{21}$$

where q_i^2 are the roots of equation (17) and a_i^*, b_i^* are coupling constants defined by ,

$$a_i^* = \frac{b_2}{b_4 - q_i^{\ 2}},\ b_i^{\ *} = \frac{b_7\left(q_i^{\ 2} - \xi^2\right)}{\left(1 + b_6\right)q_i^{\ 2} - \left(b_5 + \xi^2\right)}. \tag{22}$$

4 BOUNDARY CONDITIONS

4.1 Mechanical Force

The boundary conditions at the plane surface $z = 0$ are,

(i) $t_{33} = -F(x,t)$

(ii) $t_{31} = 0,$ (23)

(iii) $\dfrac{\partial \phi}{\partial z} = 0.$

Using equations (3), (9), and (10), in the boundary conditions (23), we obtain the boundary conditions in the dimensionless form. On suppressing the primes and applying the Laplace and Fourier transform defined by equations (11) and (12) on the dimensionless boundary conditions and using equations (19) to (21), in the resulting transformed boundary conditions, we get the transformed expressions for displacement, force stress , conductive temperature and temperature distribution in a rotating generalized thermoelastic medium with two temperature under the influence of gravity as,

$$\tilde{u}_1 = \frac{-\left(\sum_{m=1}^{3}\left(i\xi + a_m^{\ *}q_m\right)\Delta_m e^{-q_m z}\right)}{\Delta} \tag{24}$$

$$\tilde{u}_3 = \frac{-\left(\sum_{m=1}^{3}\left(q_m + i\xi a_m^{\ *}\right)\Delta_m e^{-q_m z}\right)}{\Delta} \tag{25}$$

$$\tilde{t}_{31} = \frac{\left(\sum_{m=1}^{3} s_m \Delta_m e^{-q_m z}\right)}{\Delta} \tag{26}$$

$$\tilde{t}_{33} = \frac{\left(\sum_{m=1}^{3} r_m \Delta_m e^{-q_m z}\right)}{\Delta} \tag{27}$$

$$\tilde{\phi} = \frac{\left(\sum_{m=1}^{3} b_m^{\bullet} \Delta_m e^{-q_m z}\right)}{\Delta} \tag{28}$$

$$\tilde{T} = \frac{\left(\sum_{m=1}^{3} \left\{ \left(1 + a\xi^2\right) - aq_m{}^2 \right\} b_m{}^* \Delta_m e^{-q_m z} \right)}{\Delta} \tag{29}$$

where

$$\Delta = r_1\left(s_2 g_3 - s_3 g_2\right) - r_2\left(s_1 g_3 - s_3 g_1\right) + r_3\left(s_1 g_2 - s_2 g_1\right)$$

$$\Delta_1 = -\tilde{F}(\xi, p)\left[s_2 g_3 - s_3 g_2\right],\ \Delta_2 = \tilde{F}(\xi, p)\left[s_1 g_3 - s_3 g_1\right]$$

$$\Delta_3 = -\tilde{F}(\xi, p)\left[s_1 g_2 - s_2 g_1\right],\ s_i = \frac{\mu}{\rho c_0^2}\left(2i\xi q_i - \left(q_i^2 + \xi^2\right)a_i^*\right)$$

$$r_i = q_i^2 - \frac{\lambda\xi^2}{\rho c_0^2} - \frac{2i\mu\xi a_i^{\bullet} q_i}{\rho c_0^2} - \left(1 + \tau_1 p\right)\left[\left(1 + a\xi^2\right) - aq_i{}^2\right] b_i^{\bullet}\,. \tag{30}$$

5 PARTICULAR CASES

5.1

Neglecting angular velocity (i.e. $\vec{\Omega} = 0$) in equations (24) to (29), we obtain transformed components of displacement, stress forces, conductive temperature and temperature distribution in a non-rotating generalized thermoelastic medium with two temperature under the influence of gravity.

5.2

Neglecting gravitational effect (i.e. $g = 0$) in equations (24) to (29), the expressions for displacements, force stresses, conductive temperature and temperature distribution reduces in a rotating generalized thermoelastic medium with two temperature.

