Contextualized Learning Activities (CLAs)

For the “other required credits” in the bundle of credits, students in a Specialist High Skills Major program must complete learning activities that are contextualized to the knowledge and skills relevant to the economic sector of the SHSM. Contextualized learning activities (CLAs) address curriculum expectations in these courses.

This CLA has been created by teachers for teachers.
It has not undergone an approval process by the Ministry of Education.

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Development date	August 2009	
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Specialist High Skills Major	Manufacturing, Transportation or Health and Wellness
Course code and title	MCR 3U – Grade 11 Functions – University Preparation
Name of contextualized learning activity	Modelling with Periodic Functions
Brief description of contextualized learning activity.	Students will review and consolidate skills for graphing and interpreting periodic functions. They will apply this knowledge to real-life examples from the transportation, manufacturing, and health & wellness sectors, while studying and learning about modelling in a practical environment.
Duration	Approximately 11 hours: Lesson 1 – Introduction to Periodicity (60 min) Lesson 2 – Trigonometric Functions (60 min) Lesson 3 – The CAST Rule (60 min) Lesson 4 – Trigonometric Transformations (60 min) Lesson 5 – Trigonometric Transformations (cont’d) (60 min) Lesson 6 – Trigonometric Transformations (cont’d) (60 min) Lesson 7 – Modelling Periodic Phenomena (60 min)

	Lesson 8 – Modelling Periodic Phenomena (cont'd) (60 min) Lesson 9 – Review of Trigonometric Transformations (60 min) Lesson 10 – Culminating Assessment (120 min) Appendix A – Student Worksheets 1-5 Appendix B – Summative Assessments Appendix C – Solutions to Culminating Assessment Appendix D – Scoring Rubric
Overall expectations	**Trigonometric Functions** The student will: 2. demonstrate an understanding of periodic relationships and sinusoidal functions, and make connections between the numeric, graphical, and algebraic representations of sinusoidal functions; 3. identify and represent sinusoidal functions, and solve problems involving sinusoidal functions, including problems arising from real-world applications.
Specific expectations	**Trigonometric Functions** The student will: 2.1 describe key properties of periodic functions arising from real-world applications, given a numeric or graphical representation 2.2 predict, by extrapolating, the future behaviour of a relationship modelled using a numeric or graphical representation of a periodic function 2.3 make connections between the sine ratio and the sine function and between the cosine ratio and the cosine function by graphing the relationship between angles from 0º to 360º and the corresponding sine ratios or cosine ratios, with or without technology, and defining this relationship as the function $f(x) = \sin x$ or $f(x) = \cos x$, and explaining why the relationship is a function 2.4 sketch the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ for angle measures expressed in degrees, and determine and describe their key properties 2.5 determine the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$, where $f(x) = \sin x$ or $f(x) = \cos x$ with angles expressed in degrees, and describe these roles in terms of transformations on the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ 2.6 determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations in the form $f(x) = a\sin(k(x - d)) + c$ or $f(x) = a\cos(k(x - d)) + c$ 2.7 sketch graphs of $y = af(k(x - d)) + c$ by applying one or more transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$, and state the domain and range of the transformed functions 2.8 represent a sinusoidal function with an equation, given its graph or its properties 3.1 collect data that can be modelled as a sinusoidal function, through investigation, with and without technology, from primary sources, using a variety of tools, or from secondary sources 3.2 identify periodic and sinusoidal functions, including those that arise from real-world applications involving periodic phenomena, given various representations, and explain any restrictions that the context places on the domain and range 3.3 determine, through investigation, how sinusoidal functions can be used to model periodic phenomena that do not involve angles 3.4 predict the effects on a mathematical model of an application involving periodic phenomena when the conditions in the application are varied 3.5 pose problems based on applications involving a sinusoidal function, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation

Essential Skills:	**Reading Text** Understanding text in the form of sentences or paragraphs **Document Use** Using information displays including drawings, technical readouts, and graphs **Numeracy** Use of graphs, numbers and quantities **Writing** Completing solutions of multi-step problem-solving questions **Continuous Learning** Ongoing process of learning and acquiring skills **Thinking Skills** Cognitive ability, problem solving
Work Habits:	**Teamwork** Work willingly and cooperatively with others **Initiative** Starts work with little or no prompting **Work Habits** Punctual, time effective, and able to follow directions **Organization** Written work is well laid out and neat **Working Independently** Accomplishes tasks independently

Instructional/Assessment Strategies

Teacher's notes ➢ The Math teacher should communicate with the Technology teacher on a regular basis. Both teachers should be kept up to date on developments that correspond to each other's courses. ➢ The teacher should become familiar with the use of mathematics in the Manufacturing and Transportation courses. ➢ Providing applicable real life examples from the manufacturing, transportation or health and wellness sectors can be beneficial for student learning. ➢ Constant diagnostic and formative feedback is important for consistent learning and student development (ie. through use of student worksheets). ➢ If the class is a split group (not all SHSM students) it may be advantageous to group the SHSM students together, however, this CLA has benefits for all MCR students, not just those enrolled in the SHSM program. ➢ Allow students to supplement their learning with applicable computer programs.
Context This CLA is designed for students that plan on entering an apprenticeship or college or university program in the Manufacturing or Transportation or Health and Wellness sectors.

Assessment and Evaluation of Student Achievement

Strategies/Tasks	*Purpose* *Assessment for Learning (diagnostic, formative) or Assessment of Learning (summative, evaluation)*
1. Introduction to Periodicity (worksheet #1 in Appendix A)	Diagnostic Assessment (give consistent feedback on student pre-requisite knowledge)
2. Trigonometric Functions (worksheet #2 in Appendix A)	Formative Assessment (give consistent feedback on student Progress)
3. The CAST Rule (homework)	Formative Assessment (give consistent feedback on student Progress)
4. Trigonometric Transformations (homework)	Formative Assessment (give consistent feedback on student Progress)
5. Trigonometric Transformations (worksheet #3 in Appendix A)	Formative Assessment (give consistent feedback on student Progress)
6. Trigonometric Transformations (worksheets #4, 5 in Appendix A)	Formative Assessment (give consistent feedback on student Progress)
7. Modelling Periodic Phenomena (homework)	Formative Assessment (give consistent feedback on student Progress)
8. Modelling Periodic Phenomena (homework)	Formative Assessment (give consistent feedback on student Progress)
9. Review (textbook review questions)	Diagnostic Assessment
10. Culminating Assessment	Summative Assessment (see attached rubric in Appendix D)

Additional Notes/Comments/Explanations

Both Mathematics Text books and Manufacturing, Transportation and Health & Wellness Texts and Manuals are a great source for additional questions, practice, and examples.

See Attachments for further details

Resources

Smith, Robert D. Mathematics for Machine Technology, 4th Edition, Delmar Publishers,1999

Zimmer, David et al, Nelson Mathematics 11, Nelson Thomson Learning, 2001

Accommodations

- Individual Education Plans (IEP) should be followed at all times. Be sure to consult the SERT for additional information and suggestions;
- additional time may be needed for diagnostic, formative and summative assignments;
- the activities and lessons outlined in this CLA allow for flexibility in the delivery of the material. Alternating teaching strategies can help students who are not progressing at the appropriate level;
- font can be increased for those students that have vision problems;
- class rules, behaviours, and due dates should be posted in the classroom and talked about so that all students are aware of the expectations;
- if possible, more individual instruction time can be allotted to students in need;
- can account for student work habits when considering assignments;
- provide opportunities for enrichment for exceptional students;
- provide time for peer-to-peer teaching;
- use audio aids if needed;
- provide alternate assessment opportunities that are geared towards students strengths or areas of interest;
- if available, many computer programs can be used to supplement student learning.

