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THE PREDICTION OF THE PERFORMANCE OF A MICROCIRCUIT HEAT SINK IN THE BOILING MODE

by

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The Prediction of the Performance of a Microcircuit Heat Sink in the Boiling Mode

by

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ABSTRACT

As microcircuit technology advances, there is an increased need for the dissipation of the heat which is generated. Extended surfaces are a useful tool in fulfilling the heat sink requirements for a microcircuit element. Heat transfer is very effective in the boiling mode and this thesis focuses on the analysis of a spine-shaped extended surface in a boiling liquid. Because the heat transfer in the spine and the propagation of signals on a transmission line are governed by identical differential equations, an analysis procedure based on the cascading of electrical transmission lines is used to predict the performance of the spine. The result of the analysis is a computer program that can assist the circuit designer in meeting any heat transfer requirements.

TABLE OF CONTENTS

I. INTRODUCTION

A major concern in the design of electrical equipment is the dissipation of heat inherent in the use of the equipment. Specific measures must be taken to transfer the heat from the circuit element(s) where it originates to the ultimate heat sink. This thesis investigates the performance of a spine that augments the transfer of heat from a highly dissipating circuit element to a boiling liquid.

Extended surface technology has provided a means to transfer heat from microcircuit elements effectively and efficiently. There are many extended surface geometries that provide excellent heat transfer augmentation. However, this study will concentrate on the analysis of the spine geometry.

The study focuses mainly on a computer program based on a cascade analysis deriving from the theory of transmission lines. The algorithm will enable the user to obtain a prediction of the surface temperature of an operating microcircuit element. This prediction is based on certain input parameters such as the spine geometry and thermal properties. This can potentially aid in the circuit design process where a heat sink must provide a certain overall heat transfer capability. If the rate of heat transfer is not adequate, other factors such as the material, surrounding temperature, heat flow mechanisms, and spine dimensions can be adjusted and manipulated in order to arrive at desired dissipation rates.

A primary application for a spine heat sink is in the thermal control of a microcircuit element. In an electric circuit, the power source generates the required energy so that the individual circuit elements may carry out their function. In some instances the energy required may be significant. A large amount of this electrical energy is converted to thermal energy. This thermal energy can create, because of the very small size of the microcircuit, a heat flux which approaches that of the surface of the sun ($5x10^6$ W/m^2). An extended surface such as a spine can be employed to augment the transfer of much of this heat so that the circuit will not overheat and can operate with the desired efficiency. The thrust of this thesis is to establish an analogy between the electrical transmission line and heat flow in a spine and apply the electrical theory to the analysis of a thermal system. It has already been proved that electrical transmission lines can be analyzed by subdividing a lossy line into segments and then cascading these segments using a transmission matrix. After the equations for the electro-thermal transmission line analogy are presented, it is theorized that the spine can be analyzed using a similar cascade procedure. A computer program is developed based on this cascade analysis concept for a "thermal transmission line", in this case the spine.

The electrical transmission line concepts are included solely to present enough information to allow for a comparison to the thermal transmission line. It is not intended to be a rederivation of the transmission line equations.

II. REVIEW OF EXTENDED SURFACES

The earliest studies in the extended surface technology apparently occured in the year 1922. A paper by Harper and Brown (1922) reported on the mathematical analysis of heat transfer by conduction and convection occuring in a single extended surface. This extended surface became known simply as a fin. Some early experiments included coating metal rods with wax and observing the thermal properties as the bases of the rods were heated and the wax melted. The French mathematician Fourier (1822) also published mathematical analyses of the temperature variation of thin metal bars or rods, but these were not considered as fins or extended surface. Indeed, the work by Harper and Brown (1922) is considered to be the forerunner which has resulted in the advance of this significant subject area in the general field of heat transfer. From this modest beginning, the analysis and evaluation of extended surfaces has blossomed from a simple fin to arrays of surfaces where individual components are assembled into elaborate configurations.

Some typical examples of extended surface are displayed in Figure 2.1. It can be observed that the more typical geometries are rectangular or spine shaped. Depending on the application, variations of these geometries can be formed to suit individual design needs. The extended surfaces shown are

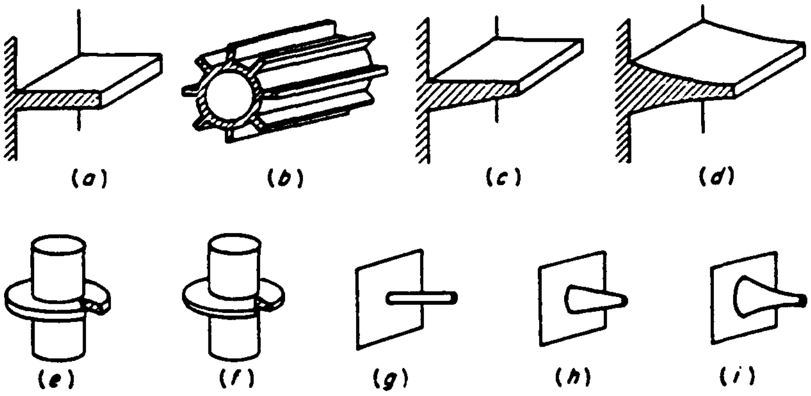


Figure 2.1. Some typical examples of extended surfaces

but a small sampling of those currently in use and/or under investigation.

Specific terminology to be used is shown in Figure 2.2 for the spine of cylindrical profile which is sometimes referred to as the pin fin. Attention should be focused on what is meant by fin height. This is the distance between the fin base and the fin tip. The fin diameter is simply the diameter of the cylindrical spine and is assumed to be constant. It is noted that the origin of the height coordinate is at the fin tip with positive orientation in a direction toward the base. If the prime surface (also called the base surface) is at a higher temperature than the environment, the heat flow in the

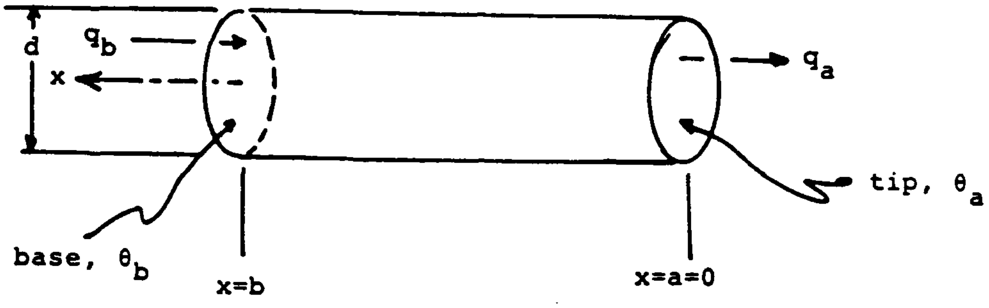


Figure 2.2. A cylindrical spine
[from Ref. 9]

spine is from base to tip in a direction that is opposed to the positive sense of the coordinate system.

It was determined by the early investigators in the extended surface technology that a set of assumptions were needed to make the mathematical analysis tractable. These have been known since 1945 as the Murray-Gardner (1945) assumptions and are taken from the papers by Murray (1938) and Gardner (1945). The assumptions have considerable importance to ongoing analyses and research because their elimination, either one at a time or in any combination, provide a series of guidelines for subsequent investigators to follow. The following assumptions regarding extended surface analysis are taken directly from the Gardner (1945) paper.

- The heat flow and temperature distribution throughout the fin are independent of time, i.e., the heat flow is steady.
- The fin material is homogeneous and isotropic.

- There are no heat sources in the fin itself.
- The heat flow to or from the fin surface at any point is directly proportional to the temperature difference between the surface at that point and the surrounding fluid.
- The thermal conductivity of the fin is constant.
- The heat transfer coefficient is the same over all the fin surface.
- The temperature of the surrounding fluid is uniform.
- The temperature of the base of the fin is uniform.
- The thickness is so small compared to its height that temperature gradients normal to the surface may be neglected.
- The heat transferred through the outermost edge of the fin is negligible compared to that passing through the sides.

Kern (1950) added another assumption to Gardner's list:

- The joint between the fin and the tube or prime surface is assumed to offer no bond resistance.

III. FIN EFFICIENCY

The fin efficiency has been defined as the ratio of the heat transferred from the fin to the heat that would be transferred by the fin if its thermal conductivity were infinite (if the entire fin were to operate at the base temperature). Fin efficiency is a function of the fin dimensions and thermal properties. Gardner (1945) defined a "fin effectiveness" as the ratio of the heat transferred into the base of the fin to the heat transferred through the same prime or base surface area if the fin were not present. He also determined a relationship to permit the conversion from fin efficiency to fin effectiveness. Although the concept of fin effectiveness has been in question, the theory is quite simple.

Using $\theta_b = T_b - T_s$ as the base temperature excess above the surroundings in the heat transfer rate equation,

$$q = hS\theta_b \tag{1}$$

the surface, S, to be used is composed of fin surface S_f and base or prime surface S_b. Because of the temperature gradient within the fin, all of the fin surface does not operate at θ_b. The fin efficiency η modifies the fin surface to allow the total surface area S to be represented by

$$S = S_b + \eta S_f \tag{2}$$

so that for ease of computation all of the surface can be presumed to operate at the base temperature excess, θ_b.