5.3

Neglecting both angular velocity and gravitational effect (i.e. $\Omega = g = 0$), we get the expressions for displacement, force stresses, conductive temperature and temperature distribution in non-rotating generalized thermoelastic medium with two temperature as (Youssef, 2008 solved the problem subjected to ramp type heating and loading),

$$\tilde{u}_1 = \frac{-\sum_{m}^{2} \left[i\xi\left(\Delta_m{}^{(3)} e^{-q_m' z}\right)\right] + q_3' \Delta_3{}^{(3)} e^{-q_3' z}}{\Delta^{(3)}}, \tag{31}$$

$$\tilde{u}_3 = \frac{-\sum_{m=1}^{2} \left(q_m' \Delta_m{}^{(3)} e^{-q_m' z}\right) + i\xi q_3' \Delta_3{}^{(3)} e^{-q_3' z}}{\Delta^{(3)}} \tag{32}$$

$$\tilde{t}_{31} = \frac{\left(\sum_{m=1}^{3} s_m{}' \Delta_m^{(3)} e^{-q_m' z}\right)}{\Delta^{(3)}} \tag{33}$$

$$\tilde{t}_{33} = \frac{\left(\sum_{m=1}^{3} r_m{}' \Delta_m^{(3)} e^{-q_m' z}\right)}{\Delta^{(3)}} \tag{34}$$

$$\tilde{\phi} = \frac{\left(\sum_{m=1}^{2} a_m{}^{**} \Delta_m^{(3)} e^{-q_m' z}\right)}{\Delta^{(3)}} \tag{35}$$

$$\tilde{T} = \frac{\left(\sum_{m=1}^{2} \left\{\left(1 + a\xi^2\right) - a q_m{}'^2\right\} a_m{}^{**} \Delta^{(3)}{}_m e^{-q_m' z}\right)}{\Delta} \tag{36}$$

Where

$$\Delta^{(3)} = a_1{}^{**} q_1{}' \left(r_2{}' s_3{}' - r_3{}' s_2{}'\right) - a_2{}^{**} q_2{}' \left(r_1{}' s_3{}' - r_3{}' s_1{}'\right),\ \Delta_1^{(3)} = \tilde{F}(\xi, p) a_2{}^{**} q_2{}' s_3{}',$$

$$\Delta_2^{(3)} = -\tilde{F}(\xi, p) a_1{}^{**} q_1{}' s_3{}',\ \Delta_2^{(3)} = \tilde{F}(\xi, p)\left(a_2{}^{**} q_2{}' s_1{}' - a_1{}^{**} q_1{}' s_2{}'\right)$$

$$q'^2_{1,2} = \frac{A_2 \pm \sqrt{A_2^2 - 4B_2}}{2},\ \ q'^2_3 = \xi^2 + \frac{p^2 \rho c_0^2}{\mu},\ \ a_{1,2}{}^{**} = \frac{q_{1,2}{}'^2 - b_1}{b_3\left[1 + a\xi^2 - a q_{1,2}{}'^2\right]},$$

$$A = \frac{-1}{(1-a) + b_6}\left[b_1\left(1 + b_6\right) + b_5 + \xi^2 + b_3 b_7\left(1 + a\xi^2\right) + b_3 b_7 a \xi^2\right],$$

$$B = \frac{-1}{(1-a) + b_6}\left[b_1\left(b_5 + \xi^2\right) + b_3 b_7 \xi^2\left(1 + a\xi^2\right)\right],$$

$$r_{1,2}{}'^2 = q_{1,2}{}'^2 - \frac{\lambda \xi^2}{\rho c_0^2} - \left(1 + \tau_1 p\right)\left[1 + a\xi^2 - a q_{1,2}{}'^2\right] a_{1,2}{}^{**},\ r_3{}' = -\frac{2\mu i \xi q_3{}'}{\rho c_0^2},$$

$$s_{1,2}{}' = \frac{2 i \xi \mu q_{1,2}{}'}{\rho c_0^2},\ \ s_3''' = \frac{\left(q_3{}'^2 + \xi^2\right)\mu}{\rho c_0^2}\ . \tag{37}$$

