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A new closed form method for design of variable bandwidth linear phase FIR filter using different polynomials

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ABSTRACT

In this paper, a new method for the design of variable bandwidth linear-phase finite impulse response (FIR) filters using different polynomials such as shifted Chebyshev polynomials, Bernstein polynomials and shifted Legendre polynomials is proposed. For this purpose, the transfer function of a variable bandwidth filter, which is a linear combination of fixed-coefficient linear-phase filters and the above polynomials are separately exploited as tuning parameters to control bandwidth of the filter. In order to determine the filter coefficients, mean squared difference between the desired variable bandwidth filter and the practical filter is minimized by differentiating it with respect to its coefficients leading to a system of linear equations. The matrix elements can be expressed in form of Toeplitz-plus-Hankel matrix, which reduces the computational complexity. Several examples are included to demonstrate effectiveness of the proposed method in terms of passband error (e_p), stopband error (e_s) and stopband attenuation (A_s).

1. Introduction

Digital filters are major basic building block of a multirate system, and are extensively exploited for numerous applications such as speech processing, image processing, biomedical signal processing, power quality measurement and multirate system design [1–5]. Therefore, design of digital filter has been extensively addressed over the past three decades. Among the various types of digital filters, a variable bandwidth filter or tunable bandwidth filter has received considerable attention because of its wide applications in numerous fields such as telecommunications, digital audio equipment, medical electronics, radar, sonar and control systems, adaptive and tracking systems, spectrum and vibration analysis, format speech synthesizers [6,7]. All applications are based on the efficient design of a tunable bandwidth filter.

During last few decades, variable digital filters have received considerable attention due to the increasing demand for reconfigurable systems required in many applications such as communication systems to support several different standards and operation modes, digital audio equipment, medical electronics, radar, sonar and control systems, adaptive and tracking systems, spectrum and vibration analyses, formant speech synthesizers and in numerous laboratory instruments [8–14]. Variable band-pass and band-stop filters are used for example to eliminate or retrieve some narrow-band or sinusoidal signal embedded in broad-band signal or noise. Other application is sample rate converter that exploits different sampling rates depending on the required signal quality and the available bandwidth of the channel and the data rate of the interfaces [8–10]. The tunable bandwidth filters are filters with variable bandwidth characteristics. These filters have controllable spectral characteristics such as variable cutoff frequency response, adjustable pass band and stop band width controllable fractional delay [6–8,15]. For example, any required magnitude characteristic can be obtained by varying a tuning parameter embedded in the filter structure, which is more flexible and efficient. For this purpose, a generalized structure as shown in Fig. 1 is employed in which overall transfer function is a weighted linear combination of fixed linear-phase FIR sub filters. In this structure, only few adjustable parameters (weights) that are directly determined by the bandwidth are required. This results in a simple updating routine [5,15–17]. Because of this, they are extensively used in various engineering applications. Literature review on variable bandwidth filters reflects that the extensive works have been done towards the design and realization of tunable bandwidth filters.

In general, the design algorithms of variable bandwidth filters can be classified into two groups: spectral transformation and spectral approximation. In transformation based approach, initially a prototype filter is designed with desirable frequency characteristics, and then a variable bandwidth filter is derived from the

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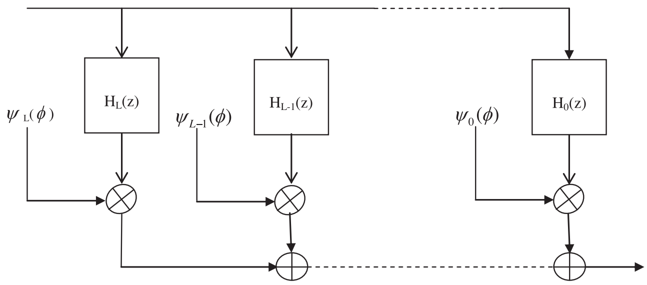

Fig. 1. A typical implementation of a variable-bandwidth filter.

prototype using suitable transformation technique. This technique is only suitable for the design of variable bandwidth filters with variable cut-off frequencies, and it is generally not used for variable bandwidth filters variable fractional delay. In early stage of the research, several methods based on transformation approach [18–22] were developed. Originally, the authors in [18,19] have developed a technique based on transformation approach in which a sub network of a zero-phase odd-length prototype FIR filter is changed by another network that performs frequency mapping of the prototype filter response [6]. However, in these algorithms, it is difficult to update, and are not exploited for designing complicated filter due to limitation of variable frequency [6,15–17].

In the spectral approximation method, a tunable bandwidth filter is considered as a weighted combination of fixed coefficient filters. Here, either impulse response or poles and zeros of the filters are polynomials of certain spectral parameter. The transfer function of a variable-bandwidth filter is a polynomial in tuning parameter that decides the bandwidth. Therefore, initially the fixed coefficient filter is designed, and then the bandwidth is controlled directly by changing the tuning parameter. Therefore, in spectral approximation approach, design of a tunable filter relies on the fixed coefficient filters. In general, the fixed-coefficient filters are designed with different computer aided technique such as linear programming [5] and the weighted least square (WLS) methods [7,23]. Originally, the spectral approximation method was proposed by Zarour and Fahmy [24]. In this method, the filter coefficients are expressed as analytical functions of the frequency specifications by using a curve-fitting technique. Thereon, several techniques [25,26] were developed based on the spectral approximation technique. The different types of linear-phase filters are presented in [27], in which the design problem has been formulated in a quadratic form. A new technique [28] was proposed based on Toeplitz and a Hankel matrix in which the fixed filter coefficients are determined by solving a system of linear equations involving a block-symmetric positive-definite matrix. A closed form method for designing tunable filters is presented in [6]. This method gives good performance and is computationally efficient. However, this method is more suitable for lower order.

In this work, a new closed form method based on different polynomials is presented for the design of linear-phase finite-impulse response (FIR) filters with adjustable bandwidth and fixed phase response. In the proposed method, different polynomials such as shifted Chebyshev polynomials, Bernstein polynomials and shifted Legendre polynomials are exploited as tuning parameters to control bandwidth of the filter. The error function is formulated by computing the difference between actual response and practical response, which is the function of tuning parameter as well as fixed coefficient filters. In the next section, an overview of the different polynomials is discussed. Section 3 reviews the analysis of variable bandwidth filter. In Section 4, the proposed method and its implementation for the design of variable bandwidth filter is presented. Finally, the simulation results are discussed in Section 5, followed by the concluding remarks in Section 6.

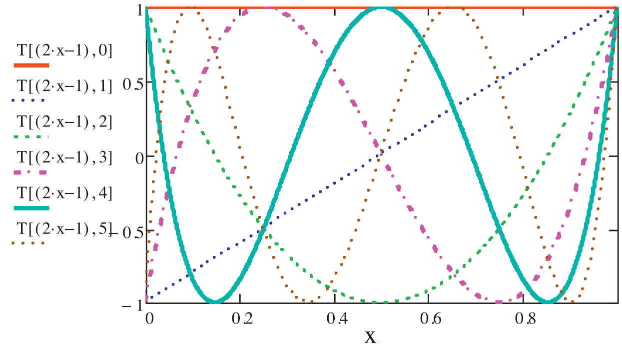

Fig. 2. First six shifted Chebyshev polynomials of first kind.

2. Overview of different types of polynomials

Polynomials are the useful mathematical tools which are simply defined. These can be computed quickly on computer systems, and represent a tremendous variety of functions. In this work, a polynomial is used as tuning parameters to control bandwidth of the filter.

