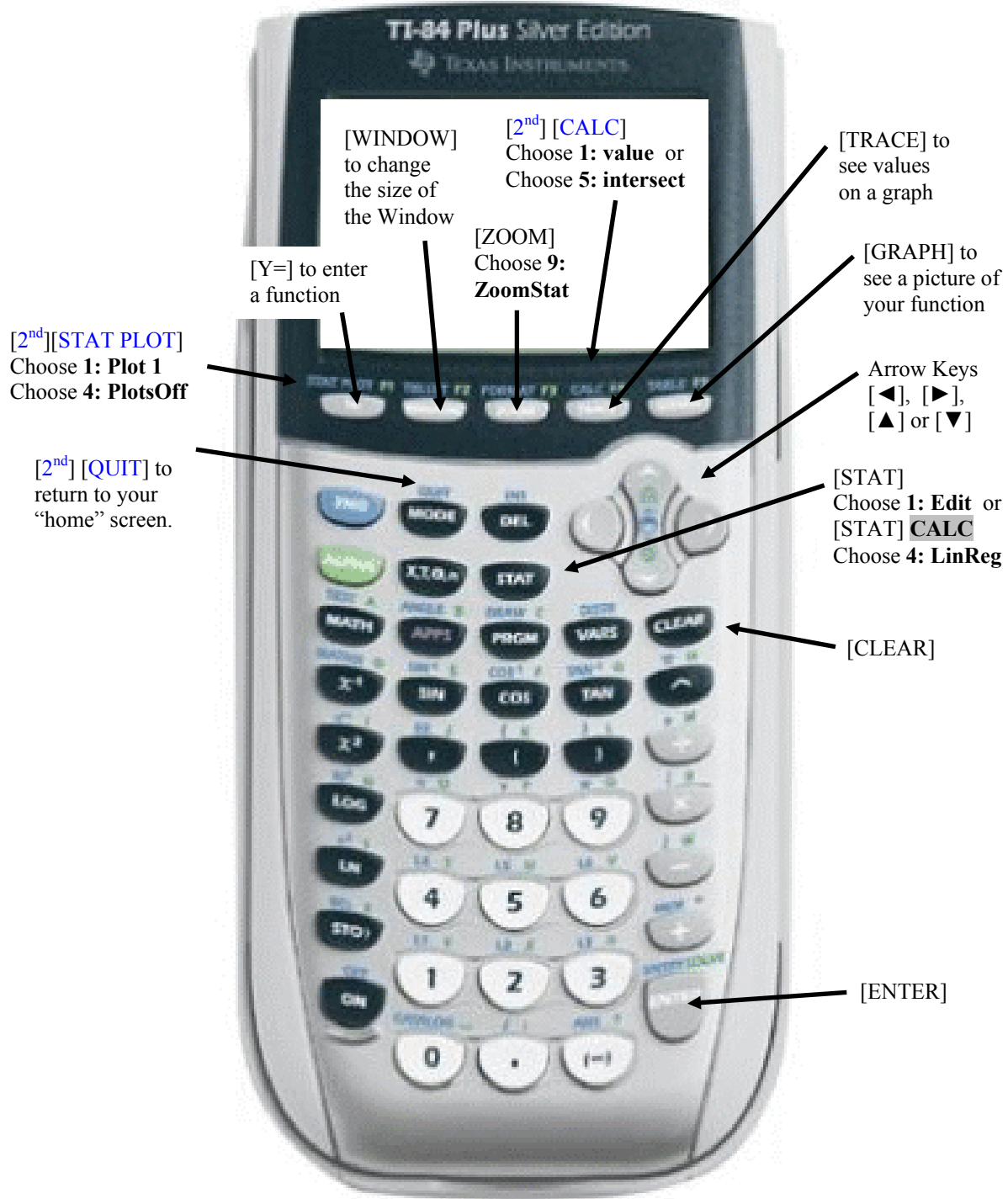


### Introduction & Calculator Help

Page 2 of these notes will walk you through the general procedures for using a graphing calculator. The diagram will show you where to locate the important keys. Refer to both pages as needed to work on Activity 1 on the Ch 2 Alg 2 L2 Activity Sheet.



**Introduction & Calculator Help**

Complete Activity 1 on the Ch 2 Alg 2 L2 Activity Sheet. All instructions on how to use the graphing calculator are given below.

**#1 Entering Data Given a Table:**

[STAT] Choose **1: Edit**

Clear any existing lists by moving the cursor up and over the list name, L1. Press [CLEAR] then [ENTER].

Enter  $x$ -data into L1 and  $y$ -data into L2.

[2<sup>nd</sup>][STAT PLOT] Choose **1: Plot 1** and turn it on by pressing [ENTER].

Type: choose scatter

Xlist: L1

Ylist: L2

Mark: choose the square.

[ZOOM] Choose **9: ZoomStat** or [WINDOW]

When you are done looking at your plot press [2<sup>nd</sup>][QUIT] to return to your “home” screen.

**#2 Entering A Function to Fit Your Data :**

Press [Y=].

Clear any equations from [Y=]. Use the [CLEAR] key.

[2<sup>nd</sup>][QUIT] to return to your “home” screen.

[STAT] Highlight **CALC** Choose **4: LinReg (ax+b)**

It prompts you. **LinReg (ax+b)**

Just press [ENTER].

Round the values the calculator gives you to three places.

[Y=] enter the equation **Y1=** substituting values into your LinReg formula.

[GRAPH] To see the graph of your plot and function.

**#5 To Stop Plotting:**

[STAT PLOT] Choose **4: PlotsOff** Press [ENTER].

Turns off the plotting but keeps your data in the lists.

**Find Values from Tracing the Graph:**

[GRAPH] To show the graph.

[TRACE] The cursor will jump from one data point to the next if you use the [◀] and [▶] keys.

Use [▲] or [▼] to jump from the plot to the line and back.

Get on the line then move using the [◀] and [▶] keys to get the  $x$ -value and  $y$ -value (shown on the bottom of the screen) as close as possible to the values you want.

**#3 Find a  $y$ -value, given an  $x$ -value:**

[GRAPH] To show the graph.

[2<sup>nd</sup>][CALC] Choose **1: value**

It prompts you. **X=** enter a number  
The calculator gives

$X = Y =$

(shown on the bottom of the screen)

**#4 Find an  $x$ -value, given a  $y$ -value:**

Press [Y=].

In **Y2 =** enter your  $y$ -value.

[GRAPH] To show the graphs.

[2<sup>nd</sup>][CALC] Choose **5: intersect**

It prompts you First curve? Use [▲] or [▼] to jump to Y1 (check the upper part of the screen). [ENTER]

It prompts you Second curve? Repeat process for Y2. [ENTER]

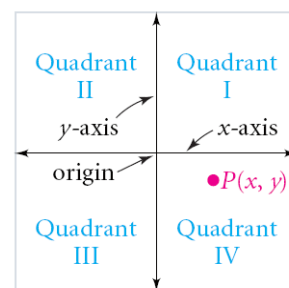
It prompts you Guess? [ENTER]

Intersection with  $X = Y =$

is shown on the bottom of the screen.

## 2.1 Relations and Functions

In Unit 2, you will move from simplifying variable expressions and solving one-variable equations and inequalities to working with two-variable equations and inequalities. If you do not remember how to graph points in the coordinate plane, do page 54. Always go horizontal then vertical.



### Graphing Relations

A **relation** is a set of pairs of input and output values. (Using set notation,  $\{ \}$ )  $\{ \}$  is read “the set of”, which means a collection of.

You can write a relation as a set of ordered pairs in the  $\{ \}$ .

Simply plot the points to graph a relation.

The **domain** of a relation is the set of all inputs, or  $x$ -coordinates of the ordered pairs.

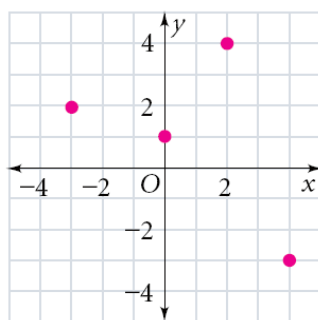
The **range** of a relation is the set of all outputs, or  $y$ -coordinates of the ordered pairs.

A **function** is a relation in which each value in the domain ( $x$ -value) is paired with exactly one value in the range ( $y$ -value).