List of Attachments

- Lessons 1 - 9
- Appendix A – Student Worksheets 1 - 5
- Appendix B – Summative Assessments
- Appendix C – Solutions for Culminating Assessment
- Appendix D – Scoring Rubric

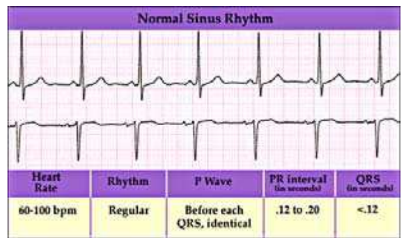
LESSON 1: Introduction to Periodicity

Periodic phenomena are situations that repeat again and again, over a period of time. Repeating relationships can be seen in:

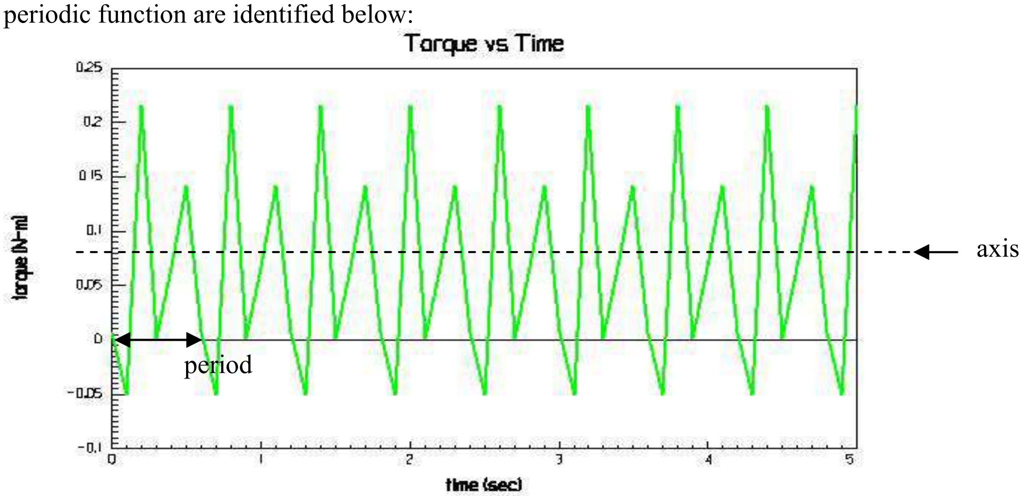
- Weather patterns.
- Astronomy (Lunar cycles, solar cycles, rotations and revolutions of planets).
- Tidal patterns.
- Traffic patterns.
- Animal migration patterns.
- Health and Wellness (blood pressure, ECG).
- Engineering and Manufacturing (pendulums, pistons, assembly lines).
- Transportation and Electronics (flow of electricity through circuits, wheel speed sensors).

Mathematics can be used to graph and assign equations to such periodic functions.

This first graph shows the periodic rhythm of a heart as it beats at regular intervals.



This second graph shows the periodic nature of the force acting on a crank shaft. Some important characteristics of the periodic function are identified below:



A **CYCLE** is the shortest repeating pattern.

The length of the cycle is the **PERIOD**. In this case the period is approximately <u>0.6 seconds.</u> (see arrow above)

The **MINIMUM** is the lowest value. In this case the minimum is <u>-0.05 N-m.</u>

The **MAXIMUM** is the highest value. In this case the maximum is <u>0.225 N-m.</u>

The **AXIS OF THE CURVE** is the line halfway between the minimum and maximum. (see dotted line above)

The equation of the axis is: $y = \frac{\max + \min}{2}$. In this case the axis is at $y = \frac{0.225 + (-0.05)}{2} = \frac{0.175}{2} = \underline{0.0875}$

The **AMPLITUDE** is the vertical distance between the axis and the minimum/maximum.

The equation of the amplitude is: $amp = \frac{\max - \min}{2}$. In this case the amplitude is $amp = \frac{0.225 + 0.05}{2} = \underline{0.1375}$

Students should complete worksheet #1, either independently or in small groups.

Lesson 2 – Trigonometric Functions

Students should add the following relevant characteristic information to the trigonometric functions that appear on the attached student handout (appendix A).

The functions $f(x) = \sin\theta$, $f(x) = \cos\theta$, $f(x) = \tan\theta$ are periodic functions.

Sine and Cosine Functions

$f(x) = \sin\theta$ and $f(x) = \cos\theta$ have a maximum of 1, a minimum of –1, an amplitude of 1, a period of 360° and an axis of $y = 0$. Graphs with the shape of $f(x) = \sin\theta$ and $f(x) = \cos\theta$ are said to be sinusoidal.

Tangent Function

$f(x) = \tan\theta$ has no minimum, maximum or amplitude but has an asymptote every 180° beginning with 90°. The period of $f(x) = \tan\theta$ is 180°.

Extending the Primary Trigonometric Ratios Beyond Triangles:

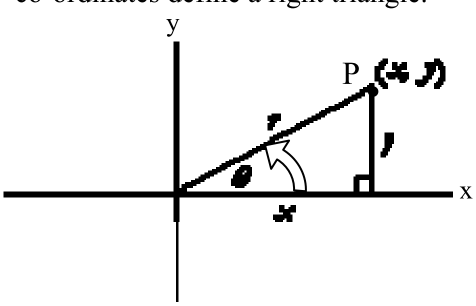
Recall the primary trig ratios for triangles: SOH CAH TOA

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

Let a straight line of length *r* sweep out an angle *θ* in standard position and let the co-ordinates of its extremity be point P (x,y).

The question is: How shall we now define the trigonometric ratios of this new angle *θ,* since it is too big to be simply an angle inside of a triangle?!?!?

We will take our cue from the first quadrant. In that quadrant, a radius *r* will terminate at a point (x,y). Those co-ordinates define a right triangle.



For any point P(x,y) on the terminal arm of an angle in standard position, the trigonometric functions are

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \qquad x \neq 0$$

As *θ* changes, the magnitude and sign of x and y also change.

On worksheet #2, students should refer to graphs for the three primary trig ratios, and consolidate skills from periodicity to complete the characteristics of these repeating functions.

Lesson 3 – The CAST Rule

A positive sign for trigonometric functions is summarized using the CAST rule.

C indicates that $\cos\theta$ is positive.

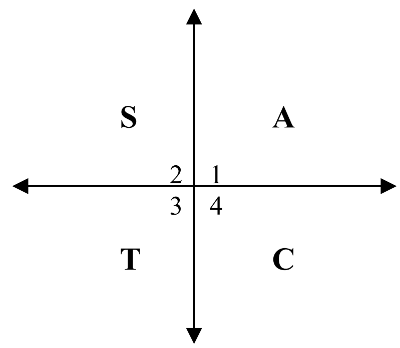
A indicates all trig ratios are positive.

S indicates that $\sin\theta$ is positive.

T indicates that $\tan\theta$ is positive.

	$\sin\theta$ is	$\cos\theta$ is	$\tan\theta$ is
In Quadrant 1	+	+	+
In Quadrant 2	+	-	-
In Quadrant 3	-	-	+
In Quadrant 4	-	+	-

The CAST Rule



Summary of Special Angles (found Using Unit circles)

	0^o	30^o	45^o	60^o	90^o	180^o	270^o	360^o
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined	0	undefined	0

Homework : from textbook (choose contextualized examples)

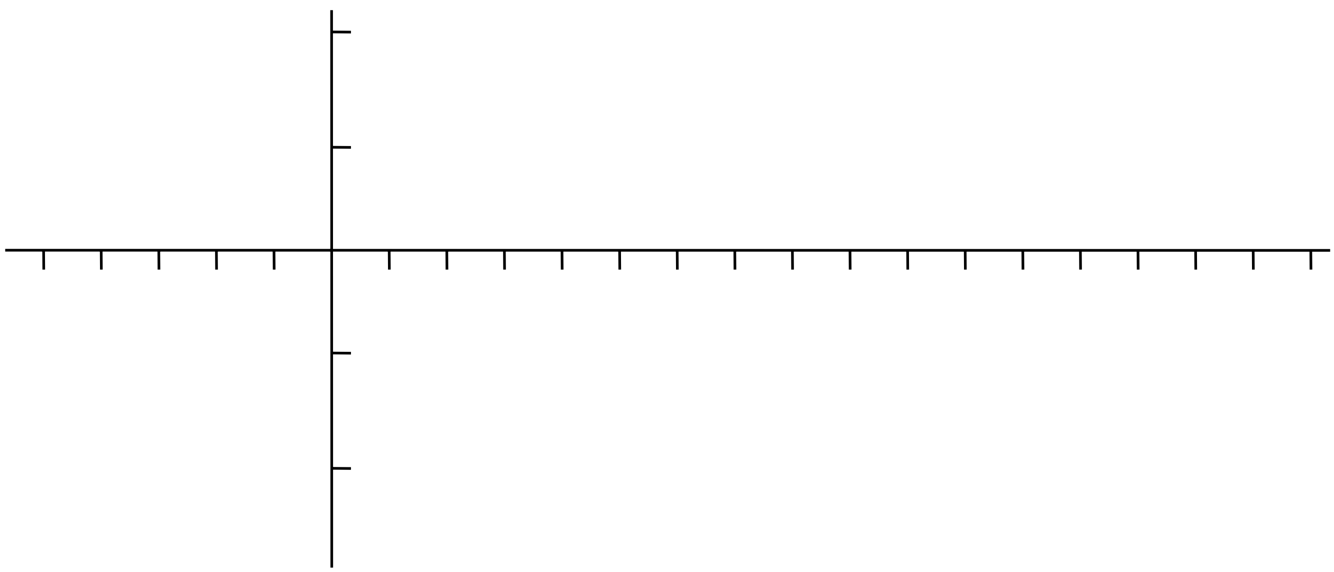
Lesson 4 – Trigonometric Transformations

Amplitude and Period

For $y = a\sin kx$ and $y = a\cos kx$,

amplitude = $|a|$; if a < 0, the graph is reflected in the x-axis;
and period = $\frac{360^\circ}{k}$.

