LOW-BETA STOCKS IN THE BRAZILIAN MARKET

Abstract

This paper aims to study the presence in the Brazilian stock market of the low-risk stocks anomaly, originally reported by Black, Jensen, and Scholes (1972), in which stocks with low beta coefficients present higher results than those of high-beta stocks, contrary to the CAPM predictions, under which the return is a direct and linear function of the investment's systematic risk. For this purpose, this study is based on the daily prices (adjusted for dividends) of stocks listed on BM&FBOVESPA for a period of 18 years (1995-2012), and consisted of several portfolio studies Evidence shows that low-beta stocks' portfolios have higher returns than high-beta stocks' portfolios, in about eighty percent of the cases, contrary to predictions of the CAPM. There was also a reversal of the anomalous behavior of the portfolios during the economic crises started in 1999, 2003 and 2007. Statistical significance of the results is rejected through t-tests, but accepted through Kolmogorov Smirnov tests.

Keywords – Asset Pricing – Efficient Markets – Anomaly – Beta Coefficient

1. INTRODUCTION

This paper investigates the existence of the Low-beta stocks anomaly (LBSA) in the Brazilian Stock Market. Several researchers report such effect throughout world's stock markets. This anomaly constitutes an important challenge to the assumptions underlying the modern financial theory, directly opposing the Market Risk/Reward Theorem (MRRT) posed by Sharpe (2008), according to which only market risk is rewarded with higher expected return.

The MRRT is one of the underlying principles that resulted in the proposition of the Capital Asset Pricing Model, presented by Sharpe (1964) and Lintner (1965). The CAPM is a model of economic that is broadly utilized both in academia and in capital markets (Levy, 2012). Through this model, academics and practitioners can estimate the cost of equity of companies, businesses and projects, consequently being able to estimate discounted cash flows that are basis for an asset's intrinsic value (Damodaran (2012).

Markowitz (2008) states that the linear relation between an asset's expected returns and its market beta is usually interpreted so that the investors are compensated for taking non-diversifiable risks. If that interpretation holds, the riskier the assets are in relation to a market portfolio, the higher the market will discount its expected cash flows. If the expected cash flows and other conditions that were relevant for asset pricing were the same for two given assets, a market investor would rationally expect that the one with higher beta would present higher returns.

Black, Jensen and Scholes (1972) and Miller and Scholes (1972) found abnormally higher returns arising from low systematic risk stocks, being this systematic risk expressed in terms of an asset's beta coefficient with the market. Recently, this anomaly was found in most of worlds markets by authors such as Frazzini and Pedersen (2011), Baker and Haugen (2012), and Baker, Bradley and Wurgler (2011). These former authors believe that the LBSA is a very strong

candidate for the post of biggest financial anomaly on capital markets, allowing investors to bet on a market inefficiency and derive abnormal returns based on the assets exposure to market risk.

We investigated the anomaly in the Brazilian stock market in a broad period (1995 to 2012), building stocks' portfolios based on the stocks' beta coefficients, considering stocks traded on BM&FBOVESPA. During the analyzed period, Brazil faced very distinct periods, experiencing the control of high inflation, monetary and Exchange rate crisis and a period of higher economic stability started on 2002. Due to that, we analyzed the anomaly's behavior through various economic crises and through the organic growth of the Brazilian Stock Market.

We found that during the analyzed period, an investor that exploited in any given day the strategy of buying low beta portfolios achieved higher abnormal returns than those obtained by investors that bet on high beta portfolios for most of the 3.454 days in which portfolio selection was possible. Our results were similar whether we restrict our sample to the most liquid stocks or consider stocks that are negotiated sparsely (only on fifty per cent of the 720 days prior to portfolio selection).

The remainder of this paper is structured as follows: section two presents a review of the relevant literature on the CAPM, market efficiency and the LBSA; section three describes thoroughly the dataset and the methodology applied in our tests; in section four, we present the results and discuss their implications; finally section five presents our conclusions.

2. RELEVANT LITERATURE REVIEW

The Low-beta Stocks Anomaly (LBSA) contradicts the expectations that arise from the Efficient Markets Hypothesis (EMH) and the CAPM. In this section, we present an overview of the literature related to EMH and its interrelation with the CAPM (as one of the possible models that must be utilized in order to perform tests of market efficiency). This section also presents a review of the relevant literature on the CAPM model and on the LBSA, drawing up a framework for the definition of the methodology that we applied in this study.

2.1 EFFICIENT MARKETS HYPOTHESIS

Shleifer (2000) argues that the efficient markets hypothesis (EMH) has been the central proposition in finance for over thirty years. Fama (1970) presented the initial formulation of the hypothesis, according to which an efficient market is one in which prices fully reflect available information.

This definition, in Fama's (1970) opinion, is so general that it does not present any implications directly suitable for empirical tests. These tests must rely on a more detailed specification of what is meant by fully reflecting available information. In the tests of market's efficiency weak form, the only information considered is the historical stock prices. In the literature review reported by Fama in 1970, empirical evidences were favorable to the weak form of the EMH, with insufficient evidence in order to declare markets inefficient.

According to Fama (1970), EMH tests of semi-strong form are those in which we verify if stock prices continuously adjust to reflect all publicly available information (earnings calls, stock splits etc.), meanwhile strong-form tests of EMH are those considering if some investor groups have monopolistic

access to certain information relevant to the pricing of shares. At the time, Fama (1970) reported that there were no substantial evidence against the EMH in its weak and semi-strong forms, and only limited evidence against its strong for.

Jensen (1978) stated that there was no other proposition in terms of economics for which there was as much solid favorable evidence as was the case of the EMH. Two decades later, updating the review of market efficiency tests, Fama (1991) concluded that the category of weak form EMH tests should include a more general concept of predictability of returns. These tests include the ability to predict future returns based on stocks past characteristics. Among these past features, one can include the beta coefficient of a given stock.

According to Fama (1970, 1991), we have the following forms in which the EMH can be tested (as shown in Table 1). According to Shleifer (2000), the main consequence of a quick and accurate price reaction to new information is that past information is of no use to obtain significant returns. Schwert (2002) reports evidence that the anomalies to the EMH reported in the literature since Fama's (1970) initial formulation of the EMH, such as the size effect, value effect, end-of-the-week effect etc., either weakened or disappeared after the publication of the papers in which the anomalies were identified. Shleifer plausibly infers that the activities of marginal investors that implement strategies for taking advantage of anomalous behavior of markets can result in the disappearance of these behaviors , making inefficient markets efficient.

Table 1 – EMH Tests

Classic Tests (1970)	Updated Tests (1991)
Weak form efficiency: How well do past returns predict future returns?	Predictability of returns tests
Semi-strong form efficiency: How quickly stock prices reflect the information publicly available?	Event study
Strong form efficiency: Investors have inside information, which is not fully reflected in market prices?	Insider's tests

Source: Adapted by the authors from Fama (1991)

This inference emphasize the importance of arbitrageurs (market agents who perform arbitraging operations). Shleifer (2000) stated that even if the feelings of non-rational investors are correlated, arbitrageurs must take the other side of the unsophisticated demand, bringing stock prices back to equilibrium in its intrinsic values. Thus , the strongest argument in favor of market efficiency depends on the effectiveness of arbitration .

The CAPM is an important tool to test the EMH, since according to Fama (1991), one can only test whether information is properly reflected in assets' market prices in the context of a pricing model that defines the meaning of "properly". As a result, Fama (1991) believes that when we find evidence of anomalies in return's behavior, a matter of ambiguous nature is faced, given that the anomaly may be due to inefficiencies of the market itself , or an inapropriatte market equilibrium model.

In an efficient market , market prices are unbiased estimates of the real value of investments, not being necessary that asset's market values are equal to

their actual values at all times, just that the differences are not biased (Damodaran, 2012). In this sense, market prices may be higher or lower than assets' actual prices, provided that such deviations are random, resulting in an equal chance that assets are under or overvalued at any time, and the deviations are not correlated with any observable variable. As a result of these random deviations , no group of investors could rely on any strategy to find undervalued or overvalued stocks consistently over time .

There are several studies that indicate market inefficiencies. In Table 2, we present some of the main criticisms of the EMH, according to a survey conducted by Shleifer (2000).

Famá, Cioffi and Coelho (2008) presented an extensive list of EMH testing on the Brazilian market, considering the CAPM model, including those presented in Table 3. According to the authors, the evidence is not sufficient for refuting the binomial CAPM and EMH.

Tonidandel and Decourt (2012) performed strong-form tests, finding evidence that insider traders earn abnormal returns in Brazil, based on the analysis of a sample of 38,141 transactions performed by insider traders of 167 companies between 2006 and 2011.

Table 2 – Main criticism to the EMH

Authors	Results
Shiller (1981)	Market prices Stocks' were more volatile than it would be justified by a simple model of discounting future dividends.
De Bondt and Thaler (1985)	Possibility of obtaining abnormal returns by betting on strong losing stocks, called extreme losers who had negative returns in the recent past
Jegadeesh and Titman (1993)	Momentum effect, when movements in stock prices over the last six or twelve months strongly suggest the trend for future moves in the same direction
De Bondt and Thaler (1987), Fama and French (1992), Lakonishok et al. (1994)	Stocks with a higher market to book value ratio have significantly lower returns than firms with lower ratio.
Roll (1984, 1988)	Analysis of Stocks' price movements together with the analysis of news about the companies showed that factors others than only the news seemed to result in movements in stock prices.

