## Inventory Management

## Discussion Questions

1. The four types of inventory are:

- Raw material-those items that are to be converted into product
- Work-in-process (WIP)-those items that are in the process of being converted
- Finished goods-those completed items for which title has not been transferred
- MRO-(maintenance, repair, and operating supplies)those items that are necessary to keep the transformation process going

2. The advent of low-cost computing should not be seen as obviating the need for the ABC inventory classification scheme. Although the cost of computing has decreased considerably, the cost of data acquisition has not decreased in a similar fashion. Business organizations still have many items for which the cost of data acquisition for a "perpetual" inventory system is still considerably higher than the cost of the item.
3. The purpose of the ABC system is to identify those items that require more attention due to cost or volume.
4. Types of costs—holding cost: cost of capital invested and space required; shortage cost: the cost of lost sales or customers who never return; the cost of lost good will; order cost: the costs associated with ordering, transporting, and receiving the items; unit cost: the actual cost of the item.
5. Assumptions of EOQ model: demand is known and constant over time; lead time is known and constant; receipt of inventory is instantaneous; quantity discounts are not possible; the only variable costs are the costs of placing an order or setting up production and the cost of holding or storing inventory over time and if orders are placed at the right time, stockouts or shortages can be completely avoided.
6. The EOQ increases as demand increases or as the setup cost increases; it decreases as the holding cost increases. The changes in the EOQ are proportional to the square root of the changes in the parameters.
7. Price times quantity is not variable in the EOQ model, but is in the discount model. When quality discounts are available, the unit purchase price of the item depends on the order quantity.
8. Advantages of cycle counting:
9. eliminating the shutdown and interruption of production necessary for annual physical inventories
10. eliminating annual inventory adjustments
11. providing trained personnel to audit the accuracy of inventory
12. allowing the cause of errors to be identified and remedial action to be taken
13. maintaining accurate inventory records
14. A decrease in setup time decreases the cost per order, encourages more and smaller orders, and thus decreases the EOQ.
15. Discount points below the $E O Q$ have higher inventory costs, and the prices are no lower than at the EOQ. Points above the EOQ have higher inventory costs than the corresponding price break point or EOQ at prices that are no lower than either of the price beaks or the EOQ. (It depends on whether or not there exists a discount point above the EOQ.)
16. Service level refers to the fraction of customers to whom the product or service is delivered when and as promised.
17. If the same costs hold, more will be ordered using an economic production quantity, because the average inventory is less than the corresponding EOQ system.
18. In a fixed-quantity inventory system, when the quantity on hand reaches the reorder point, an order is placed for the specified quantity. In a fixed-period inventory system, an order is placed at the end of the period. The quantity ordered is that needed to bring on-hand inventory up to a specified level.
19. The EOQ model gives quite good results under inexact inputs; a $10 \%$ error in actual demand alters the EOQ by less than $5 \%$.
20. Safety stock is inventory beyond average demand during lead time, held to control the level of shortages when demand and/or lead time are not constant; inventory carried to assure that the desired service level is reached.
21. The reorder point is a function of: demand per unit of time, lead time, customer service level, and standard deviation of demand.
22. Most retail stores have a computerized cash register (point-of-sale) system. At the time of purchase, the computer system simultaneously rings up the bill and reduces the inventory level in its records for the products sold.
23. Advantage of a fixed period system: there is no physical count of inventory when items are withdrawn. Disadvantage: there is a possibility of stockout during the time between orders.

## Ethical Dilemma

Setting service levels to meet inventory demand is a manager's job. Setting an $85 \%$ service level for whole blood is an important
judgment call on the part of the hospital administrator. Another major disaster means a certain shortage, yet any higher level may be hard to cost justify. Many hospitals do develop joint or regional groups to share supplies. The basic issue is how to put a price tag on lifesaving medicines. This is not an easy question to answer, but it makes for good discussion.

## ACTIVE MODEL EXERCISES

## ACTIVE MODEL 12.1: Economic Order Quantity (EOQ) Model

1. What is the EOQ and what is the lowest total cost? $E O Q=200$ units with a cost of $\$ 100$
2. What is the annual cost of CARRYING inventory at the EOQ and the annual cost of ORDERING inventory at the EOQ of 200 units.
$\mathbf{\$ 5 0}$ for carrying and also $\mathbf{\$ 5 0}$ for ordering
3. From the graph, what can you conclude about the relationship between the lowest total cost and the costs of ordering and carrying inventory?