IV. HEAT TRANSFER IN THE BOILING MODE

Boiling may occur when a surface is immersed in a liquid and is maintained at a temperature above the saturation temperature of the liquid. The heat flux will depend on the difference in temperature between the surface and the saturation temperature. Subcooled or local boiling occurs if the bulk temperature of the liquid is below the saturation temperature. If the liquid is sustained at the saturation temperature, the process is referred to as saturated, or bulk, boiling.

The boiling curve is shown in Figure 4.1 where heat flux data from an electrically heated platinum wire submerged in water are plotted against temperature excess. The heat transfer data for boiling water was first obtained by Collier (1972). The temperature excess in this case is the temperature difference between the heated wire and the surrounding liquid. In region I, the liquid near the heated surface is superheated slightly at the lower temperatures. The heated water subsequently rises to the surface where it evaporates. In region II bubbles begin to form on the surface of the wire and dissipate into the liquid as they break away from the surface. This region marks the beginning of the nucleate boiling regime. With a further increase in temperature excess, bubble

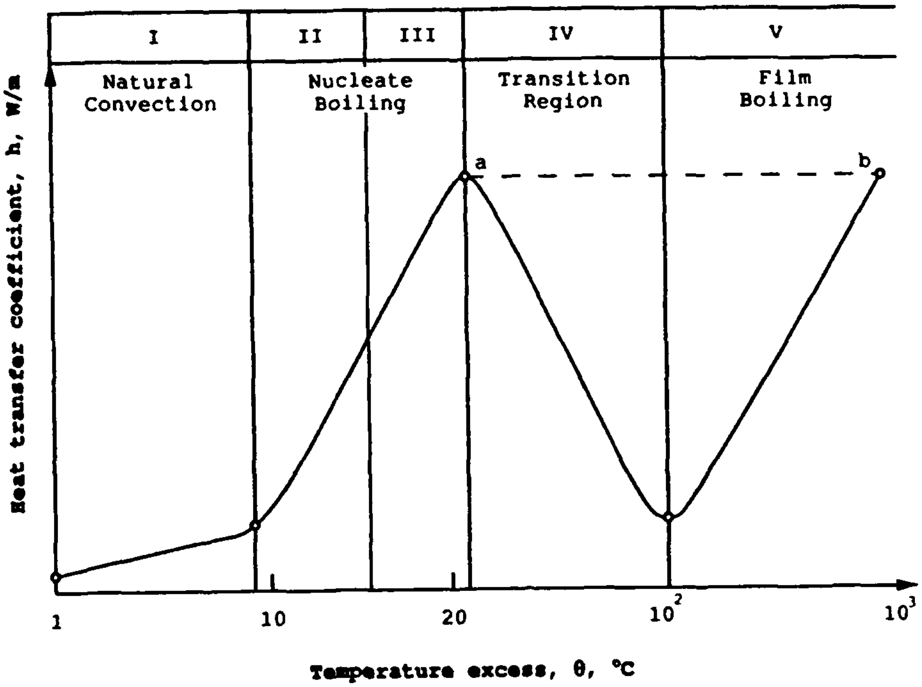


Figure 4.1 The boiling curve

formation is more rapid. These bubbles break away and rise to the surface where they are dissipated. This is a characteristic of region III.

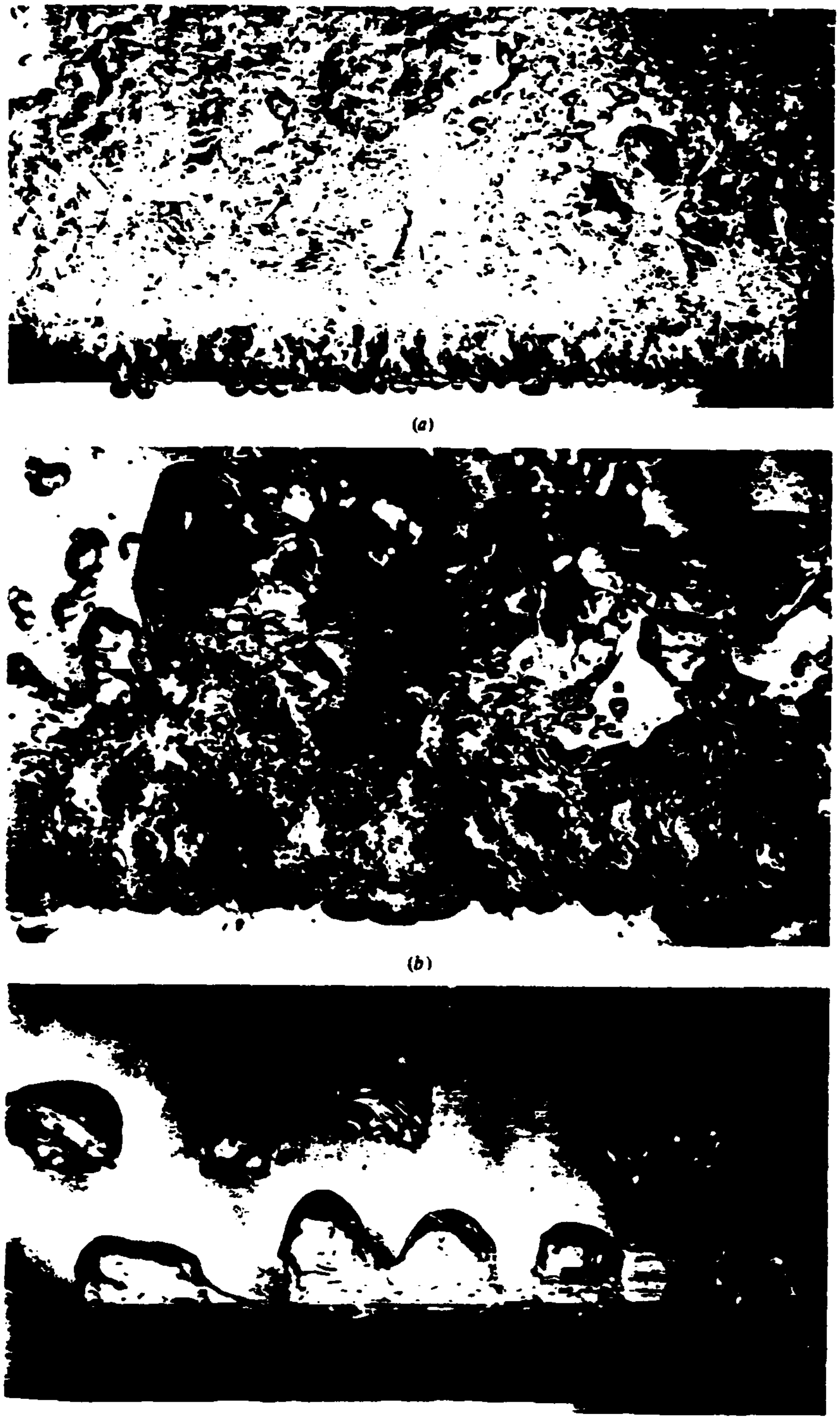
Eventually, the bubble formation becomes so rapid that they totally envelop the heating surface. This prevents any inflow of liquid from taking their place. At this point the bubbles coalesce, forming a thin vapor film which blankets the surface. This vapor film acts as an insulator between the heated surface and the liquid. The heat must be conducted through this film to the liquid in order to effect the boiling process. The thermal resistance of this insulating film causes a dramatic reduction in heat flux. This phenomenon is known as transition to film boiling and is illustrated in region IV.

Note the negative slope of this region as compared to the positive slope of region III. Region IV is the transition region from nucleate boiling to stable film boiling and is unstable.

Region V depicts stable film boiling. The surface temperatures required to maintain stable film boiling are high. Much of the heat lost in this region may be the result of thermal radiation. This region shows the return of the positive slope on the boiling curve. The curve in region V continues upward until the melting point of the heated surface is reached.

The boiling process is unstable at point "a" on the boiling curve because a decrease in the heat flux results from just a small increase in temperature excess. Because the increased heat flux requirement is not met, operation further along the boiling curve results. Eventually, equilibrium can again be attained at point "b" in the film boiling region. It can be observed that an increase in temperature excess at this point yields a corresponding increase in heat flux. However, this is rarely seen in practice because this temperature usually exceeds the melting point of the material and burnout results. It is obvious that the preferred regions of the boiling curve for heat transfer are in regions II and III where nucleate boiling exists.

Detailed photographs of the boiling modes using methanol as the liquid coolant are shown in Figure 4.2. They



(a)

(b)

(c)

Figure 4.2. Photographs of the boiling modes [from Ref. 9]

illustrate the energetic nucleate boiling regime. Transition boiling is seen in Figure 4.2b as the bubbles begin to coalesce at higher temperatures. Figure 4.2c shows film boiling and the thin vapor layer can be seen as well as the dramatic decrease in bubble formation.

A. FIN OPTIMIZATION IN THE BOILING MODE

The optimization of a cylindrical spine dissipating heat to a boiling liquid was thoroughly studied by Haley and Westwater (1966). Their determinations of preferred heat flow, temperature gradient and spine dimensions were selected for a minimum spine volume rather than a minimum spine area profile. A constant thermal conductivity was assumed even though, realistically, a variation in exterior heat transfer coefficient and fin material thermal conductivity usually exists.