Source: Adapted by the authors from Shleifer (2000).

Table 3 –EMH tests in Brazil

Authors	Results
Costa Jr. and Ceretta (2000)	Day of the week effect - The authors found no significant results for Brazil, Argentina, Chile and Mexico relating to the trend of negative results occurring on Mondays and positive results occurring on Fridays.
Famá et al. (2007b)	January Effect - the analysis of the Ibovespa index in the period from 1969 to 2006 showed no evidence of the January effect anomaly or any other month of the year.
Famá, Mussa and Santos (2007)	Size effect - the analysis of a 4-factor model, with the consequent exclusion of the size factor, resulted in a reduction in the explanatory power of the model in Brazil from 1995 to 2006.
Correia and Amaral (2002), Novis and Saito (2003); and Bruni et al. (2003)	Dividends effect - the authors argued that the systematic risk and dividend yield can explain the profitability of the Brazilian market.
Vieira and Procianoy (1998)	Dividends effect - it was concluded that the abnormal return of stocks with higher dividend yield was not significant.
Costa Jr., Leal and Lemgruber (2000)	Overreaction – in the long-term overreaction was detected in North America and Brazil, the latter with a more pronounced magnitude, but most of the analyzed Brazilian companies were small companies when compared to U.S. companies, supporting idea that the effect is more significant in small businesses.
Famá, Oda and Yoshinaga (2003)	Overreaction - the contrarian strategy had significantly higher returns for periods up to a quarter. When measuring results in semesters, however, no abnormal returns were found.
Rouwenhorst (1999), Famá, Mussa and Santos (2007)	Momentum Effect – authors found the momentum effect respectively in emerging markets (excluding Brazil, where the strategy showed no significant results) and in the Brazilian market. Three of the 16 tested strategies for Famá , Mussa and Santos had significant results.
Leal and Varanda (2000)	Price barriers and technical analysis - the authors concluded that the strategy of mobile-average intersection may indicate trends in the short term, achieving higher results than those of buy and hold strategies in Brazil. One can use this strategy to rebalance portfolios, since it indicates the direction of the market. The results, however, have no statistical significance.

Table 3 –EMH tests in Brazil

Authors	Results
Minardi (2002, 2004)	Price barriers and technical analysis - the 2002 study showed evidence against the random walk hypothesis and suggested that technical chartists cannot be refuted emphatically. By analyzing the Brazilian market, the 2004 study concluded that there is some informational content in past prices.
Famá e Trovão (2007)	Announcement effect - the authors analyzed the impact of mergers and acquisitions in the banking sector in the period 1998-2005. Their results did not reject the semi-strong form EMH.

Source: Adapted by the authors from Famá, Cioffi e Coelho (2008)

2.2 CAPM

The Capital Asset Pricing Model, attributed to Sharpe (1964) and Lintner (1965), allows the pricing of assets as a function of the return required by investors, being premised on the fundamental relationship between systematic risk and asset returns. This model is widespread both in academia and in the professional practice of managers, investors, appraisers and other economic agents.

The relationship between the asset' systematic risk and its expected return enables the determination of a rate of return that can be taken as benchmark for the assessment of investments, acting as a mechanism for price adjustment on expected cash flows. Theoretically, the more widespread the use of the model by the market players, the greater its explanatory power regarding the adjustment of asset prices as a function of their returns and perceived risk by these agents.

For any investor operating under the CAPM, the relevant risk of any asset is the risk added by this asset to the investor's portfolio, being this risk measured in terms of a beta coefficient, obtained through the regression of stock returns in relation to a market portfolio's returns (Damodaran, 2004).

In order to estimate the yield that will be required by a diversified marginal investor on a given security, its beta coefficient is multiplied by the overall market premium, being the result added to the expected return of a theoretical risk free asset, which is the required return on an investment with a fixed return (therefore riskless).

According to Sharpe (1964), the derivation of the equilibrium conditions of the capital markets depends on the assumption of a general pure interest rate, with which all investors able to borrow or lend on equal terms and on the hypothesis of homogeneity of investor expectations, under which it is assumed that investors agree on the prospects of various investments - their expected values, standard deviations and correlation coefficients.

Under the mechanism proposed by Sharpe, rational investors will only be interested in buying assets that are efficient in terms of risk and return, reducing the demand for securities offering worse conditions. This move will reduce the price of riskier assets, making their expected return (as a function of price) to increase, which will lead these securities to the condition of efficiency. The continuous movements of investors, adjusting asset prices to perceived risk, will dynamically lead this asset prices to an equilibrium state.

Sharpe (1964) emphasizes that there may be various combinations of assets that are mean-variance efficient. Hence, his theory does not imply that all investors would allocate their financial resources in the same portfolio.

Considering Sharpe's theory (1964), prices will adjust until there is a linear relationship to the magnitude of the asset's responsiveness to economic activities, being that risk non-diversifiable, and measured through the regression of the asset's and the efficient Market portfolio's returns.

As later stated by Sharpe (2007), while explaining the Market Risk/Reward Theorem, the market risk that is relevant for asset pricing in equilibrium conditions is defined as an investment's Market beta, following that:

$$E(R_i) = r + \beta_i^{f(R_m)}(E(R_M) - r)$$

where:
$E(R_i)$: Expected return on asset i
r: Risk-free rate
$E(R_M)$: Expected return of Market portfolio
β: Beta of asset i

As one can infer from the equation above, the CAPM is a model of expectations. Therefore, its inputs must be of ex ante parameters, denoting the expectations of rational diversified marginal investors. The question about the level of stationarity of betas is highly relevant for the practical applications of this model, since stable betas are supposed to provide a greater reliability on the estimation of future betas, since all one should do is to look into past data.

Since its proposition, and due to its importance for financial theory development, the CAPM has been the subject of several attempts of refutation. Fama and French (2004) present a summary of the various empirical approaches used in attempts to confirm/reject the CAPM, mostly with negative results in regard of the explanatory power of stocks betas with respect to stock returns, as presented in Table 4.

Table 4 – Main empirical tests of the CAPM

Authors	Results
Banz (1984)	Identifies that smaller companies have higher risk adjusted returns, on average, than larger firms.
Basu (1977)	When stocks are ranked in terms of their Earnings to Price ratios, the future returns of firms with higher ratios are larger than predicted by the CAPM
Bhandari (1988)	Companies with higher leverage have returns that are higher than expected on the basis of their market betas
Statman (1980), Rosenberg, Reid and Lanstein (1985)	Companies with high book-to-market equity ratios have higher returns than predicted by their betas
Fama and French (1992)	Through cross-section regressions, the authors conclude that factors such as size and profits-to-price ratio, debt-to-equity ratio and book to market ratio substantially increase the explanatory power of market betas

Source: Adapted by the authors from Fama and French (2004)

Despite the empirical criticism on the CAPM, Roll (1977) argued that the empirical rejection of the CAPM model would only be possible if we were sure about all the assets comprising the market portfolio. In the author's opinion, even a minimal specification error on the market portfolio's proxy can lead to erroneous conclusions, which have crucial importance for performing hypothesis testing.

Levy (2012) believes that the CAPM and its alpha and beta coefficients are still the financial measures most widely used by academic researchers, and are even more heavily adopted investment and finance practitioners. Similarly, a survey conducted by Graham and Harvey (2001) with CFOs of U.S. and Canadian companies indicated that 73.5 % of respondents always or almost always use the CAPM.

Elton et al. (2012) contribute to the possibility of testing the CAPM, highlighting that all of its variables are expressed in terms of future values, so that the relevant beta is the asset's future beta. Moreover, both the return of the market portfolio and the return of the minimum variance zero beta portfolio are future expected returns. As there are no systematic large-scale data on expectations, the authors argue that almost all of the CAPM validity tests have been conducted with the use of ex post variables, raising questions on how to justify testing a model of expectations considering what has already happened.

Levy (2012) emphasized that the CAPM is defined by Sharpe and Lintner in terms of ex ante parameters rather than ex post parameters. Levy (2010) concludes that the CAPM can safely continue to be used in academic research and in practice, since it can not be refuted, and is even more strongly supported experimentally using ex ante parameters. In the author's view, the great practical difficulty lies on the estimation of the ex ante parameters, but this difficulty can not be considered as a disadvantage of the model, since virtually all theoretical models suffer from the same problem.

Ray, Savin and Tiwari (2009) argue that the classical evidences in favor of CAPM's refutation are statistically weaker than suggested by the consensus view. Their opinion derives from the fact that the CAPM was not rejected in most of the periods (1965-2004) in they performed recent robust tests for heteroskedasticity and autocorrelation.

Guimarães and Guimarães (2006) conducted a joint test of market efficiency and the CAPM in the Brazilian market, concluding that there is no evidence of market inefficiency through the analysis of mutual fund's performance, assuming they IBX and Bovespa indexes as proxies of the market portfolio and the CDI rate as a representative of the risk-free asset's return rate.

