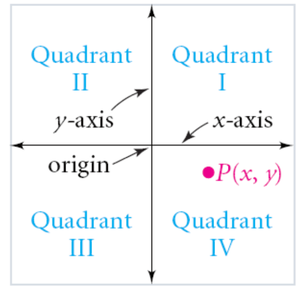
2.1 Relations and Functions



In Chapter 2, you will move from simplifying variable expressions and solving one-variable equations and inequalities to working with two-variable equations and inequalities. If you do not remember how to graph points in the coordinate plane, do page 54.

Introduction

Suppose you use a motion detector to track an egg as it drops from 10 ft above the ground. The motion detector stores input values (times) and output values (heights).

As a relation:

input (time in seconds) ⟶ {0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7} ⟵ **domain** (x-coordinates)

relation: {(0, 10), (0.1, 9.8), (0.2, 9.4), (0.3, 8.6), (0.4, 7.4), (0.5, 5.9), (0.6, 4), (0.7, 1.9)}

output (height in feet) ⟶ {10, 9.8, 9.4, 8.6, 7.4, 5.9, 4, 1.9} ⟵ **range** (y-coordinates)

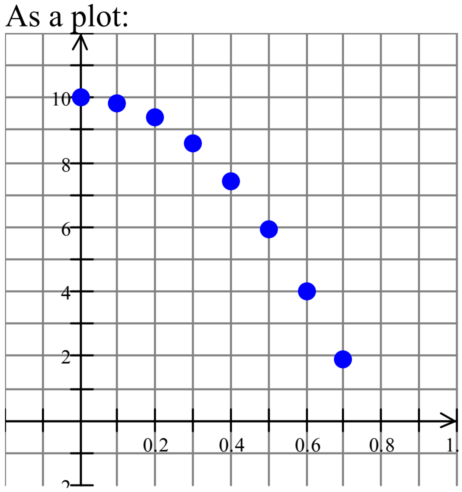
{ } is read "the set of", which means a collection of.

As a table:

time in seconds	height above ground (ft)
x	y
0	10
0.1	9.8
0.2	9.4
0.3	8.6
0.4	7.4
0.5	5.9
0.6	4
0.7	1.9

Use to organize data and find patterns.

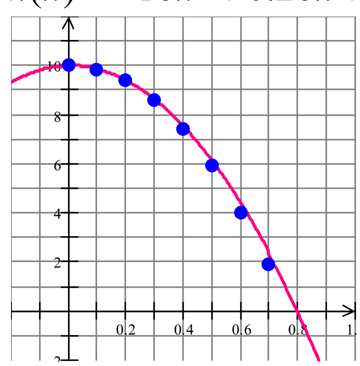
As a plot:



Use to organize data and visually find patterns. Also, use to choose a model.

Write a function to model the data. It's like a formula for your data.

$$h(x) = -16x^2 + 0.26x + 10$$



Use to summarize patterns and make predictions. You can use them to fill in gaps in data.

Now you can use the function rule to answer questions…

How high is the egg at 0.42 seconds?

$h(.42) = -16(.42)^2 + 0.26(.42) + 10 \approx 7.287$

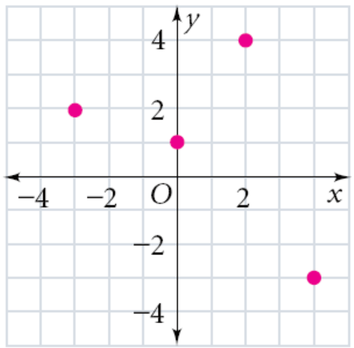
When will the egg hit the ground? It is when $h(x) = 0$, or height is 0.

or $0 = -16x^2 + 0.26x + 10$ Some algebra occurs… and $x = 0.799$ So about 0.8 seconds.

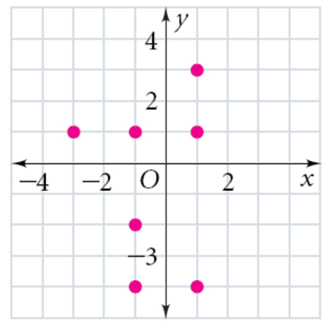
Graphing Relations

A **relation** is a set of pairs of input and output values. (Using set notation.) You can write a relation as a set of ordered pairs. The **domain** of a relation is the set of all inputs, or x-coordinates of the ordered pairs. The **range** of a relation is the set of all outputs, or y-coordinates of the ordered pairs.

Example 1alt
Graph the relation {(-3, 2), (0,1), (2, 4), (4, -3)}



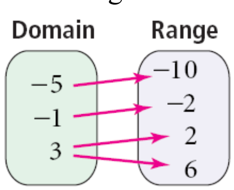
Example 2 alt
Find the domain and range of the relation shown in the graph.



Domain is {-3, -1, 1}
Range is {-4, -2, 1, 3}

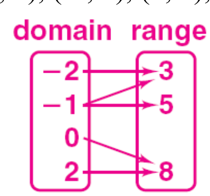
Another way to show a relation is to use a **mapping diagram**, which links elements of the domain with corresponding elements of the range. Write the elements of the domain in one region and the elements of the range in another. Draw arrows to show how each element from the domain is paired with elements from the range.

Example 3 alt
Write the relation given the mapping diagram.



{(-5,-10), (-1,-2), (3, 6), (3, 2)}
Not a function, x = 3 has 2 y's.

Example 3b alt
Make a mapping diagram for the relation
{(2, 8), (-1, 5), (0, 8), (-1, 3), (-2, 3)}

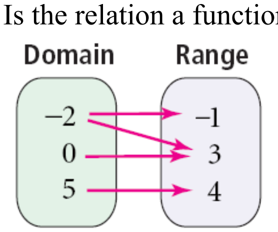


Not a function, x = -1 has 2 y's.

Identify Functions

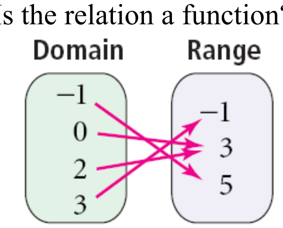
A **function** is a relation in which each element of the domain is paired with exactly one element in the range. In other words, a function is a special kind of relation in which each x-value can have only one corresponding y-value.

Example 4a
Is the relation a function?



The element -2 of the domain is paired with both -1 and 3 of the range. The relation is NOT a function.

Example 4b
Is the relation a function?

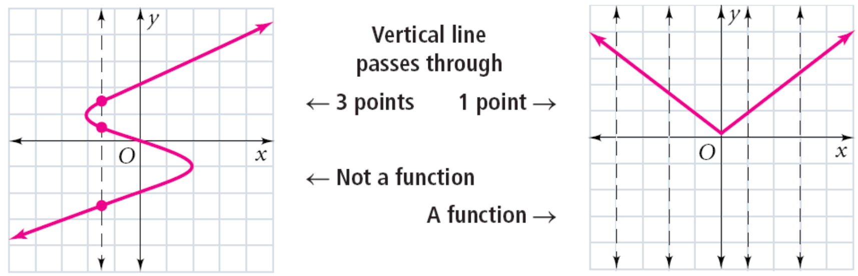


Each element of the domain is paired with exactly one element of the range. The relation is a function.