2.1. Chebyshev polynomials

Chebyshev polynomials are widely used in many applications [29]. If $x=\cos(\theta)$, for $0\le\theta\le\pi$, for the function $T_k(x)=\cos(k\theta)=\cos(k\arccos x)$ is a polynomial of x of degree $k\,(k=0, 1, 2, 3, \ldots)$. Then, $T_k(x)$ is called Chebyshev polynomial of first kind of degree k. As θ increases from 0 to π or x decreases from 1 to -1. Then, the interval $[-1, 1]$ is the domain of definition of $T_k(x)$. Outside this interval, $T_k(x)=0$. Some important properties of Chebyshev polynomials:

- Chebyshev polynomials of first kind can be defined in interval $[-1, 1]$ by following recursive relation defined as

$$T_{k+1}(x) = 2xT_k(x) - T_{k-1}(x) \tag{1}$$

- where, $T_0(x)=0$ and $T_1(x)=x$.
- Multiplication of two Chebyshev polynomials is defined as

$$2T_m(x)T_n(x) = T_{m+n}(x) + T_{|m-n|}(x) \tag{2}$$

- Chebyshev polynomial $T_k(x)$ lies between -1 and 1, when $-1\le x\le 1$.

The Chebyshev polynomial of first kind can be defined to any interval $[a, b]$ by replacing x with $(2x/(b-a)-((b+a)/(b-a))$ in $T_k(x)$ which is said to be shifted Chebyshev polynomial of first kind in the interval $[a, b]$. Hence, a shifted Chebyshev polynomial is defined as

$$T_k\left(\frac{2x}{b-a} - \frac{b+a}{b-a}\right) \tag{3}$$

For example, $T_k(2x-1)$ is called shifted Chebyshev polynomials which is defined in the interval $[0, 1]$, where $T_k(x)$ is Chebyshev polynomial defined in the interval $[-1, 1]$. The first six shifted Chebyshev polynomial of first kind is depicted in Fig. 2. A detailed discussion on Chebyshev polynomial is provided in [29] and the references therein.

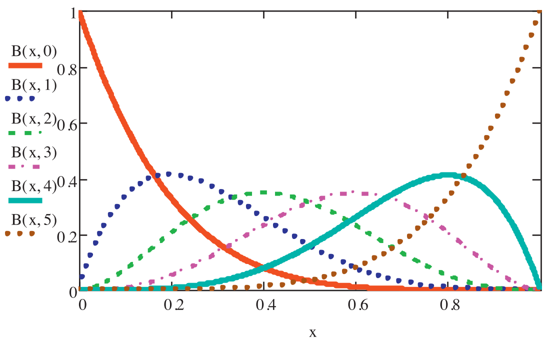

Fig. 3. First six basis Bernstein polynomials for $n = 5$.

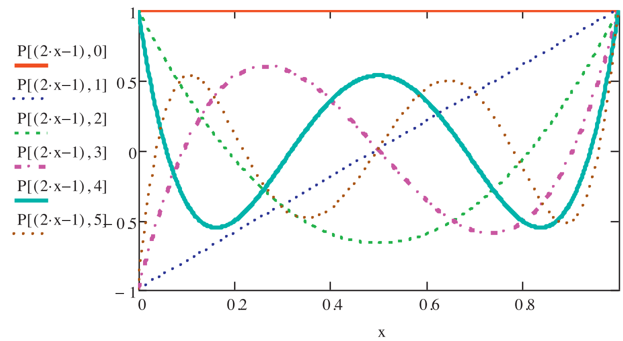

Fig. 4. First six basis shifted Legendre polynomials in the interval [0, 1] for $n = 5$.

2.2. Bernstein polynomial

Bernstein basis polynomial of degree n in the interval [0, 1] is defined as [30,31]

$$B_{k,n}(t) = \binom{n}{k} t^k (1-t)^{(n-k)}, \quad \text{for } k = 0, 1, 2, \ldots, n. \tag{4}$$

where, $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$. There are $(n+1)$ nth-degree Bernstein polynomials. For mathematical convenience, $B_{k,n}(t)$ is considered equal to zero if $k<0$ or $k>n$. The Bernstein basis polynomials are very easy expressed as the coefficients $\binom{n}{k}$ are computed from Pascal's triangle. The Bernstein polynomials can also be defined recursively by blending together two Bernstein polynomials. For example, the kth nth-degree Bernstein polynomial described by Eq. (4) can be written as

$$B_{k,n}(t) = (1-t)B_{k,n-1}(t) + tB_{k-1,n-1}(t). \tag{5}$$

These polynomials show many important properties for various applications. Some these properties are

- $B_{k,n}(0) = \delta_{k0}$ and $B_{k,n}(1) = \delta_{kn}$, here δ_{kn} is the Kronecker delta function.[$\delta_{kn} = 1$ if $k = n$ and $\delta_{kn} = 0$ if $k \neq n$].
- $B_{k,n}(0) \geq 0$ for $t \in [0, 1]$ and, $B_{k,n}(1-x) = B_{n-k,n}(x)$.
- The Bernstein polynomials form a partition of unity i.e. $\sum_{k=0}^{n} B_{k,n}(x) = 1$.

The first six basis Bernstein polynomials for $n = 5$ are graphically shown in Fig. 3. A detailed discussion on these polynomials is given in [30,31] and the reference therein.

2.3. Legendre polynomials

Legendre polynomials of degree n in the interval [−1, 1] are defined by the recursive relation [32]:

$$P_{k+1}(x) = \frac{2k+1}{k+1} x P_k(x) - \frac{k}{k+1} P_{k-1}(x), \tag{6}$$

where $P_0(x) = 0$ and $P_1(x) = 1$. Using Eq. (6), different versions of Legendre polynomials are constructed. These polynomials satisfy many important properties:

- $P_k(x) = 1$ at $x = 1$
- $P_k(-x) = (-1)^k P_k(x)$.
- $-1 \leq P_k(x) \leq 1$ when $-1 \leq x \leq 1$.

When Legendre polynomials are defined in the interval [0, 1], they are known as shifted Legendre polynomial $P_k(2x-1)$, where $P_k(x)$ is the Legendre polynomials defined in the interval [−1, 1]. The first six Legendre polynomials for $n = 5$ is displayed in Fig. 4. A detailed discussion on these polynomials is given [29,30] and the reference therein.

3. Analysis of variable bandwidth filter

Many applications require the use of digital filter with easily tunable characteristics. Based on the spectral approximation, the impulse response $h(n, \phi)$ of a digital filter having variable bandwidth characteristics, whose typical implementation is shown in Fig. 1 [6,7], can be expressed by a linear combination of a function $\psi_k(\phi)$ of the spectral parameter (ϕ):

$$h(n, \phi) = \sum_{k=0}^{L} h_{n,k} \psi_k(\phi), \tag{7}$$

where $h_{n,k}$ is the coefficients of expansion. Therefore, the design problem of variable bandwidth filter is reduced to find out $h_{n,k}$ with the specified $\psi_k(\phi)$ so that the given frequency response of $h(n, \phi)$ will approximate some desirable frequency response as a function of ϕ which controls the bandwidth. The z-transform of $h(n, \phi)$ can be given as

$$H(z, \phi) = \sum_{n=0}^{N} h(n, \phi) z^{-n} = \sum_{n=0}^{N} \sum_{k=0}^{L} h_{n,k} \psi_k(\phi) z^{-n}, \tag{8}$$

which can be rewritten as

$$H(z, \phi) = \sum_{k=0}^{L} \left[\sum_{n=0}^{N} h_{n,k} z^{-n} \right] \psi_k(\phi) = \sum_{k=0}^{L} H_k(z) \psi_k(\phi) \tag{9}$$

In Eq. (9), $H_k(z) = \sum_{n=0}^{N-1} h_{n,k} z^{-n}$, is the transfer function of the Kth Nth order fixed coefficient linear phase digital filter symmetrical or anti-symmetrical impulse response $h_{n,k}$ and $\psi_k(\phi)$ is tuning parameter or control parameter to vary the bandwidth of the filter.