If we assume the theoretical validity of the CAPM, the existence of low-beta stocks that have risk adjusted returns that are higher than those predicted by the CAPM, and also higher than those results obtained by investing in high-beta stocks is intriguing, as shown by the literature reviewed herein.

One possible explanation for that anomaly would be the temporary inability of marginal diversified investors to explore low-beta stocks anomalous stock returns and bring prices to equilibrium. If such failure were only temporary, with a decrease of the anomaly over time, we would have a strong hint of the market forces that make markets efficient.

We must emphasize, nonetheless, that due to the expectation of randomness of the deviations on stocks market prices and their actual intrinsic values (DAMODARAN, 2012), one can expected that in an efficient market

stocks with lower betas should not be more or less probably underestimated than those stocks with higher betas. In this case, one should not be able to find, under the validity of the EMH, high abnormal returns strongly correlated to low- beta stocks.

2.3 LBSA

According to the results we reported in the previous subsection, there is no definitive evidence refuting the validity of the EMH, both in international markets and in the Brazilian market. The significant adoption of the CAPM in academic and practical context, as reinforced by the literature presented, as well as its usefulness for implementing market efficiency tests, reinforces the importance of the classical Box and Draper's (1987) statement, essentially all models are wrong but some are useful. Due to that, in this subsection we review the literature that addresses specifically the LBSA in world markets.

According to Baker, Bradley and Wurgler (2011), in an efficient market, investors should get above-average returns only by taking above-average risk. Concordantly, risky stocks must have higher returns, on average, than those of safe stocks. In the authors' opinion, this simple empirical proposition has not been supported based on US stocks' history. Baker, Bradley and Wurgler's (2011) paper includes data from January 1968 to December 2008, concluding that either defining risk as stock's volatility or beta coefficient, low-risk stocks' performance was superior to high-risk stocks over the period examined.

For the period between 1931 and 1965, Black (1993) reports that studies conducted by Black, Jensen and Scholes (1972) and Miller and Scholes (1972) showed that low-beta stocks in the United States fared much better than predicted by the CAPM model, while high-beta stocks fared worse than expected. Black (1993) concluded that if the capital market line is really as flat as identified in the existing literature at the time, this fact would imply dramatic investment opportunities for those who use beta as a parameter on asset selection.

Ang et al. (2009), using the Fama and French (1993) three-factor model to measure idiosyncratic volatility (defined as the standard deviation of residues on the regression estimation of the three factors) concluded that the puzzle of why high-idiosyncratic volatility stocks have small returns is a global phenomenon, suggesting that further research investigates whether there are real economic sources of unaccounted risk behind this anomaly.

Frazzini and Pedersen (2011) found empirically that portfolios with high-beta assets have lower alphas and smaller Sharpe ratios than those of low-beta assets' portfolios, both for stocks traded in the U.S. market and in international markets. The authors also found that the security market line is flatter than predicted by the CAPM model in the U.S. market and in eighteen international markets (of the nineteen markets tested). According to the authors, this deviation from the CAPM model can be captured by investors using portfolios with Betting Against Beta Factors – BAB. These portfolios can be constructed through shorting high-beta assets and using leverage in order to invest in low-beta assets.

The theory presented by Frazzini and Pedersen (2011) states that agents with restrictions on marginal activity and leverage seek excess returns by investing more heavily in riskier assets, which eventually reduces the expected return of these assets. Investors with lower restrictions exploit this situation by

decreasing the share of risky assets in their portfolios, or even shorting risky assets.

Haugen and Baker (2012) reported a study including 33 different markets in the 1990-2011 periods, finding that low-risk stocks had superior performance than high-risk stocks in all these markets. Blitz and Van Vliet (2007) also presented empirical evidence of more significant positive abnormal returns (adjusted for risk) in stocks with low volatility, assessing the American, European and Japanese markets. According to these authors, to explore the effect of volatility in practical situations, investors should include low-risk stocks as a specific asset class, to be considered as such in the strategic asset allocation process.

Blitz et al. (2012) analyzed the results of the LBSA in emerging markets, finding that that the empirical relationship between risk and return is negative in emerging markets, and more heavily when volatility rather than the beta is taken as a measure of risk.

Regarding the specific analysis of the Brazilian market, Rostagno, Soares and Soares (2008) conducted a portfolio study based on stocks of 70 companies traded between 1995 and 2002. Portfolios were built based on historical returns, and stocks were divided in five quintiles. After the portfolio's returns were measured, the authors investigated the differentiation between quintiles of five groups of fundamental variables. For that sample, the authors identified that the portfolios that obtained higher returns showed a less-than-average systematic risk. Nevertheless, their study was not conducted to specifically analyze the LSBA, and their portfolios were not built on the basis of stocks' betas. Therefore, their results are not directly comparable to our results.

As noted earlier, the literature provides important evidence about the advantages of investing against the systematic risk of stocks (represented by the beta coefficient), both in terms of superiority of returns and in terms of persistence of the negative effect. A summary of the literature is given in Table 5.

Table 5 – Summary of LBSA empirical literature

(continues)

Authors	Periods and markets	Summarized conclusions
Baker, Bradley and Wurgler (2011)	January 1968 to December 2008 - United States	Defining the risk as a function of stocks' volatility or beta, and considering the sample including all stocks or only the 1000 largest stocks in terms of market capitalization, stocks of low risk showed superior performance than high risk stocks over the period.
Black, Jensen, and Scholes (1972)	1931 to 1965 – United States	Low beta stocks in the United States showed better results than those predicted by CAPM, while high beta stocks had worse results.

Table 5 – Summary of LBSA empirical literature

Authors	Periods and markets	Summarized conclusions
Ang et al. (2009)	23 markets of the MSCI developed country Index.1980 to 2003, except Finland, Greece, New Zealand, Portugal, Spain and Sweden, whose quotations are available from the middle of the 80s.	In the countries investigated, stocks that previously had high idiosyncratic volatility tend to have significantly lower returns in comparison to stocks with opposite historic (low volatility).
Frazzini and Pedersen (2011)	50,826 stocks from 20 countries. U.S. data from January 1926 to December 2009. International data considered 19 markets in the universe of the MSCI developed between January 1984 and December 2009.	Portfolios of assets with high and alpha and beta have lower Sharpe ratios than portfolios of stocks with low betas, both for stocks traded in the U.S. market and in international markets.
Baker and Haugen (2012)	From 1990 to 2011. 33 different markets (including Brazil).	Low-risk stocks present better performance than high-risk stocks in all markets analyzed.
Blitz and Van Vliet (2007)	From 1986 to 2006. FTSE World Developed Index (global stock indexes, considering developed countries, on average composed of 2,000 large companies by market capitalization)	Positive abnormal returns (adjusted for risk) more significant in stocks with low volatility when assessing the American, European and Japanese markets.
Blitz et al. (2012)	December 1988 to December 2010. All stocks included in the S & P / IFC Investable Emerging Markets Index (index containing only stocks considered accessible and sufficiently liquid to international investors, including Brazilian stocks.	The empirical relationship between risk and return is negative in emerging capital markets, and more heavily when volatility is the relevant measure of risk, rather than the beta coefficient.

Source: Prepared by the authors from studies presented in the literature review presented in this section.

3. APPLIED METHODOLOGY

The research presented in this paper considered the performance analysis of a betting against beta strategy while selecting assets in order to build investment portfolios. Based on the relevant literature, we expected the strategy to achieve superior returns from investing on low beta stock's portfolios.

In our sample period, between January 1998 and December 2011, there were 3,454 trading days on the São Paulo Stock Exchange (currently named BM&FBOVESPA). In each of these days, the stocks that met the selection criteria mentioned in subsection 3.3.1 were ranked in terms of their Beta coefficients. The stocks were then divided into quintiles, the first quintile corresponding to that formed by low-beta stocks, and the last quintile corresponding to those higher beta stocks.

We conducted our studies considering five alternative criteria of trading presence, and the daily building of five portfolios resulted in 86 350 portfolios for 3454 days of trading. Portfolios varied in size depending on the frequency restriction adopted in each test. Table 6, below shows the distribution of these portfolios per presence criteria:

This section describes the method utilized in order to achieve the research objectives. The procedures adopted for sample definition, collection and processing of data, definition of systematic tests to be applied, and finally the data analyses are presented in the following sub-items.

Table 6 – Number of portfolios build in the studies performed.

Presence Criterion	Days	Number of portfolios
50%	3.454	17.270
60%	3.454	17.270
70%	3.454	17.270
80%	3.454	17.270
90%	3.454	17.270
Total of portfolios		86.350

Source: prepared by the authors.

3.1 SAMPLE AND DATA COLLECTION

To estimate stock's beta coefficients on the sample stocks, and consequently calculate the returns of the portfolios built in our tests, we obtained daily prices adjusted for dividends opting for the daily closing price, in the period between January 1995 and December 2012.

Portfolio studies may present different results depending on the day of the week (day of the week effect) or month of the year (calendar effect) chosen for starting asset selection. Thus, in order to achieve a more comprehensive analysis of the frequency of the anomaly in the Brazilian market, we chose to perform daily portfolio buildings, based on historical stock's data. For each sampling day, we measured the results obtained by the portfolios in a period of one year under the buy-and-hold mode.