You can identify a function by its domain, its range, and the rule that relates the domain to the range. You can picture a function as in Example 4 or with table or a graph in the coordinate plane. When the domain values are **discrete**, the graph may be a collection of isolated points. When the domain values are **continuous**, such as the set of real numbers, the graph may be a line or curve.

In a graph, you will use the **vertical-line test** to identify repeated y-values in relations that are not functions.

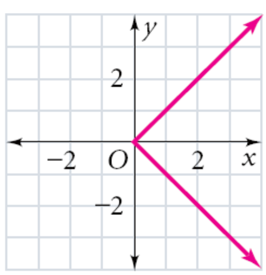
If a vertical line passes through at least two points on the graph, then one element of the domain is paired with more than one element of the range, and the relation is <u>not</u> a function.



Example 5 QC

Is the graph discrete or continuous? Is the relation a function? (Use the vertical-line test.)

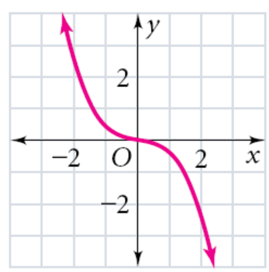
a.



continuous, not a function

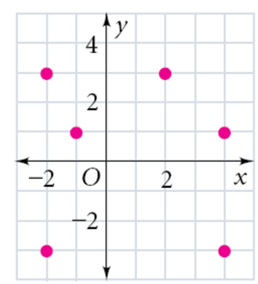
$x = 1,\ y = \pm 1$

b.



continuous, a function

c.



discrete, not a function

$x = -2,\ y = \pm 3$

A **function rule** expresses an output value in terms of an input value. The output values can be represented by a simple variable or it can be represented using function notation. (This notation came about so that the reader can see which input value yielded the output value.)

Example 6

The area of a square tile is a function of the length of a side of the square. Write a function rule for the area of a square.

Evaluate the function for a square tile with side length 3.5 in.

$$A(s) = s^2$$

$$A(3.5) = (3.5)^2 = 12.25$$

In Example 6…

The input is side length and the output is the area.

Say: Area *is a function of* the length of a side.

Write: $A(s)$.

Read: "*A* of *s*". This does NOT mean *A* times *s*!!

It means that if you know a side, you can find the area by substituting in the value for *s*.

When the value of *s* is 3.5 inches, you write…

$A(3.5)$ or $A(3.5) = (3.5)^2 = 12.25$.

Functions can be thought of as machines. You input a value and the function spits out a value based on a function rule. The inputs and outputs can be shown using function notation or as ordered pairs.

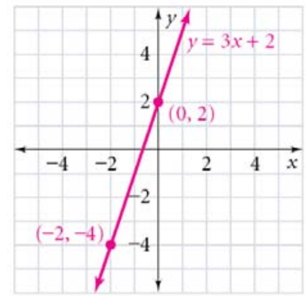
Input		Function		Output	Ordered Pair
5	⟶	Subtract 1	⟶	4	(5, 4)
a	⟶	Add 2	⟶	$a + 2$	$(a, a + 2)$
3	⟶	g	⟶	$g(3)$	$(3, g(3))$
x	⟶	f	⟶	$f(x)$	$(x, f(x))$

2.2 Linear Equations

A function whose graph is a line is a **linear function**. You can represent a linear function with a **linear equation**, such as $y = 3x + 2$. A solution of a linear equation is <u>any</u> ordered pair (x, y) that makes the equation true.

Write the solutions of the equation using set notation as $\{(x, y) \mid y = 3x + 2\}$. Read the notation as "the set of ordered pairs x, y such that $y = 3x + 2$."

Because the value of y depends on the value of x, y is called the **dependent variable** and x is called the **independent variable**.



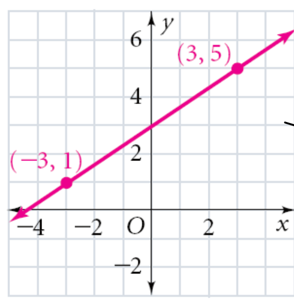
Geometry Since two points determine a line, you can use two points to graph a line. Check your line by finding a third point.

Example 1

Graph the equation $y = \frac{2}{3}x + 3$.

Choose two values for x and find the corresponding y.
Use substitution and simplification.

	substitute		graph
x	$\frac{2}{3}x + 3$	y	(x, y)
–3	$\frac{2}{3}(-3) + 3$	1	(–3, 1)
3	$\frac{2}{3}(3) + 3$	5	(3, 5)



Is (0, 3) a point on the line? Yes, $\frac{2}{3}(0) + 3 = 3$

The ***y*-intercept** of a line is the point at which the line crosses the y-axis. <u>All</u> points on the y-axis have an x-coordinate of 0.

The ***x*-intercept** of a line is the point at which the line crosses the x-axis. <u>All</u> points on the x-axis have an y-coordinate of 0.

Find the intercepts for the line $y = \frac{2}{3}x + 3$.

y-intercept, $x = 0$, so $y = \frac{2}{3}(0) + 3 = 3$

y-intercept is (0, 3)

x-intercept, $y = 0$, so $0 = \frac{2}{3}x + 3$

$\Rightarrow -3 = \frac{2}{3}x \Rightarrow -3 \cdot \frac{3}{2} = x$ or $x = -\frac{9}{2}$

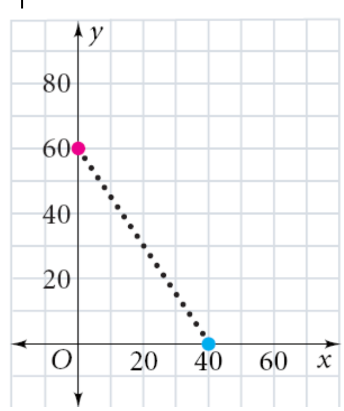
x-intercept is (–4.5, 0).

Example 2

The equation 3x + 2y = 120 models the number of passengers who can sit in a train car, where x is the number of adults and y is the number of children. Graph the equation. Explain what the x- and y-intercepts represent. Describe the domain and the range.

x-intercept, $y = 0$:
$3x + 2y = 120$
$3x + 2(0) = 120$
$3x = 120$
$x = 40$
x-intercept is (40, 0)

y-intercept, $x = 0$
$3x + 2y = 120$
$3(0) + 2y = 120$
$2y = 120$
$y = 60$
y-intercept is (0, 60)



When 40 adults are seated, no children can sit.
When no adults are seated, 60 children can sit.
The number of passengers is a whole number. The situation is discrete. The domain is limited to the whole numbers 0 to 40. The range to the whole numbers 0 to 60.