#### Example 1

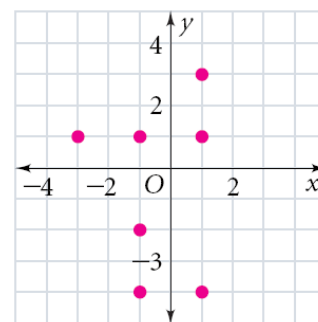
Graph the relation  $\{(-3, 2), (0, 1), (2, 4), (4, -3)\}$ .  
Is the relation a function?

Yes because every  $x$  has exactly one  $y$ -value.



#### Example 2

Find the domain and range of the relation shown in the graph.  
Is the relation a function?



Domain is  $\{-3, -1, 1\}$

Range is  $\{-4, -2, 1, 3\}$

No  $x = -1$  is paired with  $-4, -2$  and  $1$ .  
or  $x = 1$  is paired with  $-4, 1$  and  $3$ .

### Identifying Functions

Again, a **function** is a relation in which each element of the domain is paired with exactly one element in the range. A function is a special kind of relation in which each  $x$ -value can have only one corresponding  $y$ -value.

#### Example 3a

Is the relation given by the table a function?

Not a function,  
 $x = 3$  paired with  
 $y = 2$  and  $6$ .

$x$	$y$
-5	-10
-1	-2
3	2
3	6

#### Example 3b

Is the relation,  $\{(2, 8), (-1, 5), (0, 8), (-1, 3), (-2, 3)\}$  a function?

Not a function,  $x = -1$  paired with  $y = 3$   
and  $5$ . The rest is ok.

**Example 4a**

Is the relation  $\{(-2, -1), (-2, 3), (0, 3), (5, 4)\}$  a function?

The element -2 of the domain is paired with both -1 and 3 of the range. The relation is NOT a function.

Repeat x-values NOT ok.

**Example 4b**

Is the relation, given by the table, a function?

x	y
-1	5
0	3
2	3
3	-1

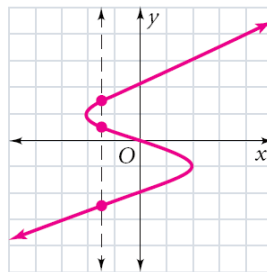
Each element of the domain is paired with exactly one element of the range. The relation is a function.

Repeat y-values ok.

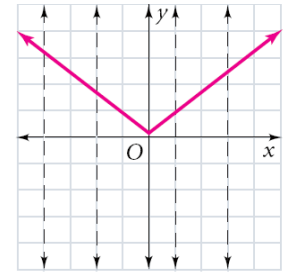
You can picture a function as a set of points, or with table or a graph in the coordinate plane. This works well when the data is discrete. When the domain values are **discrete**, the graph may be a collection of isolated points. When the domain values are **continuous**, such as every real number, the graph may be a line or curve. An easy way to see if a continuous graph is a function is to apply the vertical line test.

In a graph, you will use the **vertical-line test** to identify repeated y-values in relations that are not functions.

If a vertical line passes through at least two points on the graph, then one element of the domain is paired with more than one element of the range, and the relation is not a function.

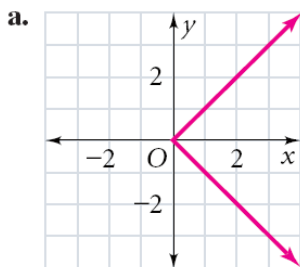


Vertical line passes through  
 ← 3 points    1 point →  
 ← Not a function  
                   A function →

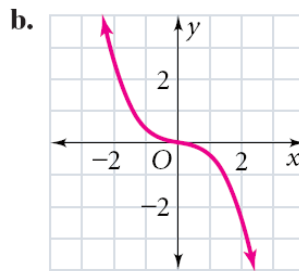


**Example 5 QC**

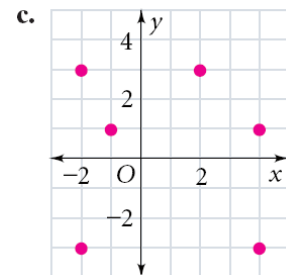
Is the graph discrete or continuous? Is the relation a function? (Use the vertical-line test.) Write the domain and range for each.



continuous  
 NOT a function  
 $x = 1, y = \pm 1$   
 Domain:  $x \geq 0$   
 Range: All real numbers



continuous  
 a function  
 Domain: All real numbers  
 Range: All real numbers



discrete  
 NOT a function  
 $x = -2$  gives  $y = \pm 3$   
 Domain:  $\{-2, -1, 2, 4\}$   
 Range:  $\{-3, 1, 3\}$

A **function rule** is an expression which gives an output value (usually  $y$ ) in terms of an input value (usually  $x$ ). The output values can be represented by a simple variable or it can be represented using function notation,  $f(x)$ , which is read "f of x". (This notation came about so that the reader can see which input value yielded the output value.)  $f(x)$  is one variable and does not mean  $f \bullet x$ !

As a table:

$t$ , time (in hours)	$d(t)$ , Distance (in miles)
input	output
0	0
1	55
2	110
3	165
4	220

Show  $\Delta x$  and  $\Delta y$ .

**Example 6alt**

If you are traveling at a constant rate of 55 mph, the distance you have traveled depends on the amount of time you have traveled. So  $d = r \bullet t$  can be written in the form  $d(t) = 55t$ , since distance depend on time.

So to denote the distance traveled in 1 hour:  $d(1) = 55(1) = 55$  miles.

To denote the distance traveled in 2 hours:  $d(2) = 55(2) = 110$  miles.

To denote the distance traveled in 3 hours:  $d(3) = 55(3) = 165$  miles.

The function notation is very descriptive,  $d(4) = 200$ , shows that when the input is 4, the output is 200, just like the table would. An equation like  $y = 200$  really does not say much about the situation, nor does it indicate what the matching input value is.

**Example 6**

The area of a square tile is a function of the length of a side of the square. Write a function rule for the area of a square.

Evaluate the function for a square tile with side length 3.5 in.

$A(s) = s^2$  or

$A(3.5) = (3.5)^2$  substitute in 3.5 for every  $s$ .  
 $= 12.25$  evaluate (or simplify)

A sentence like  $y = x^2$ , is the same relationship but is not very descriptive.

In Example 6...

The **input** is side length and the **output** is the area.

The **independent variable** is  $s$  and the **dependent variable** is  $A(s)$ , area, because the area depends on the side length.

Say: Area is a function of the length of a side.

Write:  $A(s)$ .

Read: "A of s". This does NOT mean A times s!! It means that if you know a side, you can find the area by substituting in the value for  $s$ .

When the value of  $s$  is 3.5 inches, you write...

$A(3.5)$  or  $A(3.5) = (3.5)^2 = 12.25$ .

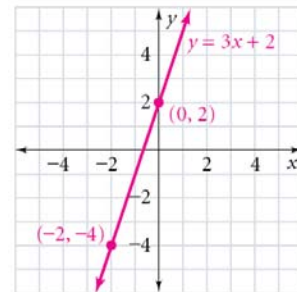
Functions can also be thought of as machines. You input a value and the function spits out a value based on a function rule.

The inputs and outputs can be shown using function notation or as ordered pairs.

Input	Function	Output	Ordered Pair
5	Subtract 1	4	(5, 4)
$a$	Add 2	$a + 2$	( $a$ , $a + 2$ )
3	$g$	$g(3)$	(3, $g(3)$ )
$x$	$f$	$f(x)$	( $x$ , $f(x)$ )

### 2.2 Linear Equations

A function whose graph is a line is a **linear function**. You can represent a linear function with a **linear equation**, such as  $y = 3x + 2$  or  $f(x) = 3x + 2$ . A solution of a linear equation is any ordered pair  $(x, y)$  that makes the equation true. There are an infinite number of these solutions that is why graphs are typically used to show all of the solutions. Tables are useful for organizing your ordered pairs.



Because the value of  $y$  depends on the value of  $x$ ,  $y$  is called the **dependent variable** and  $x$  is called the **independent variable**. In the equation for a line the  $x$  and the  $y$  represent any or all points on the line,  $(x, y)$ . So to find points on a line put a value in for one variable, say  $x$ , and use the equation to find the matching  $y$ -value.

**Part A. Graphing Lines Geometry Connection** Since two points determine a line, you can use two points to graph a line. You can check your line by finding a third point and seeing if it is on the line.