It was found that several types of boiling existed simultaneously on the spine. The boiling forms most often encountered were nucleate and film boiling. This was the result of different heat transfer coefficients for various sections of the spine. A numerical solution for the heat transfer coefficient was derived using boiling curves, Simpson's rule and a fourth order Runge-Kutta approach. The optimum spine resulting from the previous determinations possesses a spadelike shape.

The unusual spine geometry is justified when the distribution of the surface heat flux is considered. Heat transfer is greater in the nucleate and transition boiling modes than it is in the natural convection and film boiling modes. To keep the surface area in the film boiling region to a minimum, the base of the spine has a small cross section. This enables the transition from film boiling to nucleate boiling to occur over a short distance. At the point where the heat transfer coefficient increases, the spine diameter is increased dramatically. This allows for a larger surface area to be exposed to the liquid operating in the transition and nucleate boiling regions. At the tip where the heat transfer coefficient decreases with lower temperatures, the spine tapers to a point. The spine with this shape makes optimum use of the most effective regions on the boiling curve.

It was pointed out by Siman-Tov (1970) that the spade shaped spine had several undesirable attributes. Some major concerns were the difficulty and expense to manufacture such a design. Moreover, the spine would be difficult to weld and when attached to the prime surface, the small spine base could make the structure too fragile. Versions of this optimum spine were proposed that would include the basic design theory and also overcome some of these operational problems. Siman-Tov (1970) introduced a series of longitudinal fins protruding from a cylinder and a disk-type fin attached to a cylindrical spine in a "thumbtack" type of configuration. Cash, Klein, and Westwater (1971) concluded that two cones attached to a small cylindrical neck was a good approximation to the optimum turnip-shaped spine. A computer was used to predict the heat transfer capabilities of this two-cone arrangement. As expected, this arrangement had greater heat transfer rates per unit volume than those calculated for a cylindrical spine. Experiments proved that the computer analysis was conservative and that the actual fins gave higher performance than the computed prediction. This, and the other works discussed, showed that the use of extended surfaces in the boiling mode can significantly increase the heat dissipation.

V. MATHEMATICS OF EXTENDED SURFACE ANALYSIS

The heat transfer characteristics of individual fins are analyzed by a procedure summarized by Kern and Kraus (1972). The procedure begins with the differential equations relating the temperature excess and heat flow in each fin. The differential equations are solved using initial values for the temperature excess and heat flow. When many fins are physically coupled the problem then becomes one of solving a set of linear algebraic equations after the individual differential equations are integrated analytically.

The procedure developed here treats each fin in the configuration as a lumped parameter matrix. Each fin or fin section can then be combined via matrix operations. Some improvements of this algorithm over existing techniques are as follows:

- The differential equations for the individual fins are uncoupled.
- The process which mathematically assembles the characteristics of the entire configuration from the characteristics of the individual fins is simple. It may even be done on a hand calculator and the effect of each parameter is easily isolated.
- The performance of the individual fin can be measured by the ratio of the total heat dissipated to the temperature excess existing at the base of the fin.
- While the analysis evaluates heat exchange operating under the Murray-Gardner considerations, some of these may be relaxed. The two Murray-Gardner assumptions which are relaxed in this procedure are as follows:

- * There is no contact resistance between the base of the fin and the prime surface.
- * The heat transferred through the outermost edge of the fin (the fin tip) is negligible compared to that transferred through the lateral surfaces of the fin.

A. THE HEAT FLOW IN A SPINE

The heat transfer texts tell us that for the cylindrical spine displayed in Figure 2.2, the differential equation for the temperature excess, θ, above some datum level (the environmental temperature) will be

$$\frac{d\theta^2}{dx^2} - m^2\theta = 0 \tag{3}$$

where

$$m = \sqrt{4h / kd} \tag{4}$$

Because Eq(3) is an ordinary second order differential equation, it has a general solution given by

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \tag{5}$$

According to Fourier's law, the heat flow will be

$$q(x) = kA\frac{d\theta}{dx} = kAm\left[C_1 e^{mx} - C_2 e^{-mx}\right] \tag{6}$$

because the height coordinate orientation is opposed to the heat flow.

The arbitrary constants C_1 and C_2 in Eqs(5) and (6) can be evaluated from the initial conditions

$$\theta(x=0)=\theta_a=C_1+C_2 \tag{7}$$

and

$$q(x=0)=q_a=hAm[C_1-C_2] \tag{8}$$

When these are inserted into Eqs(5) and (6), two expressions containing C_1 and C_2 are obtained

$$C_1+C_2=\theta_a \tag{9}$$

and

$$C_1-C_2=\frac{q_a}{hAm} \tag{10}$$

The addition of Eqs(9) and (10) gives

$$2C_1=\theta_a+\frac{q_a}{hAm}=\theta_a+Y_0q_a \tag{11}$$

where $Y = 1/hAm = (\pi^2 hkd^3)^{1/2}/2$

Solving for C_1 and C_2 yields

$$C_1=\frac{1}{2}[\theta_a+Y_0q_a] \tag{12}$$

and

$$C_2=\theta_a-Y_0q_a \tag{13}$$

When these constants are substituted into Eqs(5) and (6), the expressions for the temperature excess and heat flow can be written as hyberbolic sines and cosines. The temperature excess and heat flow at any point "x" will be

$$\theta(x) = \theta_a \cosh mx + Y_0 q_a \sinh mx \tag{14}$$

and

$$q(x) = hAm\theta_a \sinh mx + hAmY_0 \cosh mx \tag{15}$$

In particular, when x=b a linear transformation, mapping conditions from the spine tip to the spine base, can be proposed and the matrix of this linear transformation is

$$\tau = \begin{bmatrix} \cosh mb & Z_0 \sinh mb \\ Y_0 \sinh mb & \cosh mb \end{bmatrix} \tag{16}$$

B. THE TRANSMISSION LINE ANALOGY

With the use of transmission line concepts, the distributed heat flow pattern of a spine can be represented by an analogous distributed electrical network. To analyze this system, the transmission line can be represented as a series of lumped parameter "tee" sections. Consider a line of length, L, with a length coordinate, x, having its origin at the receiving end and its positive orientation toward the sending end. Such a line is shown in Figure 5.1a where the subscripts, S and R, refer to the sending and the receiving ends respectively. An isolated differential section of transmission line is displayed in Figure 5.1b.

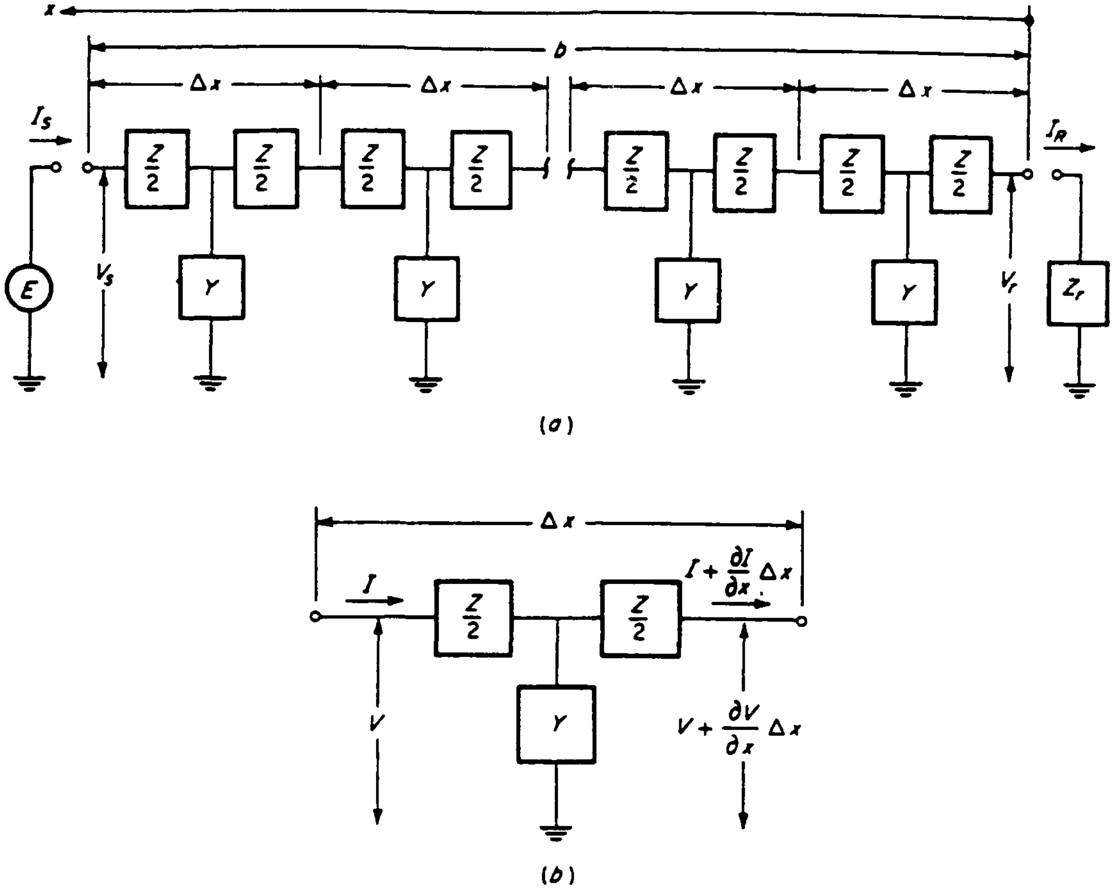


Figure 5.1. The electrical transmission line
[from Ref. 9]