Definition **Slope (or rate of change)**

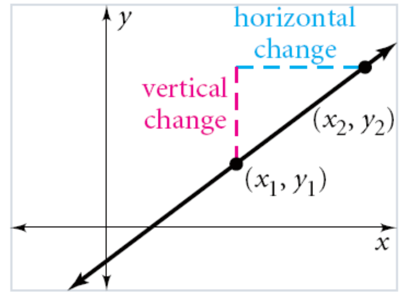
Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, be points on a nonvertical line.

The **slope** is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$$

where $x_2 - x_1 \neq 0$.

Also, slope gives the **average rate of change** is change in one quantity per change in another, like miles per hour.



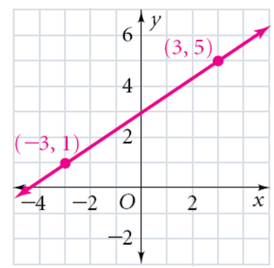
Uphill as x increases → positive slope
Downhill as x increases → negative slope
$\Delta y = 0$ → slope of 0, horizontal line
$\Delta x = 0$ → slope is undefined, vertical line

Example 3

Find the slope of the line through the points (–3, 1) and (3, 5).

$$m = \frac{5-1}{3-(-3)} = \frac{4}{6} = \frac{2}{3} \quad \text{OR} \quad m = \frac{1-5}{(-3)-(3)} = \frac{-4}{-6} = \frac{2}{3}$$

Hint: Fill in y then below matching x! Watch signs!



Forms for Equations of Lines (Simply use algebra to switch from one form to another.)

Definition **Point-Slope Form**

The equation

$$y - y_1 = m(x - x_1)$$

is the line through the point (x_1, y_1) with slope m.

Use to write an equation, given a point and slope OR given two points.
Read the form to find a point and/or slope.

Definition **Standard Form Linear Equation**

The equation

$$Ax + By = C$$

A, B and C are real numbers, and A and B not both 0.

Use to find points on a line. Is also the "simplest form" for a line.

Example 4 alt

Write in standard form an equation of the line with slope 5/6 through the point (6, 10).

$y - y_1 = m(x - x_1)$ given a point & slope

$y - 10 = \frac{5}{6}(x - 6)$

$6(y - 10) = 5(x - 6)$

$6y - 60 = 5x - 30$

or $5x - 6y = -30$

Example 5 alt

Write in standard form the equation of the line through (5, 1) and (–4, –3).

Find slope: $m = \frac{1-(-3)}{5-(-4)} = \frac{4}{9}$

$y - y_1 = m(x - x_1)$

use either point

$y - 1 = \frac{4}{9}(x - 5)$

$9(y - 1) = 4(x - 5)$

$9y - 9 = 4x - 20$

or $4x - 9y = 11$

$y - (-3) = \frac{4}{9}(x - (-4))$

$9(y + 3) = 4(x + 4)$

$9y + 27 = 4x + 16$

$11 = 4x - 9y$

How did standard form help?

Another form of the equation of a line is which you can use to find slope by examining the equation.

Definition **Slope-Intercept Form**
The equation

$$y = mx + b$$

is the line with slope m and y-intercept (0, b).
Use to write an equation, given a slope and y-intercept.
Read the form to find slope and/or y-intercept.

Example 6 QCa
Find the slope and y-intercept of the line

$3x + 2y = 1$.

$2y = -3x + 1$

$\frac{2y}{2} = \frac{-3x+1}{2}$

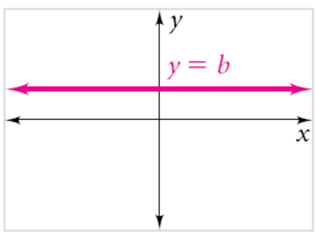
$y = -\frac{3}{2}x + \frac{1}{2}$

$y = mx + b$

So slope is –1.5 and the y-intercept is 0.5.

The slopes of horizontal, vertical, perpendicular, and parallel lines have special properties.

Horizontal Line

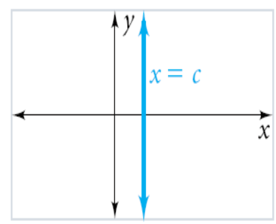


In table:

x	y
	b
	b
	b
	b

$y = mx + b$ since $m = 0$,
$y = (0)x + b$ so write $y = b$

Vertical Line

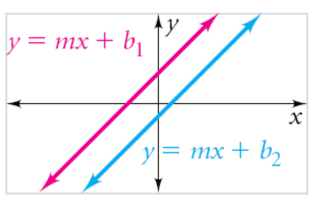


In table:

x	y
c	
c	
c	
c	

Since m = undefined,
and x constant, write $x = c$

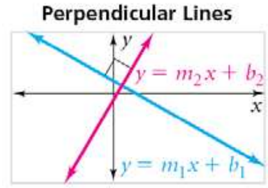
Parallel Lines



$m = m$
$b_1 \neq b_2$

Nonvertical parallel lines have the same slope.

Perpendicular Lines



$m_1 \cdot m_2 = -1$
(In other words, m_2 is the negative reciprocal of m_1.)

$m_1 = -\frac{1}{m_2}$, $m_2 = -\frac{1}{m_1}$

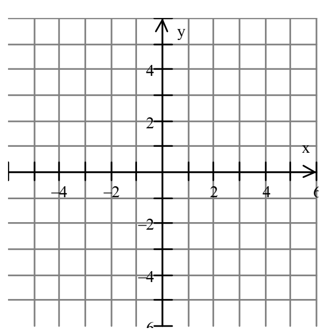
Example 7b
Write an equation for the line through (2, 1) and perpendicular to the line $y = \frac{2}{3}x + \frac{5}{8}$. Then graph the line.

$y - y_1 = m(x - x_1)$ slope = -3/2

$y - 1 = \frac{-3}{2}(x - 2)$

$-3(y-1) = 2(x-2)$

$-3y + 3 = 2x - 4$ or $2x + 3y = 7$



Example 7c
Write an equation for the vertical line through (5, –3).

slope = undefined
All points with an x-coordinate of 5.

$x = 5$

2.4 Using Linear Models

Modeling Real-World Data

Example 1 (plus)
Jacksonville, Florida has an elevation of 12 ft above sea level. A hot-air balloon taking off from Jacksonville rises 50 ft/min. Write an equation to model the balloon's elevation as a function of time. Graph the equation. Interpret the intercept at which the graph intersects the vertical axis.