The lowest total cost occurs where the ordering and inventory costs are the same.
4. How much does the total cost increase if the store manager orders 50 MORE hypodermics than the EOQ? 50 LESS hypodermics?

Ordering more increases costs by $\mathbf{\$ 2 . 5 0}$ or $\mathbf{2 . 5 \%}$. Ordering LESS increases costs by $\$ 4.17$ or $\mathbf{4 . 1 7 \%}$
5. What happens to the EOQ and total cost when demand is doubled? When carrying cost is doubled?

The EOQ rises by 82 units ( $41 \%$ ) and the total cost rises by $\$ 41$ ( $41 \%$ ) in EITHER case.
6. Scroll through lower setup cost values and describe the changes to the graph. What happens to the EOQ?

The curves seem to drop and move to the left. The EOQ decreases.
7. Comment on the sensitivity of the EOQ model to errors in demand or cost estimates.

The total cost is not very sensitive to mistakes in forecasting demand or placing orders.

## ACTIVE MODEL 12.2: Production Order Quantity Model

1. What is the optimal production run size for hubcaps? 283
2. How does this compare to the corresponding EOQ model? The run size is larger than the corresponding EOQ.
3. What is the minimal cost? $\$ 70.71$
4. How does this compare to the corresponding EOQ model? The total cost is less than the cost for the equivalent EOQ model.

End-of-Chapter Problems
12.1

| Code | Total Cost $=$ Unit <br> Cost $\times$ Demand |
| :--- | :---: |
| XX1 | $\$ 7,008$ |
| B66 | $\$ 5,994$ |
| 3CP0 | $\$ 1,003.52$ |
| 33CP | $\$ 82,292.16$ |
| R2D2 | $\$ 2,220$ |
| RMS | $\$ 1,998.88$ |

Total cost $=\$ 100,516.56$
$70 \%$ of total cost $=\$ 70,347.92$
The item that needs strict control is 33CP so it is an "A" item. Items that should not be strictly controlled are XX1, B66, 3CP0, R2D2, and RMS. The "B" items will be XX1 and B66. With so few items, an exact breakdown into the general $A, B, C$ categories is flexible.
12.2 You decide that the top $20 \%$ of the 10 items, based on a criterion of demand times cost per unit, should be A items. (In this example, the top $20 \%$ constitutes only $58 \%$ of the total inventory value, but in larger samples the value would probably approach $70 \%$ to $80 \%$.) You therefore rate items F3 and G2 as A items. The next $30 \%$ of the items are A2, C7, and D1; they represent $23 \%$ of the value and are categorized as B items. The remaining $50 \%$ of the items (items B8, E9, H2, I5, and J8) represent $19 \%$ of the value and become C items.

| Item | Annual <br> Demand | Cost (\$) | Demand $\times$ Cost | Classification |
| :--- | :---: | :---: | :---: | :---: |
| A2 | 3,000 | 50 | 150,000 | B |
| B8 | 4,000 | 12 | 48,000 | C |
| C7 | 1,500 | 45 | 67,500 | B |
| D1 | 6,000 | 10 | 60,000 | B |
| E9 | 1,000 | 20 | 20,000 | C |
| F3 | 500 | 500 | 250,000 | A |
| G2 | 300 | 1,500 | 450,000 | A |
| H2 | 600 | 20 | 12,000 | C |
| I5 | 1,750 | 10 | 17,500 | C |
| J8 | 2,500 | 5 | 12,500 | C |

12.3 First we rank the items from top to bottom on the basis of their dollar usage. Then they are partitioned off into classes.

| Item | Usage (\$) | Class |
| ---: | :---: | :--- |
| 13 | 70,800 | A: These four items (20\% of 20) |
| 15 | 57,900 | have a combined dollar usage |
| 7 | 44,000 | of \$206,100. This is $71 \%$ of |
| 3 | 33,400 | the total. |
| 19 | 19,000 | B: These six items (30\% of 20) |
| 20 | 15,500 | have a combined dollar usage |
| 12 | 10,400 | of \$69,000. This is $24 \%$ of |
| 1 | 9,200 | the total. |
| 4 | 8,100 |  |
| 14 | 6,800 |  |
| 18 | 4,800 | C: These ten items (50\% of 20) |
| 16 | 3,900 | have a combined dollar usage |
| 5 | 1,100 | of \$13,500. This is $5 \%$ of |
| 8 | 900 | the total. |
| 17 | 700 |  |
| 10 | 700 |  |
| 6 | 600 |  |
| 2 | 400 |  |
| 11 | 300 |  |
| 9 | 100 |  |