The two Kirchoff laws may be used to obtain the so-called 'telegraph equations' which relate the voltage and the current at any time and at any point along the line. These telegraph equations are

$$\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2} + (RC + LG)\frac{\partial v}{\partial t} + RGv \qquad (17)$$

and

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} + (RC + LG)\frac{\partial i}{\partial t} + RGi \qquad (18)$$

Consider a lossy line in which L=0 and C=0. In this event Eqs(17) and (18) become

$$\frac{d^2v}{dx^2} - \alpha v = 0 \tag{19}$$

and

$$\frac{d^2i}{dx^2} - \alpha i = 0 \tag{20}$$

where the partial derivative is no longer needed because neither v nor i depend on time. These ordinary second order differential equations have general solutions

$$v(x) = V_1 e^{\alpha x} + V_2 e^{-\alpha x} \tag{21}$$

and

$$i(x) = I_1 e^{\alpha x} + I_2 e^{-\alpha x} \tag{22}$$

Use of the initial conditions at the receiving end of the line where x=0 gives

$$v(0) = V_R \tag{23}$$

and

$$i(0) = I_R \tag{24}$$

At any point x along the line,

$$\frac{dv}{dx} = Ri \tag{25}$$

and

$$\frac{di}{dx} = Gv \tag{26}$$

and these, coupled with Eqs(23) and (24), yield the total solution for the voltage and current at any point along the line

$$v(x) = V_R \cosh \alpha x + Z_0 I_R \sinh \alpha x \tag{27}$$

and

$$i(x) = Y_0 V_R \sinh \alpha x + I_R \cosh \alpha x \tag{28}$$

where Z_0 is the characteristic impedance of the line

$$Z_0 = \sqrt{RG} \tag{29}$$

and where Y_0 is the characteristic admittance of the line

$$Y_0 = \frac{1}{Z_0} = \frac{1}{\sqrt{RG}} \tag{30}$$

The form of Eqs(27) and (28) suggests a linear transformation from conditions of voltage and current at the receiving end to any point along the line.

$$\begin{bmatrix} v(x) \\ i(x) \end{bmatrix} = \begin{bmatrix} \cosh \alpha x & Z_0 \sinh \alpha x \\ Y_0 \sinh \alpha x & \cosh \alpha x \end{bmatrix} \begin{bmatrix} v_R \\ I_R \end{bmatrix} \tag{31}$$

In the case at the sending end where x=b this transformation can be written as

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cosh \alpha b & Z_0 \sinh \alpha b \\ Y_0 \sinh \alpha b & \cosh \alpha b \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \tag{32}$$

A comparison of Eqs(16) and (32) shows the similarities. The linear transformation matrix for heat flow and temperature excess is almost identical to that for voltage and current in a transmission line. The only difference is that the value α in the electrical network is replaced by m in the thermal network. These parameters contain constants obtained from solutions to the respective differential equations. Therefore, there is an obvious analogy between the thermal transmission line and the electrical transmission line.

Utilizing electrical two-port theory, the entire transmission line can be segmented into a series of minute yet finite sections. These sections can then be analyzed individually with the results of each assembled via a cascade procedure. This results in the treatment of each section as a two-port system indicated by Figure 5.2. The output of one section is the input to the next. This method has proven to be useful and accurate in the analysis of electrical transmission lines.

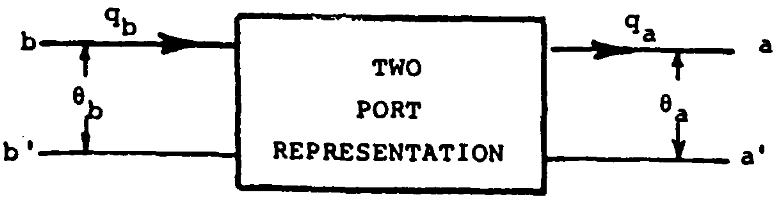


Figure 5.2. A two-port representation

It is logical that a similar cascade procedure may be successful in analyzing a fin or spine which is now referred to as a thermal transmission line. The linear transformation matrices are equivalent when α is replaced by m. In the thermal case, heat flow q and temperature excess θ are the parameters which are "transformed" in each individual segment. Certain extended surface geometries, specifically the spine, allow for direct application of the cascade procedure for analysis. Two cascaded subfins are shown in Figure 5.3. A computer program was developed to test this procedure for a spine.

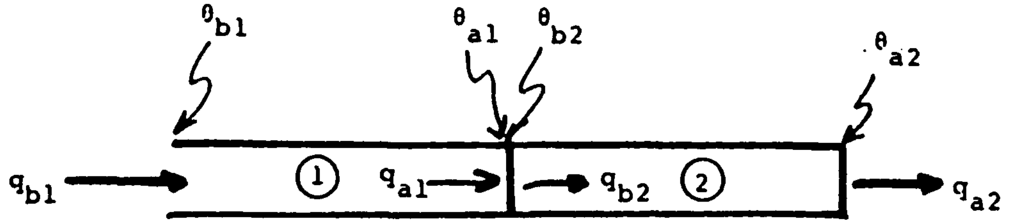


Figure 5.3. Two cascaded subfins
[from Ref. 9]

C. THE ALGORITHM FOR THE CASCADE PROCEDURE

As mentioned earlier, a cascade algorithm was developed to analyze the heat transfer properties of a spine. The spine is divided into segments, or subfins, and each section is treated as an independent two-port system. The computations begin at the tip and conclude at the section nearest to the base surface. The inputs to the program are:

- Temperature at the fin tip
- Temperature of the surrounding liquid (boiling liquid)
- Thermal conductivity of the spine material
- Diameter of the spine
- Height of the spine

A flowchart for the cascade algorithm is shown in Figure 5.4.

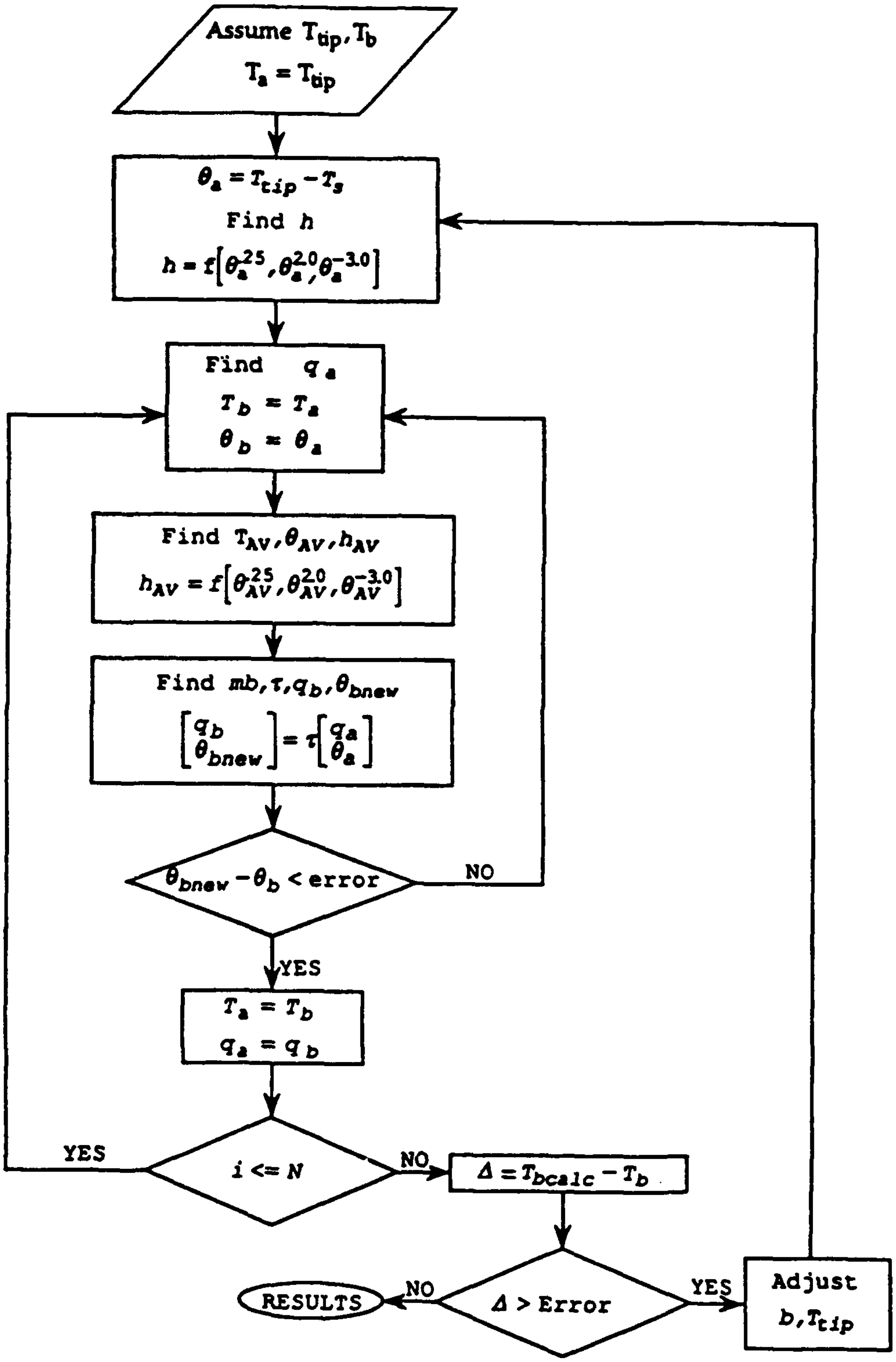


Figure 5.4. Flowchart for the cascade algorithm

The following are the significant steps in the cascade algorithm:

1) Assume the tip temperature, T_a

   This value is entered by the user and must be greater than the saturation temperature.