Analyze & Define Variables:
Independent variable: time in minutes = t

Dependent variable: elevation in feet = h, depends on time

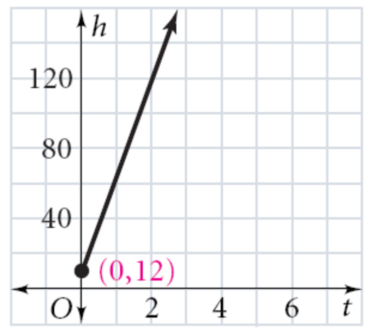
Relate the Variables (write an equation):
Increase in elevation = rate times time
Elevation = initial elevation + increase in elevation
$h = 12 + 50t$

Interpret:
h-intercept is (0, 12)
t-coordinate 0 represents the time at the start of the trip
h-coordinate 12 represents the elevation of the balloon at the start of the trip

Table:

t	h
0	12
1	62
2	112
3	162

Note: change in ***h*** is 50.



a. How high is the balloon after 10 minutes?

$h = 12 + 50t$
$h = 12 + 50(10)$
$h = 512$ ft

b. When will the balloon reach a height of 150 feet?

$h = 12 + 50t$
$150 = 12 + 50t$
$138 = 50t$
$t = \frac{138}{50} = 2\frac{19}{25} \approx 2.76$ min.

Calculator:
[Y=] enter equation
Use the [X, T, θ] key for X.
Use the (–) key for negative.
[WINDOW]
Set the values to match the grid you want to use.

WINDOW
Xmin= –2
Xmax= 8
Xscl= 1
Ymin= –20
Ymax= 160
Yscl= 20

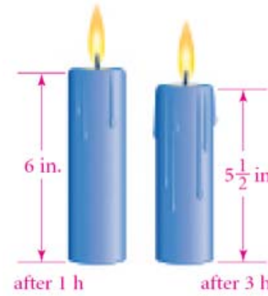
[GRAPH]
To see the graph of your function.
[TBLSET]
Put in your starting value.
Δ Tbl will set the change in the independent variable.

TABLE SETUP
TblStart= 0
Δ Tbl= 1

[TABLE]
Displays the table. Scroll up or down to find your values.

Example 2 & 3

A candle is 6 in. tall after burning for 1 h. After 3 h, it is 5.5 in. tall. Write a linear equation to model the height of the candle after burning for *x* hours. In how many hours will the candle be 4 in. tall? How tall will the candle be after 12 h?



Analyze & Define Variables:

Independent variable: time in hours = *x*

Dependent variable: height of candle, inches = *y*, depends on time

Write a Linear Equation:

Know two points, (1, 6) and (3, 5.5) so use point-slope form.

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{6-5.5}{3-1}=\frac{.5}{2}=-\frac{1}{4}$$

$$y-y_1=m(x-x_1)$$

$$y-6=-\frac{1}{4}(x-1)$$

$$y-6=-\frac{1}{4}x+\frac{1}{4}$$

$$y=-\frac{1}{4}x+6\frac{1}{4}$$

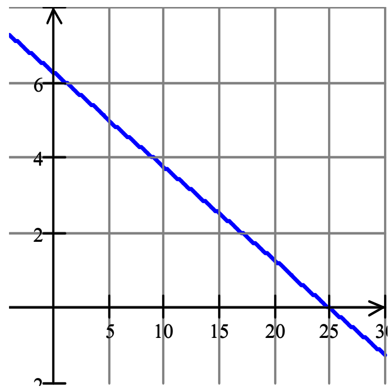


Table:

x hours	*y* height inches
1	6
3	5.5

Calculator:

[CALC] Choose **1: value**

It prompts you. **X=** enter 0 for the *y*-intercept.

Try **X=** 12, does it match your answer?

[CALC] Choose **2: zero**

It prompts you. **Left Bound?** move the cursor (using the arrow keys) to the left of the *x*-intercept. Press [ENTER].

It prompts you. **Right Bound?** move the cursor to the right of the *x*-intercept. Press [ENTER].

It prompts you. **Guess?** move the cursor near the *x*-intercept. Press [ENTER].

It gives you the coordinates for the *x*-intercept. (The candle is completely melted in 25 hours, *y* = 0.)

In how many hours will the candle be 4 in. tall?

4 inches tall means *y* = 4.

$$4=-\frac{1}{4}x+6\frac{1}{4}$$

$$-2\frac{1}{4}=-\frac{1}{4}x$$

$$\left(-\frac{4}{1}\right)\left(-\frac{9}{4}\right)=\left(-\frac{4}{1}\right)\left(-\frac{1}{4}\right)x$$

$$9=x$$

So in 9 hours, the candle will be 4 inches tall.

How tall will the candle be after 12 h?

12 hours means *x* = 12.

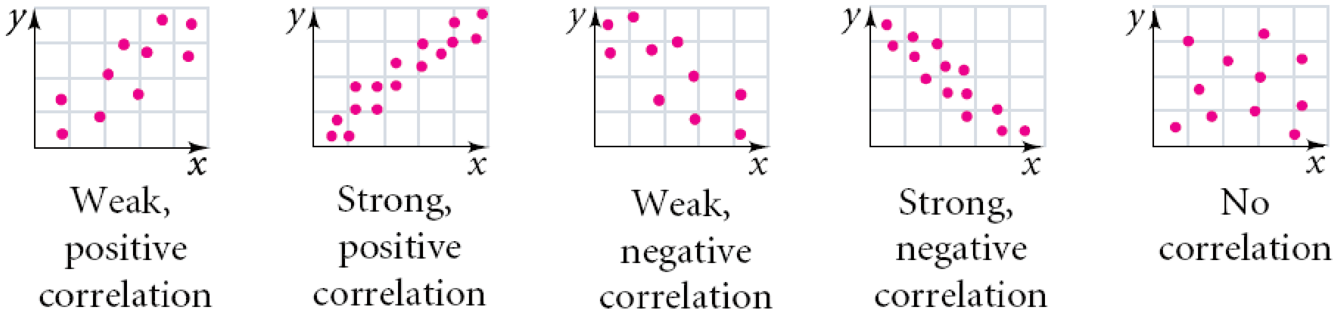
$$y=-\frac{1}{4}(12)+6\frac{1}{4}$$

$$y=-3+6\frac{1}{4}=3\frac{1}{4}$$

So in 12 hours, the candle will be 3 ¼ inches tall.

Predicting With Linear Models

A **scatter plot** is a graph that relates two different sets of data by plotting the data as ordered pairs. You can use a scatter plot to determine a relationship between the data sets.



A **trend line** is a line that approximates the relationship between the data sets of a scatter plot. You can use a trend line to make predictions.

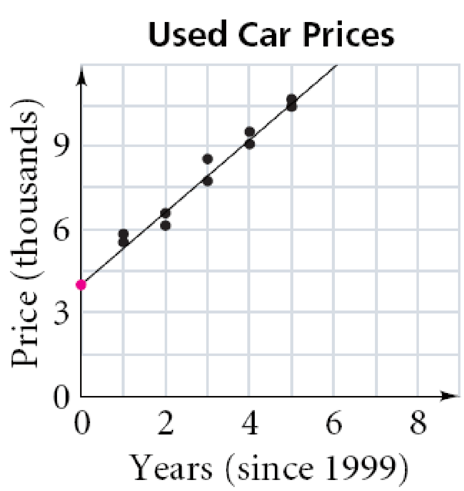
Example 4
A woman is considering buying the 1999 car. She researches prices for various years of the same model and records the data in a table.