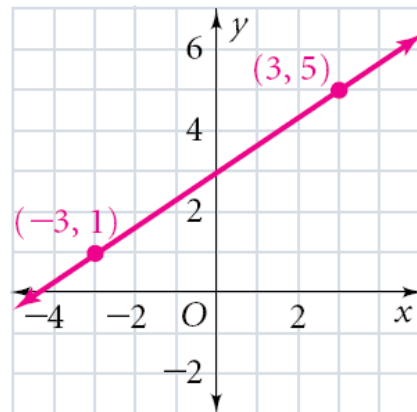
**Example 1a**

Graph the equation  $y = \frac{2}{3}x + 3$ .

Choose two values for  $x$  and find the corresponding  $y$ .

Use substitution and simplification. Plot the 2 points and draw!

	substitute		graph
$x$	$\frac{2}{3}x + 3$	$y$	$(x, y)$
-3	$\frac{2}{3}(-3) + 3 = 1$	1	$(-3, 1)$
3	$\frac{2}{3}(3) + 3 = 5$	5	$(3, 5)$



Is  $(2, 4)$  a point on the line? No,  $\frac{2}{3}(2) + 3 = 4\frac{1}{3}$  but  $(2, 4\frac{1}{3})$  is!

A great way to graph a line is by determining its intercepts.

The **y-intercept** of a line is the point at which the line crosses the  $y$ -axis. All points on the  $y$ -axis have an  $x$ -coordinate of 0.

The **x-intercept** of a line is the point at which the line crosses the  $x$ -axis. All points on the  $x$ -axis have an  $y$ -coordinate of 0.

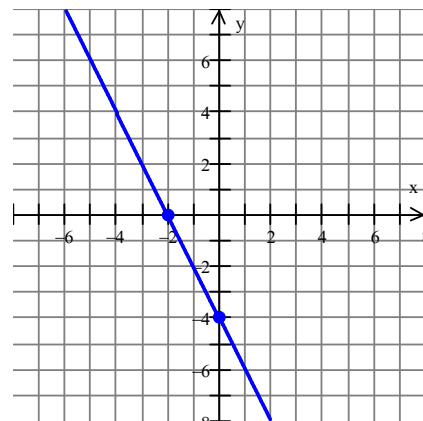
**Example 1b**

Find the intercepts for the line  $2x + y = -4$ .

Use the intercepts to graph the line.

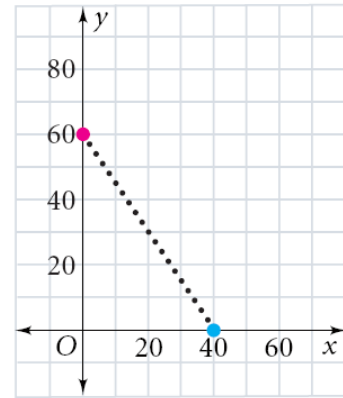
**y-intercept,  $x = 0$ ,**  
**SO  $2(0) + y = -4$**   
**y-intercept is  $(0, -4)$**

**x-intercept,  $y = 0$ ,**  
**SO  $2x + 0 = -4 \Rightarrow 2x = -4 \Rightarrow x = -2$**   
**x-intercept is  $(-2, 0)$ .**



**Example 2**

The equation  $3x + 2y = 120$  models the number of passengers who can sit in a train car, where  $x$  is the number of adults and  $y$  is the number of children. Graph the equation. Explain what the  $x$ - and  $y$ -intercepts represent. Describe the domain and the range.



$x$ -intercept,  $y = 0$ :

$$3x + 2y = 120$$

$$3x + 2(0) = 120$$

$$3x = 120$$

$$x = 40$$

$x$ -intercept is  $(40, 0)$

$y$ -intercept,  $x = 0$

$$3x + 2y = 120$$

$$3(0) + 2y = 120$$

$$2y = 120$$

$$y = 60$$

$y$ -intercept is  $(0, 60)$

When 40 adults are seated, no children can sit.

When no adults are seated, 60 children can sit.

The number of passengers is a whole number. The situation is discrete.

The domain is limited to the whole numbers 0 to 40. The range to the whole numbers 0 to 60.

**Definition Slope (or rate of change)**

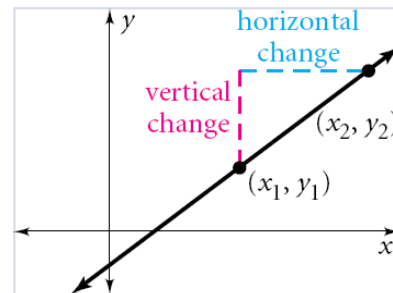
Let  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$ , be points on a nonvertical line.

The **slope** is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$$

where  $x_2 - x_1 \neq 0$ .

Also, slope gives the **average rate of change** is change in one quantity per change in another, like miles per hour.



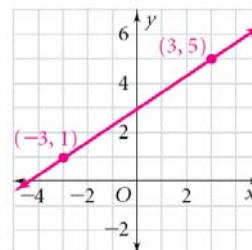
Uphill as  $x$  increases  $\rightarrow$  positive slope  
 Downhill as  $x$  increases  $\rightarrow$  negative slope  
 $\Delta y = 0 \rightarrow$  slope of 0, horizontal line  
 $\Delta x = 0 \rightarrow$  slope is undefined, vertical line

**Example 3**

Find the slope of the line through the points  $(-3, 1)$  and  $(3, 5)$ .

$$m = \frac{5-1}{3-(-3)} = \frac{4}{6} = \frac{2}{3} \quad \text{OR} \quad m = \frac{1-5}{(-3)-(3)} = \frac{-4}{-6} = \frac{2}{3}$$

Hint: Fill in  $y$  then below matching  $x$ ! Watch signs!



Show slope on graph.

**Part B. Forms for Equations of Lines**

There are three basic forms for equations for lines.

- You can use these forms to write equations of lines. Choose the form based upon what information you are given about the line.
- You can get information about the line from certain forms. So, simply use algebra to switch from one form to another. Then read the form.
- In all cases, the  $x$  and  $y$  in the equation are the points that are ON the line,  $(x, y)$ .

**I. Slope-Intercept Form**

Slope-Intercept Form is one of the easiest forms to use and to gather information from.

Watch the videos on the website if you need to and complete the examples below.

**Definition Slope-Intercept Form**

The equation

$$y = mx + b$$

is the line with slope  $m$  and  $y$ -intercept  $(0, b)$ .

Use to write an equation, given a slope and  $y$ -intercept.  
Read the form to find slope and/or  $y$ -intercept.

**Example 1a** Use slope-intercept form to graph the following lines:

a.  $y = \frac{1}{3}x - 2$

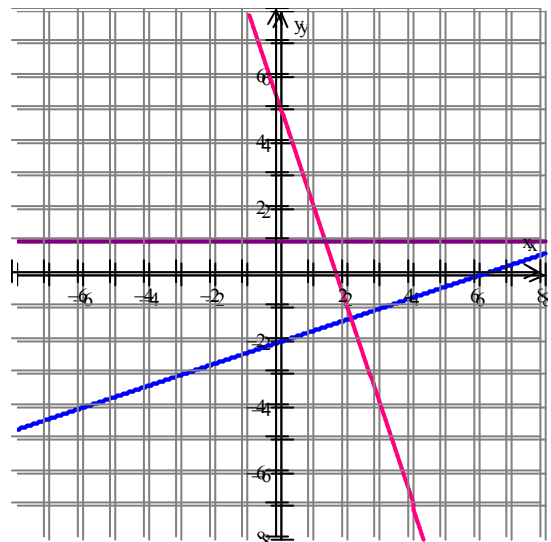
read the form:  $m = \frac{1}{3} = \frac{\Delta y}{\Delta x}$  and  $y$ -intercept  $(0, -2)$

b.  $y = -3x + 5$

read the form:  $m = \frac{-3}{1} = \frac{\Delta y}{\Delta x}$  and  $y$ -intercept  $(0, 5)$

c.  $y = 1$

read the form:  $m = 0$  and  $y$ -intercept  $(0, 1)$

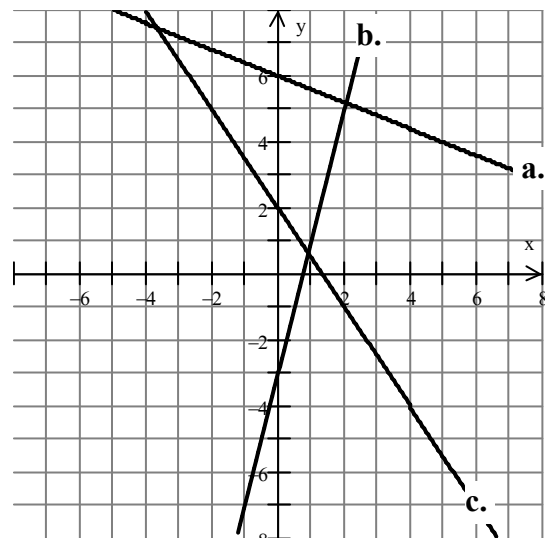


**Example 1b** Write the equation for the line in slope-intercept form from the graph.

a.  $y = -\frac{2}{5}x + 6$

b.  $y = 4x - 3$

c.  $y = -\frac{3}{2}x + 2$





Write the equation for the lines:

**Example 2a** Write the equation for the line with a slope of  $-\frac{3}{4}$  and goes through  $(0, 8)$ .

Since  $(0, 8)$  is the y-intercept;  $m = -\frac{3}{4}$

$$y = -\frac{3}{4}x + 8$$

**Example 2b** Write the equation for the line with a slope of 7 and goes through  $(-4, -11)$ .

$$m = 7$$

Substitute:

$$y = 7x + b \quad \text{now put in the point}$$

$$-11 = 7(-4) + b$$

$$-11 = -28 + b$$

$$17 = b$$

$$\text{Line: } y = 7x + 17$$

**Example 2c** Write the equation for the line that passes through  $(-1, 6)$  and  $(5, -4)$

$$\text{Find slope: } m = \frac{6 - (-4)}{-1 - 5} = \frac{10}{-6} = -\frac{5}{3}$$

Substitute:

$$y = -\frac{5}{3}x + b \quad \text{now put in a point}$$

$$6 = -\frac{5}{3}(-1) + b$$

$$6 = \frac{5}{3} + b$$

$$6 - \frac{5}{3} = b$$

$$\frac{18}{3} - \frac{5}{3} = b$$

$$\frac{13}{3} = b$$

$$\text{Line: } y = -\frac{5}{3}x + \frac{13}{3} \quad \text{or } y = -1\frac{2}{3}x + 4\frac{1}{3}$$

Convert the equations to Slope-Intercept Form. Just use algebra techniques!

**Example 3a** Write the equation in slope intercept form  $4x + 2y = -8$ . Read the form.

$$4x + 2y = -8$$

$$2y = -4x - 8$$

$$\frac{2y}{2} = \frac{-4x - 8}{2}$$

$$y = -2x - 4$$

The line has a slope of  $-2$  and a y-intercept of  $(0, -4)$

**Example 3b** Write the equation in slope intercept form  $2y = -8$ . Read the form.

$$\frac{2y}{2} = \frac{-8}{2}$$

$$y = -4$$

The line has a slope of 0 and a y-intercept of  $(0, -4)$

## II. Point-Slope Form

Is also a convenient form to use and to also gather information about the line.

**Example 1** Write the equation for the line with a slope of 7 and goes through  $(-4, -11)$ .

$$m = 7 \text{ and point is } (-4, -11)$$

Substitute:

$$y - y_1 = m(x - x_1) \text{ now put in the info.}$$

$$y - (-11) = 7(x - (-4))$$

$$y + 11 = 7(x + 4)$$

If you want slope-intercept form:

$$y + 11 = 7(x + 4)$$

$$y + 11 = 7x + 28$$

$$y = 7x + 17$$

### Example 3

Write in point-slope form an equation of the line with slope  $-2$  through the point  $(3, -5)$ .

Key is to start in point-slope form and then use algebra to convert it to slope-intercept form.

$$y - y_1 = m(x - x_1) \text{ given a point \& slope}$$

$$y - (-5) = -2(x - (3)) \text{ plug in the info.}$$

$$y + 5 = -2(x - 3) \text{ simplify.}$$

### Definition Point-Slope Form

The equation

$$y - y_1 = m(x - x_1)$$

is the line through the point  $(x_1, y_1)$  with slope  $m$ .

Use to write an equation, given a point and slope  
OR given two points.

Read the form to find a point and/or slope.

### Example 2

Determine the slope of the line.

$y + 5 = -\frac{2}{3}(x - 9)$ . Name a point that the line passes through.

Compare to the form

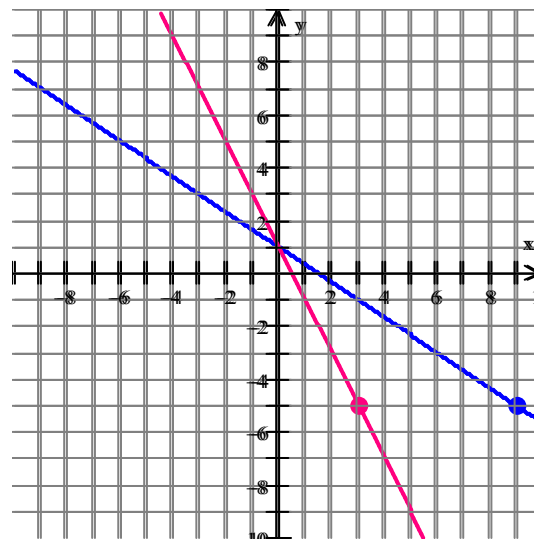
$$y - y_1 = m(x - x_1)$$

$$y + 5 = -\frac{2}{3}(x - 9)$$

$$y - (-5) = -\frac{2}{3}(x - 9)$$

So the slope is  $-2/3$  and it passes through  $(9, -5)$ .

What would you plug in for  $y_1$  to get  $+5$ ?



### III. Standard Form

Not so easy to gather information from this form nor is it easy to write equations in this form. So the trick is to use the most convenient for and use algebra techniques to write your equations in the form you need. There are good videos of this form on my website.

#### Example 1

Write in standard form an equation of the line with slope  $5/6$  through the point  $(-6, 10)$ .

Since you are given the slope and a point, start with point-slope form.

$$y - y_1 = m(x - x_1) \text{ given a point \& slope}$$

$$y - 10 = \frac{5}{6}(x - (-6)) \text{ plug in the info.}$$

Multiply both sides by 6.

$$6(y - 10) = 5(x + 6) \text{ use algebra to get it in the form}$$

$$6y - 60 = 5x + 30$$

$$-5x + 6y = 90$$

or  $5x - 6y = 90$  make it look like standard form and make the x-term positive by multiplying both sides by negative 1.

#### Definition **Standard Form Linear Equation**

The equation

$$Ax + By = C$$

$A$ ,  $B$  and  $C$  are real numbers, and  $A$  and  $B$  not both 0.

Use to find points on a line.

Is also the "simplest form" for a line.

#### Example 2

Write an equation in standard form for a line that has a slope of  $2/3$  and has y-intercept  $(0, -6)$ .

Since you are given the slope and a point that is a y-intercept, start with slope- intercept form.

$$y = mx + b$$

$$y = \frac{2}{3}x + (-6) \text{ in slope intercept form}$$

$$3(y) = 3\left(\frac{2}{3}x - 6\right)$$

$$3y = 2x - 18$$

$$-2x + 3y = -18$$

$$2x - 3y = 18$$

#### Example 3

Find the slope and y-intercept of the line  $3x + 2y = 1$ .

Get it in slope- intercept form.

$$3x + 2y = 1$$

$$y = mx + b$$

$$\frac{2y}{2} = \frac{-3x + 1}{2}$$

$$y = -\frac{3}{2}x + \frac{1}{2}$$

So slope is  $-3/2$  or  $-1.5$  and the y-intercept is  $0.5$  or  $\left(0, \frac{1}{2}\right)$ .

**Example 4**

Write in standard form the equation of the line through (5, 1) and (-4, -3).

Find slope:  $m = \frac{1 - (-3)}{5 - (-4)} = \frac{4}{9}$  use either point

Using (5, 1):

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{4}{9}(x - 5)$$

$$9(y - 1) = 4(x - 5)$$

$$9y - 9 = 4x - 20$$

$$-4x + 9y = -11$$

or  $4x - 9y = 11$

Using (-4, -3):

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \frac{4}{9}(x - (-4))$$

$$y + 3 = \frac{4}{9}(x + 4)$$

$$9(y + 3) = 4(x + 4)$$

$$9y + 27 = 4x + 16$$

$$-4x + 9y = -11$$

or  $4x - 9y = 11$

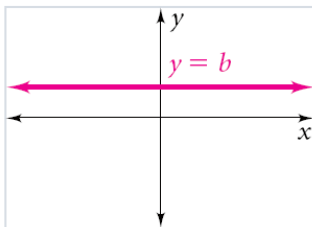
Does the point you choose to use matter when writing the equation? **No**  
 Standard Form is like the "simplest form" for writing equations.