Ex. 1. Graph y = $3\sin\theta$ and y = $-\frac{1}{2}\sin\theta$ on the same set of axes for $0 \le \theta \le 540^\circ$.
(Use transformations based on the function y = $\sin\theta$.)



	Function	Range
Note:	y = $\sin\theta$	$-1 \le y \le 1$
	y = $3\sin\theta$	$-3 \le y \le 3$
	y = $-\frac{1}{2}\sin\theta$	$-\frac{1}{2} \le y \le \frac{1}{2}$

The **AMPLITUDE** of a function in the form y = **a** $\sin\theta$ and y = **a** $\cos\theta$ is $|a|$.

Ex. 2 Graph y = $\cos 2\theta$ and y = $\cos\frac{1}{2}\theta$ on the same axes for $-180^\circ \le \theta \le 1040^\circ$.

(Use transformations based on the function y = $\cos\theta$.)

Recall: 1) The function y = f(kx) is a horizontal stretch when k < 1 and a horizontal compression when k > 1.

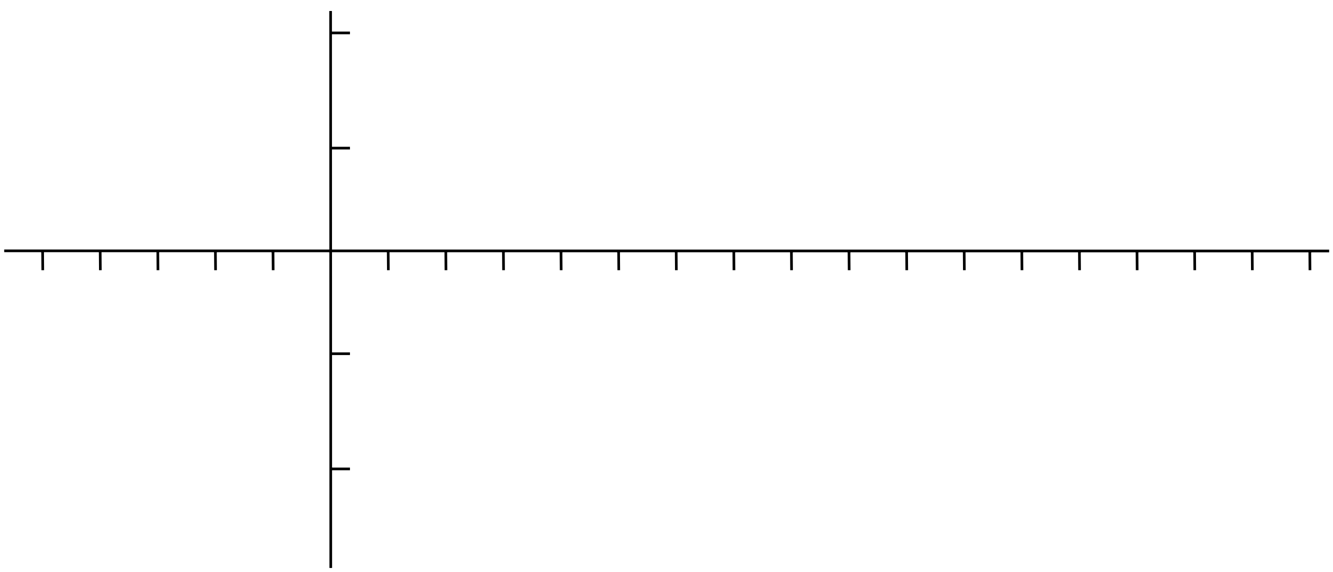
2) The period of y = $\cos\theta$ is 360°.

Note: When $\theta = 180^\circ$, $\cos 2\theta = \cos(360^\circ)$ which indicates that the function y = $\cos 2\theta$ completes one period from $\theta = 0$ to 180°.

This function's period is 180° (360 ÷ 2).

When $\theta = 720^\circ$, $\cos\frac{1}{2}\theta = \cos(360^\circ)$ which indicates that the function y = $\cos\frac{1}{2}\theta$ completes one period from $\theta = 0$ to 720°.

This function's period is 720° (360 ÷ ½).



The **PERIOD** of a function in the form y = sin **k** θ and y = cos **k** θ is $\frac{360^\circ}{k}$, $k > 0$.

Ex. 3 For $0 \le \theta \le 360^\circ$, state the amplitude and period for each function below:

a) $f(\theta) = 2\sin 3\theta$ amplitude = 2 , and period = $\dfrac{360^\circ}{k} = \dfrac{360^\circ}{3} = 120^\circ$

b) $f(\theta) = 3\cos 2\theta$ amplitude = 3 , and period = $\dfrac{360^\circ}{k} = \dfrac{360^\circ}{2} = 180^\circ$

c) $f(\theta) = -4\cos 5\theta$ amplitude = $|-4| = 4$, and period = $\dfrac{360^\circ}{k} = \dfrac{360^\circ}{5} = 72^\circ$

Homework : from textbook (choose contextualized examples)

Lesson 5 – Trigonometric Transformations (continued)

Horizontal Translation (Phase Shift) of Sine and Cosine Graphs

Recall: The graph of y = f(x - p) is the image of f(x) shifted p units right.
The graph of y = f(x + p) is the image of f(x) shifted p units left.

In other words, the graphs of y = sin(θ+ b) and y = cos(θ+ b) are the images of y = sin θ and y = cos θ horizontally translated (shifted) b units to the right when b < 0 and to the left when b > 0.

This type of horizontal shift is called a "phase shift". To determine the **phase shift** of y = sin(k θ+ p), factor out the k value to rewrite the equation in the form y = sin k(θ+ b), where b = p/k.

It is important to perform transformations in the correct order – closely following rules for BEDMAS!

Order for Application of Transformations:

1. horizontal stretch or compression
2. horizontal phase shift left or right
3. vertical stretch or compression
4. reflection in θ-axis
5. vertical translation up or down

Ex. 1 Describe, in words, the transformations for each function for $-180^{\circ} \leq \theta \leq 360^{\circ}$, then graph:
(be sure to list transformations in the correct order!)

a) $y = \sin\left(\theta - 90^{\circ}\right)$

→ horizontal shift 90º to right

b) $y = 2\cos\left(\theta + 180^{\circ}\right)$

→ horizontal shift 180º to left
→ vertical stretch by a factor of 2

c) $y = \sin 2\left(\theta + 45^{\circ}\right)$

→ horizontal compression by a factor of 2
→ horizontal shift 45º to left

d) $y = -\sin\left(2\theta + 90^{\circ}\right)$

→ horizontal compression by a factor of 2
→ horizontal shift 45º to left
→ reflection in θ-axis

Vertical Translation of Sine and Cosine Graphs

Recall: The function y = f(x) + q is the image of y = f(x) shifted q units upward when q is positive and q units downward when q is negative.

In other words, the graphs of the functions y = sinθ + d and y = cosθ + d are the images of y = sinθ and y = cosθ shifted "d" units upwards when d > 0 and "d" units downwards when d < 0.

Ex2. Describe, in words, the transformations for each function for $-180^\circ \le \theta \le 360^\circ$, then graph: (be sure to list transformations in the correct order!)

a) $y = 2\cos\theta + 3$

→ vertical stretch by a factor of 2
→ vertical translation up 3

b) $y = \sin(\theta - 60^\circ) - 2$

→ horizontal shift 60° to right
→ vertical translation down 2

c) $y = -\cos 3(\theta + 120^\circ) + 1.5$

→ horizontal compression by a factor of 3
→ horizontal shift 120° to left
→ reflection in θ-axis
→ vertical translation up 1.5

d) $y = 3\cos(2x - 180^\circ) + 2$

→ horizontal compression by a factor of 2
→ horizontal shift 90° to right
→ vertical stretch by a factor of 3
→ vertical translation up 2

Students should complete Worksheet #3, either independently, or in pairs.

Lesson 6 – Trigonometric Transformations (continued)

Students will work through related handouts (Student Worksheets #4 and 5) in pairs to consolidate understanding of trigonometric transformations.

Worksheets should be corrected and/or assessed in a formative way.

Lesson 7 – Modelling Periodic Phenomena

Any situation with "sinusoidal data" (ie. data that, when graphed, follows the general shape of a Sine or Cosine curve) can be expressed as a trigonometric function with transformations to either the sine or cosine function. In order to accomplish this, we must think about the pertinent information and make the necessary changes to one of the trigonometric functions.

For sinusoidal data, use either the sine function or the cosine function, but the resulting equation should be the simpler of the two → that is the one with fewer terms or transformations. Determine the values of *a*, *k*, *b*, and *d* for one of the functions. (The values for *k* and *d* will be the same for both the sine and cosine functions so focus on the phase shift.) Some questions to ask yourself may include: Will a translation right or left make one curve more like the model? Will simply reflecting one of the curves (i.e. changing the sign of *a*), make it more like the model?