The frequency response of $H_k(z)$ is given by

$$H_k(e^{j\omega}) = \sum_{n=0}^{N} h_{n,k} z^{-n} = e^{j\theta(\omega)} G_k(\omega), \tag{10}$$

where $G_k(\omega)$ is the zero-phase frequency response of the kth fixed-coefficient linear phase digital filter. Therefore, the zero phase frequency response of a variable-bandwidth filter can be defined as

$$H_R(\omega, \phi) = \sum_{k=0}^{L} \psi_k(\phi) G_k(\omega), \tag{11}$$

where $H_R(\omega, \phi)$ is real valued.

Generally, digital filters are categorized into two types: infinite impulse response (IIR) filter and finite impulse response (FIR) filter [23]. Both types of filters are exploited for designing the variable bandwidth filter. Some applications including channel equalization, low-delay filtering, and seismic migration [6] require filter with nonlinear phase, while others such as video processing and communication systems employ filter with linear-phase. It is always possible to design an FIR filter with an exact linear-phase response. However, IIR filter is impossible to design with an exact linear-phase. Recently, the design of variable bandwidth filter based on FIR has received considerable attention due to their simple design procedure and good filter performance.

This work, therefore, is focused on the design of variable bandwidth FIR digital filters using different polynomials. From the analysis of variable bandwidth filter [6], it is observed that the tuning parameters or control parameters (ϕ) vary as $\phi \in [0, 1]$ where as passband and stopband edge frequencies vary linearly as

$$\omega_p(\phi) = (\omega_{p_2} - \omega_{p_1})\phi + \omega_{p_1} \tag{12}$$

$$\omega_a(\phi) = (\omega_{a_2} - \omega_{a_1})\phi + \omega_{a_1} \tag{13}$$

where variable passband edge frequencies are defined in the range $[\omega_{p1}, \omega_{p2}]$, and the stopband edge frequencies have a range $[\omega_{a1}, \omega_{a2}]$. Thus, the tuning parameter is defined in the same range as polynomial. Hence, shifted Chebyshev, Bernstein and Legendre polynomials can be used as tuning parameter.

4. Design of variable bandwidth filter using polynomial approach

In digital filter design, FIR filters are classified into four types based on symmetric/antisymmetic and length of the impulse response. In Type I, the frequency response of filter is given by

$$G_k(e^{j\omega}) = e^{-j\omega N/2} G_k(\omega), \tag{14}$$

where $G_k(\omega)$ is the zero phase frequency response or amplitude response defined as

$$\begin{aligned} G_k(\omega) &= (h_{k,N/2}) + 2\sum_{n=1}^{N/2}[h_{k,((N/2)-n)}]\cos(\omega n) \\ &= \sum_{n=0}^{M} a_k(n)\cos(\omega n) \end{aligned} \tag{15}$$

where $a_k(n) = h_{k,((N/2)-n)}$ and $M = N/2$.

Then, the zero phase frequency response of a variable bandwidth linear phase filter can be rewritten as

$$H_R(\omega, \phi) = \sum_{k=0}^{k=L} \psi_k(\phi) \sum_{n=0}^{M} a_k(n)\cos(\omega n) = \mathbf{a}^T \mathbf{c} \tag{16}$$

where

$$\mathbf{a} = [a_0(0), a_0(1), \ldots, a_0(M), a_1(0), \ldots, a_1(M), \ldots, a_k(0), a_k(1), \ldots, a_k(M)]^T, \tag{17}$$

and

$$\mathbf{c} = [\psi_0(\phi)\, t_c^T, \psi_1(\phi)\, t_c^T, \ldots, \psi_L(\phi)\, t_c^T]^T. \tag{18}$$

In Eq. (18), $\mathbf{t_c}$ are defined as,

$$\mathbf{t_c} = [1, \ \cos(\omega), \ldots, \cos(M\omega)]^T. \tag{19}$$

In this work, $\psi_k(\phi)$ is taken as shifted Chebyshev polynomials or Bernstein polynomials or shifted Legendre polynomials such as

$$\text{Shifted Chebyshev polynomial}: \ \psi_k(\phi) = T_k(2x - 1), \tag{20}$$

$$\text{Bernstein polynomial}: \ \psi_k(\phi) = B_{k,n}(\phi), \tag{21}$$

$$\text{and shifted Legendre polynomials}: \ \psi_k(\phi) = P_k(2x - 1). \tag{22}$$

If a low pass filter with variable bandwidth characteristic having tunable passband and stopband edges is designed, then its ideal magnitude response is given as

$$M_I(\omega, \phi) = \begin{cases} 1, & 0 \le \omega \le \omega_p(\phi) \\ 0, & \omega_a(\phi) \le \omega \le \pi. \end{cases} \tag{23}$$

Therefore, in the proposed method, a low pass filter is designed by minimizing the quadratic measure of errors in desired response of the filter. Thus, the error function is computed from the difference between the desired variable bandwidth filter and the practical filter represented as a linear combination of fixed-coefficient filters defined as

$$E(\omega, \phi) = M_I(\omega, \phi) - \mathbf{a}^T \mathbf{c} \tag{24}$$

the mean-square error is defined as

$$\begin{aligned} E_{mse} &= \int_{\phi_l}^{\phi_k}\int_{\Omega_s}^{\Omega_p} W(\omega,\phi)(M_I^2(\omega,\phi) - 2M_I(\omega,\phi)\mathbf{a}^T\mathbf{c} + \mathbf{a}^T\mathbf{cc}^T\mathbf{a})\, d\omega d\phi, \\ &= \int_{\phi_l}^{\phi_u}[\int_0^{\omega_p(\phi)} K_p(M_I^2(\omega,\phi) - 2M_I(\omega,\phi)\mathbf{a}^T\mathbf{c} + \mathbf{a}^T\mathbf{cc}^T\mathbf{a}) \\ &\quad + \int_{\phi_l}^{\phi_u}\int_{\omega_p(\phi)}^{\pi} K_a(M_I^2(\omega,\phi) - 2M_I(\omega,\phi)\mathbf{a}^T\mathbf{c} + \mathbf{a}^T\mathbf{cc}^T\mathbf{a})]\, d\omega d\phi, \end{aligned} \tag{25}$$

where $\Omega_s \in [0, \omega_p(\phi)] \cup [\omega_a(\phi), \pi]$ and $W(\omega, \phi)$ is the weighting function defined as

$$W(\omega, \phi) = \begin{cases} K_p, & 0 \le \omega \le \omega_p(\phi) \\ K_a, & \omega_a(\phi) \le \omega \le \pi. \end{cases} \tag{26}$$

So, E_{mse} can be rewritten as,

$$\begin{aligned} E_{mse} &= \int_{\phi_l}^{\phi_u}[\int_0^{\omega_p(\phi)} K_p(M_I^2(\omega,\phi) - 2M_I(\omega,\phi)\mathbf{a}^T\mathbf{c} + \mathbf{a}^T\mathbf{cc}^T\mathbf{a}) \\ &\quad + \int_{\phi_l}^{\phi_u}\int_{\omega_p(\phi)}^{\pi} K_a(M_I^2(\omega,\phi) - 2M_I(\omega,\phi)\mathbf{a}^T\mathbf{c} + \mathbf{a}^T\mathbf{cc}^T\mathbf{a})]\, d\omega d\phi, \end{aligned} \tag{27}$$