Model Year	2000	2001	2002	2003	2004
Prices	\$5784 \$5435	\$6810 \$6207	\$8237 \$7751	\$9660 \$9127	\$10,948 \$10,455

Analyze & Define Variables:
Independent variable: x = the model year, since 1999 would be the starting year, let $x = 0$ for 1999, then $x = 1$ for 2000, etc…

Dependent variable: y = prices in \$, converted to thousands ($\div$ 1000)

Make a scatter plot and examine:
Draw a scatter plot. Decide whether a linear model is reasonable.



A linear model seems reasonable, since the points fall close to a line.

Draw a trend line then write an equation:
Draw a line that has about the same number of data points above and below it. (See graph.)

Write the equation of the line. Choose two possible points like (1, 5.4) and (6, 10.7).

$$m = \frac{10.7 - 5.4}{6 - 1} = \frac{5.3}{5} = 1.06$$

$$y - y_1 = m(x - x_1)$$

$$y - 5.4 = 1.06(x - 1)$$

$y = 1.06x + 4.34$ equation of the trend line

Determine whether the asking price is reasonable.
A fair price would be the value of y for $x = 0$, $y = 1.06(0) + 4.34 = 4.34$ so about \$4,340. The asking price of \$4200 is reasonable.

Finding a Line of Best Fit Pages 86 – 87.

Activity 1: Finding the LinReg Line of Best Fit

When a honeybee finds a source of food, it returns to the beehive and communicates to other bees the direction and distance of the source. The bee makes a loop, waggles its belly along a line, makes another loop, and then another waggle. The bee repeats its dance several times. The time for each cycle (one loop and waggle) reveals the approximate distance to the food source.

Enter the given data on your calculator. Show a scatter plot of the data. Find the LinReg line of best fit. Then show a graph of the line.

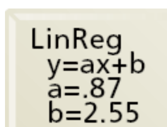
Distance to Food (km)	1.35	1.5	1.5	2	2.15	2.65	2.75	3.5	4	5	6
Cycle Time (seconds)	3.5	3.8	3.9	4.4	4.3	5	5.1	5.6	6	7	7.6

Calculator Activity 1:

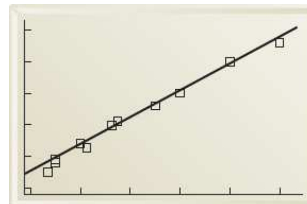
[STAT] Choose **1: Edit**
Clear any existing lists.
Move the cursor over the list name, L1. Press [CLEAR] then [ENTER].
Clear any equations from [Y=]. Use the [CLEAR] key.

[STAT] Choose **1: Edit**
Enter distance data into L1 and cycle time into L2.
[STAT PLOT] Choose **1: Plot 1** and turn it on by pressing [ENTER].
[ZOOM] Choose **9: ZoomStat**

[STAT] Highlight **CALC** Choose **4: LinReg (ax+b)**
It prompts you. **LinReg (ax+b)**
Just press [ENTER].
Round the values the calculator gives you to two places.



[Y=] enter the equation **Y1=** .87X+2.55
[GRAPH] To see the graph of your plot and function.

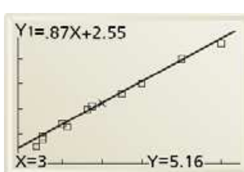


Activity 2: Using the Line of Best Fit to Predict

A bee finds food 3 km from the hive. Use the line of best fit and the CALC feature to predict the time of a waggle dance cycle.

Calculator Activity 2:

[GRAPH] To show the graph.
[CALC] Choose **1: value**
It prompts you. **X=** enter 3
The calculator gives
X = 3 Y = 5.16



A waggle dance cycle should take 5.2 seconds.

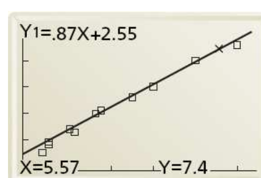
Algebraically: $y = .87(3) + 2.55 = 5.16$

Activity 3: Using the Line of Best Fit to Estimate

A waggle dance cycle is 7.4 s. Use the line of best fit and the TRACE feature to estimate the distance to the food source.

Calculator Activity 3:

[GRAPH] To show the graph.
[TRACE] The cursor will jump from one data point to the next if you use the [◀] and [▶] keys.
Use [▲] or [▼] to jump from the plot to the line.
Get on the line then move using the [◀] and [▶] keys to get the *y*-value (shown on the bottom of the screen) as close to 7.4 as possible.
Read the corresponding *x*-value.



The food source is approximately 5.6 km from the beehive.

To Stop Plotting:

[STAT PLOT] Choose **4: PlotsOff** Press [ENTER].
Turns off the plotting but keeps your data in the lists.

2.6 Families of Functions

Intro: The Absolute Value Function

Example 1

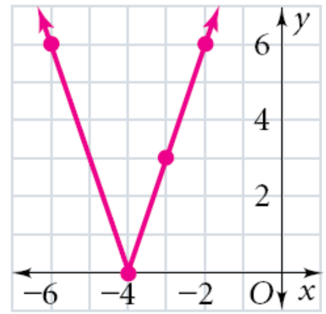
Graph $y = |3x + 12|$

Complete an $x|y$ table by evaluating the equation for several values of x.

Find the location of the "corner".

(–4, 0)

x	y
–6	6
–5	3
–4	0
–3	3
–2	6



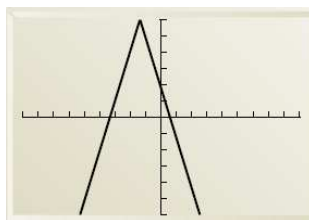
Example 2

Graph $y = -|2x + 4| + 6$ on a graphing calculator.

Use the absolute value function.
Sketch a graph the equation.

Use the table to find the location of the "corner". Look for matching y-values to determine.

(–2, 6)



Calculator:
To get the absolute value function on the calculator press [MATH], highlight NUM, choose **1: abs(**

[Y=] enter equation
$\text{Y1} = -\text{abs}(2\text{X} + 4) + 6$

[ZOOM] choose **6: ZStandard**
Sets it to a –10 to 10 by –10 to 10 viewing window. You get distortion.

[ZOOM] choose **5: ZSquare**
Sets it to a square viewing window, just like graph paper, no distortion.

A function of the form $f(x) = |mx + b| + c$, where $m \neq 0$, is an absolute value function. The related equation $y = |mx + b| + c$ is an absolute value equation in two variables.
Graphs of absolute value equations in two variables look like angles.