**Part C. Special Types of Lines**

Do the "Investigation: Interesting Lines" on your activity sheet.

**The slopes of horizontal and vertical lines.**

**Horizontal Line**

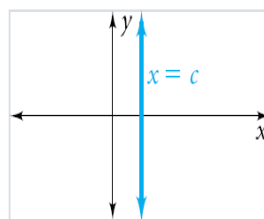


$y = mx + b$  since  $m = 0$ ,  
 $y = (0)x + b$  so write  $y = b$

In table:

$x$	$y$
	$b$
	$b$
	$b$
	$b$

**Vertical Line**



Since  $m = \text{undefined}$ ,  
 and  $x$  constant, write  $x = c$

In table:

$x$	$y$
$c$	
$c$	
$c$	
$c$	

**Example 1a**

Write the equation of the horizontal line through (5, -3).

The y-coordinate never changes, it is always -3,  
 so  
 $y = -3$

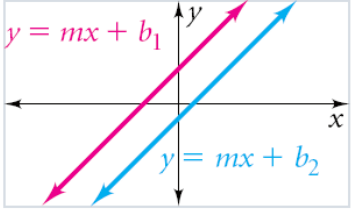
**Example 1b**

Write the equation of the vertical line through (5, -3).

The x-coordinate never changes, it is always 5,  
 so  
 $x = 5$

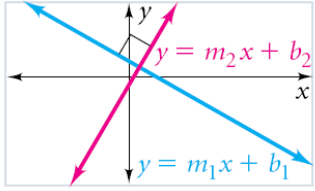
The slopes of perpendicular, and parallel lines have special properties.

Non-vertical **parallel** lines have the **same slope**.



$m = m$

**Perpendicular** lines have the slopes that are **opposites and reciprocals**.



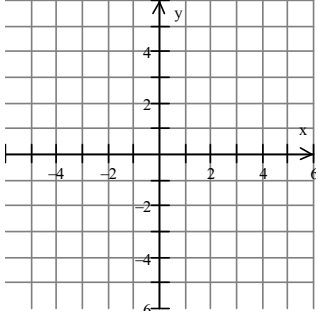
$m_1 = -\frac{a}{b}$   
 $m_2 = \frac{b}{a}$   
 $-\frac{a}{b} \cdot \frac{b}{a} = -1$

$m_1 \cdot m_2 = -1$

**Example 2**  
 Write an equation for the line through (2, 1) and perpendicular to the line  $y = \frac{2}{3}x + \frac{5}{8}$ . Graph the lines if it helps.

$y - y_1 = m(x - x_1)$     slope =  $-\frac{3}{2}$   
 $y - 1 = -\frac{3}{2}(x - 2)$   
 $2(y - 1) = -3(x - 2)$   
 $2y - 2 = -3x + 6$   
 $3x + 2y = 8$     OR

$2y = -3x + 8$   
 $\frac{2y}{2} = \frac{-3x}{2} + \frac{8}{2}$   
 $y = -\frac{3}{2}x + 4$



**Example 3**  
 Write an equation for the line through (-2, 1) and parallel to the line  $3x - 2y = -8$ . Graph the lines if it helps.

$3x - 2y = -8$   
 $-2y = -3x - 8$   
 $\frac{-2y}{-2} = \frac{-3x}{-2} - \frac{8}{-2}$   
 $y = \frac{3}{2}x + 4$

Slope  $\frac{3}{2}$

Slope  $\frac{3}{2}$  through (-2, 1)  
 $y - y_1 = m(x - x_1)$

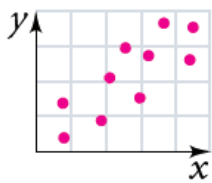
$y - 1 = \frac{3}{2}(x - (-2))$   
 $y - 1 = \frac{3}{2}(x + 2)$   
 $2(y - 1) = 3(x + 2)$   
 $2y - 2 = 3x + 6$   
 $2y = 3x + 8$   
 $y = \frac{3}{2}x + 4$

### 2.4 Using Linear Models

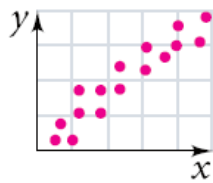
#### Predicting With Linear Models

A **scatter plot** is a graph that relates two different sets of data by plotting the data as ordered pairs.

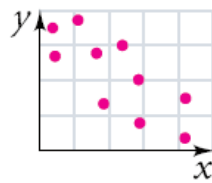
You can use a scatter plot to determine a relationship between the data sets.



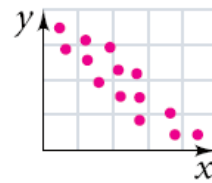
Weak, positive correlation



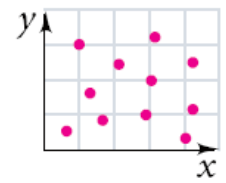
Strong, positive correlation



Weak, negative correlation



Strong, negative correlation



No correlation

A **trend line** is a line that approximates the relationship between the data sets of a scatter plot. You can use a trend line to make predictions.

#### Example 1

A woman is considering buying the 1999 car. She researches prices for various years of the same model and records the data in a table. The asking price is \$4200. Is the asking price reasonable?

Years after 1999	1	2	3	4	5
Model Year	2000	2001	2002	2003	2004
Prices	\$5784	\$6810	\$8237	\$9660	\$10,948
	\$5435	\$6207	\$7751	\$9127	\$10,455

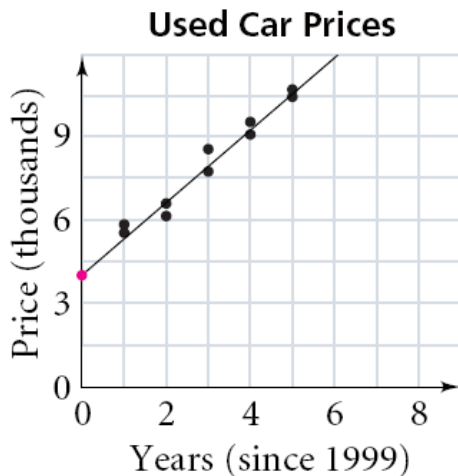
#### Analyze & Define Variables:

Independent variable:  $x =$  the model year, since 1999 would be the starting year, let  $x = 0$  for 1999, then  $x = 1$  for 2000, etc...

Dependent variable:  $y =$  prices in \$, converted to thousands ( $\div 1000$ )

#### Make a scatter plot and examine:

Draw a scatter plot. Decide whether a linear model is reasonable.



#### Draw a trend line then write an equation:

Draw a line that has about the same number of data points above and below it. (See graph.)

Write the equation of the line. Choose two points on the line like (1, 5.4) and (5, 10.5).

$$m = \frac{10.5 - 5.4}{5 - 1} = \frac{5.1}{4} = 1.275$$

$$y - y_1 = m(x - x_1)$$

$$y - 5.4 = 1.275(x - 1)$$

$$y = 1.275x + 4.125 \text{ equation of the trend line}$$

Determine whether the asking price is reasonable.

A fair price would be the value of  $y$  for  $x = 0$ ,  $y = 1.275(0) + 4.125$  so about \$4,125. The asking price of \$4200 is reasonable, but you could talk it down.

A linear model seems reasonable, since the points fall close to a line.

### Calculator Walkthrough

Review page 1 & 2 on these notes before proceeding. Leave your L1 and L2 data in the calculator from the previous problem. I will show you in this example how to use other lists and other plots on the calculator.

As a table:

year	years passed 1997	National Health Expend. (billions \$)
1997	0	1093
1998	1	1150
1999	2	1223
2000	3	1309
2001	4	1421
2002	5	1553

#### Example

Given the data in the table for National Health Expenditures, predict the health expenditures for 2010.

#### Analyze & Define Variables:

Independent variable:  $x = \text{years passed 1997}$ .

Dependent variable:  $y = \text{expenditures in billions \$}$

#### Make a scatter plot and examine:

Draw a scatter plot. Decide whether a linear model is reasonable.

[STAT] Choose **1: Edit**

Clear L3 and L4 and turn plots off:

[2<sup>nd</sup>][STAT PLOT] Choose **4: PlotsOff**

Enter x-data into L3 and y-data into L4.

[2<sup>nd</sup>][STAT PLOT] Choose **1: Plot 2** and turn it on by pressing [ENTER].