Similar to algorithm given in [6,7], in this method, E_{mse} is minimized by differentiating with respect to the coefficients of each $H_k(z)$,

$$\begin{aligned} \frac{dE_{mse}}{da} &= \int_{\phi_l}^{\phi_u}\int_{\Omega_s}^{\Omega_p} W(\omega,\phi)(-2M_I(\omega,\phi)c(a')^T + [(a')^T cc^T a + a^T cc^T a']\, d\omega d\phi \\ &= \int_{\phi_l}^{\phi_u}\int_{\Omega_s}^{\Omega_p} W(\omega,\phi)(-2M_I(\omega,\phi)c(a')^T + [acc^T(a')^T + acc^T(a')^T]\, d\omega d\phi, \end{aligned} \tag{28}$$

where a' is the differentiation of coefficients w.r.t. each coefficients. For minimization $dE_{mse}/da = 0$, of leads to,

$$\begin{aligned}
&\int_{\phi_l}^{\phi_u}\int_{\Omega_s}^{\Omega_p} W(\omega,\phi)[(-2M_I(\omega,\phi))c + cc^T a + cc^T a]\, d\omega d\phi = 0\\
&\int_{\phi_l}^{\phi_u}\int_{\Omega_s}^{\Omega_p} W(\omega,\phi)[cc^T a + cc^T a]\ d\omega d\phi = \int_{\phi_l}^{\phi_u}\int_{\Omega_s}^{\Omega_p} W(\omega,\phi)(2M_I(\omega,\phi))c\, d\omega d\phi\\
&\int_{\phi_l}^{\phi_u}\int_{\Omega_s}^{\Omega_p} 2W(\omega,\phi)(cc^T a)\, d\omega d\phi = \int_{\phi_l}^{\phi_u}\int_{\Omega_s}^{\Omega_p} W(\omega,\phi)(2M_I(\omega,\phi))c\, d\omega d\phi,\\
&\int_{\phi_l}^{\phi_u}\int_{0}^{\omega_p(\phi)} K_p(cc^T a)\, d\omega d\phi + \int_{\phi_l}^{\phi_u}\int_{\omega_a(\phi)}^{\pi} K_a(cc^T a)\, d\omega d\phi = \int_{\phi_l}^{\phi_u}\int_{\Omega_s}^{\Omega_p} W(\omega,\phi)M_I(\omega,\phi)c\, d\omega d\phi
\end{aligned} \tag{29}$$

This results in a system of linear equations defined as $\mathbf{Qa} = \mathbf{b}$. From $\mathbf{a} = \mathbf{Q}^{-1}\mathbf{b}$ and using Eq. (17), the coefficient matrix can be obtained. Here, $\mathbf{Q}$ and $\mathbf{b}$ are given, respectively as

$$Q[k(M+1)+n, l(M+1)+m] = \int_{\phi_l}^{\phi_u} \psi_k(\phi)\psi_l(\phi) \times \left[K_p \int_0^{\omega_p(\phi)} \cos(n\omega)\cos(m\omega)\, d\omega + K_a \int_{\omega_a(\phi)}^{\pi} \cos(n\omega)\cos(m\omega)\, d\omega\right], \tag{30}$$

and

$$\mathbf{b}[k(M+1)+n] = \int_{\phi_l}^{\phi_u} \psi_k(\phi)\left[K_p \int_0^{\omega_p(\phi)} M_I(\omega,\phi)\cos(n\omega)\, d\omega\right] d\phi \tag{31}$$

where $0 \le n \le M$, $0 \le m \le M$ and $0 \le k, l \le L$.

The entries of matrix Q can also be written in Toeplitz-plus-Hankel matrix, which reduces the computational complexity because not all entries of the matrix need to be evaluated.

$$\mathbf{Q}[k(M+1)+n, l(M+1)+m] = \frac{1}{2}[T[t_{k,l}(n,m)] + H[g_{k,l}(n), t_{k,l}(n)], \tag{32}$$

For $0 \le m, n \le M$ and for $0 \le k, l \le L$. In Eq. (32), $t_{k,l}(n)$ is

$$t_{k,l}(n) = \begin{cases} K_p \int_{\phi_l}^{\phi_l} \psi_k(\phi)\psi_l(\phi)[\omega_p(\phi)]\, d\phi + K_a \int_{\phi_l}^{\phi_l} \psi_k(\phi)\psi_l(\phi)[\pi - \omega_a(\phi)]\, d\phi, & \text{for } n = 0\\ \dfrac{K_p \int_{\phi_l}^{\phi_l} \psi_k(\phi)\psi_l(\phi)\sin[n\omega_p(\phi)]\, d\phi - K_a \int_{\phi_l}^{\phi_l} \psi_k(\phi)\psi_l(\phi)\sin[\omega_a(\phi)] d\phi,}{n} & \text{for } 1 \le n \le M \end{cases} \tag{33}$$

and

$$g_{k,l}(n) = \frac{K_p \int_{\phi_l}^{\phi_u} \psi_k(\phi)\psi_l(\phi)\sin[(n+M)(\omega_p(\phi))]\ d\phi - K_a \int_{\phi_l}^{\phi_u} \psi_k(\phi)\psi_l(\phi)\sin[(n+M)(\omega_p(\phi))]\ d\phi}{n+M}, \tag{34}$$

For $0 \le n \le M$. In Eq. (32), T is Toeplitz matrix whose elements are arranged as the relation $T_{k,l} = T_{k+1,l+1}$. Now for a given vector t, the Toeplitz matrix is obtained from the entries of this vector as

$$T[t](n,m) = t(|n-m|), \quad \text{for } 0 \le n,\ m \le M. \tag{35}$$

In Eq. (32), $T_k(x) = \cos(k\theta) = \cos(k \arccos x)$ is Hankel matrix whose elements are arranged as $H_{k,l} = H_{k-1,l+1}$. Assume two vectors c and r, a new vector p is constructed as

$$\mathbf{p} = [c(0)c(1)\ldots c(P)r(1)r(2)\ldots r(P)]^T \tag{36}$$

Then, Hankel matrix is obtained from the above vectors as

$$H[c,r](n,m) = p(n+m) \quad \text{for } 0 \le n,\ m \le M. \tag{37}$$

Finally, the matrix a is computed using Eqs. (29) and (30).

This method has been implemented in MathCAD on Genuine Intel (R) Core(TM) 2 Duo CPU T6670 @ 2.20 GHz, 4 GB RAM.

5. Results and discussion

In this section, the proposed methodology has been used for designing the tunable bandwidth FIR filter using different polynomials. To examine the efficacy of this algorithm for designing tunable digital filters, several significant parameters such as stop band energy (ϕ_s), error in passband (ϕ_p), and stopband attenuation (A_s) are employed. These parameters are computed using following equations:

$$\text{Stopband error}: \ e_s = \frac{1}{\pi}\int_{\omega_s(\phi)}^{\pi} [M_I(\omega,\phi) - H_R(\omega,\phi)]^2\, d\omega, \tag{38}$$

$$\text{Passband error}: \ e_p = \frac{1}{\pi}\int_{0}^{\omega_a(\phi)} [M_I(\omega,\phi) - H_R(\omega,\phi)]^2\, d\omega, \tag{39}$$

$$\text{Stopband attenuation}: \ A_s = -20\log_{10}(H_R(\omega,\phi)) \quad \text{at } \omega = \omega_a(\phi), \tag{40}$$

In Eqs. (38) and (39),

$$M_I(\omega,\phi) = \begin{cases} 1, & 0 \le \omega \le \omega_p(\phi)\\ 0, & \omega_a(\phi) \le \omega \le \pi. \end{cases} \tag{41}$$

First of all, performance comparison of the different polynomials for designing tunable bandwidth filter is carried out. Thereon, the comparison of the proposed method is done with the other known methods.

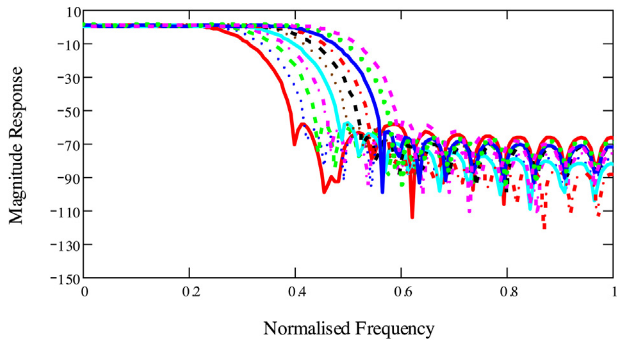

Fig. 5. Tunable bandwidth FIR filter designed using the proposed method with shifted Chebyshev polynomials, $N=32$: amplitude response of a low pass filter Type I in dB.