The amplitude is half the difference between the maximum and minimum values and is always positive, but the value of *a* can be positive or negative. A negative value indicates that the curve is reflected about the x-axis.

The value of k is determined from the period of the curve. It tells how many cycles there are for each full cycle of $y = \sin\theta$ or $y = \cos\theta$. So look for one "rotated S shape" () in the graph and compare it to one complete cycle of the base sine curve.

So, k can be calculated using the equation $k = \dfrac{360^o}{period}$.

Read the value of the phase shift, *b*, from the graph, by first deciding where a typical sine or cosine curve begins.

The value of *d* is the axis of the curve. The axis of the curve is the average of the maximum and minimum values, which can be read from the graph.

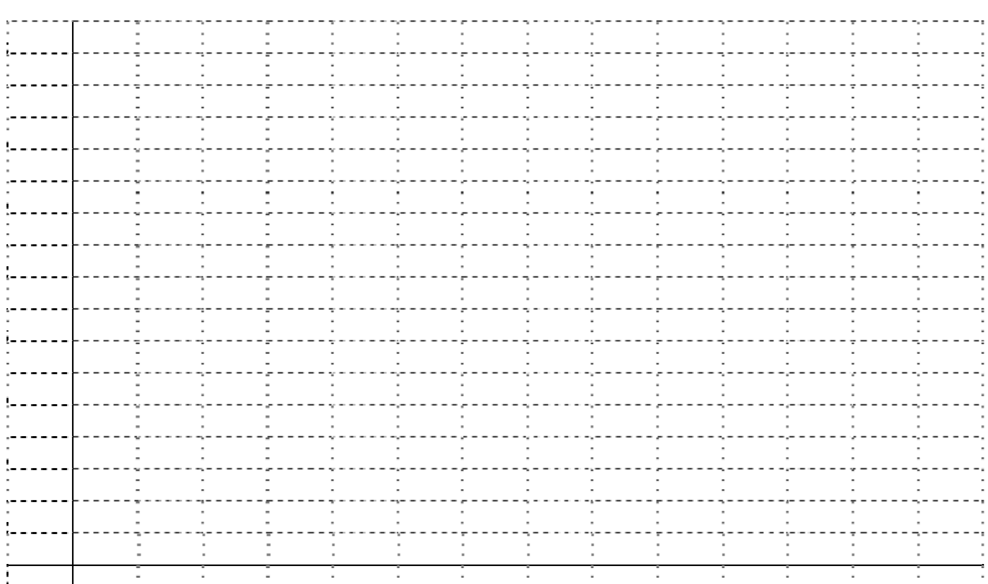
Ex 1. The depth of water in a harbour on the Bay of Fundy that faces the ocean changes each hour as shown.

Time (hours)	0	1	2	3	4	5	6	7	8	9	10	11	12
Depth (meters)	5.5	6.3	8.5	11.5	14.5	16.7	17.5	16.7	14.5	11.5	8.5	6.3	5.5

a) Determine an equation that models the situation.

b) Use the equation to determine the depth of water at 10: 30 a.m. Verify the answer using the graph.

c) When is the water 7 m deep?



$$\text{axis of curve} = \frac{\max + \min}{2} = \frac{17.5 + 5.5}{2} = 11.5$$

amplitude = 6

phase shift = 3 (if using sine curve)

OR

phase shift = 6 (if using cosine curve)

period = 12 h → $\frac{360t}{12} = 30t$

a) $y = 6\sin 30(t-3) + 11.5$

$y = 6\sin(30t - 90) + 11.5$

OR

$y = 6\cos 30(t-6) + 11.5$

$y = 6\cos(30t - 180) + 11.5$

b) Substitute t = 10.5 h into equation:

$$y = 6\sin 30(10.5 - 3) + 11.5$$
$$y = 6\sin 30(7.5) + 11.5$$
$$y = 6\sin(225) + 11.5$$
$$y = -4.2426 + 11.5$$
$$y = 7.2574 \text{ m}$$

c) Set equation (y) equal to 7:

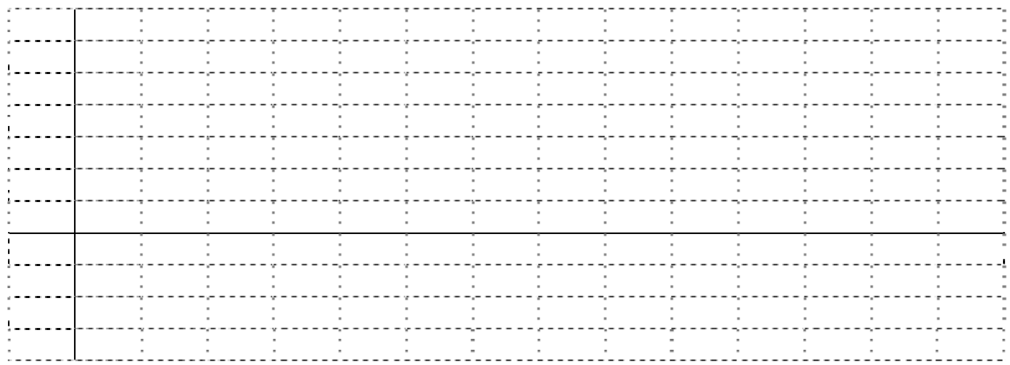
$$7 = 6\sin 30(t-3) + 11.5$$
$$7 - 11.5 = 6\sin 30(t-3)$$
$$-4.5 = 6\sin 30(t-3)$$
$$\frac{-4.5}{6} = \sin 30(t-3)$$
$$\sin^{-1}\left(\frac{-4.5}{6}\right) = 30(t-3)$$
$$\frac{\sin^{-1}\left(\frac{-4.5}{6}\right)}{30} = (t-3)$$
$$t = \frac{\sin^{-1}\left(\frac{-4.5}{6}\right)}{30} + 3$$
$$t \cong 1.38 \text{ h}$$

Homework : from textbook (choose contextualized examples)

Lesson 8 – Modelling Periodic Phenomena (continued)

Ex.1 A Ferris wheel has a radius of 4 m and rotates once every 24 s.
The bottom of the wheel is 2 m above the ground.

a) Draw a graph that shows one complete rotation of the Ferris wheel. It should illustrate a person's height above the ground, starting from a position that is level with the centre axle of the wheel.



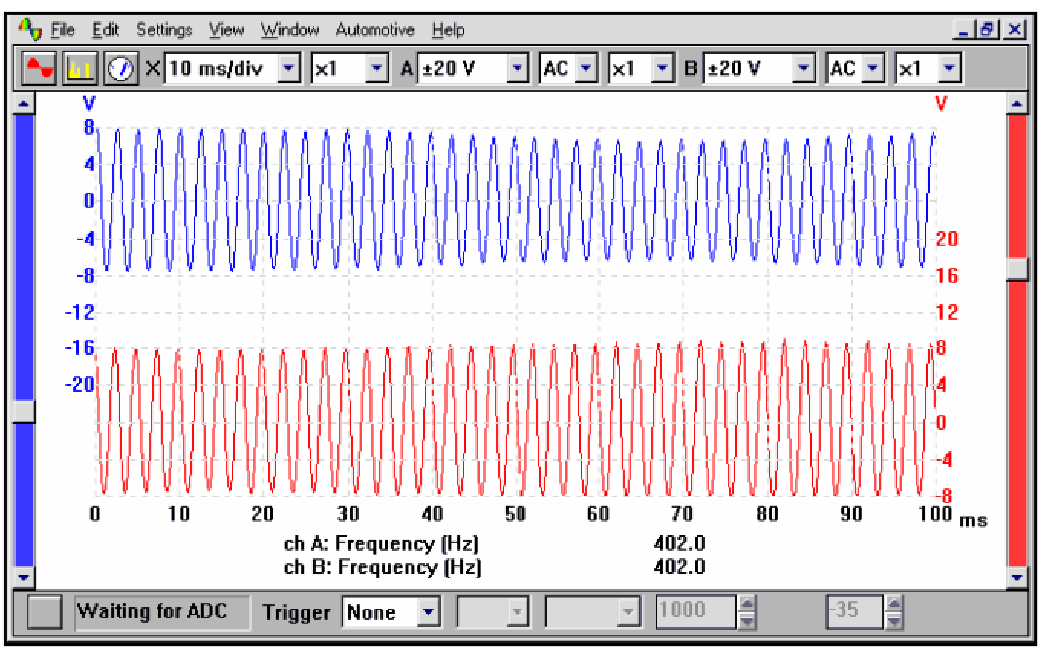
b) Find an equation for a) using a sine function.

$$y = 4\sin\left(360^{o} \cdot \frac{t}{24}\right) + 6$$

24 seconds for one cycle

$$y = 4\sin(15t) + 6$$

Ex. 2 While servicing a car, the mechanic notices the following display on the diagnostic screen.
Find an equation for the bottom curve, using a cosine function. (NOTE: the vertical scale for the bottom curve is on the right-hand side of the graph.)