The dollar usage percentages do not exactly match the predictions of ABC analysis. For example, Class A items only account for $71 \%$ of the total, rather than $80 \%$. Nonetheless, the important finding is that ABC analysis did find the "significant few." For the items sampled, particularly close control is needed for items 3,7 , 13 , and 15 .
12.4
$7,000 \times 0.10=700 \quad 700 \div 20=35 \quad 35 \mathrm{~A}$ items per day
$7,000 \times 0.35=2,450 \quad 2450 \div 60=40.83 \quad 41 \mathrm{~B}$ items per day $7,000 \times 0.55=3,850 \quad 3850 \div 120=32 \quad \frac{32}{108}$ items items per day
12.5
(a) $\mathrm{EOQ}=Q=\sqrt{\frac{2(19,500)(25)}{4}}=493.71=494$ units
(b) Annual holdings costs $=[Q / 2] H=[494 / 2](4)=\$ 988$
(c) Annual ordering costs $=[D / Q] S=[19500 / 494](25)=\$ 987$
12.6 $\mathrm{EOQ}=\sqrt{\frac{2(8,000) 45}{2}}=600$ units
12.7 This problem reverses the unknown of a standard EOQ problem.

$$
\begin{aligned}
& 60=\sqrt{\frac{2 \times 240 \times S}{.4 \times 10}} ; \text { or, } 60=\sqrt{\frac{480 S}{4}}, \text { or, } \\
& 60=\sqrt{120 S}, \text { so solving for } S \text { results in } S=\$ 30 .
\end{aligned}
$$

That is, if $S$ were $\$ 30$, then the EOQ would be 60 . If the true ordering cost turns out to be much greater than $\$ 30$, then the firm's order policy is ordering too little at a time.
12.8 (a) Economic Order Quantity (Holding cost $=\$ 5$ per year):

$$
Q=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2 \times 400 \times 40}{5}}=80 \text { units }
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost
(b) Economic Order Quantity (Holding cost $=\$ 6$ per year):

$$
Q=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2 \times 400 \times 40}{6}}=73 \text { units }
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost
$12.9 D=15,000, H=\$ 25 /$ unit $/$ year, $S=\$ 75$
(a) $\mathrm{EOQ}=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2 \times 15,000 \times 75}{25}}=300$ units
(b) Annual holding costs $=(Q / 2) \times H=(300 / 2)$

$$
\times 25=\$ 3,750
$$

(c) Annual ordering costs $=(D / Q) \times S=(15,000 / 300)$

$$
\times 75=\$ 3,750
$$

(d) ROP $=d \times L=\left(\frac{15,000 \text { units }}{300 \text { days }}\right) \times 8$ days $=400$ units
12.10 Reorder point $=$ demand during lead time

$$
=100 \text { units } / \text { day } \times 21 \text { days }=2,100 \text { units }
$$

$12.11 D=10,000$
Number of business days $=300$
Lead time $=5$ days
ROP $=[$ Demand $/$ Day $]($ Lead time $)=[10,000 / 300](5)$
$=166.67 \cong 167$ units.
$\mathbf{1 2 . 1 2}$ (a) Economic Order Quantity:

$$
Q=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2 \times 4,000 \times 25}{0.10 \times 90}}=149.1 \text { or } 149 \text { valves }
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost
(b) Average inventory $=74.5$ valves
(c) Number of orders per year $=\frac{\text { Demand }}{\text { EOQ }}=\frac{4,000}{149}$

$$
=26.8 \text { or } 27 \text { orders }
$$

(d) Assuming 250 business days per year, the optimal number of business days between orders is given by:
Optimal number of days $=\frac{250}{27}=9 \frac{1}{4}$ days
(e) Total annual inventory cost $=$ Order cost + holding cost
$=\frac{D S}{Q}+\frac{Q H}{2}=\frac{4,000 \times 25}{149}+\frac{149 \times 0.1 \times 90}{2}$
$=671.14+670.50=\$ 1,341.64$
Note: Order and carrying costs are not equal due to rounding of the EOQ to a whole number.
(f) Reorder point $=$ demand during lead time

$$
=16 \text { units } / \text { day } \times 5 \text { days }=80 \text { valves }
$$