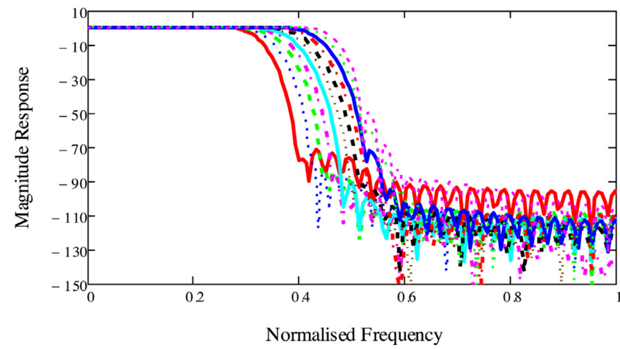

Fig. 6. Tunable bandwidth FIR filter designed using the proposed method with shifted Chebyshev polynomials, $N=64$: amplitude response of a low pass filter Type I in dB.

5.1. Design examples

A variable-bandwidth low pass filter with Type I has been designed with the same design specifications as reported in [6,7] with $\omega_{p1}=0.2\pi$, $\omega_{p2}=0.4\pi$, $\omega_{a1}=0.4\pi$, $\omega_{a2}=0.2\pi$, $\phi\in[0, 1]$, $N=32$, $L=5, K_p=1$ and $K_a=10$ using the proposed methodology with shifted Chebyshev polynomials, Bernstein polynomials and shifted Legendre polynomials. Here, these polynomials are applied separately one by one as tuning parameter. Then, corresponding Q and b matrix is found for each. Finally, filter coefficients $a=Q^{-1}b$ is obtained in order to get magnitude response. The magnitude response is simulated for different values of tuning parameter $\phi=0$, $\phi=0.1$, $\phi=0.2$, . . ., $\phi=1$, which clearly demonstrates the variable bandwidth characteristics.

5.1.1. Using shifted Chebyshev polynomials

As discussed above, shifted Chebyshev polynomials are used as control or tuning parameter for varying the bandwidth of the filter $\psi_k(\phi)=T_k(2\phi-1)$. Since $L=5$, hence first six shifted Chebyshev polynomials are used. Then, the elements of the matrixes Q and b are given as:

$$Q[k(M+1)+n, l(M+1)+m] = \int_{\phi_l}^{\phi_u} T_k(2\phi-1)T_l(2\phi-1)$$
$$\times \left[K_p \int_0^{\omega_p(\phi)} \cos(n\omega)\cos(m\omega)\,d\omega\right.$$
$$\left.+K_a \int_{\omega_a(\phi)}^{\pi} \cos(n\omega)\cos(m\omega)d\omega\, d\phi,\right. \tag{42}$$

and

$$\mathbf{b}[k(M+1)+n] = \int_{\phi_l}^{\phi_u} T_k(2\phi-1)$$
$$\times \left[K_p \int_0^{\omega_p(\phi)} M_I(\omega,\phi)\cos(n\omega)d\omega\right] d\phi \tag{43}$$

$0\le m,\ n\le M$ and $0\le k,\ l\le L$. The simulation results obtained for designing variable bandwidth filter with $N=32$ when $\phi\in[0, 1]$ are graphically shown in Fig. 5, and are also listed in Table 2 for ready reference. In this case, ϕ is divided into 11 equally spaced values between 0 and 1 ($\phi=0, 0.1, 0.2, \ldots, 1$). The method was also tested for tunable bandwidth filter with $N=64$. The design results obtained are depicted in Fig. 6 and also given in Table 1 for ready reference.

5.1.2. Using Bernstein polynomials

In this subsection, the same examples are designed using Bernstein polynomials discussed above in Section 2.2 as tuning parameter for varying the bandwidth of the filter, $\psi_k(\phi)=B_{k,5}(\phi)$.

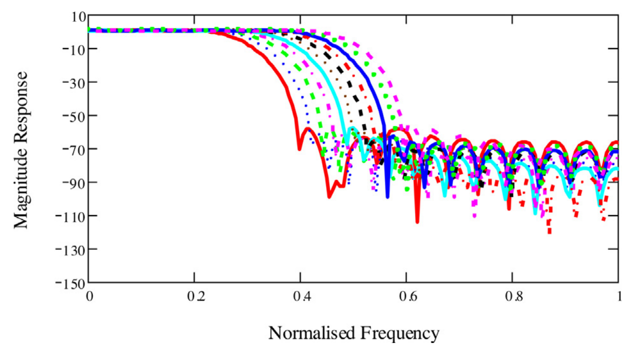

Fig. 7. Tunable bandwidth FIR filter designed using the proposed method with Bernstein polynomials, $N=32$: amplitude response of a low pass filter Type I in dB.

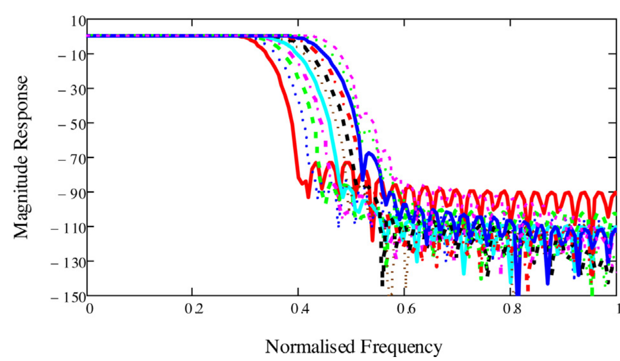

Fig. 8. Tunable bandwidth FIR filter designed using the proposed method with Bernstein polynomials, $N=64$: amplitude response of a low pass filter Type I in dB.

Here, the first six Bernstein polynomials are exploited due to $L=5$. In this case, the elements of matrix Q and b are computed as

$$Q[k(M+1)+n, l(M+1)+m] = \int_{\phi_l}^{\phi_u} B_{k,5}(\phi)B_{l,5}(\phi)$$
$$\times \left[K_p \int_0^{\omega_p(\phi)} \cos(n\omega)\cos(m\omega)\,d\omega\right.$$
$$\left.+K_a \int_{\omega_a(\phi)}^{\pi} \cos(n\omega)\cos(m\omega)\,d\omega\right] d\phi, \tag{44}$$

and $$\mathbf{b}[k(M+1)+n] = \int_{\phi_l}^{\phi_u} B_{k,5}(\phi)$$
$$\times \left[K_p \int_0^{\omega_p(\phi)} M_I(\omega,\phi)\cos(n\omega)\,d\omega\right] d\phi, \tag{45}$$

Table 1
Performance of the proposed algorithm for designing tunable bandwidth FIR filter with Type I.