The **vertex** of a function is a point where the function reaches a maximum or minimum. In an absolute value function is the "corner". In general, the vertex of $y = |mx + b| + c$ is located at $\left(-\frac{b}{m}, c\right)$.

Example 1 Part deux.
Find the vertex of $y = |3x + 12|$.

Compare $y = |mx + b| + c$ to $y = |3x + 12| + 0$

$x = -\frac{b}{m} = -\frac{12}{3} = -4$

$y = 0$ because $c = 0$

The vertex is (–4, 0).

Example 2 Part deux.
Find the vertex of $y = -|2x + 4| + 6$.

Compare $y = |mx + b| + c$ to $y = -|2x + 4| + 6$

$x = -\frac{b}{m} = -\frac{4}{2} = -2$

$y = 6$ because $c = 6$

vertex is (–2, 6)

2.6 Families of Functions

A family of functions is made up of functions with certain common characteristics.

A **parent function** is the simplest function with these characteristics. The equations of the functions in a family resemble each other. So do the graphs. Offspring of parent functions include translations, stretches, and shrinks.

The parent function

$y = |x|$

Complete an $x|y$ table.

The vertex is at (0, 0). Look at the symmetry in the y-values!

x	y
–6	6
–2	2
0	0
2	2
6	6

Technology Activity: A Family of Functions

The four functions in each group are related. Graph each set of functions in the same viewing window. Explain how they are related.

1. $y_1(x) = |x|$ $y_2(x) = |x| + 2$ $y_3(x) = |x| + 4$ $y_4(x) = |x| - 2$

2. $y_1(x) = |x|$ $y_2(x) = |x + 2|$ $y_3(x) = |x + 4|$ $y_4(x) = |x - 2|$

3. $y_1(x) = |x|$ $y_2(x) = 2|x|$ $y_3(x) = 4|x|$ $y_4(x) = \frac{1}{2}|x|$

4. $y_1(x) = |x|$ $y_2(x) = -|x|$ $y_3(x) = 2|x|$ $y_4(x) = -2|x|$

1. $y_2 \rightarrow$ shift up 2
   $y_3 \rightarrow$ shift up 4
   $y_4 \rightarrow$ shift down 2

2. $y_2 \rightarrow$ shift left 2
   $y_3 \rightarrow$ shift left 4
   $y_4 \rightarrow$ shift right 2

3. $y_2 \rightarrow$ stretch factor of 2
   $y_3 \rightarrow$ stretch factor of 4
   $y_4 \rightarrow$ shrink factor of ½

4. $y_2 \rightarrow$ reflect over x-axis
   $y_3 \rightarrow$ stretch factor of 2
   $y_4 \rightarrow$ reflect over x-axis & stretch factor of 2

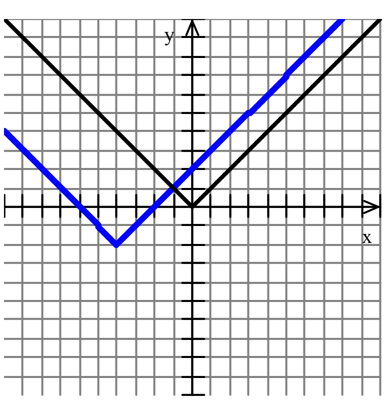
Use the graphs from parts 1–4 to predict the graph of each function below. On graph paper and without plotting points, sketch what you think the graph of f should be. Then check your sketch on a graphing calculator.

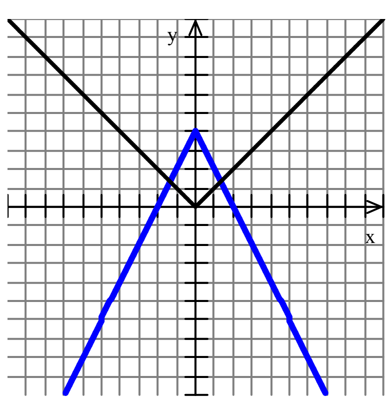
5. $f(x) = |x + 4| - 2$ **6.** $f(x) = -2|x| + 4$

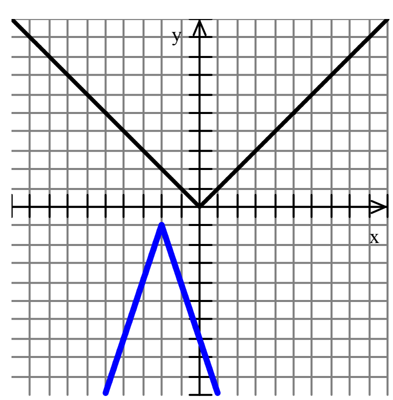
7. Sketch what you think the graph of $f(x) = -3|x + 2| - 1$ should be. Then check your sketch on a graphing calculator.

8. Considering all the functions in Exercises 1–7, which function would you call the simplest? Explain.

8. simplest is $y = |x|$







 Name ________________________

Translations

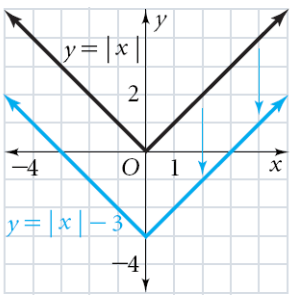
A **translation** shifts a graph horizontally, vertically, or both. It results in a graph of the same shape and size but possibly in a different position.

Summary
Parent Function: $y = |x|$

Vertical Translations
Translate **up** k units (k positive): $y = |x| + k$
Translate **down** k units (k positive): $y = |x| - k$

Horizontal Translations *(counter intuitive)*
Translate **right** h units (h positive): $y = |x - h|$
Translate **left** h units (h positive): $y = |x + h|$

Example 1a Vertical Translation
Draw the graph of $y = |x| - 3$. Describe the translations (graphically) and describe how the y-values of $y = |x| - 3$ compare to the parent function.



	Parent	Trans
x	y	y
–4	4	1
–2	2	–1
0	0	–3
2	2	–1
4	4	1

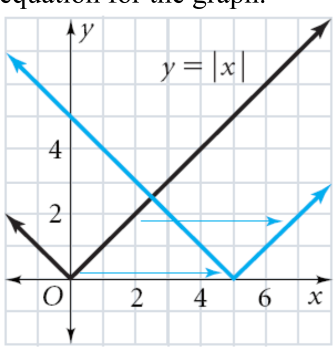
$y = |x| - 3$ is a translation of $y = |x|$ by 3 units downward

Each y-value for $y = |x| - 3$ is 3 less than the corresponding y-value for $y = |x|$.

Example 1b Vertical Translation
Write an equation to translate $y = |x|$ up ½ unit.

An equation that translates up unit is $y = |x| + \frac{1}{2}$.