6 TYPES OF SOURCES

6.1 Concentrated Source

For a concentrated source we take

$$F(x,t)=\delta(x)\delta(t)$$

such that $\tilde{F}(\xi,p)=1.$ (38)

6.2 Distributed Sources

6.2.1 Uniformly Distributed Source

The solution due to a uniformly distributed source in normal direction is obtained by setting

$F(x,t)=\phi(x)\delta(t)$ and $\tilde{F}(\xi,p)=\tilde{\phi}(\xi)$

where

$$\phi(x)=\begin{bmatrix} 1 & if \quad |x|\le a, \\ 0 & if \quad |x|>a, \end{bmatrix} \tag{39}$$

in equation (23) The Fourier transform with respect to the pair (x,ξ) for the case of a uniform strip load of unit amplitude and width 2a applied at the origin of the coordinate system $(x=y=0)$ becomes

$$\tilde{\phi}(\xi)=\left[\frac{2\sin(\xi)}{\xi}\right], \qquad \xi\neq 0. \tag{40}$$

6.2.2 Linearly Distributed Source

The solution due to a linearly distributed source in normal direction is obtained by substituting

$$\phi(x)=\begin{bmatrix} 1-\dfrac{|x|}{a} & if \ |x|\le a, \\ 0 & if \ |x|>a, \end{bmatrix} \tag{41}$$

in equation (23). The Fourier transform of $\phi(x)$ is

$$\tilde{\phi}(\xi)=\frac{2[1-\cos(\xi)]}{\xi^2}. \tag{42}$$

The expressions for the components of displacement, force stress, conductive temperature and temperature distribution are obtained as in equations (24) to (29) and (31) to (36), by replacing $\tilde{\phi}(\xi)$ as $\left[2\sin(\xi)\big/\xi\right]$ and $\frac{2[1-\cos(\xi)]}{\xi^2}$, in the case of a uniformly distributed force and linearly distributed force in equations (30) and (37) for load in normal direction.

6.3 Moving Source

In case of a source moving along the $x-$axis with uniform velocity U at the plane surface $y=0$, we have

$$F(x,t)=H(t)\delta(x-Ut),$$

where

$$\tilde{F}(\xi,p)=\frac{1}{p-i\xi U}. \tag{43}$$

7 NUMERICAL RESULTS AND CONCLUSION

With a view to illustrating the analytical procedure presented earlier, we now consider a numerical example for which computational results are given. The results depict the variations of temperature, displacement and stress fields in the context of G-L theory. For this purpose magnesium crystal like material is taken as the thermoelastic material for which, we take the following values of physical constant as Dhaliwal and Singh (Dhaliwal and Singh 1980)

$$\mu=3.278\times10^{10}\,Nm^{-2},\qquad \lambda=2.17\times10^{10}\,Nm^{-2},\quad T_0=298^0\,K,$$

$$C_E=1.04\times10^3\,JKg^{-1}\deg^{-1},\quad \rho=1.74\times10^3\,Kgm^{-3},$$

$$K^{\bullet}=1.7\times10^2\,Wm^{-1}\,s^{-1}\deg^{-1},\qquad \upsilon=2.68\times10^6\,Nm^{-2}\deg^{-1}.$$

The computations are carried out on the surface $z=1.0$ at t =1.0. The graphically results for normal displacement u_3, normal force stress t_{33}, conductive temperature ϕ and temperature distribution T are shown in figure (1) to (12) for $\Omega=0.5$.

(a) Generalized thermoelastic solid with rotation and under the influence of gravity (GTES) by solid line (———).

(b) Generalized thermoelastic solid without rotation and under the influence of gravity (GTESWR) by dashed line (*—*—*).

(c) Generalized thermoelastic solid with rotation and without gravity (GTESWG) by solid line with centered symbol (-------).

(d) Generalized thermoelastic solid without rotation and without gravity (GTESWRWG) by dashed line with centered symbol (*---*---*).

These graphical results represent the solutions obtained by using generalized theory with two relaxation times (G-L theory by taking $\tau_0 = 0.03$ *and* $\tau_1 = 0.05.$) for dimensionless two temperature parameter $a = 0.015$.