12.13
(a) $Q=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2(2500) 18.75}{1.50}}$

$$
=250 \text { brackets per order }
$$

(b) Average inventory $=\frac{Q}{2}=\frac{250}{2}=125$ units

Annual holding cost $=\frac{Q}{2} H=125(1.50)=\$ 187.50$
(c) Number of orders $=\frac{D}{Q}=\frac{2500}{250}=10$ orders/year

Annual order cost $=\frac{D}{Q} S=10(18.75)=\$ 187.50$
(d) $\mathrm{TC}=\frac{Q}{2} H+\frac{D}{Q} S=187.50+187.50=\$ 375 /$ year
(e) Time between orders $=\frac{\text { working days }}{(D / Q)}$

$$
=\frac{250}{10}=25 \text { days }
$$

(f) $\mathrm{ROP}=d L=10(2)=20$ units (where $10=$ daily demand) $d=\frac{2500}{250}=10$
12.14 (a) Total cost $=$ order cost + holding cost $=\frac{D S}{Q}+\frac{Q H}{2}$

For $Q=25:=\frac{1,200 \times 25}{25}+\frac{25 \times 24}{2}=\$ 1,500$
For $Q=40:=\frac{1,200 \times 25}{40}+\frac{40 \times 24}{2}=\$ 1,230$
For $Q=50:=\frac{1,200 \times 25}{50}+\frac{50 \times 24}{2}=\$ 1,200$
For $Q=60:=\frac{1,200 \times 25}{60}+\frac{60 \times 24}{2}=\$ 1,220$
For $Q=100:=\frac{1,200 \times 25}{100}+\frac{100 \times 24}{2}=\$ 1,500$
As expected, small variations in order quantity will not have a significant effect on total costs.
(b) Economic Order Quantity:
$Q=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2 \times 1,200 \times 25}{24}}=50$ units
where: $D=$ annual demand, $S=$ setup or order cost, $H$ = holding cost
12.15 (a) The EOQ assumptions are met, so the optimal order quantity is
$\mathrm{EOQ}=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2(250) 20}{1}}=100$ units
(b) Number of orders per year $=D / Q=250 / 100=2.5$ orders per year.

Note that this would mean in one year the company places 3 orders and in the next it would only need 2 orders since some inventory would be carried over from the previous year. It averages 2.5 orders per year.
(c) Average inventory $=Q / 2=100 / 2=50$ units
(d) Given an annual demand of 250 , a carrying cost of $\$ 1$, and an order quantity of 150 , Patterson Electronics must determine what the ordering cost would have to be for the order policy of 150 units to be optimal. To find the answer to this problem, we must solve the traditional economic order quantity equation for the ordering cost. As you can see in the calculations that follow, an ordering cost of $\$ 45$ is needed for the order quantity of 150 units to be optimal.

$$
\begin{aligned}
Q & =\sqrt{\frac{2 D S}{H}} \\
S & =Q^{2} \frac{H}{2 D} \\
& =\frac{(150)^{2}(1)}{2(250)} \\
& =\frac{22,500}{500}=\$ 45
\end{aligned}
$$

12.16 Production Order Quantity, noninstantaneous delivery:

$$
Q=\sqrt{\frac{2 D S}{H\left(1-\frac{d}{p}\right)}}=\sqrt{\frac{2 \times 10,000 \times 200}{1.00\left(1-\frac{50}{200}\right)}}=2309.4 \text { or } 2,309 \text { units }
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost, $d=$ daily demand rate, $p=$ daily production rate
12.17 Production order quantity, noninstantaneous delivery.
(a) $D=12,000 / \mathrm{yr}$.

$$
H=\$ .10 / \text { light-yr. }
$$

$S=\$ 50 /$ setup
$P=\$ 1.00 /$ light
$p=100 /$ day
$d=\frac{12,000 / \mathrm{yr} .}{300 \text { days } / \mathrm{yr} .}=40 /$ day
$Q=\sqrt{\frac{2 D S}{H\left(1-\frac{d}{p}\right)}}=\sqrt{\frac{2(12,000) 50}{.10\left(1-\frac{40}{100}\right)}}$
$=4,472$ lights per run
(b) Average holding cost/year $=\frac{Q}{2}\left[1-\left(\frac{d}{p}\right)\right] H$

$$
=\frac{4,472}{2}\left[1-\left(\frac{40}{100}\right)\right](.10)=\frac{\$ 26,832}{200}=\$ 134.16
$$