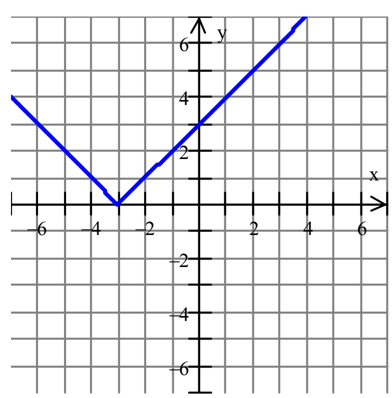
Example 2a Horizontal Translation
The blue graph is a translation of $y = |x|$. Write an equation for the graph.



The graph of $y = |x|$ is translated 5 units to the right. An equation for the graph is $y = |x - 5|$.

Example 2b Horizontal Translation
Describe the translation $y = |x + 3|$ and draw its graph.

$y = |x + 3|$ is a translation of by 3 units to the left.



 Name ________________________

Stretches, Shrinks, and Reflections

A **vertical stretch** multiplies all y-values by the same factor greater than 1, thereby stretching a graph vertically.

A **vertical shrink** reduces y-values by a factor between 0 and 1, thereby compressing the graph vertically.

A **reflection** in the x-axis changes y-values to their opposites. When you change the y-values of a graph to their opposites, the graph reflects across the x-axis. Multiplying by –1 gives a reflection across the x-axis.

Summary

Parent Function: $y = |x|$

$y = a|x|$

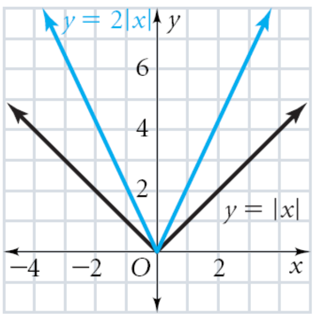
Vertical Stretch $a > 1$
Stretch away from x-axis by a factor of a.

Vertical Shrink (*fraction of*) $0 < a < 1$
Stretch away from x-axis by a factor of a.

Reflection in x-axis (*negative*) $a < 0$
Reflects over the x-axis and stretches or shrinks.

Example 4a Vertical Stretch

Draw the graph of $y = 2|x|$. Describe the transformation (graphically) and describe how the y-values of $y = 2|x|$ compare to the parent function.

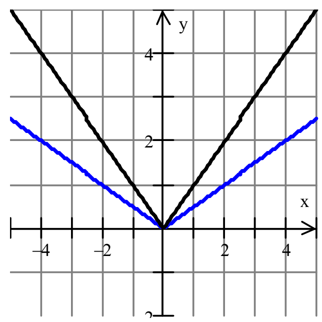


	Parent	Trans
x	y	y
–3	3	6
–2	2	4
0	0	0
2	2	4
4	4	8

$y = 2|x|$ is a vertical stretch of by a factor of 2.
Each y-value for $y = 2|x|$ is twice the corresponding y-value for $y = |x|$.

Example 4b Vertical Shrink

Write an equation for a vertical shrink of $y = |x|$ by a factor of ½ . Graph the parent function and the transformed function.

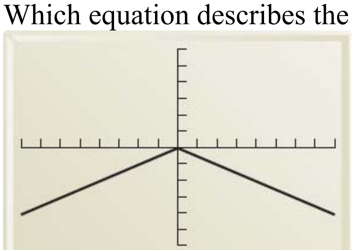


	Parent	Trans
x	y	y
–4	4	2
–2	2	1
0	0	0
2	2	1
3	3	1.5

$y = \frac{1}{2}|x|$ is a vertical shrink of by a factor of 1/2.

Example 5 Vertical Reflection

Which equation describes the graph? Both scales are by 1.



(A) $y = \frac{1}{2}|x|$

(B) $y = \frac{1}{2}x$

(C) $y = -\frac{1}{2}|x|$

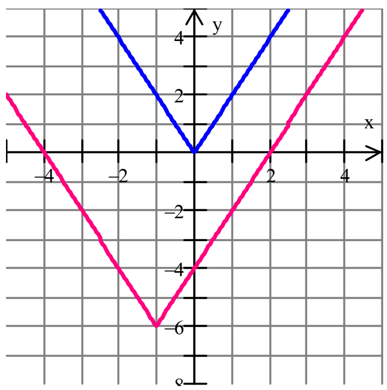
(D) $y = -\frac{1}{2}x$

C because it's a vertical reflection of an absolute value.
Also when x = 2, y = 1 so that explains the factor of ½.

You can combine the transformations investigated before. It is best to do any stretching, shrinking or reflections first. Then follow with any translations. Try the following…

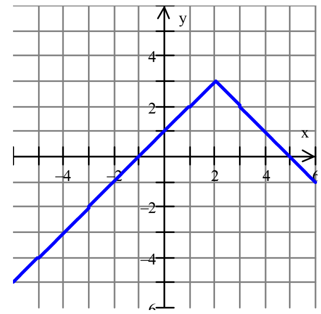
Example 6a **Combined Transformations**

Draw the graph of $y = 2|x+1| - 6$. Hint: graph $y = 2|x|$ then translate it.



Example 6b **Combined Transformations**

Write an equation for the function in the graph.



Slopes of parts are 1 and –1, so no stretch or shrink.
Translated over 2, up 3 & reflected over the *x*-axis.

$y = -|x-2| + 3$

What you have learned about the absolute value function extends to functions in general.

Each member of a family of functions is a **transformation**, or change, of a parent function.

Algebraically, the transformations take the same form using **parameters**, like h, k, and a.

Graphically, the results are similar—shifts, stretches, shrinks, and reflections of the parent function.

Summary (Transformations in general.)

Parent Function: $y = f(x)$

Vertical Translations

Translate **up** k units (k positive): $y = f(x) + k$

Translate **down** k units (k positive): $y = f(x) - k$

Horizontal Translations *(counter intuitive)*

Translate **right** h units (h positive): $y = f\left(x-h\right)$

Translate **left** h units (h positive): $y = f\left(x+h\right)$

$y = a \bullet f(x)$

Vertical Stretch $a > 1$
Stretch away from x-axis by a factor of a.

Vertical Shrink (*fraction of*) $0 < a < 1$
Stretch away from x-axis by a factor of a.

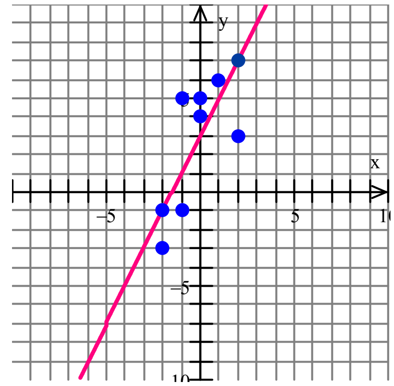
Reflection in *x*-axis (*negative*) $a < 0$
Reflects over the x-axis and stretches or shrinks.

