Name _____ Class _____ Date _____

4-6

Triangle Congruence: ASA, AAS, and HL

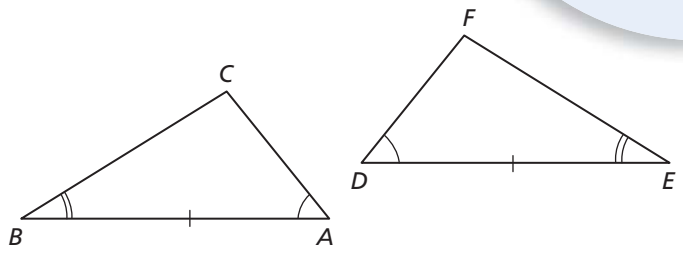
Going Deeper

Essential question: *How can you establish and use the ASA and AAS triangle congruence criteria?*

G-CO.2.8

1 PROOF **ASA Congruence Criterion**

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

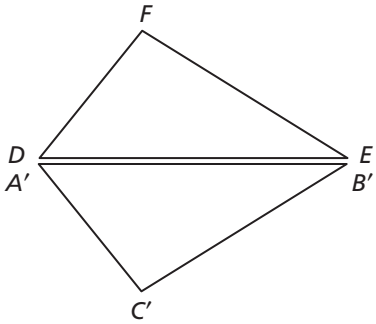


Given: $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\angle B \cong \angle E$.

Prove: $\triangle ABC \cong \triangle DEF$

To prove the triangles are congruent, you will find a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Complete the following steps of the proof.

A The first step is the same as the first step in the proof of the SSS Congruence Criterion. In particular the fact that $\overline{AB} \cong \overline{DE}$, means there is a sequence of rigid motions that results in the figure at right.



B As in the previous proofs, you can use the fact that rigid motions preserve angle measure and transitivity of congruence to show the following:

$\angle C'A'B' \cong$ _____ and $\angle C'B'A' \cong$ _____.

This means $\overline{DE}$ bisects both $\angle FDC'$ and _____.

By the Angle Bisection Theorem, under a reflection across $\overline{DE}$, $\overrightarrow{A'C'}$ maps to $\overrightarrow{DF}$, and $\overrightarrow{B'C'}$ maps to $\overrightarrow{EF}$. Since the image of C' lies on both $\overrightarrow{DF}$ and $\overrightarrow{EF}$, the image of C' must be F.

The proof shows that there is a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Therefore, $\triangle ABC \cong \triangle DEF$.

REFLECT

1a. Explain how knowing that the image of C' lies on both $\overrightarrow{DF}$ and $\overrightarrow{EF}$ allows you to conclude that the image of C' is F.

_____

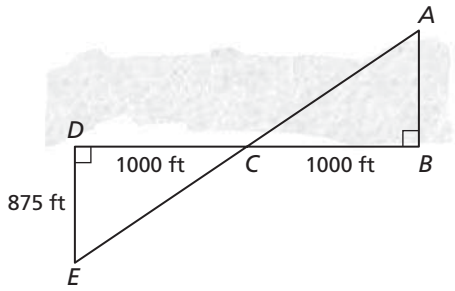
Once you have shown that two triangles are congruent, you can use the fact that corresponding parts of congruent triangles are congruent (CPCTC) to draw conclusions about side lengths and angle measures.

G-SRT.2.5

2 EXAMPLE Using the ASA Congruence Criterion

Solve the following problem.

You want to find the distance across a river. In order to find the distance AB, you locate points as described below. Explain how to use this information and the figure to find AB.



1. Identify a landmark, such as a tree, at A. Place a marker (B) directly across the river from A.

2. At B, turn 90° away from A and walk 1000 feet in a straight line. Place a marker (C) at this location.

3. Continue walking another 1000 feet. Place a marker (D) at this location.

4. Turn 90° away from the river and walk until the marker C aligns with A. Place a marker (E) at this location. Measure $\overline{DE}$.

A Show $\triangle ABC \cong \triangle EDC$.

- Based on the information marked in the figure, which pairs of sides or pairs of angles do you know to be congruent?

_____

- What additional pair of sides or pair of angles do you know to be congruent? Why?

_____

- How can you conclude that $\triangle ABC \cong \triangle EDC$?

_____

B Use corresponding parts of congruent triangles.

- Which side of $\triangle EDC$ corresponds to $\overline{AB}$? _____
- What is the length of $\overline{AB}$? Why?

_____

REFLECT

2a. Suppose you had walked 500 feet from B to C and then walked another 500 feet from C to D. Would that have changed the distance ED? Explain.

_____

_____

You have already used three triangle congruence criteria: SSS, SAS, and ASA. There is another criterion that is useful in proofs, the AAS Congruence Criterion.