(c) Average setup cost/year $=\left(\frac{D}{Q}\right) S=\left(\frac{12,000}{4,472}\right) 50$

$$
=\$ 134.16
$$

(d) Total cost (including cost of goods)

$$
\begin{aligned}
& =\mathrm{PD}+\$ 134.16+\$ 134.16 \\
& =(\$ 1 \times 12,000)+\$ 134.16+\$ 134.16 \\
& =\$ 12,268.32 / \text { year }
\end{aligned}
$$

12.18 (a) Production Order Quantity, noninstantaneous delivery:

$$
Q=\sqrt{\frac{2 D S}{H\left(1-\frac{d}{p}\right)}}=\sqrt{\frac{2 \times 10,000 \times 40}{0.60\left(1-\frac{50}{500}\right)}}
$$

$$
=1217.2 \text { or } 1,217 \text { units }
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost, $d=$ daily demand rate, $p=$ daily production rate
(b) $I_{\max }=Q\left(1-\frac{d}{p}\right)=1,095$
(c) $\frac{D}{Q}=\frac{10,000}{1,217}=8.22$
(d) $T C=\frac{I_{\max }}{2} H+\frac{D}{Q} S=328.50+328.80=\$ 657.30$
12.19 At the Economic Order Quantity, we have:

$$
\mathrm{EOQ}=\sqrt{(2 \times 36,000 \times 25) / 0.45}=2,000 \text { units. }
$$

The total costs at this quantity are:
Holding cost $=Q / 2 \times H=1,000 \times .45=\$ 450$
Ordering cost $=D / Q \times S=36,000 / 2,000 \times 25=\$ 450$
Purchase cost $=D \times P=36,000 \times 0.85=\$ 30,600$
Total cost $=\$ 900+\$ 30,600=\$ 31,500$
At the quantity discount, we have:

$$
\begin{aligned}
& \text { Holding cost }=Q / 2 \times H=3,000 \times .45=\$ 1,350 \\
& \text { Ordering cost }=D / Q \times \mathrm{S}=36,000 / 6,000 \times 25=\$ 150 \\
& \text { Purchase cost }=D \times P=36,000 \times 0.82=\$ 29,520 \\
& \text { Total cost }=\$ 1,500+\$ 29,520=\$ 31,020
\end{aligned}
$$

The quantity discount will save $\$ 480$ on this item. The company should also consider some qualitative aspects of the decision, such as available space, the risk of obsolescence of disks, and the risk of deterioration of the storage medium over time, as 6,000 represents one sixth of the year's needs.
12.20 Under present price of $\$ 50.00$ per unit, Economic Order Quantity:

$$
\begin{aligned}
& Q=\sqrt{\frac{2 D S}{H}} \\
& Q=\sqrt{\frac{2 \times 1,000 \times 40}{0.25 \times 50}}=80 \text { units }
\end{aligned}
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost, $P=$ price/unit

Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
& =\frac{D S}{Q}+\frac{Q H}{2}+P D \\
& =\frac{1,000 \times 40}{80}+\frac{80 \times 0.25 \times 50}{2}+(1,000 \times 50) \\
& =500.00+500.00+50,000=\$ 51,000
\end{aligned}
$$

Under the quantity discount price reduction of $3 \%$ :
Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
= & \frac{D S}{Q}+\frac{Q H}{2}+P D \\
= & \frac{1,000 \times 40}{200}+\frac{200 \times 0.25 \times 50 \times 0.97}{2} \\
& +1,000 \times 50 \times 0.97 \\
= & 200.00+1212.50+48,500=\$ 49,912.50
\end{aligned}
$$

Therefore, the pumps should be ordered in batches of 200 units and the quantity discount taken.
12.21 The solution to any quantity discount model involves determining the total cost of each alternative after quantities have been computed and adjusted for the original problem and every discount.