Axis = 0   Amplitude = 8

Phase shift = 0   Period = 2.5 ms

$$y = 8\cos\left(360° \cdot \frac{t}{0.0025}\right)$$

$$y = 8\cos(144000t)$$

Ex. 3 The function $P(t) = -20\cos(300t) + 100$ models the blood pressure, P, measured in millimeters of mercury, at time, t, in seconds, of a person at rest.

a) What is the period of the function? What does the period represent for an individual?
b) How many times does this person's heart beat each minute?
c) What is the range of the function? Explain the meaning of the range in terms of a person's blood pressure.

a) The period (time for one beat of the heart) is $\frac{360}{300} = \frac{6}{5}$, which is this fraction of a second (or 1.2 sec)
This value represents the time between one beat of a person's heart and the next.

b) Since the heart beats 1 time each 1.2 seconds, the person's heart rate (beats per minute) can be calculated by multiplying the period by 60 sec/min: 1.2 × 60 = 72 beats/min.

c) The range of this function is $80 \le P(t) \le 120$.
These values are the lowest blood pressure, which is 80 mmHg, and the highest, which is 120 mmHg.

Homework : from textbook (choose contextualized examples)

Lesson 9 - Review of Trigonometric Transformations

Vertical Stretch (multiply y)
Used to find amplitude

Vertical Shift

$$y = a\,\mathbf{f}(k(x-d))+c$$

Horizontal Shift

Horizontal Stretch (divide x)
Used to find the period

To graph a trig function of the above form, apply the transformations in this order:

1. Horizontal stretch or compression (k)
2. Horizontal translation or phase shift, left or right (d)
3. Vertical stretch or compression (a)
4. Reflection about the x-axis (if a < 0)
5. Vertical translation, up or down (c)

To graph $y = 3\sin 2(x-45)$, complete the following steps:

1. Sketch the function $y = \sin x$.
2. Use the period. This is $\frac{360}{2}$, which is $180°$. The point at the end of the cycle is $180°$ to the right of the first point.
3. Use the phase shift. This is $45°$, and it is the x-coordinate where the sine curve begins its cycle. Sketch new curve on the graph.
4. Use the amplitude. This is 3, so the sine curve will be stretched to have a max of 3 units and a min of 3 units. Sketch new curve on the graph.
5. Since a > 0, then the graph will not be reflected about the x-axis.
6. Use the vertical translation. This is 0 (no value is added or subtracted at the end of the equation). So the graph will not move vertically from the x-axis.

Homework : review questions from textbook
(choose contextualized examples)

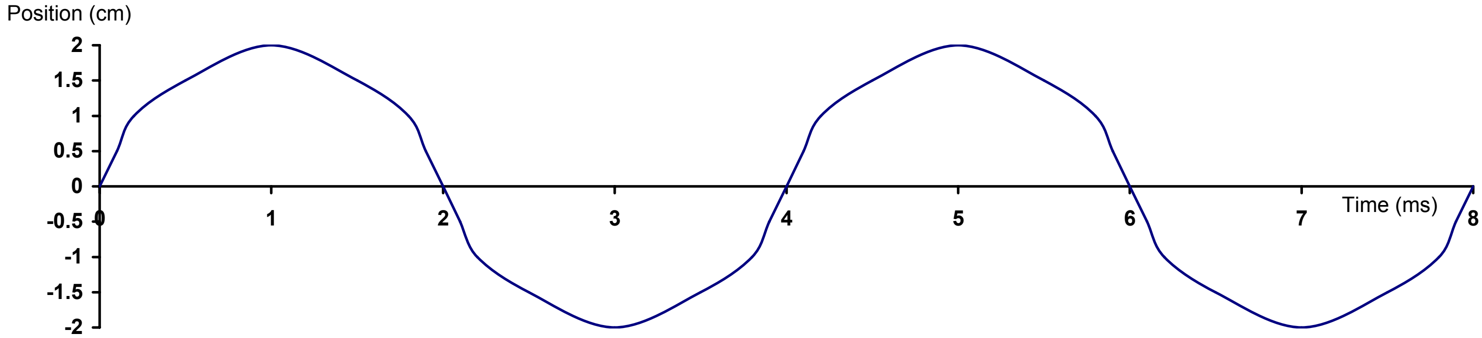
Appendix A

Student Worksheet #1 – Periodic Functions

1. The following data represents the position of a piston in an engine.

Time (ms)	0	0.1	0.2	0.5	1.0	1.5	1.8	1.9	2.0	2.1	2.2	2.5	3.0	3.5	3.8	3.9	4.0
Position (cm)	0	0.5	1	1.5	2	1.5	1	0.5	0	-0.5	-1	-1.5	-2	-1.5	-1	-0.5	0

Time (ms)	4.1	4.2	4.5	5.0	5.5	5.8	5.9	6.0	6.1	6.2	6.5	7.0	7.5	7.8	7.9	8.0
Position (cm)	0.5	1	1.5	2	1.5	1	0.5	0	-0.5	-1	-1.5	-2	-1.5	-1	-0.5	0



a) the period is _______________.

b) the minimum is ______________.

c) the maximum is _____________.

d) the axis is y = _________________.

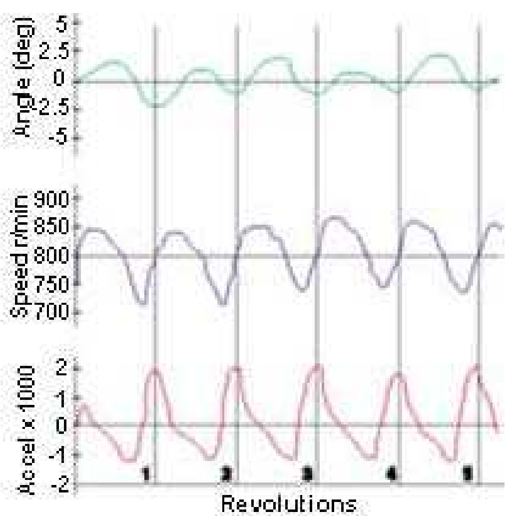
e) the amplitude is amp = ________________ .

2. Consider the following graphs. Do they represent periodic functions? Why or why not?

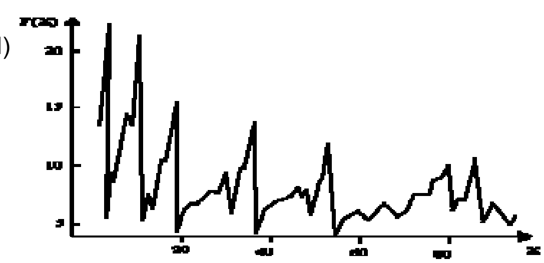
a)

b)

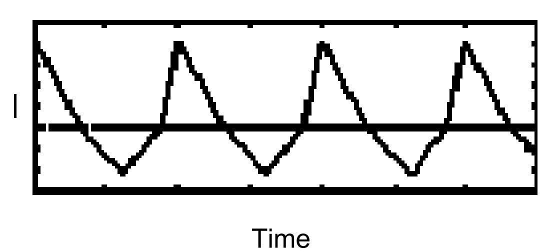
c)



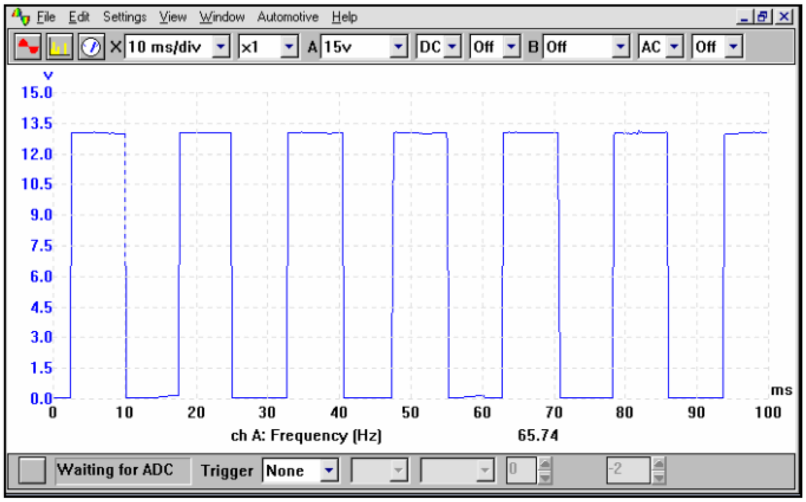
d)

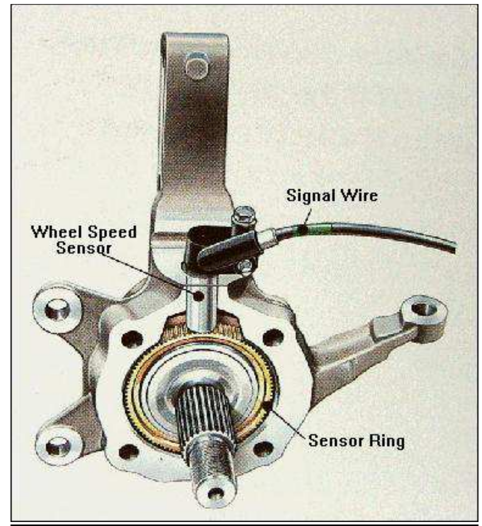


e)



3. A wheel speed sensor consists of a toothed ring positioned in the wheel hub on your car and a stationary electronic sensor. As the wheel turns, the teeth on the ring pass the sensor. As each tooth passes, the sensor causes a spike in voltage, as illustrated in the graph below of wheel speed (measured in volts) vs. time (in milliseconds).





a) What is the period of the function?

b) What is the amplitude of the function?