2) Compute the spine tip temperature excess, θ_a

   The temperature excess is found by subtracting the surrounding temperature from the tip temperature,

   $\theta_a = T_a - T_s$.

3) Find the heat-transfer coeficient, h

   The heat-transfer coefficient is dependent on the temperature excess, θ_a determined in step 2. The following are based on data given by Collier (1972).

   - If θ_a is between 0.0 and 7.0 degrees, $h = 1300.0(\theta_a)^{0.25}$,

     and the subfin is in the natural convection mode.

   - If θ_a is between 7.0 and 25.0 degrees, $h = 76.8(\theta_a)^{2.0}$,

     and the subfin is in the pool-nucleate boiling mode.

   - If θ_a is between 25.0 and 280.0 degrees, $h = 7.5 \times 10^6/(\theta_a)^{3.0}$,

     and the subfin is in the transition boiling mode.

   - If θ_a is greater than 280.0 degrees, $h = 800.0(\theta_a)^{0.2}$,

     and the subfin is in the film boiling mode.

4) Compute the heat flow, q_a

   The heat flow, q_a , is derived from the relation, $q_a = h(\pi/4)d^2\theta_a$.

5) Assume T_b and calculate θ_b

T_b is given an initial value of T_a
θ_b is then equal to θ_a

6) Compute the average temperature for the subfin, T_{AV}

This is found by averaging T_a and T_b
$T_{AV} = (T_a + T_b)/2$

7) Compute the average tmperature excess, θ_{AV}

The average temperature excess is found using T_{AV} from step 6, $\theta_{AV} = T_{AV} - T_s$

8) Compute the average heat-transfer coefficient, h_{AV}

The average heat-transfer coefficient is a function of θ_{AV} from step 7. As shown in step 3, the equation for calculating h_{AV} depends on which boiling mode the subfin is experiencing. To find h_{AV}, the equations in step 3 are used with θ_{AV} replacing θ_a.

9) Calculate mb and Y_o

This value is found using the relation $mb = \sqrt{4h_{AV}/kd}(L/N)$ where L is the length of the entire spine and N is the number of subfins.

The subfin admittance, Y_o is found using the relation $Y_o = \sqrt{\pi^2 hkd^2/2}$

10) Find the values for the transmission matrix, 'A,B,C,D' in

$$\begin{bmatrix} q_a \\ \theta_a \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} q_b \\ \theta_b \end{bmatrix}$$

where

$A = \cosh mb$
$B = 1/Y_o \sinh mb$
$C = Y_o \sinh mb$
$D = \cosh mb$

11) Update $\theta_b = \theta_{bnew}$ and compute q_b

The temperature excess, θ_{bnew} is updated and heat flow, q_b is calculated using the transformation matrix from step 10.

12) Compare the temperature excesses, θ_b and θ_{bnew}

The updated temperature excess, θ_{bnew}, from step 11 is compared to the temperature excess, θ_b, from step 5. If the values are equal or within a margin of error, θ_{bnew} becomes θ_a and q_b becomes q_a for the next subfin. The algorithm then returns to step 6 and continues. If the values are not equal, θ_b is given the value θ_{bnew}. A new value for T_b is then computed and the algorithm returns to step 6 and continues. The last calculation in the subfin under analysis is q_b/θ_b, the input admittance, Y_{in}.

$$Y_{in} = q_b/\theta_b = [C + D(q_a/\theta_a)]/[A + B(q_a/\theta_a)]$$

VI. THE CASCADE PROGRAM

As previously indicated, the cascade program is based on a thermal transmission line model. The program that was developed is a menu-driven, user-friendly software package which will assist the user in analyzing heat transfer requirements. It is intended that the program be used as a tool to ascertain whether or not a spine is a suitable heat sink to meet the circuit design needs.

The results of the program address two general design considerations. One is the adequacy of the heat transfer and the other is the compatability of the spine with regard to size limitations. Given the miniaturization of modern microcircuit packages, it is obvious that an effective heat sink may have strict size limitations. This chapter considers these aspects of the cascade program. Several sample problems are provided in order to give an assessment of the program's attributes.

A. FEATURES

The program is menu driven providing the user with a series of parameter inputs. Upon entering the program, the user will be provided with the option of viewing an optional overview. First-time users should consider reading this overview. The program is designed to operate with either upper or lower-case letters. All units used in the algorithm are SI units.

Following the overview option, the user is asked to enter the geometric parameters for the spine under analysis. The first input is the spine height. It is recommended that a height of approximately 25 mm be entered initially for base temperatures between 115°C and 200°C. The algorithm adjusts the spine height to the minimum required in order to meet the desired base temperature. The user then inputs the spine diameter which is not adjusted by the program.

Spines may be manufactured from an assortment of materials, each with its own thermal conductivity. The program provides the option to test spines of different materials by entering the appropriate values.

The user is then prompted to input the saturation temperature of the fluid. This value should be at least 100°C because the spine is analyzed in the boiling mode. After selecting the saturation temperature, the user must specify the desired base temperature. This is the surface temperature of the microcircuit element. The final entry is the desired spine tip temperature. The program may adjust this value slightly in order to meet the specified base temperature.

With all of the data entered by the user, the program calculates key parameters concerning the spine's operational characteristics. The spine base temperature is predicted within 0.1°C of the user's desired base temperature. The tip temperature may be adjusted slightly by the program and the temperature which is predicted is presented as output for observation by the user. The heat flow rate in watts is also provided as output.

The spine is operating in the boiling mode and the user may wish to ascertain the boiling regimes that are predicted. The algorithm has the capability to distinguish the percent of the spine operating in natural convection, nucleate boiling, and transition boiling. The film boiling mode is not considered in the program because microcircuit applications do not require film boiling as a means to dissipate heat. However, a simple modification to the program will permit film boiling to be analyzed.