We start the analysis with no discount:

$$
\begin{aligned}
\mathrm{EOQ}(\text { no discount }) & =\sqrt{\frac{2(1,400)(25)}{0.2(400)}} \\
& =29.6 \text { units }
\end{aligned}
$$

Total cost (no discount) $=$ material cost + ordering cost

+ carrying cost
$=\$ 400(1,400)+\frac{1,400(25)}{29.6}$
$+\frac{29.6(\$ 400)(0.2)}{2}$
$=\$ 560,000+\$ 1,183+\$ 1,183$
$=\$ 562,366$
The next step is to compute the total cost for the discount:

$$
\begin{aligned}
\text { EOQ }(\text { with discount }) & =\sqrt{\frac{2(1,400)(25)}{0.2(\$ 380)}} \\
& =30.3 \text { units } \\
\text { EOQ }(\text { adjusted }) & =300 \text { units }
\end{aligned}
$$

Because this last economic order quantity is below the discounted price, we must adjust the order quantity to 300 units. The next step is to compute total cost.

Total cost $($ with discount $)=$ material cost + ordering cost

$$
\begin{aligned}
& + \text { carrying cost } \\
= & \$ 380(1,400)+\frac{1,400(25)}{300} \\
& +\frac{300(\$ 380)(0.2)}{2} \\
= & \$ 532,000+\$ 117+\$ 11,400 \\
= & \$ 543,517
\end{aligned}
$$

The optimal strategy is to order 300 units at a total cost of \$543,517.
12.22 Economic Order Quantity:

$$
Q=\sqrt{\frac{2 D S}{H}}
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost, price/unit

- Economic Order Quantity, standard price:
$Q=\sqrt{\frac{2 \times 45 \times 10}{0.05 \times 20}}=30$ units
Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
& =\frac{D S}{Q}+\frac{Q H}{2}+P D \\
& =\frac{45 \times 10}{30}+\frac{30 \times 0.05 \times 20}{2}+(45 \times 20) \\
& =15+15+900=\$ 930
\end{aligned}
$$

- Quantity Discount, 75 units or more. Economic Order Quantity, discount over 75 units:

$$
Q=\sqrt{\frac{2 \times 45 \times 10}{0.05 \times 18.50}}=31.19 \text { or } 31 \text { units }
$$

Because $\mathrm{EOQ}=31$ and a discount is given only on orders of 75 or more, we must calculate the total cost using a $75-$ unit order quantity:
Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
& =\frac{D S}{Q}+\frac{Q H}{2}+P D \\
& =\frac{45 \times 10}{75}+\frac{75 \times 0.05 \times 18.50}{2}+(45 \times 18.50) \\
& =6+34.69+832.50=\$ 873.19
\end{aligned}
$$

- Quantity Discount, 100 units or more; Economic Order Quantity, discount over 100 units:

$$
Q=\sqrt{\frac{2 \times 45 \times 10}{0.05 \times 15.75}}=33.81 \text { or } 34 \text { units }
$$

$\mathrm{EOQ}=34$ and a discount is given only on orders of 100 or more, thus we must calculate the total cost using a 100unit order quantity. Calculate total cost using 100 as order quantity:
Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
& =\frac{D S}{Q}+\frac{Q H}{2}+P D \\
& =\frac{45 \times 10}{100}+\frac{100 \times 0.05 \times 15.75}{2}+(45 \times 15.75) \\
& =4.5+39.38+708.75=\$ 752.63
\end{aligned}
$$

Based purely upon cost, the decision should be made to order in quantities of 100 , for a total cost of $\$ 752.63$.
It should be noted, however, that an order quantity of 100 implies that an order will be placed roughly every two years. When orders are placed that infrequently, obsolescence may become a problem.
12.23 $D=20,000 / \mathrm{yr}$.
$I=20$ percent of purchase price per year in holding costs, where $H=I P$
$S=\$ 40 /$ order
$P=\$ 20 /$ tire if fewer than 500 are ordered;
$\$ 18 /$ tire if between 500 and 999 are ordered; and $\$ 17 /$ tire if 1,000 or more are ordered

$$
\begin{aligned}
Q_{20} & =\sqrt{2 D S / H} \\
& =\sqrt{(2 \times 20,000 \times 40) /(.2 \times 20)} \\
& =632.5(\text { not valid }) \\
Q_{18} & =\sqrt{2 D S / H} \\
& =\sqrt{(2 \times 20,000 \times 40) /(.2 \times 18)} \\
& =666.7(\text { valid }) \\
Q_{17} & =\sqrt{2 D S / H} \\
& =\sqrt{(2 \times 20,000 \times 40) /(.2 \times 17)} \\
& =686(\text { not valid })
\end{aligned}
$$