Phi (ϕ)	$N=32$			$N=64$		
	Pass band error (e_p)	Stop band error (e_s)	Stopband attenuation (A_s) in dB	Pass band error (e_p)	Stop band error (e_s)	Stopband attenuation (A_s) dB
Shifted Chebyshev polynomial						
0	6.21×10^{-7}	1.37×10^{-8}	65.20	2.98×10^{-9}	2.52×10^{-9}	77.42
0.1	3.70×10^{-7}	4.78×10^{-8}	52.93	3.42×10^{-10}	3.31×10^{-10}	69.04
0.2	3.98×10^{-7}	7.78×10^{-8}	53.65	2.78×10^{-10}	1.85×10^{-10}	71.33
0.3	2.59×10^{-7}	5.21×10^{-8}	50.36	2.23×10^{-10}	5.44×10^{-11}	78.03
0.4	3.27×10^{-7}	5.68×10^{-8}	50.97	2.35×10^{-10}	1.55×10^{-11}	87.69
0.5	5.37×10^{-7}	5.64×10^{-8}	53.37	1.58×10^{-10}	1.79×10^{-11}	100.60
0.6	4.64×10^{-7}	3.41×10^{-8}	52.38	3.94×10^{-10}	1.40×10^{-11}	92.79
0.7	5.04×10^{-7}	2.61×10^{-8}	53.70	6.25×10^{-10}	8.31×10^{-12}	106.70
0.8	8.05×10^{-7}	3.31×10^{-8}	56.50	2.50×10^{-9}	8.41×10^{-12}	98.62
0.9	4.77×10^{-7}	4.06×10^{-8}	54.33	4.96×10^{-9}	6.48×10^{-12}	91.98
1	9.12×10^{-7}	10×10^{-8}	70.15	3.48×10^{-9}	1.07×10^{-10}	111.06
Bernstein polynomials						
0	6.21×10^{-7}	1.37×10^{-8}	65.20	7.42×10^{-9}	2.77×10^{-9}	78.86
0.1	3.70×10^{-7}	4.78×10^{-8}	52.93	6.84×10^{-10}	3.52×10^{-10}	70.27
0.2	3.98×10^{-7}	7.78×10^{-8}	53.65	3.82×10^{-10}	1.56×10^{-10}	73.80
0.3	2.59×10^{-7}	5.21×10^{-8}	50.36	2.10×10^{-10}	4.70×10^{-11}	87.33
0.4	3.27×10^{-7}	5.68×10^{-8}	50.97	3.41×10^{-10}	3.92×10^{-11}	90.32
0.5	5.37×10^{-7}	5.64×10^{-8}	53.37	2.76×10^{-10}	4.90×10^{-11}	94.14
0.6	4.64×10^{-7}	3.41×10^{-8}	52.38	5.14×10^{-10}	4.47×10^{-11}	88.03
0.7	5.04×10^{-7}	2.61×10^{-8}	53.70	8.87×10^{-10}	2.13×10^{-11}	100.89
0.8	8.05×10^{-7}	3.31×10^{-8}	56.50	2.05×10^{-9}	2.28×10^{-11}	92.90
0.9	4.77×10^{-7}	4.06×10^{-8}	54.33	5.10×10^{-9}	1.99×10^{-11}	86.43
1	9.12×10^{-7}	10×10^{-8}	70.15	5.09×10^{-8}	1.71×10^{-10}	103.7
Shifted Legendre polynomials						
0	6.21×10^{-7}	1.37×10^{-8}	65.20	2.98×10^{-9}	2.52×10^{-9}	77.46
0.1	3.70×10^{-7}	4.78×10^{-8}	52.93	3.43×10^{-10}	3.32×10^{-10}	69.04
0.2	3.98×10^{-7}	7.78×10^{-8}	53.65	2.78×10^{-10}	1.84×10^{-10}	71.34
0.3	2.59×10^{-7}	5.21×10^{-8}	50.36	2.23×10^{-10}	5.22×10^{-11}	78.05
0.4	3.27×10^{-7}	5.68×10^{-8}	50.97	2.35×10^{-10}	1.55×10^{-11}	87.73
0.5	5.37×10^{-7}	5.64×10^{-8}	53.37	1.58×10^{-10}	1.78×10^{-11}	100.72
0.6	4.64×10^{-7}	3.41×10^{-8}	52.38	3.94×10^{-10}	1.40×10^{-11}	92.80
0.7	5.04×10^{-7}	2.61×10^{-8}	53.70	6.25×10^{-10}	8.36×10^{-12}	106.53
0.8	8.05×10^{-7}	3.31×10^{-8}	56.50	2.50×10^{-9}	8.43×10^{-12}	98.69
0.9	4.77×10^{-7}	4.06×10^{-8}	54.33	4.96×10^{-9}	6.49×10^{-12}	92.01
1	9.12×10^{-7}	10×10^{-8}	70.15	3.48×10^{-8}	1.08×10^{-10}	111.19

For $0\le m, n\le M$ and $0\le k, l\le L$. The simulation results obtained in the both cases are shown in Figs. 7 and 8, respectively.

5.1.3. Using shifted Legendre polynomials

Similarly, in this subsection, the shifted Legendre polynomials discussed in Section 2.3 are employed as tuning parameter for varying the bandwidth of filter $\psi_k(\phi)=P_k(2\phi-1)$. In this, the first six shifted Legendre polynomials are used due to $L=5$. Therefore, the elements of matrix Q and b are determined as

$$\mathbf{Q}[k(M+1)+n, l(M+1)+m]=\int_{\phi_l}^{\phi_u} P_k(2\phi-1)P_l(2\phi-1) \times\left[K_p\int_0^{\omega_p(\phi)}\cos(n\omega)\cos(m\omega)\,d\omega +K_a\int_{\omega_a(\phi)}^{\pi}\cos(n\omega)n\omega\cos(m\omega)n\omega d\omega\right]d\phi, \tag{46}$$

and

$$\mathbf{b}[k(M+1)+n]=\int_{\phi_l}^{\phi_u} T_k(2\phi-1) \times\left[K_p\int_0^{\omega_p(\phi)} M_I(\omega,\phi)\cos(n\omega)\,d\omega\right]d\phi, \tag{47}$$

For $0\le m{,}n\le M$ and $0\le k, l\le L$. The design results obtained in case of $N=32$ and 64 are displayed in Figs. 9 and 10, respectively. The design results are also listed in Table 1.

5.2. Comparison of different polynomials

In Tables 1 and 2, results of a comparative study of the proposed method with three polynomials such as shifted Chebyshev, Bernstein polynomials, and Shifted Legendre polynomials are depicted. For this, variable bandwidth FIR filter with Types I and II with similar specifications are designed. Here, $\phi\in[0, 1]$ is equally divided in 11 subparts between 0 and 1 ($\phi=0, 0.1, \ldots, 1$). As it can be seen from the tables, the proposed method yields good fidelity parameters with all these polynomials. When they are compared, the shifted Chebyshev polynomials give better overall performance as compared to other two polynomials. Average A_S obtained with shifted

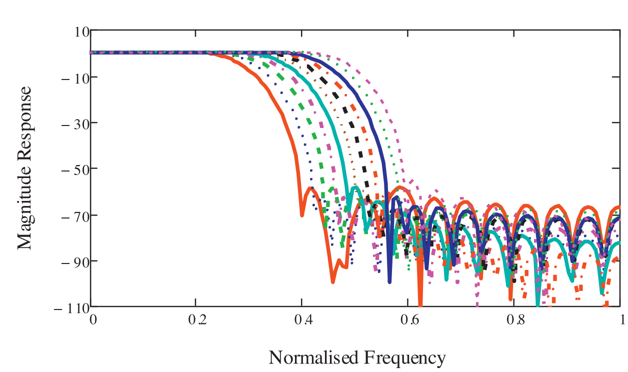

Fig. 9. Tunable bandwidth FIR filter designed using the proposed method with shifted Legendre polynomials, $N=32$: amplitude response of a low pass filter Type I in dB.

Table 2
Performance of the proposed algorithm for designing tunable bandwidth FIR filter with Type II.