3.2 ESTIMATING BETA COEFFICIENTS

The original work of Sharpe (1964) does not specifically address a methodology for the estimation of ex ante betas. In this paper, we chose to estimate stock's historical betas for each portfolio-building day, on a rolling basis, rather than attributing based on estimation a single beta for each stock, valid for the whole period. Therefore, stocks that on a given day were selected for portfolio Q1 may be part of that portfolio or any if the other portfolios (Q2, Q3, Q4 and Q5) on the next portfolio building day.

Following Daves et al (2000), daily returns were the basis of our beta estimations, because according to the authors, in periods of up to three years of data, daily intervals produce smaller standard errors and higher accuracy in estimating the coefficients. The results found by Daves et al (2000) show that an estimation period of three years captures the greatest reduction in standard error of estimation, considering estimation periods between one and eight years, and less than 50 % of the studied companies experienced significant changes in beta in a period of three years. Consequently, we decided in this paper to perform the daily beta estimates over a trailing period of approximately three years (750 days).

We opt for estimating betas based on logarithmic returns, considering the rationale presented by Fama (1965), performing bivariate linear regression according to the method of ordinary least squares (OLS) for each period of 750 days prior to each date of portfolio building. Our explanatory variable was the logarithmic returns of a theoretical market portfolio. We consider the IBOVESPA [1] Index to be representative of the Brazilian market portfolio.

The estimates of the beta coefficients were limited to the estimation based on periods of 750 trading days, using bivariate regression in which the dependent variable (stock returns) relates to a single explanatory variable (return of the market portfolio), following the definition of Gujarati and Porter (2011, p. 59), depending on the method of ordinary least squares (Gujarati and Porter, 2011, p. 78-83)

During the sampling period, many shares were not traded on all days of the period. According to Dimson (1979), estimating the beta coefficients of companies that are not traded every day results in coefficients biased downwards. Based on that conclusion, we opted to perform our tests both using the Repeat Last Quotation (RLQ) and the Trade To Trade (TT) methods for Beta estimation. We opt to report results within the RLQ method, but these results are very similar to those we achieve under the TT method.

3.3 PORTFOLIO BUILDING AND MEASUREMENT OF RESULTS

In each of the 3454 days in which trading took place in the São Paulo Stock Exchange since January 1998, we formed five investment portfolios considering the grouping of stocks into quintiles ordered by the size of their beta coefficients. Therefore, in each presence criterion, this paper monitors the buy-and-hold results (following Lyon, Barber, and Tsai, 1999) obtained by the 17,270 portfolios in a period of one year, totaling 86 350 portfolios over 14 years, as identified in Table 5.

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[1] (<http://www.bmfbovespa.com.br/indices/ResumoIndice.aspx?Indice=Ibovespa&Idioma=pt-br>)

At the end of that buy-and-hold year, the daily average returns of each portfolio and their standard deviations were analyzed and compared.
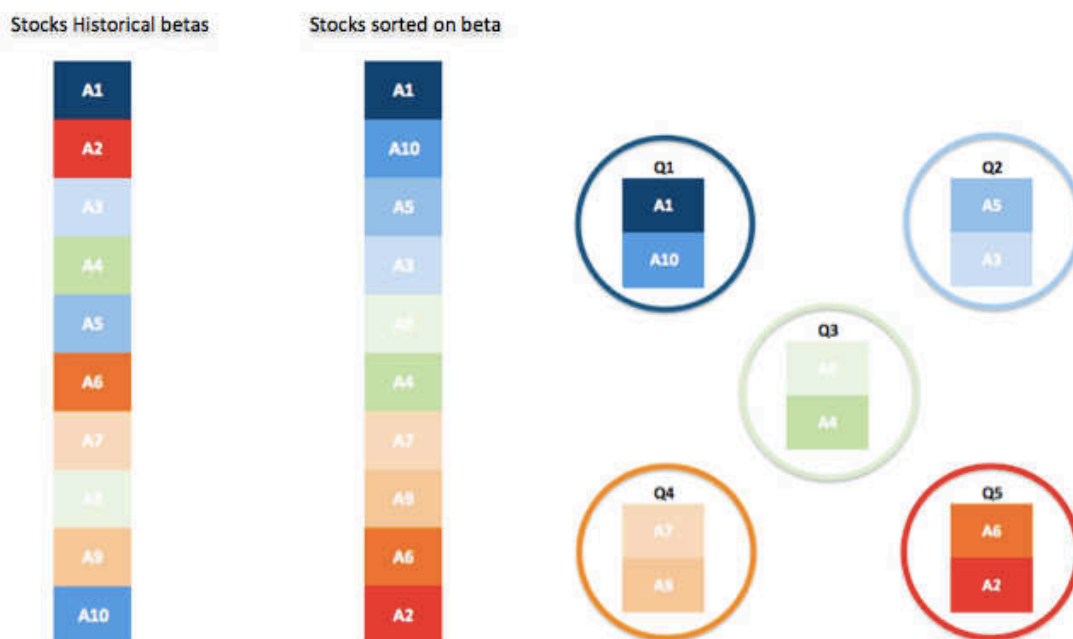
In each portfolio-building day, our sample was comprised of those stocks that were traded on that day, and therefore presented an effective quote at that date. To allow an analysis of the sensitivity of our results to the stocks frequency of trading, we opt to consider five alternative criteria of minimal presence of the shares in the last 750 days of trading, in order to allow the estimation of the beta coefficients with a reasonable number of observations. These criteria correspond to the presence of the stock, verified when the stock was traded in 50%, 60 %, 70%, 80% or 90% of the 750 days prior to portfolio building day.

For the stocks that qualified under the criteria above, we preceded the estimation of their beta coefficients against the IBOVESPA index, based on the logarithmic returns of the 750 days prior to portfolio building day. Subsequently, we proceeded to the ordering of stocks, from the smallest to the largest beta coefficient. Assets were then divided into quintiles, from quintile number one (Portfolio Q1) composed of firms whose betas were among the 20% smaller, up to the quintile number five (Portfolio Q5) comprising companies whose betas were among the largest 20%, as explained in Figure 1.

Figure 1 explains the process of portfolio building for a hypothetical sample containing 10 selectable stocks, which are ranked in terms of their beta coefficient (the darker the shade of blue, the lower the beta coefficient, the stronger the shade of red the higher the beta, and green shades representing betas near the sample mean).

The portfolios were built considering an R$ 1,000 investment, divided equally among the assets composing each quintile portfolio. Thus, the weight of each stock in the initial portfolios was the same in order to avoid a greater weight to stocks of higher nominal value, which could be a potential confounding factor for the desired analysis.

Figure 1 – Sorting stocks by size of their beta coefficients, and selecting stocks for each portfolio (five quintiles) according to beta's size.

Source: Prepared by the authors.

Assuming the premise of the stocks divisibility, the amount of each stock to be acquired to form the portfolios stems from the following division:

$$Q = \frac{InvestedValue}{VA_t}$$

where:

Q: Quantity of each stock in the portfolio

InvestedValue: R$ 1.000 divided by the quantity of stocks in the portfolio

$VA_{(t)}$: Stock price at the portfolio-building day.

The value of the portfolios in the 250 subsequent days was obtained from the daily unit prices of the stocks multiplied by the number of stocks held in each portfolio. In this case, we chose to keep the last price quote available for stocks that were not traded on any of the result measurement days. Next we performed the calculation of the daily logarithmic returns obtained in each portfolio, the average logarithmic returns and the standard deviation of their returns.

Figure 2 shows the visual representation of the routine portfolio selection adopted for each of the 3,454 portfolio building periods. The *t* dates correspond to 3,454 dates on which the assets were ordered according to their historical betas (750 days prior) and divided into five portfolios (quintiles). The whole process is repeated for each of the five criteria adopted presence (from 50 % to 90 %).

Figure 2 – Portfolio-building days

Source: Prepared by the authors.

Our tests were scripted into the R programming language (R Development Core Team, 2010), allowing us to automatically conduct the estimates of logarithmic returns and the selection of stocks that met the selection criteria of being traded on the portfolio building day and having minimum presence (50%, 60%, 70%, 80% and 90%) in the 750 days prior to the portfolio building day.

In order to test the efficiency of an investment strategy focused on identifying and betting on stocks that presented low beta coefficients (**β)**, we chose to measure the percentage excess returns earned by portfolios Q1 to Q5 on: (a) the returns to market portfolio (Bovespa) and (b) expected returns of Q1 to Q5 portfolios due to a risk free rate added to the market risk premium (market portfolio return less the risk-free rate) multiplied by the beta of the portfolio Qn (n = 1 to 5) upon its building. Formula (2) was adopted to report the percentage difference in the three cases, as follows:

$$R_{ex}(Q_n) = \left(\left(\frac{1+RQ_n}{1+R_{proxy}} \right) - 1 \right) * 100 \qquad (2)$$

Where:

$R_{ex}(Q_n)$ = Excess return of portfolio Q_n (n = 1 to 5)
RQ_n = Actual return of portfolio Q_n
R_{proxy} = Proxy returns (items (a) and (b) of the last paragraph)

3.4 STUDY LIMITATIONS

The main limitations of this study are listed in this subsection. The study's objective was not to identify the reasons that may explain the presence of the anomaly. In this sense, our research is limited to measuring the returns on portfolios of stocks grouped according to the magnitude of their beta coefficients, and therefore verifies whether the Brazilian market returns on investments in stocks are properly described by the MRRT (Sharpe, 2008), so that investors are rewarded for taking risks.