2.7 Two-Variable Inequalities

Graphing Linear Inequalities

Activity: Linear Inequalities

1. Graph the line $y = 2x + 3$ on graph paper.
2. a. Plot each point listed below.
   $(-2, -3), (-2, -1), (-1, -1), (-1, 5), (0, 4), (0, 5), (1, 6), (2, 3), (2, 7)$
   b. Classify each point as *on the line*, *above the line*, or *below the line*.
3. Are all the points that satisfy the inequality $y > 2x + 3$ *above*, *below*, or *on* the line?



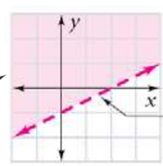
2.
On: (–2, –1), (2, 7)
Above: (–1, 5), (0, 4), (0, 5), (1, 6)
Below: (–2, –3), (–1, –1), (2, 3)

3. above the line

A **linear inequality** is an inequality in two variables whose graph is a region of the coordinate plane that is bounded by a line. To graph a linear inequality, first graph the boundary line. Then decide which side of the line contains solutions to the inequality, by testing points, and whether the boundary line is included, look at "is equal to".

$y > \frac{1}{2}x - 1$

For an inequality with $y <$ or $y \le$, shade below the line. For an inequality with $y >$ or $y \ge$, shade above the line.

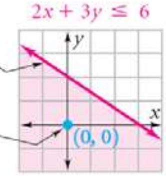


A *dashed* boundary line indicates that the line is not part of the solution.

$2x + 3y \le 6$

A *solid* boundary line indicates that the line is part of the solution.

Choose a test point above or below the boundary line. The test point (0, 0) makes the inequality true. Shade the region containing this point.



Example 1 **Graphing a Linear Inequality**

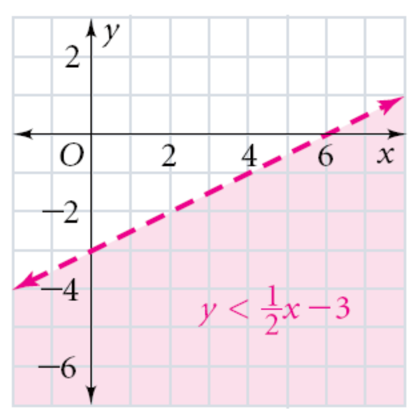
Graph the inequality $y < \frac{1}{2}x - 3$

Graph the boundary line $y = \frac{1}{2}x - 3$

Inequality is less than, *y*-values ,
so shade below the boundary line.

Test point (0, 0): $0 < \frac{1}{2}(0) - 3 \Rightarrow 0 < -3$

False, so shade away.



Example 2
At least 35 performers of the Big Tent Circus are in the grand finale. Some pile into cars, while others balance on bicycles. Seven performers are in each car, and five performers are on each bicycle. Draw a graph showing all the combinations of cars and bicycles possible for the finale.

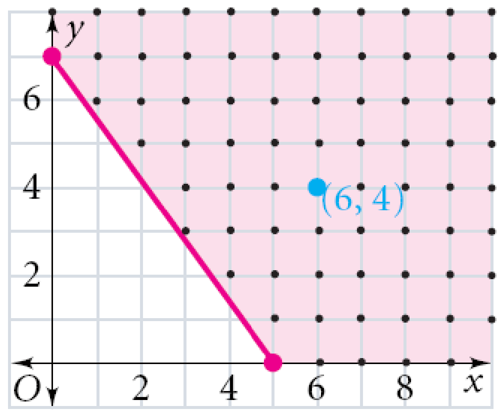
Analyze & Define Variables:
x = number of cars
y = number of bicycles

Relate the Variables (write an inequality):
people in cars + # people in bicycles ≥ 35
$7x + 5y \geq 35$

Graph: Find the intercepts.

$7(0) + 5y = 35$ $\quad$ $7x + 5(0) = 35$
$y = 7$ $\quad$ $x = 5$
(0, 7) $\quad$ (5, 0)

Use a solid boundary line.
Test point (6, 4): $7(6) + 5(4) \geq 35$ True, shade toward.



The numbers of cars x and bicycles y are whole numbers. The situation is **discrete**. In the shaded region, all points with whole number coordinates (show by dots) represent possible combinations of cars and bicycles for the grand finale.

a. Find the minimum number of bicycles that will be needed if three cars are available. Then determine three other possible combinations of bicycles and cars.

(3, y) so minimum (3, 3), 3 bicycles

b. What inequalities describe the region bounded by the x- and y-axes and the line in the graph above?

$7x + 5y \leq 35$, $y \geq 0$ and $x \geq 0$.

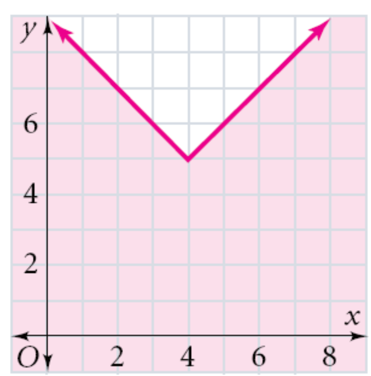
Graphing Two-Variable Absolute Value Inequalities

Example 3a
Graph the inequality $y \leq |x - 4| + 5$

Vertex: $x = -\dfrac{b}{m} = -\dfrac{-4}{1} = 4$

$y = |4 - 4| + 5 = 5$

Inequality is less than or equal, y-values, so solid and shade below.
Test point (0, 0): $0 \leq |0 - 4| + 5 \Rightarrow$ True

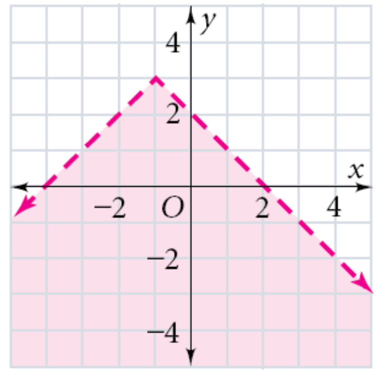


Example 3a

Graph the inequality $-y+3>|x+1|$

$-y>|x+1|-3$

$y<-|x+1|+3$



Vertex: $x=-\frac{b}{m}=-\frac{1}{1}=-1$;

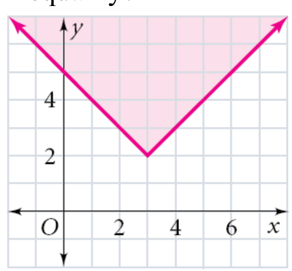
$y=-|(-1)+1|+3=3$

Inequality is less than, *y*-values,
so dashed and shade below.
Test point (0, 0): $-(0)+3>|0+1| \Rightarrow$ True

Example 4

The graph is the solution of which inequality?

Ⓐ $y>|x-3|+2$
Ⓑ $y<|x-3|+2$
Ⓒ $y\geq|x-3|+2$
Ⓓ $y\leq|x-3|+2$



The boundary is solid.
The shaded region is above the boundary.
So $y\geq|x-3|+2$
The correct choice is C.