Phi (ϕ)	N = 31			N = 63		
	Pass band error (e_p)	Stop band error (e_s)	Stopband attenuation (A_s) dB	Pass band error (e_p)	Stop band error (e_s)	Stopband attenuation (A_s) dB
Shifted Chebyshev polynomial						
0	8.71×10^{-7}	1.51×10^{-7}	49.71	8.35×10^{-9}	2.97×10^{-9}	64.76
0.1	3.30×10^{-7}	4.76×10^{-8}	52.67	6.43×10^{-10}	2.53×10^{-10}	69.82
0.2	6.30×10^{-7}	9.30×10^{-8}	52.20	2.05×10^{-10}	1.92×10^{-10}	71.21
0.3	5.15×10^{-7}	8.39×10^{-8}	50.11	2.01×10^{-10}	6.89×10^{-11}	77.50
0.4	4.02×10^{-7}	6.14×10^{-8}	49.18	4.23×10^{-10}	2.05×10^{-11}	85.90
0.5	6.47×10^{-7}	7.52×10^{-8}	51.83	2.79×10^{-10}	2.58×10^{-11}	102.55
0.6	8.60×10^{-7}	6.09×10^{-8}	53.26	2.94×10^{-10}	2.36×10^{-11}	92.93
0.7	6.76×10^{-7}	3.85×10^{-8}	52.81	7.51×10^{-10}	1.23×10^{-11}	101.22
0.8	8.46×10^{-7}	3.30×10^{-8}	52.87	2.63×10^{-9}	2.58×10^{-11}	98.77
0.9	9.19×10^{-7}	6.99×10^{-8}	51.81	5.11×10^{-9}	1.62×10^{-11}	90.07
1	8.73×10^{-7}	1.22×10^{-7}	88.05	2.96×10^{-8}	4.48×10^{-10}	111.73
Bernstein polynomials						
0	8.71×10^{-7}	1.51×10^{-7}	49.71	3.67×10^{-9}	2.30×10^{-9}	66.97
0.1	3.30×10^{-7}	4.76×10^{-8}	52.67	3.19×10^{-10}	1.61×10^{-10}	72.73
0.2	6.30×10^{-7}	9.30×10^{-8}	52.20	3.40×10^{-10}	1.51×10^{-10}	73.55
0.3	5.15×10^{-7}	8.39×10^{-8}	50.11	4.16×10^{-10}	6.25×10^{-11}	84.79
0.4	4.02×10^{-7}	6.14×10^{-8}	49.18	3.61×10^{-10}	4.27×10^{-11}	88.95
0.5	6.47×10^{-7}	7.52×10^{-8}	51.83	2.35×10^{-10}	5.63×10^{-11}	95.26
0.6	8.60×10^{-7}	6.09×10^{-8}	53.26	5.35×10^{-10}	6.19×10^{-11}	86.65
0.7	6.76×10^{-7}	3.85×10^{-8}	52.81	8.04×10^{-10}	3.59×10^{-11}	95.44
0.8	8.46×10^{-7}	3.30×10^{-8}	52.87	3.48×10^{-9}	6.27×10^{-11}	99.75
0.9	9.19×10^{-7}	6.99×10^{-8}	51.81	6.26×10^{-9}	4.41×10^{-11}	90.07
1	8.73×10^{-7}	1.22×10^{-7}	88.05	4.60×10^{-8s}	7.56×10^{-10}	98.39
Shifted Legendre polynomials						
0	8.71×10^{-7}	1.51×10^{-7}	49.71	7.19×10^{-9}	2.97×10^{-9}	64.84
0.1	3.30×10^{-7}	4.76×10^{-8}	52.67	6.39×10^{-10}	2.37×10^{-10}	70.08
0.2	6.30×10^{-7}	9.30×10^{-8}	52.20	2.28×10^{-10}	2.02×10^{-10}	71.12
0.3	5.15×10^{-7}	8.39×10^{-8}	50.11	2.13×10^{-10}	7.49×10^{-11}	77.15
0.4	4.02×10^{-7}	6.14×10^{-8}	49.18	4.27×10^{-10}	2.20×10^{-11}	85.17
0.5	6.47×10^{-7}	7.52×10^{-8}	51.83	2.81×10^{-10}	2.87×10^{-11}	100.18
0.6	8.60×10^{-7}	6.09×10^{-8}	53.26	3.05×10^{-10}	2.44×10^{-11}	91.61
0.7	6.76×10^{-7}	3.85×10^{-8}	52.81	7.75×10^{-10}	1.14×10^{-11}	106.11
0.8	8.46×10^{-7}	3.30×10^{-8}	52.87	2.74×10^{-10}	2.55×10^{-11}	96.13
0.9	9.19×10^{-7}	6.99×10^{-8}	51.81	5.05×10^{-9}	1.76×10^{-11}	89.32
1	8.73×10^{-7}	1.22×10^{-7}	88.05	3.48×10^{-8}	1.08×10^{-10}	111.19

Chebyshev polynomials is 55.77 dB and 54.95 dB in Types I and II for N = 32 and 31, respectively. While it is 89.57 dB and 87.51 dB for N = 64 and 63. The average stopband error obtained with the shifted Chebyshev polynomials for N = 32 and 31 is 4.89×10^{-8} and 5.12×10^{-8}, respectively in Types I and II. While for N = 64 and 63, it is 9.956×10^{-11}, and 3.64×10^{-10}. The average passband error obtained is 5.15×10^{-7} and 6.88×10^{-7} in Types I and II for N = 32 and 31. While for N = 64 and 63, it is 1.43×10^{-9}, and 4.41×10^{-9}. In case of Bernstein polynomials, average A_S obtained is 55.77 dB and 54.95 dB in Types I and II for the respective N = 32 and 31 and, it is 87.88 dB and 86.61 dB for N = 64 and 63. The average errors in passband and stopband are reduced to 5.15×10^{-7} and 4.89×10^{-8} with Bernstein polynomials when N = 32, while these are 6.88×10^{-7} and 5.12×10^{-8} in N = 31.

Average A_S obtained with the shifted Legendre polynomials is 55.77 dB and 54.95 dB in Types I and II for N = 32 and 31, respectively. While it is 89.62 dB and 87.86 dB for N = 64 and 63. The average stopband error obtained with the shifted Legendre

Table 3
Comparison of the proposed method with earlier published results.

Type of algorithm	Phi (ϕ)	Pass band error (e_p)	Stop band error (e_s)	Stopband attenuation (A_s) dB
Proposed method shifted Chebyshev polynomials	0	2.98×10^{-9}	2.52×10^{-9}	77.42
	0.3	2.23×10^{-10}	5.44×10^{-11}	78.03
	0.6	3.94×10^{-10}	1.40×10^{-11}	92.79
	0.9	4.96×10^{-9}	6.48×10^{-12}	91.98
Proposed method Bernstein polynomials	0	7.42×10^{-9}	2.77×10^{-9}	78.06
	0.3	2.10×10^{-10}	4.70×10^{-11}	87.33
	0.6	5.14×10^{-10}	4.47×10^{-11}	88.03
	0.9	5.10×10^{-9}	1.99×10^{-11}	86.43
Proposed method shifted Legendre polynomials	0	2.98×10^{-9}	2.52×10^{-9}	77.46
	0.3	2.23×10^{-10}	5.22×10^{-11}	78.05
	0.6	3.94×10^{-10}	1.40×10^{-11}	92.08
	0.9	4.96×10^{-9}	6.49×10^{-12}	92.01
Algorithm in [6]	0	1.79×10^{-7}	3.24×10^{-8}	57.04
	0.3	2.00×10^{-8}	4.52×10^{-10}	72.36
	0.6	4.70×10^{-8}	1.45×10^{-10}	89.12
	0.9	7.99×10^{-8}	2.18×10^{-10}	76.98