This research was limited to stocks that were traded on the Brazilian stock market during the chosen period, being considered only those stocks that were traded at least in 50% of the prior 750 days (about three years). We analyzed different presence thresholds (50%, 60%, 70%, 80% and 90%) without evaluating the possibility of optimizing the presence range that could maximize or minimize the effect of the LBSA.

We do not intend, therefore, to study the results we would find if we choose different periods of trading intervals for estimating beta coefficients, different methods for estimating betas, different periods for measuring results, as well as alternative criteria of presence or market liquidity.

To address the problem of estimating betas for companies whose stocks were not traded every day, we chose to estimate coefficients by adopting repeat last quotation (RLQ) and trade-to-trade (TT) methods, as shown by Martelanc and Serra (2013).

4.   RESULTS

As a result criteria we adopted for portfolio building, we show in Table 7 the distribution of stocks that were considered in the 3.454 portfolio studies performed for each of the presence thresholds:

Table 7 Description of the distribution of the quantities of 17,270
companies considered in the portfolios studies conducted, broken
down by presence criterion.

Presence	Maximum	Minimum	Median	Average
50%	286	86	146	171
60%	269	81	134	156
70%	252	74	121	140
80%	233	63	113	126
90%	210	53	101	108

Source: Prepared by the authors.

Figure 3 presents the evolution in the number of stocks that were considered in every portfolio-building day, per frequency threshold. There has been a significant increase in the number of stocks available since the end of 2009, as a result of the significant number of IPOs on Bovespa.

In all of the presence thresholds, returns obtained from Q1 portfolios, those formed by stocks with the 20% lowest betas, were higher than those achieved by the Q5 portfolios (80% highest betas). Similarly to the results reported by Baker, Wurgler and Bradley (2011), Haugen and Baker (2012), Blitz, Van Vliet and Pang (2012) and Frazzini and Pedersen (2011), investing in stock portfolios formed by low beta stocks result in returns that are higher than those obtaining from investing in high beta stocks.

Figura3. The evolution in the number of stocks that were considered in every portfolio-building day, per frequency threshold
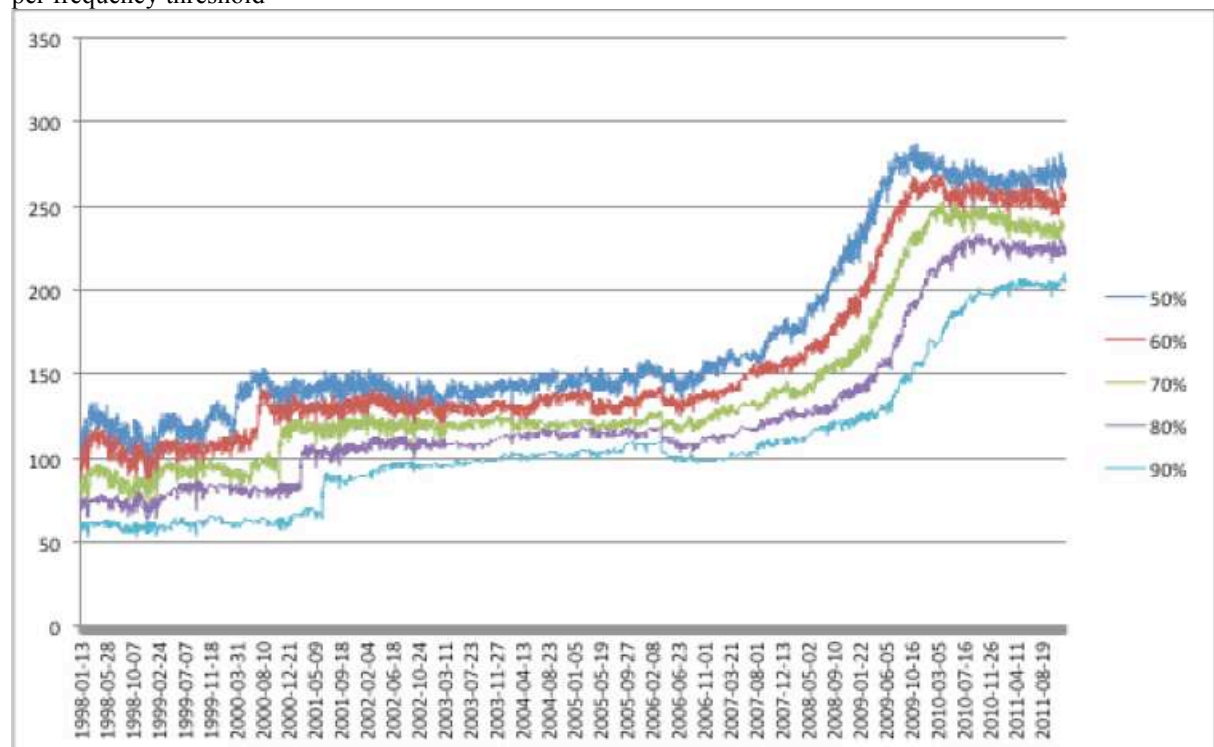
Source: Prepared by the authors.

Correlation studies performed between portfolios Q1 to Q5 historic (750 days prior to portfolio building) and future (250 days following portfolio building) betas,

according to Levy (1971) showed high average correlations, according to Table 8. Based on those high average correlations, we assume that our results do not arise from changes in portfolio's betas during our buy-and hold period.

Figure 4 shows the excess returns obtained by Q1 portfolio over Q5 portfolio, for all of the adopted presence thresholds. While this graphical representation does not provide the detailed analysis of these excess returns, considering its large amount of data points, one can check the similarity between the distributions of excesses over time. The exception in this similarity is observed in the period between 1998 and 1999, in which there is no apparent trend.

Table 8: Summary of the 3453 correlation studies conducted for each presence criterion, in which the portfolios' historical betas (based on 750 days earlier) are compared with these portfolios' future betas (based on the 250 days following the portfolio building day). The table presents the descriptive analysis of correlation coefficients between historical and future betas of the portfolios.

Presence		Maximum	Minimum	Median	Average	Standard Deviation
	50%	0,787918	0,999977	0,986952	0,983371	0,015187
	60%	0,779615	0,999969	0,985146	0,980288	0,019941
	70%	0,747147	0,999890	0,979580	0,971636	0,026680
	80%	0,836888	0,999808	0,977778	0,967795	0,027776
	90%	0,688832	0,999909	0,975474	0,955934	0,048520

Source: Prepared by the authors.

Figure 4. Excess returns obtained by Q1 portfolio over Q5 portfolio, for all of the adopted presence thresholds.
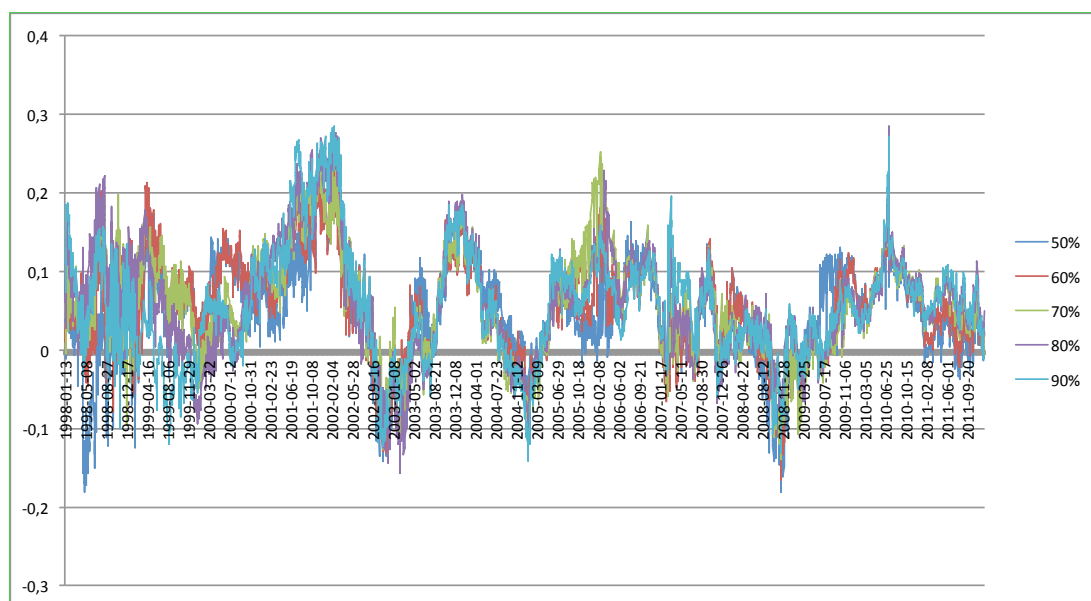
Source: Prepared by the authors.