B. SAMPLE PROBLEMS

The following terminal session is a typical example of the cascade program and its capabilities. Six sets of parameters are tested and the respective outputs can be observed.

```
>>>>>>> RUN #1:

Enter the height of the spine in millimeters:  20

Enter the diameter of the spine in millimeters:  2

Enter the thermal conductivity of the spine material
in watts per meter per deg C:  202

Enter the saturation temperature of the
fluid in deg C:  100
```

```
Enter the temperature of base surface in deg C:  110

Enter the desired tip temperature in deg C:  102


You have entered the following parameters:

Spine height(mm):           20.0
Spine diameter(mm):          2.0
Thermal conductivity:      202.0
Saturation temperature:    100.0
Base temperature:          110.0
Tip temperature:           102.0


Do you want to make any changes?:

Enter "Y" for yes, "N" for no.

N


The computed spine base temperature is (deg C): 109.8

The spine tip temperature is (deg C):           101.8

The heat dissipated by the spine is (watts):      1.2

The revised spine height is (mm):                19.0

The spine diameter (user input) is (mm):          2.0

Percentage of spine in natural convection is:    89.0

Percentage of spine in nucleate boiling is:      11.0

Percentage of spine in transition boiling is:     0.0


Do you want to enter another data set?
Enter "Y" for yes, "N" for no.

Y

>>>>>>> RUN #2:


Enter the height of the spine in millimeters:  20
```

```
Enter the diameter of the spine in millimeters:  2

Enter the thermal conductivity of the spine material
in watts per meter per deg C:  202

Enter the saturation temperature of the
fluid in deg C:  100

Enter the temperature of base surface in deg C:  115

Enter the desired tip temperature in deg C:  102


You have entered the following parameters:

Spine height(mm):           20.0
Spine diameter(mm):          2.0
Thermal conductivity:      202.0
Saturation temperature:    100.0
Base temperature:          115.0
Tip temperature:           102.0


Do you want to make any changes?:

Enter "Y" for yes, "N" for no.

N


The computed spine base temperature is (deg C):   115.0

The spine tip temperature is (deg C):             101.9

The heat dissipated by the spine is (watts):        2.8

The revised spine height is (mm):                  20.0

The spine diameter (user input) is (mm):            2.0

Percentage of spine in natural convection is:      81.0

Percentage of spine in nucleate boiling is:        19.0

Percentage of spine in transition boiling is:       0.0


Do you want to enter another data set?
Enter "Y" for yes, "N" for no.
Y
```

```
>>>>>>> RUN #3:

Enter the height of the spine in millimeters:  20

Enter the diameter of the spine in millimeters:  2

Enter the thermal conductivity of the spine material
in watts per meter per deg C:  202

Enter the saturation temperature of the
fluid in deg C:  100

Enter the temperature of base surface in deg C:  125

Enter the desired tip temperature in deg C:  102


You have entered the following parameters:

Spine height(mm):          20.0
Spine diameter(mm):         2.0
Thermal conductivity:     202.0
Saturation temperature:   100.0
Base temperature:         125.0
Tip temperature:          102.0


Do you want to make any changes?:

Enter "Y" for yes, "N" for no.

N

The computed spine base temperature is (deg C):   125.1

The spine tip temperature is (deg C):             102.0

The heat dissipated by the spine is (watts):        7.8

The revised spine height is (mm):                  21.0

The spine diameter (user input) is (mm):            2.0

Percentage of spine in natural convection is:      75.0

Percentage of spine in nucleate boiling is:        25.0

Percentage of spine in transition boiling is:       0.0
```

```
Do you want to enter another data set?
Enter "Y" for yes, "N" for no.

Y


>>>>>>> RUN #4:


Enter the height of the spine in millimeters:  30

Enter the diameter of the spine in millimeters:  2

Enter the thermal conductivity of the spine material
in watts per meter per deg C:  202

Enter the saturation temperature of the
fluid in deg C:  100

Enter the temperature of base surface in deg C:  125

Enter the desired tip temperature in deg C:  102


You have entered the following parameters:

Spine height(mm):            30.0
Spine diameter(mm):           2.0
Thermal conductivity:       202.0
Saturation temperature:     100.0
Base temperature:           125.0
Tip temperature:            102.0


Do you want to make any changes?:

Enter "Y" for yes, "N" for no.

N


The computed spine base temperature is (deg C):   125.1

The spine tip temperature is (deg C):             102.0

The heat dissipated by the spine is (watts):        7.8

The revised spine height is (mm):                  21.0
```

```
The spine diameter (user input) is (mm):           2.0

Percentage of spine in natural convection is:     75.0

Percentage of spine in nucleate boiling is:       25.0

Percentage of spine in transition boiling is:      0.0


Do you want to enter another data set?
Enter "Y" for yes, "N" for no.

Y


>>>>>>> RUN #5:


Enter the height of the spine in millimeters:  20

Enter the diameter of the spine in millimeters:  3

Enter the thermal conductivity of the spine material
in watts per meter per deg C:  202

Enter the saturation temperature of the
fluid in deg C:  100

Enter the temperature of base surface in deg C:  115

Enter the desired tip temperature in deg C:  102


You have entered the following parameters:

Spine height(mm):          20.0
Spine diameter(mm):         3.0
Thermal conductivity:     202.0
Saturation temperature:   100.0
Base temperature:         115.0
Tip temperature:          102.0


Do you want to make any changes?:

Enter "Y" for yes, "N" for no.

N
```

```
The computed spine base temperature is (deg C):  114.8

The spine tip temperature is (deg C):            102.0

The heat dissipated by the spine is (watts):       5.0

The revised spine height is (mm):                 25.0

The spine diameter (user input) is (mm):           3.0

Percentage of spine in natural convection is:     81.0

Percentage of spine in nucleate boiling is:       19.0

Percentage of spine in transition boiling is:      0.0


Do you want to enter another data set?
Enter "Y" for yes, "N" for no.

Y

>>>>>>> RUN #6:


Enter the height of the spine in millimeters:  20

Enter the diameter of the spine in millimeters:  3

Enter the thermal conductivity of the spine material
in watts per meter per deg C:  202

Enter the saturation temperature of the
fluid in deg C:  100

Enter the temperature of base surface in deg C:  125

Enter the desired tip temperature in deg C:  102


You have entered the following parameters:

Spine height(mm):          20.0
Spine diameter(mm):         3.0
Thermal conductivity:     202.0
Saturation temperature:   100.0
Base temperature:         125.0
Tip temperature:          102.0
```

```
Do you want to make any changes?:

Enter "Y" for yes, "N" for no.

N


The computed spine base temperature is (deg C):  124.8

The spine tip temperature is (deg C):            102.0

The heat dissipated by the spine is (watts):      14.0

The revised spine height is (mm):                 27.0

The spine diameter (user input) is (mm):           3.0

Percentage of spine in natural convection is:     76.0

Percentage of spine in nucleate boiling is:       24.0

Percentage of spine in transition boiling is:       .0


Do you want to enter another data set?
Enter "Y" for yes, "N" for no.

N

Stop - Program terminated.
```

It is observed that in all of the sample tests, the desired base temperature was attained with an accuracy of 0.2°C. The spine height was adjusted by the computer code as necessary to obtain the ideal geometry for the desired heat dissipation. In some cases the spine tip temperature was adjusted slightly to assist in reaching the desired base temperature.

Also apparent is the increase in heat flux in the spine as the base temperature increased. As expected, the heat flow rate in the 3 mm diameter spine is greater than the heat flow in the 2 mm diameter spine for the same base temperature.

The program output also shows the boiling modes which are predicted for the spine being tested. It is observed that, for the sample problems, most of the spine was in natural convection. A manipulation of the input parameters will permit a larger portion of the spine to operate in the nucleate boiling mode.

The sample runs give an example of how the cascade program can assist a circuit designer in analyzing specific heat transfer requirements. The simple entries and quick results allow the user to test many spine geometries for an assortment of heat transfer needs.

VII. CONCLUSION

The purpose of this thesis is to develop a cascade algorithm to analyze a spine in the boiling mode. This cascade approach is based on an analogy between the heat flow equations in the spine and the equations of the electrical transmission line.

The current version of the algorithm shows that cascading a series of subfins is an accurate and efficient method for analyzing an extended surface such as a spine. The results show that analyses can be made for a spine heat sink in a boiling environment without the need for time-consuming manual calculations.

The expansion possibilities for the cascade program are significant. Presently, the algorithm is modeled for water as the boiling liquid. With some modifications, a user could be able to input data for other boiling liquids and the spine would be analyzed accordingly. Other potential features include the addition of graphics capabilities and the introduction of user-friendly peripherals such as mouse drivers.

The possible applications for the cascade program are substantial. As the size of microcircuits decreases and the power requirements increase, the need for efficient and compact heat sinks is great. There is a real need for a tool that allows the designer to accurately perform thermal analysis of microcircuits during the design process. The cascade program to analyze a spine is the first step in enabling the designer to effectively determine a heat sink that meets specified requirements.

APPENDIX A
NOMENCLATURE

A	Cross sectional area, (m^2)
A	An element of the transmission parameter matrix, dimensionless
a	Coordinate of fin tip, (m)
B	An element of the transmission parameter matrix, $(^\circ C/W)$
b	Coordinate of fin base, (m)
C	An element of the transmission parameter matrix, $(W/^\circ C)$
D	An element of the transmission parameter matrix, dimensionless
d	Diameter, (m)
h	Heat transfer coefficient, $(W/m^2 - {}^\circ C)$
k	Thermal conductivity, $(W/m - {}^\circ C)$
L	Fin length, (m)
m	Fin performance factor, (m^{-1})
q	Heat flow, (W)
S	Surface area, (m^2)
T	Temperature, $(^\circ C)$
Y	Admittance, $(W/^\circ C)$
Z	Impedance, $(^\circ C/W)$

Matrix

[T]	Transmission matrix

Greek

α	Transmission line attenuation, (m^{-1})
η	Fin efficiency, dimensionless
θ	Temperature difference or temperature excess, $(^\circ C)$

Subscripts

a	Designates fin tip
b	Designates fin base
f	Designates fin
s	Designates surroundings