Type: choose scatter

Xlist: L3

Ylist: L4

Mark: choose the square.

To change the lists, press [2<sup>nd</sup>][3] for L3. Press [2<sup>nd</sup>][4] for L4.

#### Draw a trend line then write an equation:

Start from home screen: [2<sup>nd</sup>][QUIT]

[STAT] Highlight **CALC** Choose **4: LinReg (ax+b)**

It prompts you. **LinReg (ax+b)**

You will then type L3, L4 then [ENTER]

Round the values the calculator gives you to three places.

$$y = 91.4x + 1063$$

[Y=] enter the equation **Y1=** substituting values into your LinReg formula.

[GRAPH] To see the graph of your plot and function.

[ZOOM] Choose **9: ZoomStat** or [WINDOW]

When you are done looking at your plot press [2<sup>nd</sup>][QUIT] to return to your "home" screen.

[WINDOW] Set the values to match the grid you want to use.

WINDOW
Xmin= -2
Xmax= 14
Xscl= 1
Ymin= 1000
Ymax= 3000
Yscl= 100

#### To find an expenditure value for the year 2010:

Figure out x-value:  $2010 - 1997 = 13$

Before you try to calculate the value, you will need to expand your viewing window. Choose amounts greater than what you need.

[2<sup>nd</sup>][CALC] Choose **1: value**

It prompts you. **X=** enter a number

The calculator gives

$$X = 13 \quad Y = 2251.2$$

(shown on the bottom of the screen)

The national health expenditures in 2010 should be around 2251 billion dollars.

**2.4 Using Linear Models**  
**Modeling Real-World Data**

**Example 1 (plus)**

Jacksonville, Florida has an elevation of 12 ft above sea level. A hot-air balloon taking off from Jacksonville rises 50 ft/min. Write an equation to model the balloon’s elevation as a function of time. Graph the equation. Interpret the intercept at which the graph intersects the vertical axis.

**Analyze & Define Variables:**

Independent variable: **time in minutes =  $t$**

Dependent variable: **elevation in feet =  $h$ , depends on time**

**Relate the Variables (write an equation):**

**Increase in elevation = rate times time**

**Elevation = initial elevation + increase in elevation**

**$h(t) = 12 + 50t$**

**Interpret:**

**$h(t)$ -intercept is  $(0, 12)$**

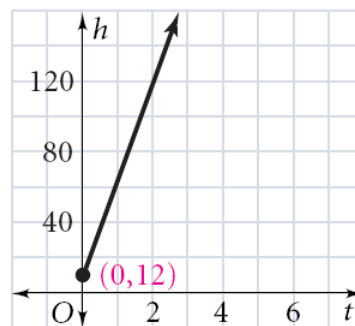
**$t$ -coordinate, 0, represents the time at the start of the trip**

**$h(t) = 12$  represents the elevation of the balloon at the start of the trip**

**Table:**

$t$	$h$
0	12
1	62
2	112
3	162

Note: change in  $h$  is 50, for every 1 minute.



a. How high is the balloon after 10 minutes?

$h = 12 + 50t$   
 $h = 12 + 50(10)$   
 $h = 512$  ft

b. When will the balloon reach a height of 150 feet?

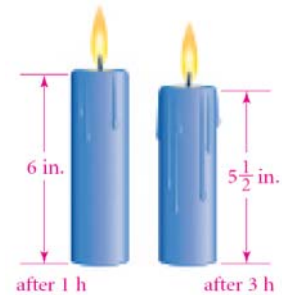
$h = 12 + 50t$   
 $150 = 12 + 50t$   
 $138 = 50t$   
 $t = \frac{138}{50} = 2\frac{19}{25} \approx 2.76$  min.

**Calculator:**  
 [Y=] enter equation  
 Use the [X, T,  $\theta$ ] key for X.  
 Use the (-) key for negative.  
 [WINDOW]  
 Set the values to match the grid you want to use.  
 WINDOW  
 Xmin= -2  
 Xmax= 8  
 Xscl= 1  
 Ymin= -20  
 Ymax= 160  
 Yscl= 20  
 [GRAPH]  
 To see the graph of your function.  
 [TBLSET]  
 Put in your starting value.  
 $\Delta$  Tbl will set the change in the independent variable.  
 TABLE SETUP  
 TblStart= 0  
 $\Delta$  Tbl= 1  
 [TABLE]  
 Displays the table. Scroll up or down to find your values.



**Example 2 & 3**

A candle is 6 in. tall after burning for 1 h. After 3 h, it is 5.5 in. tall. Write a linear equation to model the height of the candle after burning for  $x$  hours. In how many hours will the candle be 4 in. tall? How tall will the candle be after 12 h?



**Analyze & Define Variables:**

Independent variable: **time in hours =  $x$**

Dependent variable: **height of candle, inches =  $y$ , depends on time**

**Write a Linear Equation:**

Know two points, (1, 6) and (3, 5.5) so use point-slope form.

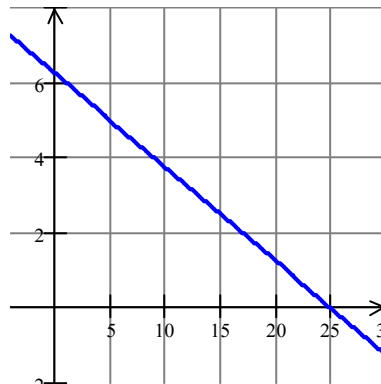
$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{6 - 5.5}{3 - 1} = \frac{.5}{2} = -\frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 6 = -\frac{1}{4}(x - 1)$$

$$y - 6 = -\frac{1}{4}x + \frac{1}{4}$$

$$y = -\frac{1}{4}x + 6\frac{1}{4}$$



**Table:**

$x$	$y$
hours	height inches
1	6
3	5.5

**Calculator:**

- [CALC] Choose **1: value**
- It prompts you. **X=** enter 0 for the  $y$ -intercept.
- Try **X= 12**, does it match your answer?
- [CALC] Choose **2: zero**
- It prompts you. **Left Bound?** move the cursor (using the arrow keys) to the left of the  $x$ -intercept. Press [ENTER].
- It prompts you. **Right Bound?** move the cursor to the right of the  $x$ -intercept. Press [ENTER].
- It prompts you. **Guess?** move the cursor near the  $x$ -intercept. Press [ENTER].
- It gives you the coordinates for the  $x$ -intercept. (The candle is completely melted in 25 hours,  $y = 0$ .)

In how many hours will the candle be 4 in. tall?

4 inches tall means  $y = 4$ .

$$4 = -\frac{1}{4}x + 6\frac{1}{4}$$

$$-2\frac{1}{4} = -\frac{1}{4}x$$

$$\left(-\frac{4}{1}\right)\left(-\frac{9}{4}\right) = \left(-\frac{4}{1}\right)\left(-\frac{1}{4}\right)x$$

$$9 = x$$

So in 9 hours, the candle will be 4 inches tall.

How tall will the candle be after 12 h?

12 hours means  $x = 12$ .

$$y = -\frac{1}{4}(12) + 6\frac{1}{4}$$

$$y = -3 + 6\frac{1}{4} = 3\frac{1}{4}$$

So in 12 hours, the candle will be  $3\frac{1}{4}$  inches tall.

### 2.6 Families of Functions

#### Intro: The Absolute Value Function

**Example 1**

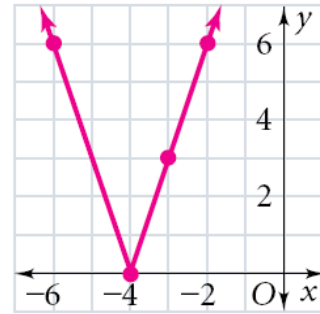
Graph  $y = |3x + 12|$

Complete an  $x|y$  table by evaluating the equation for several values of  $x$ .

Find the location of the “corner”.

$(-4, 0)$

$x$	$y$
-6	6
-5	3
-4	0
-3	3
-2	6



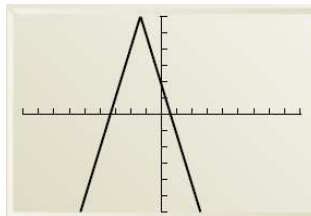
**Example 2**

Graph  $y = -|2x + 4| + 6$  on a graphing calculator.

Use the absolute value function. Sketch a graph the equation.

Use the table to find the location of the “corner”. Look for matching  $y$ -values to determine.