8 INVERSION OF THE TRANSFORM

The transformed displacements, microrotation and stresses are functions of z, the parameters of Laplace and Fourier transforms p and ξ respectively and hence are of the form $\tilde{f}(\xi, z, p)$. To get the function in the physical domain $f(x, z, t)$, first we first invert the Fourier transform and then Laplace transform by forming a computer generated program in FORTRAN earlier applied by Sharma and Kumar (Sharma and Kumar 1997).

9 SPECIAL CASES OF THERMOELASTIC THEORY

9.1

If $\tau_1 = 0, \eta = 1.0$, we obtain the corresponding expressions for components of displacement, stress, conductive temperature and temperature distribution for L-S theory.

9.2

For $\tau_1 > 0, \eta = 1.0$, we obtain the corresponding expressions for components of displacement, stress, conductive temperature and temperature distribution for G-L theory.

9.3

If $a = 0$ in equations (24) to (29) and (31) to (36), we obtain the corresponding expressions for components of displacement, stress, conductive temperature and temperature distribution in generalized thermoelasticity.

10 DISCUSSIONS

10.1 Concentrated Force

Near the point of application of source, the value of normal displacement is negligible for a thermoelastic medium without rotation (GTESWR and GTESWRWG). The value at the same point for a thermoelastic medium under the effect of rotation is very large. Also, the values of normal displacement for GTES and GTESWG decrease sharply with increase in horizontal distance. These variations of normal are shown in figure 1. As compared to the variations of normal displacement, the variations of normal force stress are more oscillatory in nature as shown in figure 2. The variations are similar for both GTES and GTESWG with difference in magnitude.

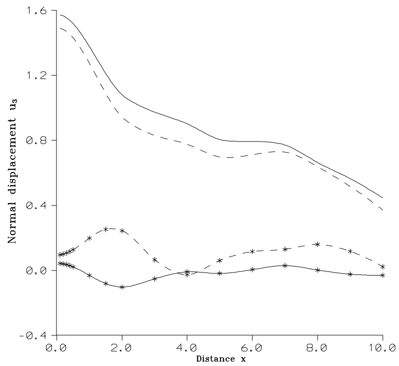

Figure 1: Variation of Normal displacement u_3 with horizontal distance x for concentrated force (————) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

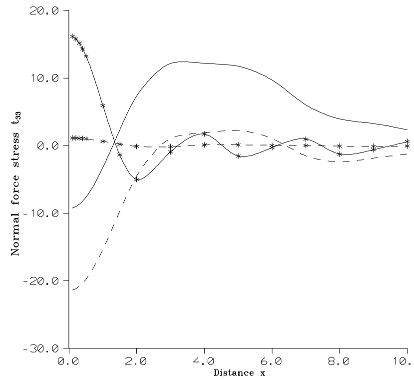

Figure 2: Variation of Normal force stress t_{33} with horizontal distance x for concentrated force. (————) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

It is quite interesting to observe from figure 3 that contrary to the variations of normal displacement, the variations of conductive temperature for GTES and GTESWG increase sharply with oscillating behaviour. However, the variations for other medium (GTESWR and GTESWRWG) are oscillatory and decrease in magnitude with increase in distance *x*. The variations of temperature distribution are opposite in nature to the variations of normal force stress to a large extent for a particular medium. These variations of temperature distribution may be observed form figure 4.

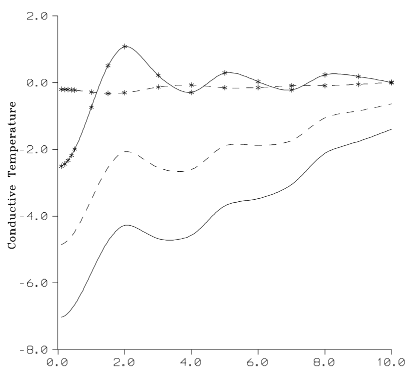

Figure 3: Variation of Conductive temperature ϕ with horizontal distance x for concentrated force. (————) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