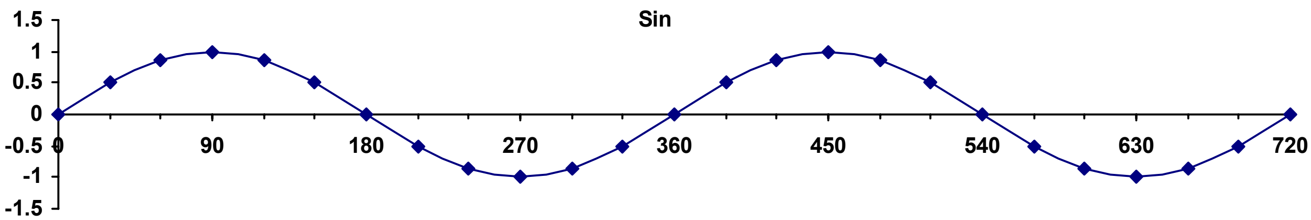
c) What is the axis of the function?

Student Worksheet #2 - Trigonometric Functions

The following table was generated using a calculator and used to make the graphs.

θ	**0**	**30**	**60**	**90**	**120**	**150**	**180**	**210**	**240**	**270**	**300**	**330**	**360**
sin θ	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0
cos θ	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1
tan θ	0	0.58	1.73	-	-1.73	-0.58	0	0.58	1.73	-	-1.73	-0.58	0

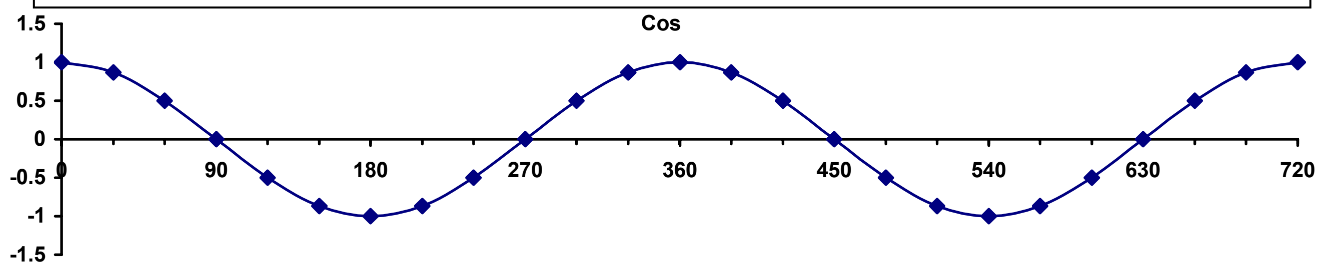
θ	**390**	**420**	**450**	**480**	**510**	**540**	**570**	**600**	**630**	**660**	**690**	**720**
sin θ	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0
cos θ	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1
tan θ	0.58	1.73	-	-1.73	-0.58	0	0.58	1.73	-	-1.73	-0.58	0



Period = ______________________ Minimum = ______________________

Maximum = ____________________ Amplitude = ______________________

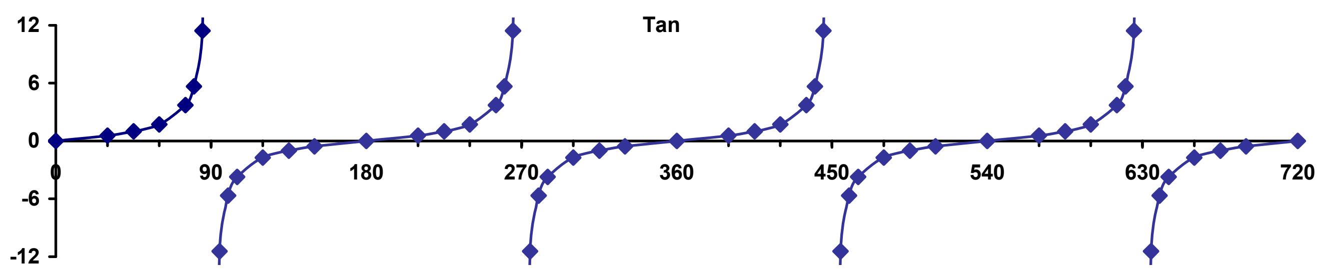
Axis = ________________________



Period = ______________________ Minimum = ______________________

Maximum = ____________________ Amplitude = ______________________

Axis = ________________________



Period = ______________________ Minimum = ______________________

Maximum = ____________________ Amplitude = ______________________

Axis = ________________________

Student Worksheet #3 – Trig Transformations

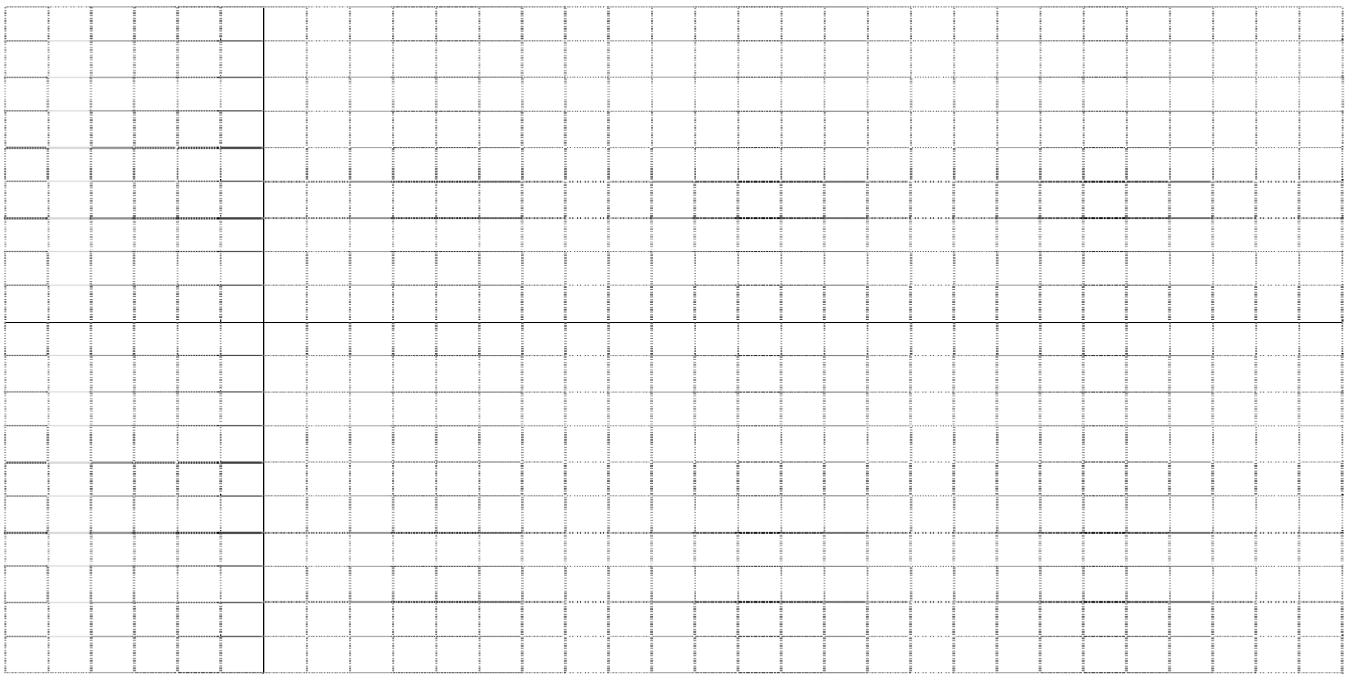
Amplitude and Period

For $y = a \cdot \sin kx$ and $y = a \cdot \cos kx$:

- amplitude = $|a|$, if $a < 0$, the function is reflected in the *x*-axis
- period = $\dfrac{360^\circ}{k}$.