APPENDIX B PROGRAM LISTING

The following is a listing of the cascade program used to analyze a spine operating in the boiling mode. It consists of a main program followed by three subroutines. The key variables used in the program are given as well as comments throughout the algorithm.

```
C
C
C
C
C
C
C
C     TITLE:    CASCADED SPINE ANALYSIS -- MAIN PROGRAM
C     AUTHOR:   LT JOHN W LARKIN
C     DATE:     03 MAR 1992
C     COMPILER: MICROSOFT VERSION 3.2
C     LINKER:   MICROSOFT VERSION 3.2
C
C
C     PURPOSE:
C
C        THIS PROGRAM PREDICTS THE PERFORMANCE OF A SPINE IN
C        THE BOILING MODE
C
C
C     DESCRIPTION OF MAIN PARAMETERS:
C
C        ELINP -- SPINE HEIGHT (mm)
C        EL -- SPINE HEIGHT (m)
C        DIAM  -- SPINE DIAMETER (mm)
C        DEE -- SPINE DIAMETER (m)
C        CONDTH -- CONDUCTIVITY OF THE SPINE MATERIAL
C        TSAT -- SATURATION TEMPERATURE OF THE LIQUID
C        TBASE -- DESIRED BASE TEMPERATURE
C        TBCALC -- PREDICTED BASE TEMPERATURE
C        TTIP -- DESIRED TIP TEMPERATURE
C        THA -- TIP TEMPERATURE EXCESS
C        THAV -- AVERAGE TEMPERATURE EXCESS FOR SUBFIN
C        THBN,THBO -- BASE TEMPERATURE EXCESS
C        TIPSUR -- AREA OF SPINE CROSS SECTION (m^2)
C        H -- HEAT TRANSFER COEFICIENT
C        QA -- HEAT FLOW AT THE TIP
C        QB -- HEAT FLOW AT THE BASE
C        A -- 'A' TRANSMISSION PARAMETER
C        B -- 'B' TRANSMISSION PARAMETER
C        C -- 'C' TRANSMISSION PARAMETER
C        D -- 'D' TRANSMISSION PARAMETER
C        EMB -- FIN PERFORMANCE FACTOR (m) x FIN BASE
C               COORDINATE (b)
C        YO -- ADMITTANCE
C        ZO -- IMPEDANCE
C        PERNAT -- PERCENTAGE OF SPINE IN NATURAL CONVECTION
C        PERNUC -- PERCENTAGE OF SPINE IN NUCLEATE BOILING
C        PERTRN -- PERCENTAGE OF SPINE IN TRANSITION BOILING
C
C
C*****************BEGINNING OF MAIN PROGRAM****************
      DIMENSION  YIN(100)
```

```
      CHARACTER*3  CHNG

 4000 CALL CLS
C
C     DISPLAY A PROGRAM OVERVIEW IF DESIRED
C

 4001 WRITE(IOT,3008)
      READ(IN,2840) ANS

      IF (ANS .EQ. 'Y' .OR. ANS .EQ. 'y') THEN
        CALL OPTION
        GO TO 4002
      ENDIF
      IF (ANS .EQ. 'N' .OR. ANS .EQ. 'n') GO TO 4002
        GO TO 4001

C
C     READ THE INITIAL PARAMETERS AND CALCULATE THE FIN
C     TIP CONDITIONS
C
 4002 CALL CLS
      WRITE(IOT,2803)
      READ(IN,2802) ELINP
      WRITE(IOT,2804)
      READ(IN,2802) DIAM
      WRITE(IOT,2805)
      WRITE(IOT,2811)
      READ(IN,2802) CONDTH
      WRITE(IN,2806)
      READ(IN,2802) TSAT
      WRITE(IOT,2807)
      READ(IN,2802) TBASE
      WRITE(IOT,3006)
      READ(IN,2802) TTIP

C
C     CORRECT THE INPUT VALUES IF THEY WERE ENTERED
C     INCORRECTLY
C
C
  501 CALL CLS
      WRITE(*,3000) ELINP,DIAM,CONDTH,TSAT,TBASE,TTIP
      READ(*,3003) CHNG

      IF (CHNG .EQ. 'Y'.OR. CHNG .EQ. 'y') THEN
  502    CALL CORRECT
        READ(*,3004) INPUT
       IF (INPUT .EQ. 1 .OR. INPUT .EQ. 2 .OR. INPUT .EQ. 3
     +          .OR. INPUT .EQ. 4 .OR. INPUT .EQ. 5 .OR. INPUT
     + .EQ. 6) THEN
        CALL UPDATE(INPUT,ELINP,DIAM,CONDTH,TSAT,TBASE,TTIP)
        GO TO 501
      ELSE
        GO TO 502
       ENDIF
      ENDIF
      IF (CHNG .EQ. 'N' .OR. CHNG .EQ. 'n') GO TO 500
      GO TO 501

C
C     SET THE INITIAL VALUES
C

  500 EL = ELINP/1000.
      DEE = DIAM/1000.
```

```
      NAT = 0.
      NUC = 0.
      NTRANS = 0.
      PERNAT = 0.
      PERNUC = 0.
      PERTRN = 0.
      NCOUNT = 1
      FLAG = 0
      FLAGA = 0
      FLAGB = 0
      CHKR = 0.
      VALUE = 0.
      ADD = 0
      PI = 3.14159265
      BEE = EL/100.

C
C     FIRST, OBTAIN THE TIP CONDITIONS
C

      THA = TTIP - TSAT
   10 THA1 = THA
      TIPSUR = PI*(DEE**2.)/4.

      IF(THA.GT.7.) GO TO 60
      H = 1300*(THA**.25)*((1./DIAM)**.25)
      MODE = 1
      GO TO 80
      60 IF(THA.GT.40.) GO TO 70
      H = 76.8*(THA**2.)
      MODE = 2
      GO TO 80
   70 H = 7.5*(10.**8.)/(THA**3.)
      MODE = 3

   80 QA = H*TIPSUR*THA
      N = 1
C
C     THE TIP PARAMETERS HAVE BEEN CALCULATED,
C     PROCEED TO THE 100 SUBFINS
C
      THREF = THA
   90 THBO = THA
   95 THAV = (THBO + THA)/2.

      IF(THAV.GT.7.) GO TO 120
      H = 1300*(THAV**.25)*((1./DIAM)**.25)
      MODE = 1
      GO TO 140
  120 IF(THAV.GT.40.) GO TO 130
      H = 76.8*(THAV**2.)
      MODE = 2
      GO TO 140
  130 H = 7.5*(10.**8.)/(THAV**3.)
      MODE = 3

  140 EMB = SQRT(4.*H/(CONDTH*DEE))*BEE
      YO = SQRT(PI*PI*H*CONDTH*DEE*DEE*DEE)/2.
      ZO = 1./YO

      EPLUS = EXP(EMB)
      EMINUS = 1/EPLUS
      A = (EPLUS + EMINUS)/2.
      D = A
      DUMMY = (EPLUS - EMINUS)/2.
      B = ZO*DUMMY
```

```
      C = YO*DUMMY
C
C     WE NOW HAVE THE T MATRIX FOR EACH SUBFIN
C
      THBN = A*THA + B*QA
      ZETZ = ABS(THBN - THREF)
      IF(ZETZ .LE. .01) GO TO 175
      ADD = ADD + 1
      THBO = THBN
      N = N + 1
      IF (N.GT.50.AND.ZETZ.LE.6.5) GO TO 175
       IF (N.GT.50 .AND. ZETZ .GT. 6.5) THEN
       EL = EL - .01
       BEE = EL/100
       THA = THA1
       GO TO 10
       ENDIF

      THREF = THBN
      GO TO 95

  175 QB = C*THA + D*QA
      YIN(N) = QB/THBN
      IF (NCOUNT .EQ. 100) GO TO 300
      NCOUNT = NCOUNT + 1
      N = 1
      QA = QB
      THA = THBN

      IF (MODE.GT.1) GO TO 182
      NAT = NAT + 1.
      GO TO 90
  182 IF (MODE.GT.2) GO TO 184
      NUC = NUC + 1.
      GO TO 90
  184 NTRANS = NTRANS + 1.
      GO TO 90

C
C     COMPARE THE PREDICTED BASE TEMPERATURE TO THE DESIRED
C     BASE TEMPERATURE - ADJUST THE SPINE HEIGHT AS NECESARY
C

  300 TBCALC = THBN + TSAT
      ERR = TBCALC - TBASE
      ERRABS = ABS(ERR)
      IF (FLAGA .NE. 0 .OR. FLAGB .NE.0 ) GO TO 305
      IF (ERRABS .LE. 3) GO TO 305
      IF (ERR .GT. 3 .AND. FLAG .EQ. 0) THEN
       EL = EL - .001
       BEE = EL/100.
      ENDIF
      IF (ERR .GT. 3 .AND. FLAG .NE. 0) THEN
       VALUE = VALUE + 1.
       EL = EL - .001/VALUE
       BEE = EL/100
      ENDIF
      IF (ERR .LT. 3) THEN
       EL = EL + .001
       BEE = EL/100.
       FLAG = 1
      ENDIF

      THA = THA1
C
C     RESET VALUES FOR THE NEXT LOOP
```

```
C
  306 NCOUNT = 0
      ADD = 0
      NAT = 0.
      NUC = 0.
      NTRANS = 0.
      GO TO 10

C
C     IF THE COMPUTED BASE TEMPERATURE IS WITHIN 3 DEGREES C
C     OF THE DESIRED BASE TEMPERATURE, ADJUST THE TIP
C     TEMPERATURE UNTIL THE ERROR IS LESS THAN 0.2 DEGREES C
C

  305 IF(ERRABS .LE. .186) GO TO 310
      IF (ERR .GT. .186) THEN
        IF (FLAGB .NE. 0) THEN
          CHKR = CHKR+.15
        ENDIF
       THA = THA1 - .05/CHKR
       THA1 = THA
       FLAGA = FLAGA + 1
      IF (FLAGA .GT. 100) GO TO 555

      GO TO 306
      ENDIF

      IF (ERR .LT. .186) THEN
       IF (FLAGA.GT.1) THEN
         CHKR = CHKR+.15
       ENDIF
       THA = THA1 + .05/CHKR
       THA1 = THA
       FLAGB = 1
       GO TO 306
      ENDIF
C
C     DETERMINE THE SPINE'S BOILING REGIMES
C

  310 PERNAT = (NAT/100.)*100.
      PERNUC = (NUC/100.)*100.
      PERTRN = (NTRANS/100.)*100.

C
C     OUTPUT THE RESULTS
C

      CALL CLS
      WRITE(IOT,2808)
      WRITE(IOT,2809) TBCALC
      WRITE(IOT,3007)
      WRITE(IOT,2809) THA1 + TSAT
      WRITE(IOT,2810)
      WRITE(IOT,2809) QB
      WRITE(IOT,2813)
      WRITE(IOT,2809) EL*1000.
      WRITE(IOT,2819)
      WRITE(IOT,2809) DIAM
      WRITE(IOT,2815)
      WRITE(IOT,2816) PERNAT
      WRITE(IOT,2817)
      WRITE(IOT,2816) PERNUC
      WRITE(IOT,2818)
      WRITE(IOT,2816) PERTRN
```

```
  520 WRITE(IOT,3005)
      READ(IN,2840) ANS
      IF (ANS .EQ. ’Y’ .OR. ANS .EQ. ’y’) GO TO 4002
      IF (ANS .EQ. ’N’ .OR. ANS .EQ. ’n’) GO TO 555
      GO TO 520

C
C       FORMAT STATEMENTS
C
C
 2801 FORMAT (BN, A1)
 2802 FORMAT (BN, F15.0)
 2803 FORMAT (/’ Enter the height of the spine in