As shown in Table 9, the Q1 investment portfolio is not the main annual winner in calendar years 1999, 2003, 2004, 2008 and 2009, in which the winners are the Q2 (1999) and Q3 portfolios for the other periods. It is noteworthy that in none of the annual periods that we analyzed the Q5 portfolio, that with the highest beta stocks and therefore the portfolio with higher expected returns higher, was the main annual winner.

Table 9 - Percentage distribution of Winning portfolios in each of the 3454 days of portfolio building. The winning portfolio is the one that showed the highest average daily logarithmic returns in the 250 days subsequent to the portfolio building dates . Presence Criteria of 90%.

Year	Q1	Q2	Q3	Q4	Q5
1998	61,09%	37,66%	0,00%	0,00%	1,26%
1999	0,81%	67,89%	8,54%	22,76%	0,00%
2000	53,23%	2,82%	0,00%	33,47%	10,48%
2001	79,27%	20,73%	0,00%	0,00%	0,00%
2002	59,84%	18,07%	13,25%	0,00%	8,84%
2003	16,80%	2,40%	66,80%	14,00%	0,00%
2004	22,09%	0,40%	35,74%	40,16%	1,61%
2005	61,04%	17,27%	2,81%	7,63%	11,24%
2006	45,12%	40,24%	11,79%	2,85%	0,00%
2007	61,63%	24,08%	2,86%	3,67%	7,76%
2008	22,09%	19,68%	21,29%	14,86%	22,09%
2009	5,69%	0,00%	86,59%	0,00%	7,72%
2010	85,83%	0,00%	10,12%	4,05%	0,00%
2011	81,63%	8,98%	0,00%	9,39%	0,00%
Total	46,79%	18,50%	18,65%	10,97%	5,10%

Source: Prepared by the authors.

Table 10 - Percentage distribution of losing portfolios in each of the 3454 days of portfolio building. The losing portfolio is the one that showed the lowest average daily logarithmic returns in the 250 days subsequent to the portfolio building dates. Presence Criteria of 90%.

Year	Q1	Q2	Q3	Q4	Q5
1998	0,84%	0,00%	1,67%	92,05%	5,44%
1999	57,32%	0,00%	8,94%	6,10%	27,64%
2000	0,00%	6,45%	81,05%	8,47%	4,03%
2001	0,00%	0,00%	26,83%	29,27%	43,90%
2002	25,30%	0,80%	3,61%	40,56%	29,72%
2003	44,40%	0,00%	0,00%	0,00%	55,60%
2004	24,90%	8,84%	0,00%	0,00%	66,27%
2005	23,69%	10,44%	7,23%	0,40%	58,23%
2006	0,00%	0,00%	0,00%	2,03%	97,97%
2007	0,00%	40,41%	35,10%	17,14%	7,35%
2008	10,04%	18,47%	38,55%	9,24%	23,69%
2009	1,22%	53,66%	0,00%	8,94%	36,18%
2010	0,00%	2,02%	0,40%	1,62%	95,95%
2011	0,00%	2,86%	38,78%	0,41%	57,96%
Total	13,49%	10,28%	17,31%	15,26%	43,66%

Source: Prepared by the authors.

With regard to the losing investment portfolios, the Q5 portfolios were the major losers in seven of fourteen years of analysis, as presented in Table 10. The portfolio Q5 was the main loser in 43% of 3454 days analyzed.

In 1999 (Brazilian currency crisis), 2003 (Brazilian liquidity crisis, according to the BNDES, 2009) and 2008/2009 (global financial crisis initiated by the subprime mortgage market in the United States), the Q1 portfolio was not main winner. This coincidence raises questions to be investigated in future research on the behavior of

betas and of the LBSA in times of crisis, considering also that the Q1 portfolio was the main loser in 1999 and 2003.

Figures 5 and 6 display the graphical representation of the percentage of wins and losses of each portfolio for each of the presence thresholds. It is noteworthy that the superiority of the Q1 portfolio increases significantly along with threshold increases. In the 50% and 60% thresholds Q2 portfolios have higher average returns than those obtained by the Q1 portfolio.

Table 11 presents the percentage of total and wins and losses obtained by the Q1 to Q5 portfolio per presence threshold.

Regarding the Q5 portfolios' performances, their loss percentage decreases as the presence threshold is increased. These results suggest that factors related to liquidity can affect the intensity of the anomaly (LBSA), and should therefore be the object of future research.

Table 12 shows the comparative analysis of Q1 and Q5 wins over each other per presence threshold. The superiority of the Q1 portfolio over Q5 portfolio occurs at least in 79% of the portfolio building days, with small variations depending on adopted threshold.

Table 11 - - Analysis of percentage of days of wins and losses for each portfolio in 3454 moments of asset selection, per criterion of presence

| Wins	Q1	Q2	Q3	Q4	Q5
50%	30,0232%	42,1830%	10,7412%	12,8547%	4,1980%
60%	35,2056%	36,0162%	9,4962%	16,4737%	2,8083%
70%	41,1407%	22,0903%	11,3781%	22,2061%	3,1847%
80%	42,3277%	16,2999%	18,6161%	18,7609%	3,9954%
90%	46,7863%	18,5003%	18,6450%	10,9728%	5,0955%
Losses	Q1	Q2	Q3	Q4	Q5
50%	11,6966%	6,3405%	12,7678%	11,1175%	58,0776%
60%	9,5831%	5,5588%	16,3289%	15,8946%	52,6346%
70%	10,3358%	8,1065%	17,1106%	15,3445%	49,1025%
80%	10,3937%	7,4696%	17,6896%	16,9079%	47,5391%
90%	13,4916%	10,2779%	17,3133%	15,2577%	43,6595%

Source: Prepared by the authors.

Figure 5 – Histogram of percentual frequency of winning portfolios in the 3454 days of asset selection.
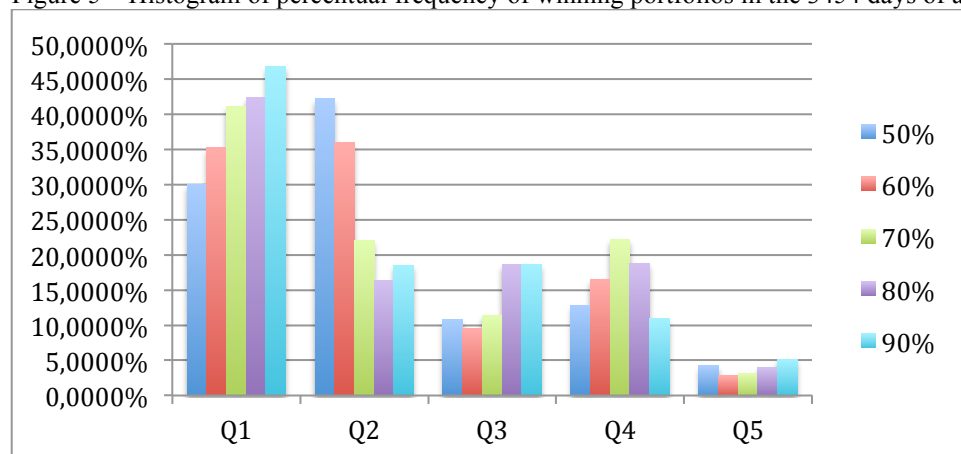
Source: Prepared by the authors.

Figure 6 – Histogram of percentual frequency of losing portfolios in the 3454 days of asset selection.
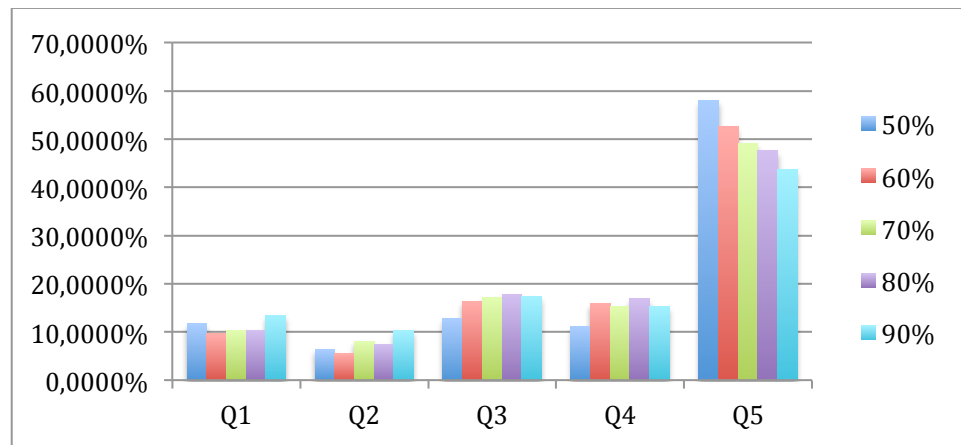
Source: Prepared by the authors.

Table 12 - Direct comparison between Q1 and Q5 portfolio presenting the total number of days in which Q1 showed a higher return compared to Q5 (Q1 being deemed the winner) and vice versa.