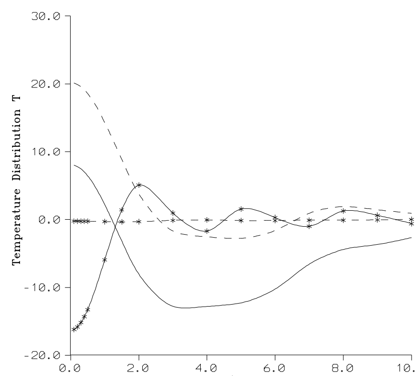

Figure 4: Variation of Temperature distribution T with horizontal distance x for concentrated force . (————) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

10.2 Distributed Force

The variations of all the quantities are similar in nature in case of concentrated force and distributed force (Linearly or uniformly) with difference in magnitude. The graphical results depicting the variations of normal displacement, normal force stress, conductive temperature and temperature distribution on application of uniformly distributed force are shown in figures 5 to 8 respectively.

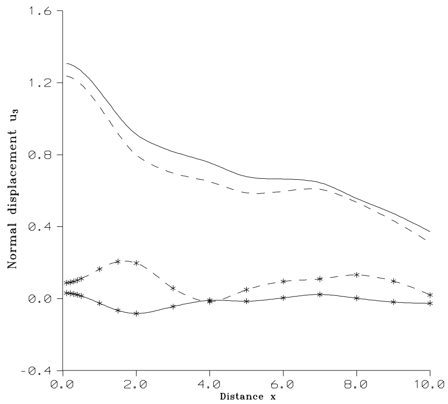

Figure 5: Variation of Normal displacement u_3 with horizontal distance x for uniformly distributed force (———) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

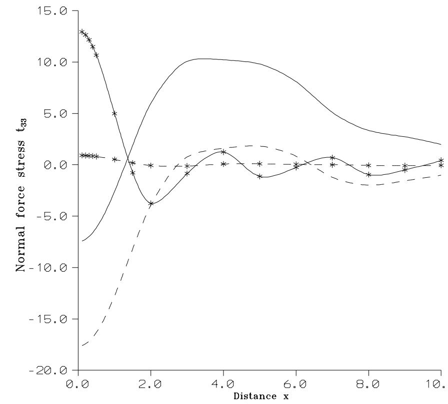

Figure 6: Variation of Normal force stress t_{33} with horizontal distance x for uniformly distributed force . (———) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

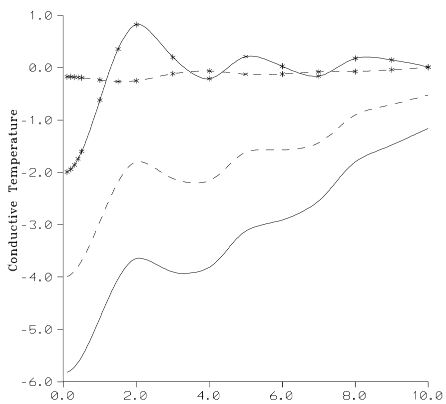

Figure 7: Variation of Conductive temperature ϕ with horizontal distance x for uniformly distributed force. (———) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

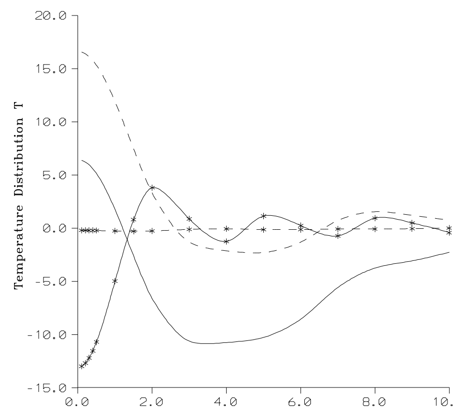

Figure 8: Variation of Temperature distribution T with horizontal distance x for uniformly distributed force. (———) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

10.3 Moving Source

Similar to the discussions given above, the variations of all the quantities are similar in nature for GTES and GTESWG. The values of all these quantities for GTESWR lie in a very short range. However, these variations for GTESWRWG are highly oscillatory in nature in comparison to the variations obtained on application of concentrated force or distributed force. These variations are observed form figures 9 to 12 respectively for normal displacement, normal force stress, conductive temperature and temperature distribution.