AAS Congruence Criterion

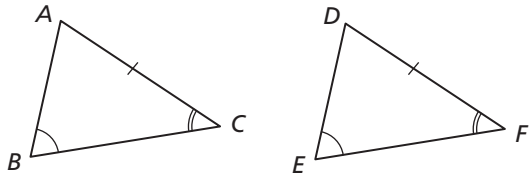
If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle, then the triangles are congruent.

G-CO.3.10

3 PROOF ASA Congruence Criterion

Given: $\angle B \cong \angle E, \angle C \cong \angle F, \overline{AC} \cong \overline{DF}$

Prove: $\triangle ABC \cong \triangle DEF$



To prove the triangles are congruent, you can use the Triangle Sum Theorem and reasoning about the angles of the triangles to show that $\angle A \cong \angle D$. Then you can show the triangles are congruent by using ASA.

Complete the proof by filling in the missing statements and reasons.

Statements	Reasons
1. $\angle B \cong \angle E, \angle C \cong \angle F$	**1.** Given
2. $m\angle A + m\angle B + m\angle C = 180°$	**2.** Triangle Sum Theorem
3. $m\angle A = 180° - m\angle B - m\angle C$	**3.**
4. $m\angle D + m\angle E + m\angle F = 180°$	**4.**
5.	**5.** Subtraction Property of Equality
6. $m\angle B = m\angle E, m\angle C = m\angle F$	**6.** Definition of congruent angles
7. $m\angle D = 180° - m\angle B - m\angle C$	**7.**
8. $m\angle A = m\angle D$	**8.** Transitive Property of Equality
9. $\angle A \cong \angle D$	**9.**
10. $\overline{AC} \cong \overline{DF}$	**10.** Given
11.	**11.** ASA Congruence Criterion

REFLECT

3a. Which prior steps of the proof are used in step 8?

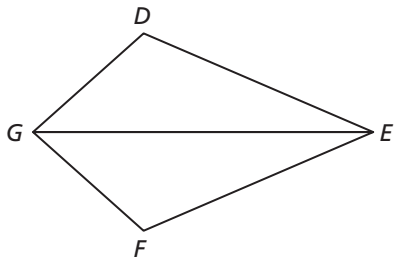
3b. Which prior steps of the proof are used in the last step? Explain.

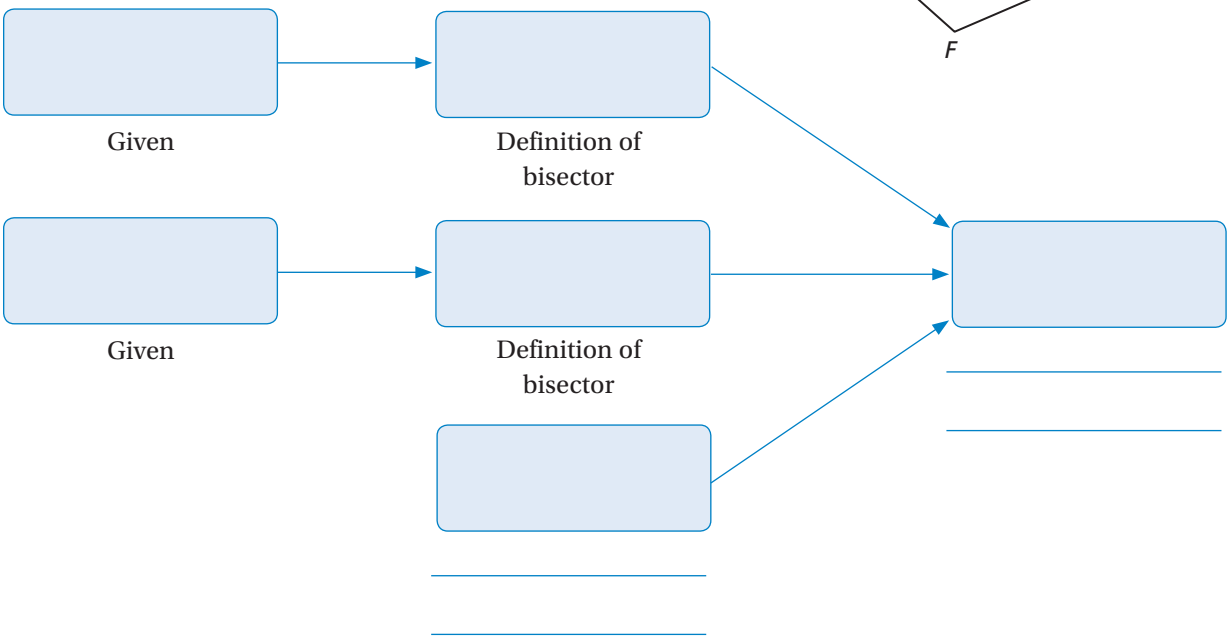
PRACTICE

1. Complete the proof.

Given: $\overline{GE}$ bisects $\angle DGF$ and $\angle DEF$.