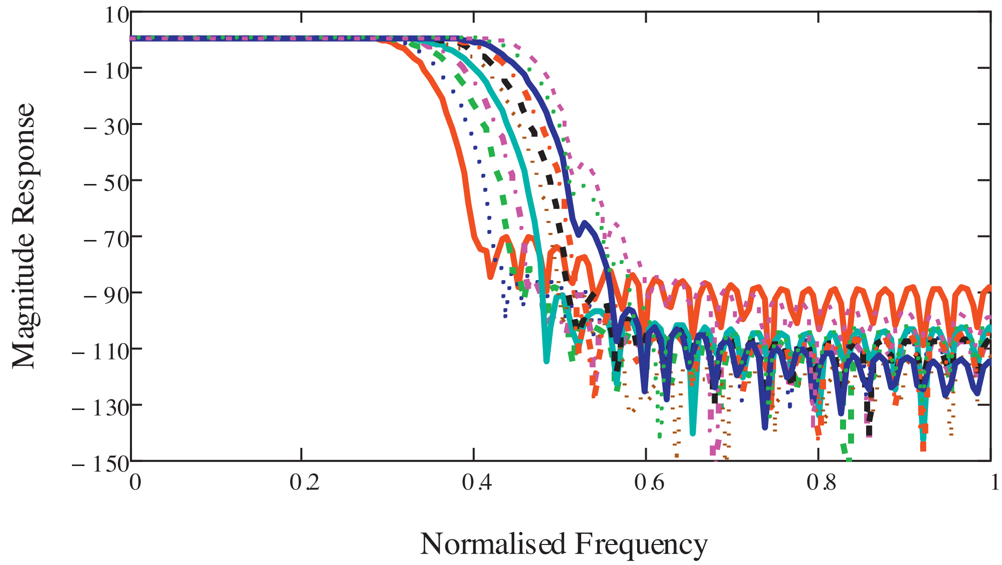

Fig. 10. Tunable bandwidth FIR filter designed using the proposed method with shifted Legendre polynomials, $N = 64$: amplitude response of a low pass filter Type I in dB.

polynomials for $N = 32$ and 31 is 4.89×10^{-8} and 5.12×10^{-8} in Types I and II. While for $N = 64$ and 63, it is 9.95×10^{-11}, and 3.68×10^{-10}. The average passband error obtained is 5.15×10^{-7} and 6.88×10^{-7} in Types I and II, respectively for $N = 32$ and 31. While for $N = 64$ and 63, it is 1.43×10^{-9}, and 4.47×10^{-9}. Therefore, this technique can be effectively utilized for the design of variable bandwidth filters.

Similar to Chebyshev polynomials, the multiplication property is also applicable for shifted Chebyshev polynomials, and defined as

$$2T_k(x)T_l(x) = T_{k+l}(x) + T_{|k-l|}(x) \tag{48}$$

$$2T_k(2x-1)T_l(2x-1) = T_{k+l}(2x-1) + T_{|k-l|}(2x-1), \quad \text{for } 0 \le k \le L,\ 0 \le l \le L. \tag{49}$$

Due to multiplication property, the computational complexity is significantly reduced when shifted Chebyshev polynomial is used as tuning parameter as all elements need not to be computed. Among these polynomials, shifted Chebyshev polynomial provides least computational complexity while Bernstein polynomials provide maximum computational complexity.

5.3. Comparison of different algorithms

Results of the comparative studies given in Table 3 clearly show superiority of the proposed methodology over other exiting algorithms [6]. For this, tunable bandwidth FIR filters with Type I with same design specifications have been designed and compared. It can be seen that performance of the proposed method are significantly improved. The average errors in passband and stopband region obtained with the proposed method is 1.432×10^{-9} and 956×10^{-11} respectively in case of shifted Chebyshev polynomials, 1.706×10^{-9} and 1.159×10^{-11} in case of Bernstein polynomials and 1.432×10^{-9} and 958×10^{-11} in case of shifted Legendre polynomials. While in other algorithms [6], it is 4.178×10^{-8} and 1.172×10^{-9}. The average stopband attenuation obtained in [6] is 79.53 dB, while with the proposed method; it is 89.57, 87.88, and 89.62, in respective polynomials. Similar behavior is obtained with the shifted Legendre polynomials.

A comparative study of the performance of the proposed method with algorithm given in [6] is also graphically shown in Fig. 11. In this figure, the variation of fidelity parameters are shown for the proposed method and the algorithm given in [6] for different values of $\phi \in [0, 1]$. Here, '$C(\phi)$', '$B(\phi)$', and '$L(\phi)$' denote the stopband attenuation when the shifted Chebyshev polynomials, Bernstein polynomials, and the shifted Legendre polynomials respectively are used as tuning parameter. And '$S(\phi)$' stands A_S for the algorithm given in [6]. The constants 'ep$C(\phi)$', 'ep$B(\phi)$' and 'ep$L(\phi)$' represent the passband error when these polynomials are used as tuning parameter. 'ep$S(\phi)$' is the passband error for the algorithm given in [6]. Similarly, 'es$C(\phi)$', 'es$B(\phi)$' and 'es$L(\phi)$' stand for

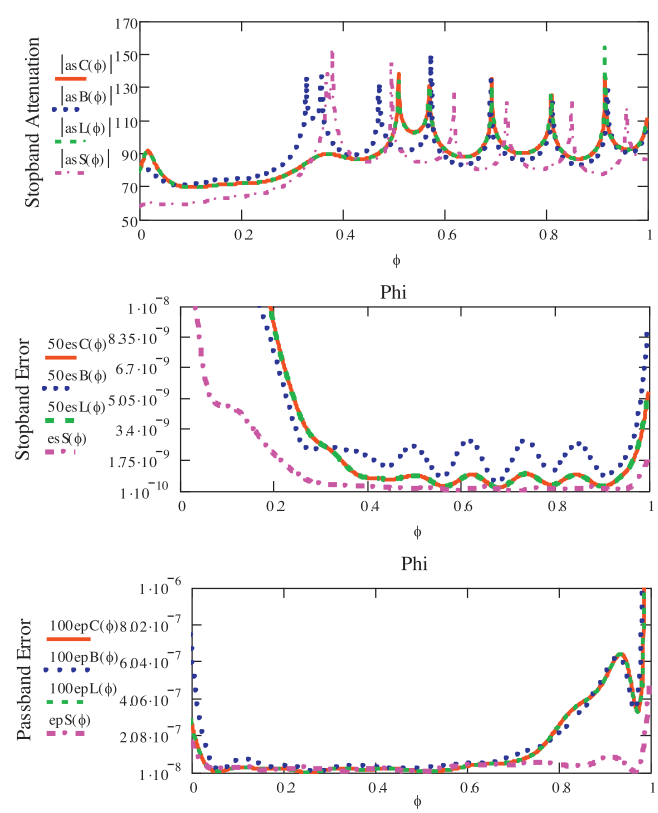

Fig. 11. Variation of fidelity parameters obtained with the proposed method and algorithm in [6] for different values of $\phi \in [0, 1]$.

the stopband error when the shifted Chebyshev polynomials, Bernstein polynomials and the shifted Legendre polynomials are used as tuning parameter. 'es$S(\phi)$' symbolizes the stopband error for the algorithm given in [6]. As it is evident from this figure, the proposed method gives better performance for the higher filter taps as compared to the algorithm given in [6]. For Type I: $N=32$, the worst case stopband attenuation in algorithms [23,33] is 42.885 dB and 43.03 dB, respectively, whereas in the proposed method it is 50.36 dB (approximately for all the three polynomials). For Type II: $N=31$, the worst case stopband attenuation is 46.1 dB in algorithm [26], while it is 49.71 dB in the presented method approximately for all the three polynomials. Therefore, the proposed method can be successfully and efficiently exploited for the design of variable bandwidth filter for various applications. The proposed method has more computational complexity as number of variables to be optimized is more and has complex objective function.

6. Conclusion

A new method based on different polynomials such as shifted Chebyshev polynomials, Bernstein polynomials and the shifted Legendre polynomials is presented for the design of variable bandwidth filters. These polynomials are used as tuning parameter, which controls the bandwidth. Simulation results demonstrate the variable bandwidth characteristics of the designed filter. Good fidelity parameters stopband attenuation, stopband energy and error in passband are obtained by the proposed method. In addition, the proposed method also yields improved performance for tunable bandwidth filters with larger filter taps as compared to earlier published results.

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