$(-2, 6)$



**Calculator:**

To get the absolute value function on the calculator press [MATH], highlight NUM, choose 1: abs(

[Y=] enter equation

$$Y1 = -\text{abs}(2X + 4) + 6$$

[ZOOM] choose 6: ZStandard

Sets it to a -10 to 10 by -10 to 10 viewing window. But, you get distortion, so...

[ZOOM] choose 5: ZSquare

Sets it to a square viewing window, just like graph paper, no distortion.

Graphs of absolute value equations in two variables look like angles. Notice that each part of the absolute value graph is a ray (a part of a line) and that each part has the same steepness, but opposite slope. So you can use slope to graph each side.

The **vertex** of a function is a point where the function reaches a maximum or minimum. In an absolute value function, the vertex is the “corner”.

A **family of functions** is made up of functions with certain common characteristics. We will be examining the V-shaped function absolute value.

A **parent function** is the simplest function with these characteristics. The equations of the functions in a family resemble each other. So do the graphs (in their general shape). Offspring of parent functions include **translations** (shifts), **stretches**, and **shrinks**.

**The parent function**

$$y = |x|$$

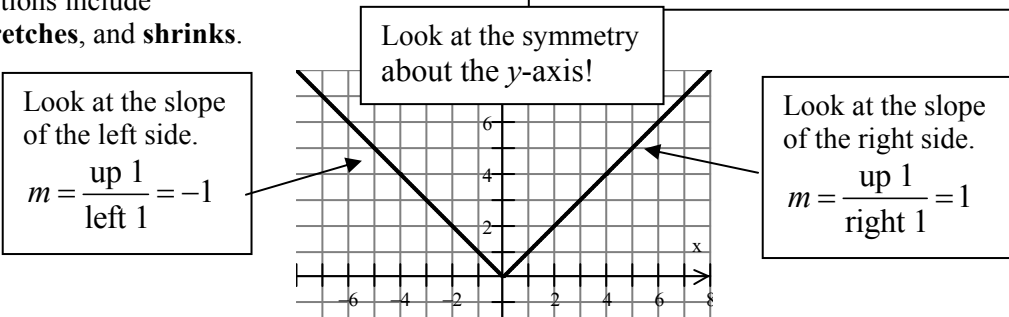
Complete an  $x|y$  table.

$x$	$y$
-6	6
-2	2
0	0
2	2
6	6

The **vertex** is at  $(0, 0)$ .

Look at the symmetry in the  $y$ -values!

**Now complete the investigation.**



### 2.6 Families of Functions

A family of functions is made up of functions with certain common characteristics.

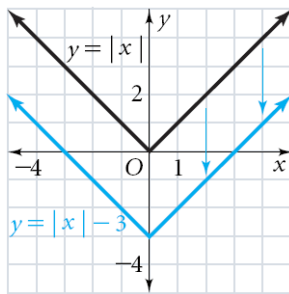
A **parent function** is the simplest function with these characteristics. The equations of the functions in a family resemble each other. So do the graphs. Offspring of parent functions include translations, stretches, and shrinks.

#### Translations

A **translation** shifts a graph horizontally, vertically, or both. It results in a graph of the same shape and size but possibly in a different position.

#### Example 1a Vertical Translation

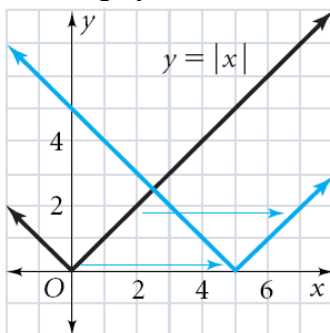
Draw the graph of  $y = |x| - 3$ . Describe the translations (graphically) and describe how the  $y$ -values of  $y = |x| - 3$  compare to the parent function.



$y = |x| - 3$  is a translation of  $y = |x|$  by 3 units downward  
 Each  $y$ -value for  $y = |x| - 3$  is 3 less than the corresponding  $y$ -value for  $y = |x|$ .

#### Example 2a Horizontal Translation

The blue graph is a translation of  $y = |x|$ . Write an equation for the graph.



The graph of  $y = |x|$  is translated 5 units to the right. An equation for the graph is  $y = |x - 5|$ .

#### Summary

Parent Function:  $y = |x|$

#### Vertical Translations

Translate **up**  $k$  units ( $k$  positive):  $y = |x| + k$

Translate **down**  $k$  units ( $k$  positive):  $y = |x| - k$

#### Horizontal Translations (counter intuitive)

Translate **right**  $h$  units ( $h$  positive):  $y = |x - h|$

Translate **left**  $h$  units ( $h$  positive):  $y = |x + h|$

#### Example 1b Vertical Translation

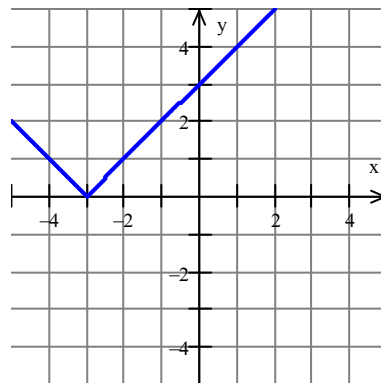
Write an equation to translate  $y = |x|$  up  $\frac{1}{2}$  unit.

An equation that translates  $y = |x|$  up  $\frac{1}{2}$  unit is  $y = |x| + \frac{1}{2}$ .

#### Example 2b Horizontal Translation

Describe the translation  $y = |x + 3|$  and draw its graph.

$y = |x + 3|$  is a translation of  $y = |x|$  3 units to the left.



**Stretches, Shrinks, and Reflections**

A **vertical stretch** multiplies all  $y$ -values by the same factor greater than 1, thereby stretching a graph vertically.

A **vertical shrink** reduces  $y$ -values by a factor between 0 and 1, thereby compressing the graph vertically.

A **reflection** in the  $x$ -axis changes  $y$ -values to their opposites. When you change the  $y$ -values of a graph to their opposites, the graph reflects across the  $x$ -axis. Multiplying by  $-1$  gives a reflection over the  $x$ -axis.

**Summary**

Parent Function:  $y = |x|$

$$y = a|x|$$

**Vertical Stretch  $a > 1$**

Stretch away from  $x$ -axis by a factor of  $a$ .

**Vertical Shrink (fraction of)  $0 < a < 1$**

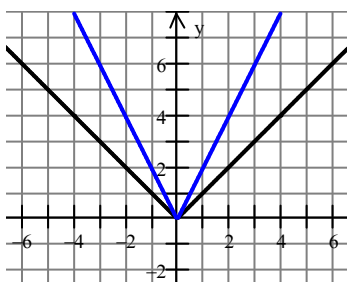
Shrink toward  $x$ -axis by a factor of  $a$ .

**Reflection in  $x$ -axis (negative)  $a < 0$**

Reflects over the  $x$ -axis and stretches or shrinks.

**Example 4a Vertical Stretch**

Draw the graph of  $y = 2|x|$ . Describe the transformation (graphically) and describe how the  $y$ -values of  $y = 2|x|$  compare to the parent function.

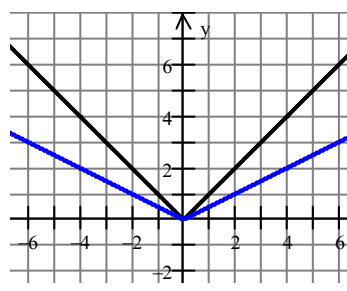


	Parent	Transf
$x$	$y1$	$y2$
-3	3	6
-2	2	4
0	0	0
2	2	4
4	4	8

$y = 2|x|$  is a vertical stretch of by a factor of 2.  
Each  $y$ -value for  $y = 2|x|$  is twice the corresponding  $y$ -value for  $y = |x|$ .

**Example 4b Vertical Shrink**

Write an equation for a vertical shrink of  $y = |x|$  by a factor of  $\frac{1}{2}$ . Graph the parent function and the transformed function.

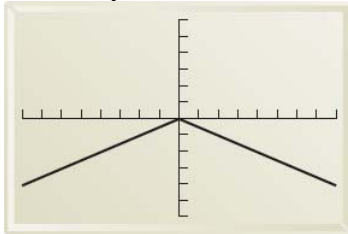


	Parent	Transf
$x$	$y1$	$y2$
-4	4	2
-2	2	1
0	0	0
2	2	1
3	3	1.5

$y = \frac{1}{2}|x|$  is a vertical shrink of by a factor of  $\frac{1}{2}$ . Each  $y$ -value for  $y = \frac{1}{2}|x|$  is half of the corresponding  $y$ -value for  $y = |x|$ .