Prove: $\triangle GDE \cong \triangle GFE$

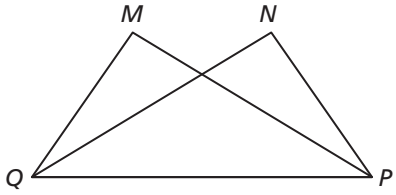




2. **a.** Write a two-column proof in the table provided at right. You may not need to use all the rows of the table for your proof.

Given: $\angle QMP \cong \angle PNQ$, $\angle MPQ \cong \angle NQP$

Prove: $\triangle MQP \cong \triangle NPQ$

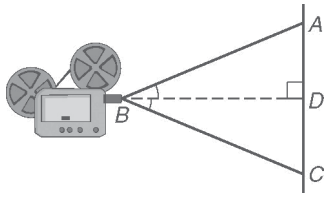


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

b. What additional congruence statements can you write using CPCTC?

Additional Practice

Students in Mrs. Marquez's class are watching a film on the uses of geometry in architecture. The film projector casts the image on a flat screen as shown in the figure. The dotted line is the bisector of $\angle ABC$. Tell whether you can use each congruence theorem to prove that $\triangle ABD \cong \triangle CBD$. If not, tell what else you need to know.



1. Hypotenuse-Leg

_____

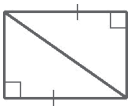
2. Angle-Side-Angle

_____

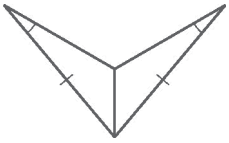
3. Angle-Angle-Side

_____

Write which postulate, if any, can be used to prove the pair of triangles congruent.

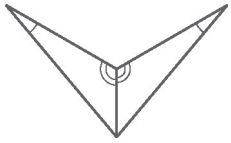


4. _____

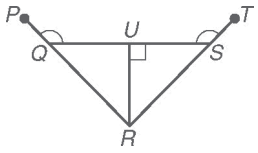


6. _____

5. _____



7. _____



Write a paragraph proof.

8. **Given:** $\angle PQU \cong \angle TSU$, $\angle QUR$ and $\angle SUR$ are right angles.

   Prove: $\triangle RUQ \cong \triangle RUS$

Problem Solving

Use the following information for Exercises 1 and 2.

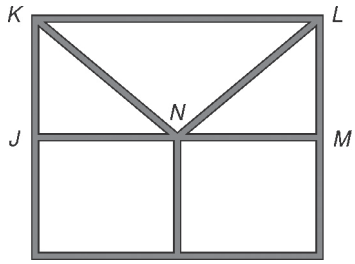
Melanie is at hole 6 on a miniature golf course. She walks east 7.5 meters to hole 7. She then faces south, turns 67° west, and walks to hole 8. From hole 8, she faces north, turns 35° west, and walks to hole 6.

1. Draw the section of the golf course described. Label the measures of the angles in the triangle.

2. Is there enough information given to determine the location of holes 6, 7, and 8? Explain.

_____

3. A section of the front of an English Tudor home is shown in the diagram. If you know that $\overline{KN} \cong \overline{LN}$ and $\overline{JN} \cong \overline{MN}$, can you use HL to conclude that $\triangle JKN \cong \triangle MLN$? Explain.



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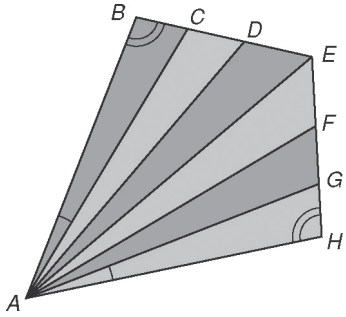
Use the diagram of a kite for Exercises 4 and 5.

$\overline{AE}$ is the angle bisector of $\angle DAF$ and $\angle DEF$.



4. What can you conclude about $\triangle DEA$ and $\triangle FEA$?

   A $\triangle DEA \cong \triangle FEA$ by HL.

   B $\triangle DEA \cong \triangle FEA$ by AAA.

   C $\triangle DEA \cong \triangle FEA$ by ASA.

   D $\triangle DEA \cong \triangle FEA$ by SAS.

5. Based on the diagram, what can you conclude about $\triangle BCA$ and $\triangle HGA$?

   F $\triangle BCA \cong \triangle HGA$ by HL.

   G $\triangle BCA \cong \triangle HGA$ by AAS.

   H $\triangle BCA \cong \triangle HGA$ by ASA.

   J It cannot be shown using the given information that $\triangle BCA \cong \triangle HGA$.