     +        ,millimeters:’,2X,\)
 2804 FORMAT (/’ Enter the diameter of the spine in
     +        ,millimeters:’,2X,\)
 2805 FORMAT (/’ Enter the thermal conductivity of the
     +        ,spine material ’)
 2811 FORMAT (’ in watts per meter per deg C:’,2X,\)
 2806 FORMAT (/’ Enter the saturation temperature of the
     +        ’,/, ’ fluid in deg C:’,2X,\)
 2807 FORMAT (/’ Enter the temperature of base surface
     +        ,in deg C:’,2X,\)
 2808 FORMAT (/’ The computed spine base temperature is
     +        , (deg C):’,5X,\)
 2809 FORMAT (F5.1)
 2810 FORMAT (/’ The heat dissipated by the spine is
     +        ,(watts):’,8X,\)
 2813 FORMAT (/’ The revised spine height is (mm):’,20X,\)
 2815 FORMAT (/’ Percentage of spine in natural convection
     +        ,is:’,4X,\)
 2816 FORMAT (F9.1)
 2817 FORMAT (/’ Percentage of spine in nucleate boiling
     +        , is:’,6X,\)
 2818 FORMAT (/’ Percentage of spine in transition boiling
     +        , is:’,4X,\)
 2819 FORMAT (/’ The spine diameter (user input) is
     +        ,(mm):’,13X,\)
 2840 FORMAT (BN,A1)
 3000 FORMAT (////, 5X,’You have entered the following
     +        ,parameters:’,//,
     +        5X,’Spine height(mm):’,8X,F6.12,/,5X,
     +        ’Spine diameter(mm):’
     +        ,7X,F5.1,/,5X,’Thermal conductivity:’,4X,
     +        F6.1,/,5X,
     +        ’Saturation temperature:’,2X,F6.1,/,5X,
     +        ’Base temperature:’,8X,F6.1,/,5X,’Tip
     +        temperature:’,
     +        9X,F6.1,////,5X,’ Do you want to make any
     +        ,changes?:’,
     +        //,5X,’Enter "Y" for’,’ yes, "N" for no.’,/)
 3001 FORMAT (F6.1)
 3003 FORMAT (BN,A1)
 3004 FORMAT (BN,I1)
 3005 FORMAT (////,5X,’Do you want to enter another data
     +        ,set?’,/,
     +        5X,’Enter "Y" for yes, "N" for no.’,/)

 3006 FORMAT (/,’ Enter the desired tip temperature in deg
     +        ,C:’,2X,\)
 3007 FORMAT (/,’ The spine tip temperature is (deg C):’,
     +        15X,\)
 3008 FORMAT (////,5X,’Do you want an overview of the
     +        ,program?’,/,
     +       5X,’Enter "Y" for yes, "N" for no.’,/)
```

```
 555  CALL CLS
      STOP
      END

C
C************************END OF MAIN PROGRAM****************
C
C************************************************************
C************************************************************
C
C
C     THIS SUBROUTINE DISPLAYS THE CHOICES OF PARAMETERS SO
C     THAT THE USER MAY CORRECT ANY INPUTS
C
C
C

      SUBROUTINE CORRECT

      CALL CLS
      WRITE(IOT,5000)
 5000 FORMAT(////,5X,’(1) Spine height’,/,5X,’(2)
     +       ,Spine diameter’,/,
     +      5X,’(3) Thermal conductivity’,/,5X,’
     +      , (4) Saturation’
     +      ’ temperature’,/,5X,’(5) Desired base
     +      , temperature’,/,
     +      5X,’(6) Spine tip temperature’,
     +      /////,5X,’Enter the number of the parameter
     +      , you want to’,
     +      ’ change.’,/)

      RETURN
      END
C
C
C************************************************************
C************************************************************
C
C
C     THIS SUBROUTINE INPUTS ANY CORRECTIONS THE USER
C     DECIDES TO MAKE
C
C
C


      SUBROUTINE UPDATE(INPUT,ELINP,DIAM,CONDTH,TSAT,TBASE,
     +                  TTIP)

      CALL CLS
      IF (INPUT .EQ. 1) THEN
       WRITE(IOT,5001)
5001   FORMAT(//,5X,’Enter the height of the spine in
     +       millimeters:’,2X,\)
       READ(IN,5010) ELINP
      ELSEIF (INPUT .EQ. 2) THEN
       WRITE(IOT,5002)
5002   FORMAT(//,5X,’Enter the diameter of the spine in’
     +         ’ millimeters:’,2X,\)
       READ(IN,5010) DIAM
      ELSEIF (INPUT .EQ. 3) THEN
       WRITE(IOT,5003)
5003   FORMAT(//,5X,’Enter the thermal conductivity of the
     + spine material in watts per meter per deg C:’,2X,\)
       READ(IN,5010) CONDTH
```

```
      ELSEIF (INPUT .EQ. 4) THEN
       WRITE(IOT,5004)
5004   FORMAT(//,5X,'Enter the saturation temperature of the
     +        , fluid in deg C:',2X,\)
       READ(IN,5010) TSAT
      ELSEIF (INPUT .EQ. 5) THEN
       WRITE(IOT,5005)
5005   FORMAT(//,5X,'Enter the temperature of the base
     +        ,surface in deg C:',2X,\)
       READ(IN,5010) TBASE
      ELSEIF (INPUT .EQ. 6) THEN
       WRITE(IOT,5006)
5006   FORMAT(//,5X,'Enter the spine tip temperature in deg
     +        ,C:',2X,
     +         \)
       READ(IN,5010) TTIP
      ENDIF

 5010 FORMAT(BN,F15.0)

      RETURN
      END
C
C
C***************************************************************
C***************************************************************
C
C
C     THIS SUBROUTINE DISPLAYS THE PROGRAM OVERVIEW
C
C
C
      SUBROUTINE OPTION

      CALL CLS


      WRITE(IOT,6000)
 6000 FORMAT(///,20X,'>>>>>>>>> PROGRAM OVERVIEW <<<<<<<<<'
     +       ,//,10X,'This program will predict the
     +       ,performance of a spine',
     +       ' in the',/,8X,'boiling mode. The boiling
     +       ,liquid for this program model is',/,8X,'water.
     +       , The algorithm divides the spine into 100
     +       ,subfins',/,8X,'and uses a cascade procedure to
     +       ,analyze the performance of, /,8X,'the spine.
     +       ,The cascade begins at the fin tip',
     +       ' and concludes at the',/,8X,'base surface. The
     +       ,user will be asked to enter the',/,8X,
     +       , 'following in SI units:',//,10X,
     +       '> Spine height (mm)',/,10X,'> Spine diameter
     +       ,(mm)',/,10X,'> Thermal conductivity of the
     +       , spine (W/m - deg C)',
     +       /,10X,'> Saturation temperature (deg C)',/,10X,
     +       '> Desired base temperature (deg C)',/,10X,'>
     +       ,Desired tip temperature (deg C)',//,8X,
     +       'The program may adjust the spine height input
     +       ,in order',/,8X,'to meet the specified base
     +       , temperature. The spine',
     +       /,8X,'diameter is not adjusted.')


      WRITE(IOT,6001)
 6001 FORMAT(//'--------PRESS ENTER TO CONTINUE-------',2X,/)
      READ(IN,6002) RESPON
 6002 FORMAT(BN,A1)
```

```
      CALL CLS
      WRITE(IOT,6003)
 6003 FORMAT(///,10X,'The model assumes a saturation
     +       ,temperature',
     +       ' above 100 deg C.',/,
     +       8X,'It is suggested that the tip temperature be
     +       ,at least 2 deg C',/,8X,'higher than the
     +       ,saturation temperature. A numerical
     +       ,instability',/,8X,'may occur',
     +       ' with a low tip temperature excess.',/,10X,
     +       'If any value is entered incorrectly, the user
     +       ,is given',/,8X,'the opportunity to change any
     +       ,or even all of the values',/,8X,'before the
     +       ,program is run.',/,
     +       10X,'The following are the outputs displayed by
     +       ,the program:',//,10X,'> Adjusted spine height
     +       ,(mm)',/,10X,
     +       '> Spine diameter (mm)',/,10X,'> Heat flow
     +       , (Watts)',/,
     +       10X,'> Predicted base temperature (deg C)',/,
     +       10X,'> Adjusted tip temperature',/,10X,
     +       '> Percent of spine in natural convection',/,10X,
     +       '> Percent of spine in nucleate boiling',/,10X,
     +       '> Percent of spine in transition boiling')

      WRITE(IOT,6001)
      READ(IN,6002) RESPON

      CALL CLS
      WRITE(IOT,6004)
  6004 FORMAT(///,10X,'All values are rounded off to the',
      +       ' tenth decimal place.',/,10X,'At the
      +        ,conclusion',
      +       ' of the program, the user is given the option
      +       ,to',/,8X,'either enter another data set or to
      +       ,terminate.')

      WRITE(IOT,6001)
      READ(IN,6002) RESPON

      CALL CLS
      RETURN
      END

C***********************END OF SUBROUTINES********************
```

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