	Absolute		Percentage	
Year	Q1	Q5	Q1	Q5
50%	2762	692	79,9653%	20,0347%
60%	2803	651	81,1523%	18,8477%
70%	2835	619	82,0787%	17,9213%
80%	2774	680	80,3127%	19,6873%
90%	2728	726	78,9809%	21,0191%

Source: Prepared by the authors.

Regarding the daily logarithmic returns of the Q1 to Q5 portfolios in the 250 days following portfolio-building days, we performed the Jarque-Bera normality test (Jarque-Bera test) on the distribution of daily logarithmic returns. Table 13 shows the percentage of portfolios whose p-values of the Jarque-Bera test were greater than 0.1, indicating non-normality of returns. As a result, a standard statistical comparison between the results of Q1 and Q5 portfolios, such as Student's T test, may result in comparing returns arising from different statistical distributions.

Table 13 - Percentage of portfolios whose Jarque-Bera normality tests' p-values were greater than 0.1, denoting no normality of returns.

	Q1	Q2	Q3	Q4	Q5
50%	16,76%	17,69%	21,57%	26,69%	34,45%
60%	16,53%	18,33%	21,95%	26,58%	35,41%
70%	16,44%	12,83%	19,98%	27,45%	35,44%
80%	15,89%	17,08%	15,95%	29,85%	37,55%
90%	15,14%	24,23%	23,51%	34,80%	40,27%

Source: Prepared by the authors.

Considering the non-normality of returns obtained in some of the portfolios we built, we chose to perform two statistical tests in order to compare the returns of portfolios Q1 and Q5.

First we performed the t-test (Gujarati and Porter, 2011, p. 135-139), under the assumption that Q1's daily returns minus Q5's the returns are bigger than zero. This t-test is of parametric nature, based on the assumption that the analyzed data-points are normally distributed. The null hypothesis is that the excess returns obtained by Q1 portfolios over Q5 portfolios are not bigger than zero.

As an alternative procedure, we performed the non-parametric Kolmogorov-Smirnov test (KS Test) of difference in distributions. This test was performed though a specific function of R's Stats package. Using the ks.test function, we run a two-sample test of the null hypothesis that Q1 and Q5 returns were drawn from the same distribution.

Table 14 presents the results of the statistical tests we performed. The average acceptance of the null hypothesis under the t-test is of 85,42% under the five presence criteria we adopted. Under the KS Test, however, we find the rejection of the null hypothesis that both Q1 results and Q5 results were drawn from the same distribution on about 93,26% of the tests.

Table 14 –T-test and KS Test – Rejection of the null hypoteses

	T-test		KS-test	
	Rejection	Acceptance	Rejection	Acceptance
50%	10,71%	89,29%	94,30%	5,70%
60%	14,36%	85,64%	95,37%	4,63%
70%	16,88%	83,12%	95,31%	4,69%
80%	17,46%	82,54%	91,02%	8,98%
90%	13,49%	86,51%	90,30%	9,70%
Average	14,58%	85,42%	93,26%	6,74%

Source: Prepared by the authors.

It is possible that the results we found while performing the t-test result from significant standard deviations of the portfolios' returns, whereas the present study was based on daily returns of investment portfolios. Gujarati and Porter (2011) stated that as sample sizes become very large, statistical significance loses some of its power, while the aspects of economic significance become essential. In these authors' opinion, there are very large samples in which practically no null hypothesis could be rejected.

Given our doubts regarding the best statistical method to be applied in order to test the differences in returns, we suggest further studies to be performed regarding the best way to test the statistical differences between the daily returns of portfolios assembled by beta order on the Brazilian market, considering alternatives to the choices made in the present study such as different intervals for measuring results and different methods for normality testing of the portfolio returns.

We present in Figures 7 to 9 the results obtained by the portfolios Q1 to Q5 under the five presence criteria adopted, when these results are risk adjusted. Initially, in Figure 7 we present (in percentage) the average daily returns each portfolio obtained in excess when comparing to an investment in IBOVESPA index, considering all of the 3,454 portfolio building dates.

Figure 7 – - Average percentage of excesses of daily logarithmic returns of the portfolios relative to the average of the daily daily logarithmic returns of the Bovespa index.
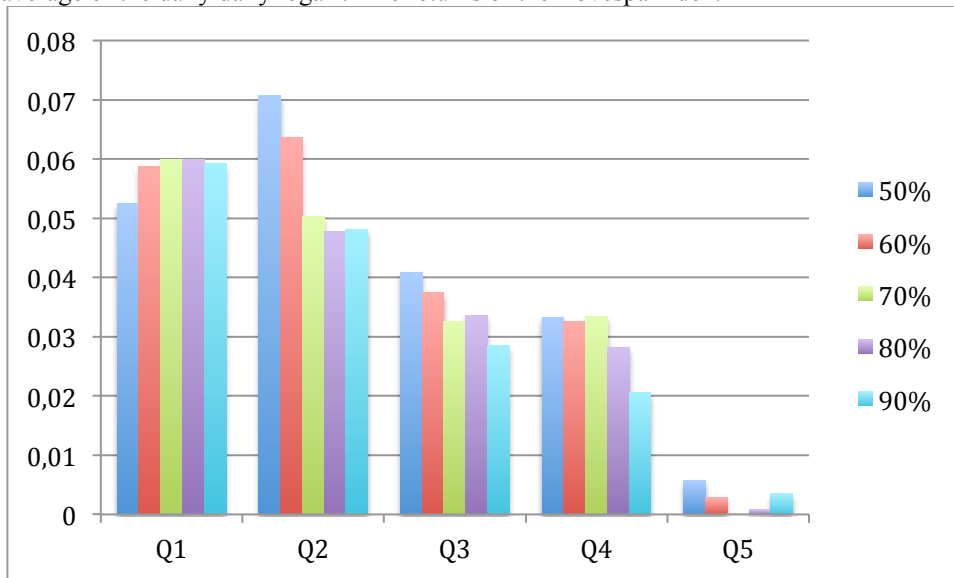
Source: Prepared by the authors.

Finally, adjusting the expected returns for the risk taken through formula (4), considering a risk-free asset, we found the excess returns shown in Figure 8 for a constant risk-free rate of 6% p.a. throughout the period. Figure 9 presents the results we find should we select the CDI rate (Brazilian Banking Prime Rate) as a proxy for the risk free rate.

$$Ex(R_i) = \left(\left(\frac{1+R_i}{1+\left(R_f + \beta_i^{f(R_m)} * (R_M - R_f)\right)} \right) - 1 \right) * 100 \qquad (4)$$

where:

$Ex(R_i)$: Percentage excess return of the asset i

$R_{i:}$ Asset i's actual return

$R_{f:}$ Risk-free asset's expected return

R_M: Market portfolio's expected return

$\beta_i^{f(R_m)}$: Asset i''s Beta coefficient

The average daily excess returns above the IBOVESPA index returns, presented in Figure 7, show quite clearly the superiority of the Q1 portfolio relative Q5 portfolio. Annualizing the returns above the IBOVESPA, Q1 portfolio presents an average excess return of 14.5% p.a. (based on the average of the five presence criteria) against Q5's average excess return of 0.6% p.a. (also based on the average of the five criteria cut). Since the excess returns reported in Figure 7 are not adjusted for risk, risk adjusting them in theory would increase the magnitude of the differences considering that the low-beta stocks (included in portfolio Q1) have lower expected returns than those of high-beta stocks (included in the portfolio Q5), according to the postulates of the MRRT and of the CAPM.

Regarding the results presented in Figures 8 and 9, the annualized differences in excess returns, risk-adjusted according to formula (4) with a constant 6% p.a. risk-free rate, the Q1 portfolio presents an average excess return of 20.3% p.a. (taking the average of the five presence criteria) and Q5 portfolio has negative average excess return of 0.90 % p.a. under the same conditions, as presented in Figure 8. In Figure 9, considering the CDI rate as a proxy for the risk-free rate, annualized excess returns of Q1 portfolio were 14% higher than expected for the portfolio on average (per presence criteria), meanwhile Q5 results were 0.20% lower than its expected returns.

Overall, the results we identified are similar to those reported in the international literature (Frazzini and Pedersen, 2011; Baker and Haugen, 2012; Blitz and Van Vliet, 2007; Blitz, Pang and Van Vliet, 2012), in which the LBSA is identified in many markets and periods, contradicting the main hypothesis of the MRRT and the intuition arising from the CAPM, that investors are rewarded for taking risks.

In connection with the results reported by Baker, Bradley and Wurgler (2011), low-risk stocks that were considered in the present study were a superior investment when compared to high-risk stocks, during the analyzed period. Similarly to what has been reported by Black, Jensen, and Scholes (1972), low-beta portfolios performed better than expected according to the CAPM, while high-beta portfolios were worse than predicted by CAPM in about 80% of the 3.454 portfolio building days.

Baker and Haugen (2012) identified the LBSA in Brazilian stocks and concluded that low risk stocks exhibit superior performance when compared to high-risk stocks in all analyzed markets, including Brazil, rebalancing portfolios each 30 days. The adoption of their strategy, however, could be more costly than the one adopted in the present study, in which we maintained portfolios for 250 days, with an assumed lower incidence of transaction costs.