We compare the cost of ordering 667 with the cost of ordering 1,000 .

$$
\begin{aligned}
T C_{667}= & P D+H Q / 2+S D / Q \\
= & \$ 18 \times 20,000+(.2 \times \$ 18 \times 667) / 2 \\
& +(\$ 40 \times 20,000) / 667 \\
= & \$ 360,000+\$ 1,200+\$ 1,200 \\
= & \$ 362,400 \text { per year }
\end{aligned}
$$

$$
\begin{aligned}
T C_{1,000}= & P D+H Q / 2+S D / Q \\
= & \$ 17 \times 20,000+(.2 \times \$ 17 \times 1,000) / 2 \\
& +(\$ 40 \times 20,000) / 1,000 \\
= & \$ 340,000+\$ 1,700+\$ 800 \\
= & \$ 342,500 \text { per year }
\end{aligned}
$$

Rocky Mountain should order 1,000 tires each time.
$12.24 D=700 \times 12, H=5, S=50$

| Allen |  |
| :--- | :--- |
| $1-499$ | $\$ 16.00$ |
| $500-999$ | $\$ 15.50$ |
| $1000+$ | $\$ 15.00$ |
|  |  |
| Baker |  |
| $1-399$ | $\$ 16.10$ |
| $400-799$ | $\$ 15.60$ |
| $800+$ | $\$ 15.10$ |

(a) $Q=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2(8,400) 50}{5}}=409.88 \rightarrow 410$
(b,c) Vendor: Allen

$$
\begin{aligned}
\text { at } 410, T C & =\frac{410}{2}(5)+\frac{8,400}{410}(50)+8,400(16)=\$ 136,449.36 \\
\text { at } 500, T C & =\frac{500}{2}(5)+\frac{8,400}{500}(50)+8,400(15.5)=\$ 132,290 \\
\text { at } 1,000, T C & =\frac{1,000}{2}(5)+\frac{8,400}{1,000}(50)+8,400(15) \\
& =\$ 128,920 \mathrm{BEST}
\end{aligned}
$$

Vendor: Baker
at $410, T C=\frac{410}{2}(5)+\frac{8,400}{410}(50)+8,400(15.60)=\$ 133,089.39$
at $800, T C=\frac{800}{2}(5)+\frac{8,400}{800}(50)+8,400(15.10)=\$ 129,365$
$12.25 S=10, H=3.33, D=2,400$

|  |  | Costs |  |  |  |  |
| :--- | :---: | ---: | ---: | ---: | ---: | :--- |
| Qty | Price | Holding | Ordering | Purchase | Total |  |
| 120 | $\$ 33.55$ | $\$ 199.80$ | $\$ 200.00$ | $\$ 80,520.00$ | $\$ 80,919.80$ | Vendor A |
| 150 | $\$ 32.35$ | $\$ 249.75$ | $\$ 160.00$ | $\$ 77,640.00$ | $\$ 78,049.75$ |  |
| 300 | $\$ 31.15$ | $\$ 499.50$ | $\$ 80.00$ | $\$ 74,760.00$ | $\$ 75,339.50$ |  |
| 500 | $\$ 30.75$ | $\$ 832.50$ | $\$ 48.00$ | $\$ 33,800.00$ | $\$ 74,680.50$ |  |
| 120 | $\$ 34.00$ | $\$ 199.80$ | $\$ 200.00$ | $\$ 81,600.00$ | $\$ 81,999.80$ | Vendor B |
| 150 | $\$ 32.80$ | $\$ 249.75$ | $\$ 160.00$ | $\$ 78,720.00$ | $\$ 79,129.75$ |  |
| 300 | $\$ 31.60$ | $\$ 499.50$ | $\$ 80.00$ | $\$ 75,840.00$ | $\$ 76,419.50$ |  |
| 500 | $\$ 30.50$ | $\$ 832.50$ | $\$ 48.00$ | $\$ 73,200.00$ | $\$ 74,080.50$ | BEST |
| 120 | $\$ 33.75$ | $\$ 199.80$ | $\$ 200.00$ | $\$ 81,000.00$ | $\$ 81,399.80$ | Vendor C |
| 200 | $\$ 32.50$ | $\$ 333.00$ | $\$ 120.00$ | $\$ 78,000.00$ | $\$ 78,453.00$ |  |
| 400 | $\$ 31.10$ | $\$ 666.00$ | $\$ 60.00$ | $\$ 74,640.00$ | $\$ 75,366.00$ |  |
| 120 | $\$ 34.25$ | $\$ 199.80$ | $\$ 200.00$ | $\$ 82,200.00$ | $\$ 82,599.80$ | Vendor D |
| 200 | $\$ 33.00$ | $\$ 333.00$ | $\$ 120.00$ | $\$ 79,200.00$ | $\$ 79,653.00$ |  |
| 400 | $\$ 31.00$ | $\$ 666.00$ | $\$ 60.00$ | $\$ 74,400.00$ | $\$ 75,126.00$ |  |