Ex. 1: Graph $y = 3\sin\theta$ and $y = -\frac{1}{2}\sin\theta$ on the same set of axes for $0 \le \theta \le 540^\circ$.

(Use transformations based on the function $y = \sin\theta$).



Note:

Function	Range
$y = \sin\theta$	$-1 \le y \le 1$
$y = 3\sin\theta$	$-3 \le y \le 3$
$y = -\frac{1}{2}\sin\theta$	$-\frac{1}{2} \le y \le \frac{1}{2}$

The **AMPLITUDE** of a function in the form y = **a** $\sin\theta$ and y = **a** $\cos\theta$ is $|a|$.

Ex. 2: Graph $y = \cos 2\theta$ and $y = \cos\frac{1}{2}\theta$ on the same axes for $-180^\circ \leq \theta \leq 1040^\circ$.

(Use transformations based on the function $y = \cos\theta$).

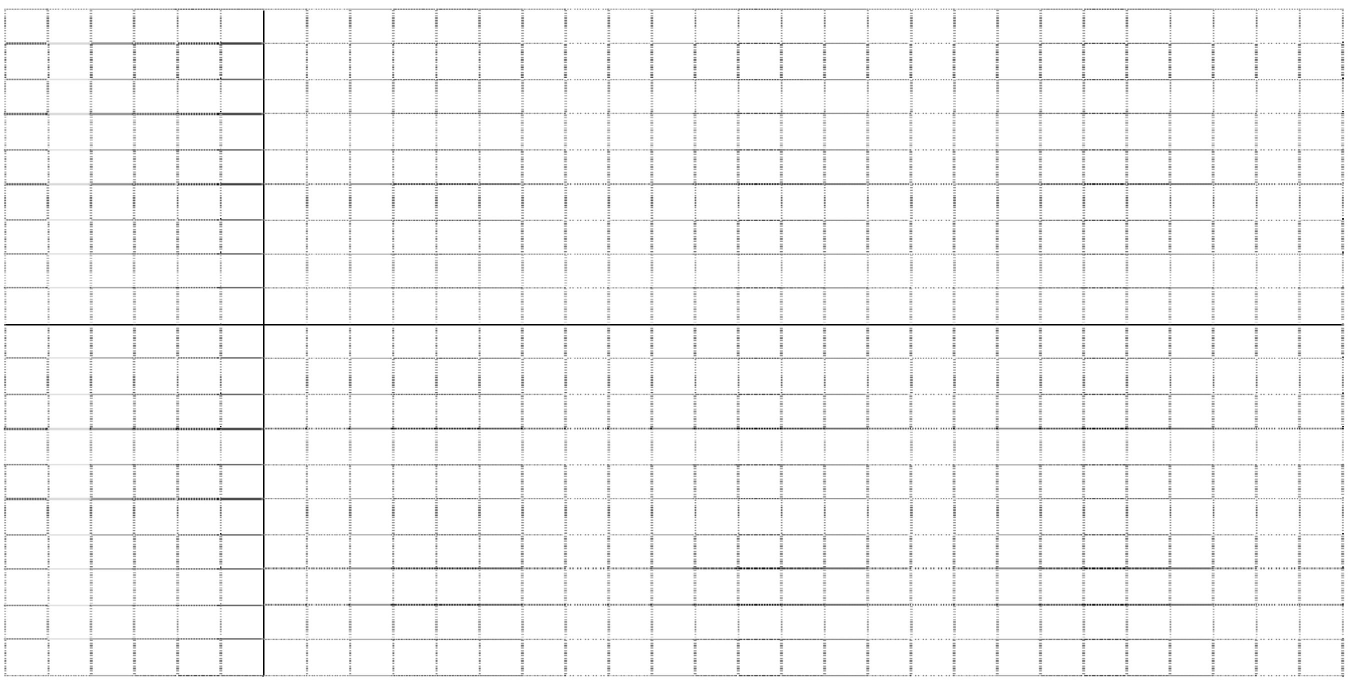
Recall: 1) The function y = f(kx) is a horizontal stretch when k < 1, and a horizontal compression when k > 1.
2) The period of $y = \cos\theta$ is 360°.

Note: When $\theta = 180^\circ$, $\cos 2\theta = \cos(360^\circ)$ which indicates that the function $y = \cos 2\theta$ completes one period from $\theta = 0$ to 180°.

⇨ This function's period is 180° (360 ÷ 2).

When $\theta = 720^\circ$, $\cos\frac{1}{2}\theta = \cos(360^\circ)$ which indicates that the function $y = \cos\frac{1}{2}\theta$ completes one period from $\theta = 0$ to 720°.

⇨ This function's period is 720° (360 ÷ ½).



The **PERIOD** of a function in the form $y = \sin \mathbf{k}\,\theta$ and $y = \cos \mathbf{k}\,\theta$ is $\frac{360^\circ}{k}$, $k > 0$.

Student Worksheet #4 - Trigonometric Transformations

1. Sketch the graphs of the following:

   a) $y = 3\sin 2\left(\theta - 45^\circ\right)$, $-180^\circ \le \theta \le 360^\circ$

   b) $y = \cos\left(3\theta + 90^\circ\right) + 1$, $-180^\circ \le \theta \le 180^\circ$

   c) $y = \frac{1}{2}\cos\left(2\theta - 90\right)$, $-180^\circ \le \theta \le 180^\circ$

   d) $y = \cos 2\theta + 3$, $-180^\circ \le \theta \le 360^\circ$

2. Complete the following chart.

Function	Vertical Translation with direction	Phase Shift with direction	Amplitude	Period
$y = -12\sin 3\theta + 2$				
$y = \frac{5}{2}\sin\left(2t - 150\right) - 1$				

Student Worksheet #4 - Trigonometric Transformations

1. Sketch the graphs of the following:

   a) $y = 3\sin 2\left(\theta - 45^\circ\right)$, $-180^\circ \le \theta \le 360^\circ$

   b) $y = \cos\left(3\theta + 90^\circ\right) + 1$, $-180^\circ \le \theta \le 180^\circ$

   c) $y = \frac{1}{2}\cos\left(2\theta - 90\right)$, $-180^\circ \le \theta \le 180^\circ$

   d) $y = \cos 2\theta + 3$, $-180^\circ \le \theta \le 360^\circ$

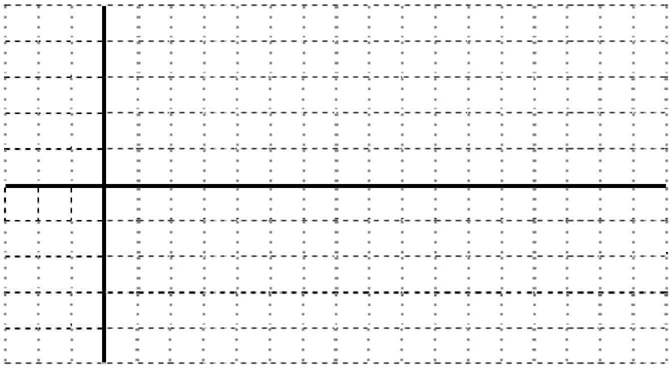
2. Complete the following chart.

Function	Vertical Translation with direction	Phase Shift with direction	Amplitude	Period
$y = -12\sin 3\theta + 2$				
$y = \frac{5}{2}\sin\left(2t - 150\right) - 1$				

Student Worksheet #5 - Trig Transformations Practice

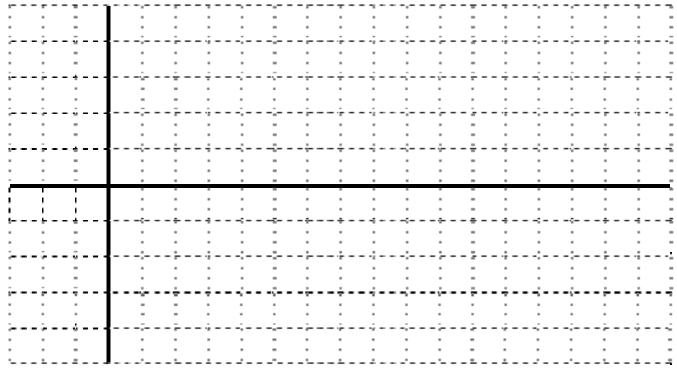
1. Graph each function and complete the information that follows each graph.