**Example 5 Vertical Reflection**

Which equation describes the graph? Both scales are by 1.



- (A)  $y = \frac{1}{2}|x|$
- (B)  $y = \frac{1}{2}x$
- (C)  $y = -\frac{1}{2}|x|$
- (D)  $y = -\frac{1}{2}x$

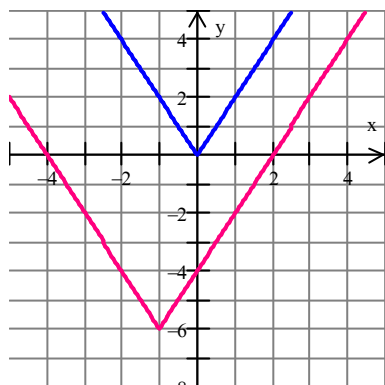
**C** because it's a vertical reflection of an absolute value.  
Also when  $x = 2$ ,  $y = 1$  so that explains the factor of  $\frac{1}{2}$ .

You can combine the transformations investigated before. It is best to do any stretching, shrinking or reflections first. Then follow with any translations. Try the following...

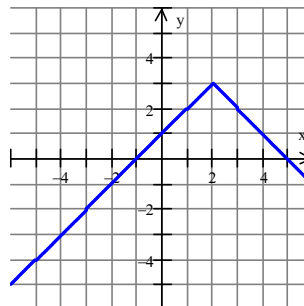
**Example 6a Combined Transformations**

Draw the graph of  $y = 2|x+1| - 6$ . Hint: graph

$y = 2|x|$  then translate it.

**Example 6b Combined Transformations**

Write an equation for the function in the graph.



Slopes of parts are 1 and  $-1$ , so **no** stretch or shrink.

Translated over 2, up 3 & reflected over the  $x$ -axis.

$$y = -|x-2| + 3$$

What you have learned about the absolute value function extends to functions in general.

Each member of a family of functions is a **transformation**, or change, of a parent function.

Algebraically, the transformations take the same form using **parameters**, like  $h$ ,  $k$ , and  $a$ .

Graphically, the results are similar—shifts, stretches, shrinks, and reflections of the parent function.

**Summary** (Transformations in general.)

Parent Function:  $y = f(x)$

**Vertical Translations**

Translate **up**  $k$  units ( $k$  positive):  $y = f(x) + k$

Translate **down**  $k$  units ( $k$  positive):  $y = f(x) - k$

**Horizontal Translations** (*counter intuitive*)

Translate **right**  $h$  units ( $h$  positive):  $y = f(x - h)$

Translate **left**  $h$  units ( $h$  positive):  $y = f(x + h)$

$$y = a \cdot f(x)$$

**Vertical Stretch**  $a > 1$ 

Stretch away from  $x$ -axis by a factor of  $a$ .

**Vertical Shrink** (*fraction of*)  $0 < a < 1$ 

Shrink toward  $x$ -axis by a factor of  $a$ .

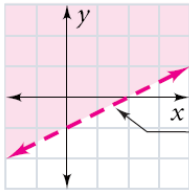
**Reflection in  $x$ -axis** (*negative*)  $a < 0$ 

Reflects over the  $x$ -axis and stretches or shrinks.

### 2.7 Two-Variable Inequalities

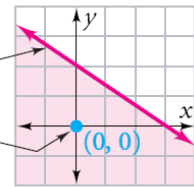
A **linear inequality** is an inequality in two variables whose graph is a region of the coordinate plane that is bounded by a line. To graph a linear inequality, first graph the boundary line. Then decide which side of the line contains solutions to the inequality, by testing points, and whether the boundary line is included, look at “is equal to”.

$$y > \frac{1}{2}x - 1$$



A *dashed* boundary line indicates that the line is not part of the solution.

$$2x + 3y \leq 6$$



A *solid* boundary line indicates that the line is part of the solution.

Choose a test point above or below the boundary line. The test point (0, 0) makes the inequality true. Shade the region containing this point.

#### Example 1 Graphing a Linear Inequality

Graph the inequality  $y < \frac{1}{2}x - 3$

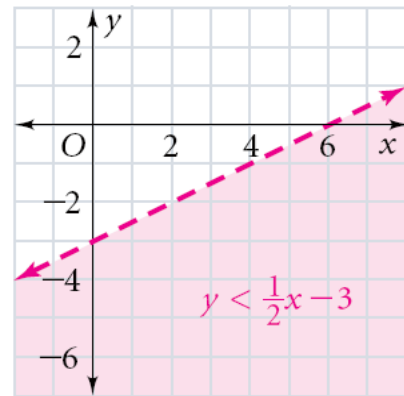
Graph the boundary line  $y = \frac{1}{2}x - 3$   
(use slope-intercept form)

Test point (0, 0):

$$0 < \frac{1}{2}(0) - 3$$

$$0 < -3$$

False, so shade region that does not include (0, 0).



#### Example 2

Graph the inequality  $-3x - 4y \geq 12$

Graph the boundary line

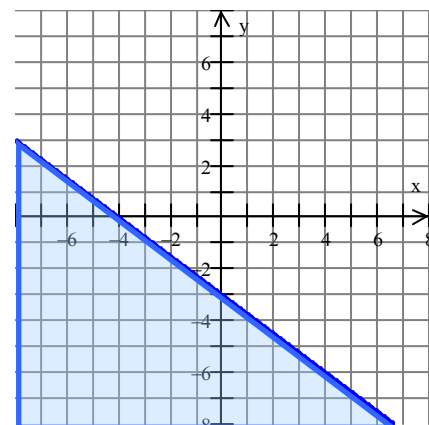
x	y
0	-3
-4	0

Test point (0, 0):

$$-3(0) - 4(0) \geq 12$$

$$0 \geq 12$$

False, so shade region that does not include (0, 0).



**Graphing Two-Variable Absolute Value Inequalities**

**Example 3a**

Graph the inequality  $y \leq |x - 4| + 5$

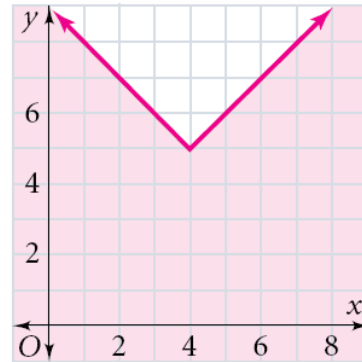
Use transformations  $y = |x - 4| + 5$ , absolute value shifted right 4 and up 5.

Inequality, so use a test point (0, 0):

$$0 \leq |0 - 4| + 5$$

$$0 \leq 4 + 5$$

True, so shade region that includes (0, 0).



**Example 3b**

Graph the inequality  $-y + 3 > |x + 1|$  get in a form you can graph

$$-y > |x + 1| - 3$$

$$y < -|x + 1| + 3$$

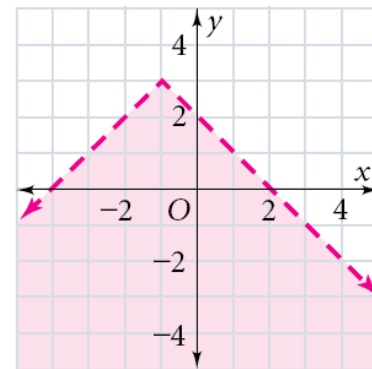
Use transformations  $y = -|x + 1| + 3$ , absolute value flipped over x-axis, shifted left 1 and up 3.

Inequality, so use a test point (0, 0):

$$-(0) + 3 > |0 + 1|$$

$$3 > 1$$

True, so shade region that includes (0, 0).



**Example 4**

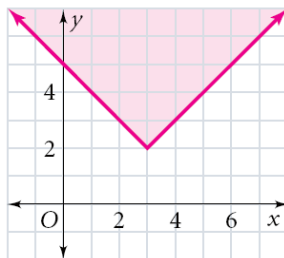
The graph is the solution of which inequality?

(A)  $y > |x - 3| + 2$

(B)  $y < |x - 3| + 2$

(C)  $y \geq |x - 3| + 2$

(D)  $y \leq |x - 3| + 2$



The boundary is solid, so C or D.

The shaded region is above the boundary.

So  $y \geq |x - 3| + 2$

The correct choice is C.

Test (0, 0) if necessary, C is false.