Figure 8 – Average daily logarithmic returns of the portfolios in excess of the average of its expected returns, defined as the daily daily logarithmic returns of the Bovespa index in excess of the risk free rate for the period, multiplied by the beta of the portfolio at the time of its building, and added to a risk free rate of 6 % p.a.
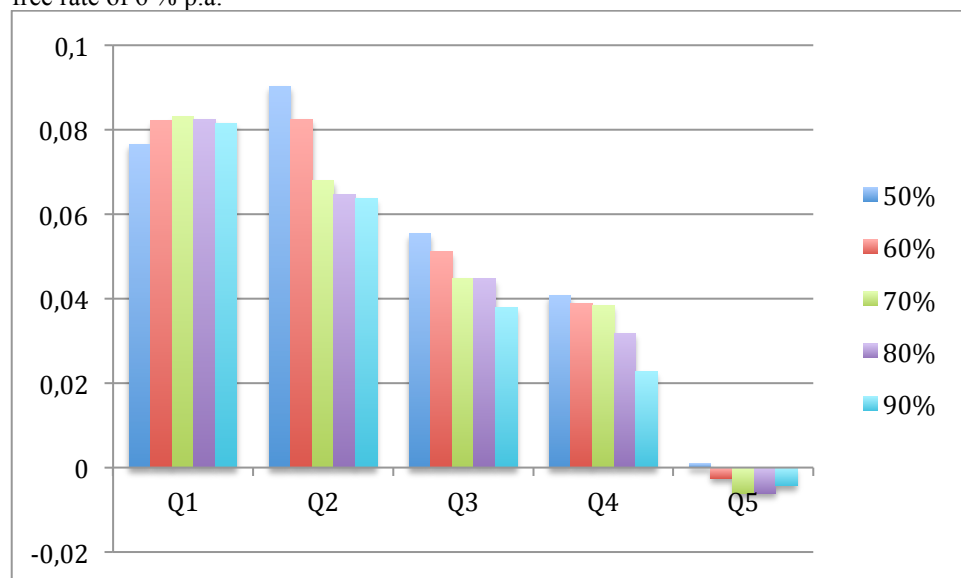
Source: Prepared by the authors.

Figure 9 – Average daily logarithmic returns of the portfolios in excess of the average of its expected returns, defined as the daily daily logarithmic returns of the Bovespa index in excess of the risk free rate for the period, multiplied by the beta of the portfolio at the time of its building, and added to a risk free rate based on the returns of an investment on the CDI rate.
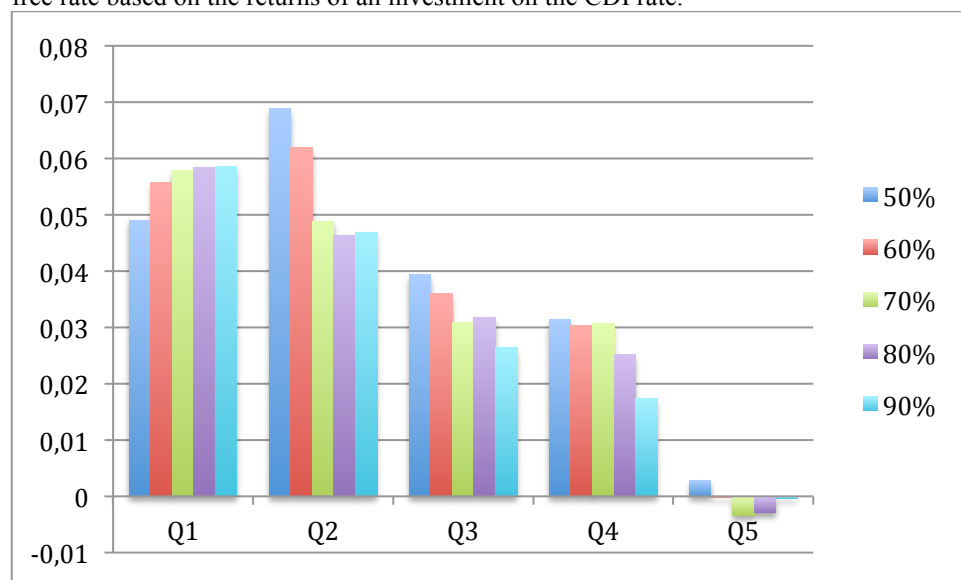
Source: Prepared by the authors.

5. CONCLUSIONS

Betting Against beta while selecting assets for portfolio building in the Brazilian Stock Market presented abnormal positive returns in most of the analyzed period. Our evidence corroborates the results found empirically by Baker, Bradley and Wurgler (2011, Baker and Haugen (2012), Blitz, Pang and Van Vliet (2012) and Frazzini and Pedersen (2011) for several of world's markets.

The exploitation of the LBSA constitutes a market inefficiency (even if we consider it a temporary one) that allows the prediction of higher returns based on stocks' low CAPM beta. We find this anomaly to be important practical evidence against the MRRT proposed by Sharpe (2008), according to which investors are rewarded for assuming higher market risks.

Our paper dealt with the study of the LBSA, reported in the literature since Black Jensen and Scholes (1972), being such study applied to the Brazilian stock market based on a large sample, both in terms of the temporal dimension of the analyzed data (1995-2012), and in terms of different liquidity restrictions for the building of the 86,350 portfolios whose results were analyzed and statistically compared.

Starting with the estimation of 2,419,011 beta coefficients, using the distribution of these coefficients as a criterion for forming portfolios of stocks traded in BM&FBOVESPA, we found that the LBSA was present with relevant practical strength during the period. In about 80% of the 3.454 portfolio building days, the portfolio comprised of low-beta stocks (including those stocks whose betas were part of the stock's betas first quintile - percentile equal to 20 % of the total stocks available for selection) outperformed the high beta stocks' portfolio (80 % percentile). In addition to that, the portfolio of low-beta proved the main winner among all of the five portfolios in 39 % of the portfolio building days.

The daily winning percentage of the portfolio Q1 increases with the level of minimum negotiation restriction, reaching 46% of wins on the criterion that requires a minimum of 90% presence, suggesting that the anomaly is not directly explained by liquidity problems and frequency of stocks 'trading. Future research is needed in order to directly assess the relationship between the LBSA and asset liquidity/trading frequency.

We identified specific periods in which the anomaly weakened with higher returns on portfolios of intermediate or higher betas. These periods were concomitant with economic crises regarding exchange rates, market liquidity, and market credit, which were suffered by Brazil in 1999, 2002/2003 and 2007/2008. We believe that these results indicates the need for further investigation of the role of betas as predictors of future returns in times of financial and economical crisis. However, the anomaly showed no significant trend of reduction over the analyzed period.

Even if marginal investors exploited the LBSA during the analyzed period, these marginal investors activities did not eliminate the effects of the anomaly, bringing prices back to equilibrium, restoring the MRRT. Consequently, the anomaly could be a result from limiting factors on the marginal investors' activities, a subject we recommend to be studied deeply in future research.

Unlike the results reported by Frazzini and Pedersen (2011) and Blitz, Pang and Van Vliet (2012), through Student's t-test we found in the present study the prevalence of statistical rejection of the hypothesis that the returns of the low-beta portfolio are higher than those of the high-beta portfolio if we incorporate assumptions of normality of returns of portfolios and homogeneity of their variance, assumptions that are a condition for the t-test.

Nevertheless, the returns of the portfolios we built were non-normally distributed in more than 30% of the high beta portfolios formed according to the Jarque-Bera normality test. Due to that non-normal behavior of returns, we believe that the t-test may not be adequate to assess statistically the difference between low-beta and high-beta portfolios. Alternatively, we performed the Kolmogorov-Smirnov test (a nonparametric test of difference in distributions) under the hypothesis that the distribution of returns of low beta portfolio was higher than the distribution of the returns of high beta portfolio. The results showed the predominant acceptance of difference between the two distributions, indicating the need for future studies of alternative approaches to test for differences in observed returns between portfolios of high and low beta.

Regardless of the statistical significance of the differences between returns arising from low beta and high beta portfolios, the practical significance of the LBSA during the researched period seems quite high, contradicting the logic proposed by financial theory according to which investors are reward for taking greater risk. Over the past 14 years, betting on a low-beta portfolio resulted in higher returns for an investor in about 80 % of the days in which he could have built stock portfolios.

The present paper presents results based on the authors' choices in what refers to factors such as beta estimation period and method, portfolios containing equal amounts of shares. We note that different ranges and methods of estimation of beta coefficients can be used to mitigate estimation problems arising from the existence of asynchronous negotiations. Thus we emphasize the importance of conducting further studies on the effect of alternative forms of tests considering different choices of beta estimation procedures and portfolio building, including portfolio optimization in terms of mean returns and variance, or weighting by the market value of the shares.

Finally, we hope to have contributed to the knowledge of the price and returns dynamics governing assets traded in Bovespa, fostering further investigations on the LBSA causes, that take into account robust statistical methods to reject, with greater significance, the hypothesis that the anomaly is just the result of a random - but persistent - fluctuation of asset prices, and also evaluate the role of behavioral and demographic aspects of investors that dynamically influence asset pricing, consistently with the anomaly under scrutiny.

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