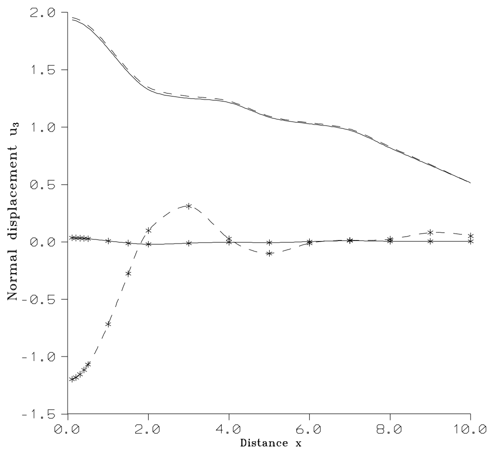

Figure 9: Variation of Normal displacement u_3 with horizontal distance x for moving source (———) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

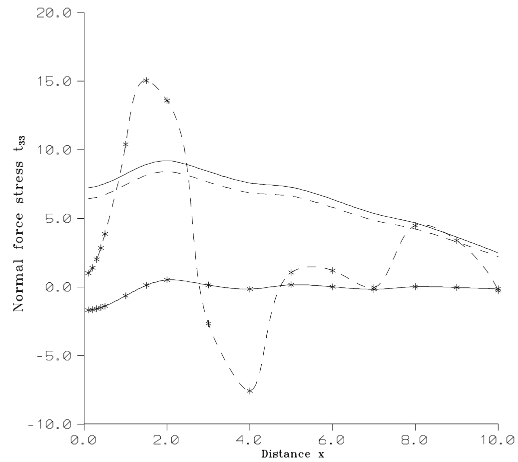

Figure 10: Variation of Normal force stress t_{33} with horizontal distance x for moving source. (———) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

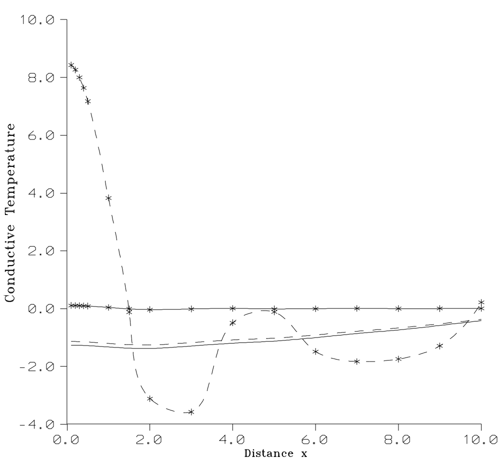

Figure 11: Variation of Conductive temperature ϕ with horizontal distance x for moving source. (———) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

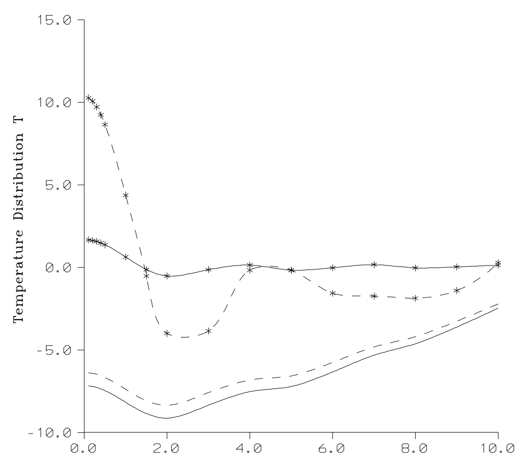

Figure 12: Variation of Temperature distribution T with horizontal distance x for moving source. (———) GTES, (*—*——*—*) GTESWR, (--------) GTESWG and (*---*-----*) GTESWRWG.

11 CONCLUSION

The equations of motion are not decoupled in the presence of rotation and/or gravity even after using the Helmholtz representation. When both the effects of rotation and gravity are neglected, the equations get decoupled. Also, the variations of all the quantities are similar in nature for GTES and GTESWG. Significant effect of rotation and gravity is observed on all the quantities.

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