a) $f(\theta) = 3\sin(\theta - 45°) + 1$



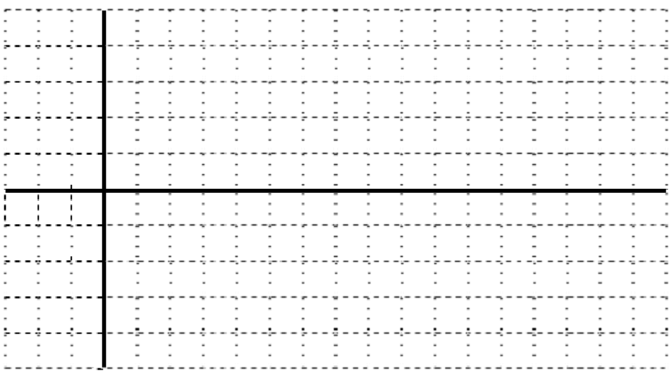
Period:
Amplitude:
Equation of the Axis:

b) $f(\theta) = -\cos(\theta + 30°) - 2$



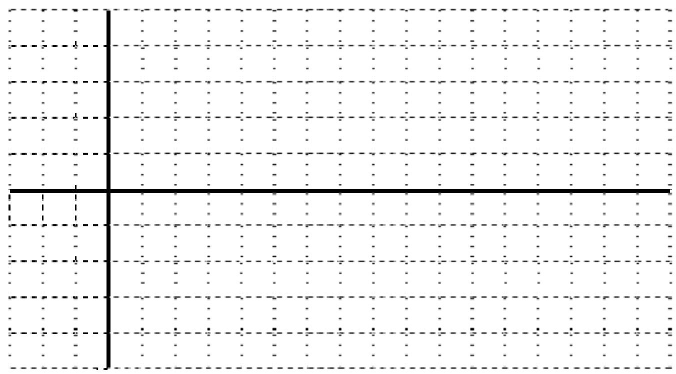
Period:
Amplitude:
Equation of the Axis:

c) $f(\theta) = -2\cos(2\theta + 120°)$



Period:
Amplitude:
Equation of the Axis:

d) $f(\theta) = 0.5\cos(3\theta + 90°) - 4$

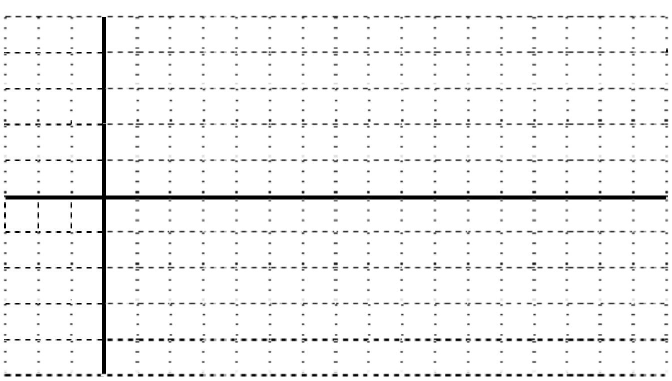


Period:
Amplitude:
Equation of the Axis:

2. Graph the data in each table. Then find the equation for each using $f(\theta) = a\sin(k\theta + c) + d$.

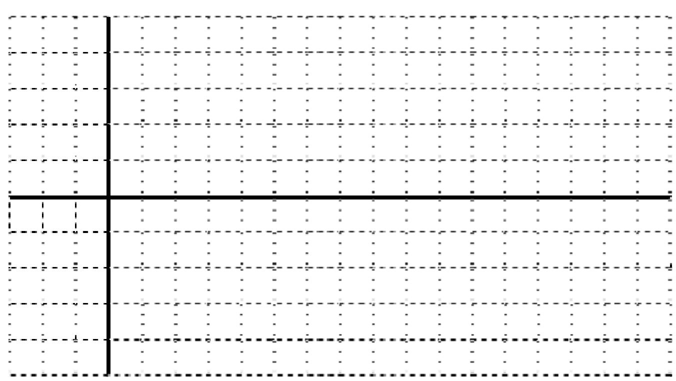
a) $f(\theta)$ = ___________ ___________ _____

θ°	-60	30	120	210	300
f(θ)	-1	0	-1	-2	-1



b) $f(\theta)$ = ___________ ___________ _____

θ°	45	135	225	315	405
f(θ)	2	-1	2	5	2

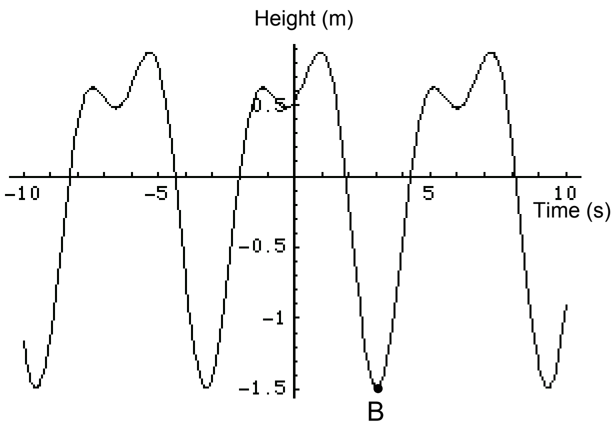


Appendix B

MCR 3UI – Periodic Functions Quiz

Name: ____________________

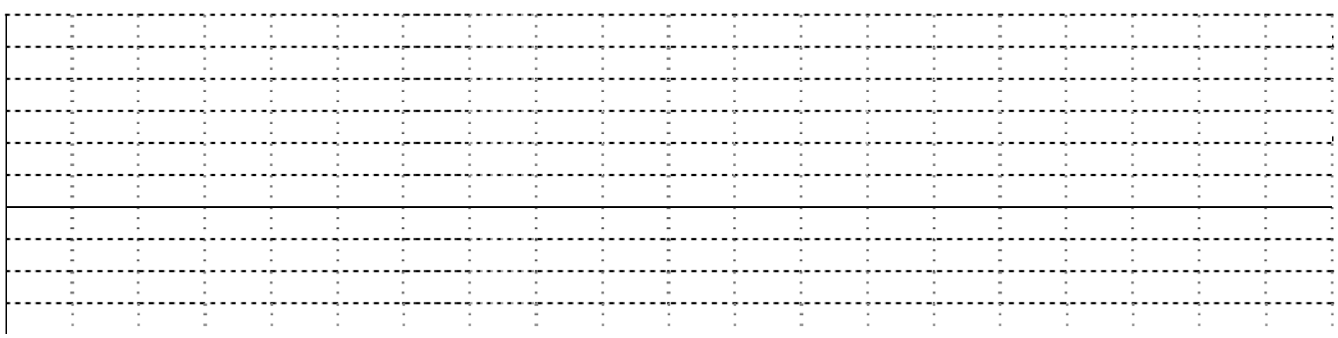
1) This graph displays the height of a buoy in the water on a wavy day.



a) Is the function periodic? Explain.

b) Explain what is happening to the buoy at 3 seconds. (ie. at point B)

2) Draw 3 cycles of any periodic function with a period of 2, an amplitude of 3 and a minimum of -2.



3) If $\sin\theta = 0.5$ and $-360^{o} \le \theta \le 360^{o}$, determine all values of θ to the nearest degree.

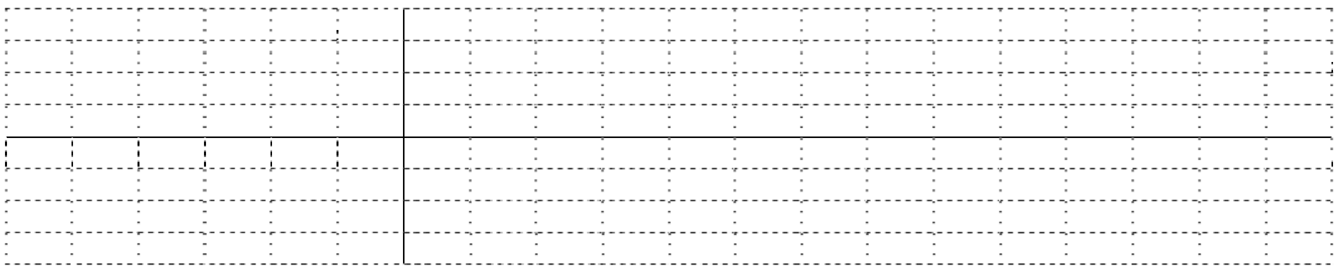
4) If $\cos\theta = -0.5$ and $-360^{o} \le \theta \le 360^{o}$, determine all values of θ to the nearest degree.

MCR 3UI – Trig Transformations Quiz

Name: ____________________

1. a) In words, describe the transformations applied to the graph of $y = \cos\theta$ in order to obtain the graph of the function $y = 3\cos 2\theta - 1$.

b) Graph $y = 3\cos 2\theta - 1$ for $-90^{\circ} \le \theta \le 450^{\circ}$. (Hint: You may want to start by graphing $y = \cos\theta$.)



2. a) In words, describe the transformations applied to the graph of $y = \sin\theta$ in order to obtain the graph of the function $y = \sin\left(\frac{1}{2}\theta - 180^{\circ}\right)$.

b) Graph $y = \sin\left(\frac{1}{2}\theta - 180^{\circ}\right)$ for $-180^{\circ} \le \theta \le 540^{\circ}$. (Hint: You may want to start by graphing $y = \sin\theta$.)