$\mathrm{EOQ}=120$ with slight rounding
12.26 Calculation for EOQ: $S=\$ 50, H=50 \%, D=9,600$
(a)

| Price | EOQ |  | Vendor |
| :--- | :--- | :--- | :---: |
| $\$ 17.00$ | 336.0672 | feasible | A |
| $\$ 16.75$ | 338.5659 | not feasible |  |
| $\$ 16.50$ | 341.1211 | not feasible |  |
| $\$ 17.10$ | 335.0831 | feasible | B |
| $\$ 16.85$ | 337.5598 | not feasible |  |
| $\$ 16.60$ | 340.0921 | not feasible |  |

(b), (c)

|  |  | Costs |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Qty | Price | Holding | Ordering | Purchase | Total |  |
| 336 | $\$ 17.00$ | $\$ 1,428.00$ | $\$ 1,428.57$ | $\$ 163,200.00$ | $\$ 166,056.57$ | Vendor A |
| 500 | $\$ 16.75$ | $\$ 2,093.75$ | $\$ 960.00$ | $\$ 160,800.00$ | $\$ 163,853.75$ |  |
| 1000 | $\$ 16.50$ | $\$ 4,125.00$ | $\$ 480.00$ | $\$ 158,400.00$ | $\$ 163,005.00$ |  |
| 335 | $\$ 17.10$ | $\$ 1,432.13$ | $\$ 1,432.84$ | $\$ 164,160.00$ | $\$ 167,024.97$ | Vendor B |
| 400 | $\$ 16.85$ | $\$ 1,685.00$ | $\$ 1,200.00$ | $\$ 161,760.00$ | $\$ 164,645.00$ |  |
| 800 | $\$ 16.60$ | $\$ 3,320.00$ | $\$ 600.00$ | $\$ 159,360.00$ | $\$ 163,280.00$ |  |
| 1200 | $\$ 16.25$ | $\$ 4,875.00$ | $\$ 400.00$ | $\$ 156,000.00$ | $\$ 161,275.00$ | BEST |

(d) Other considerations include the perishability of the chemical and whether or not there is adequate space in the controlled environment to handle 1,200 pounds of the chemical at one time.
12.27 (a) $\mu=60 ; \sigma=7$

Safety stock for $90 \%$ service level $=\sigma Z($ at 0.90$)$ $=7 \times 1.28=8.96 \approx 9$
(b) $\mathrm{ROP}=60+9=69 \mathrm{BX}-5$ bandages.
12.28 (a) $Z=1.88$
(b) Safety stock $=Z \sigma=1.88(5)=9.4$ drives
(c) $\mathrm{ROP}=50+9.4=59.4$ drives
12.29

|  | Incremental Costs |  |  |
| :---: | ---: | :---: | :---: |
| Safety Stock | Carrying Cost | Stockout Cost | Total Cost |
| 0 | 0 | $70 \times(100 \times 0.2+200 \times 0.2)=4,200$ | 4,200 |
| 100 | $100 \times 15=1,500$ | $(100 \times 0.2) \times 70=1,400$ | 2,900 |
| 200 | $200 \times 15=3,000$ |  | 0 |

The safety stock that minimizes total incremental cost is 100 units. The reorder point then becomes 200 units +100 units or, 300 units.
12.30

| Demand during Reorder Period | Probability |
| :---: | :---: |
| 0 | 0.1 |
| 50 | 0.2 |
| 100 | 0.4 |
| 150 | 0.2 |
| 200 | $\frac{0.1}{1.0}$ |


|  | Incremental Costs |  |  |
| :---: | ---: | :---: | ---: |
| Safety Stock | Carrying Cost | Stockout Cost | Total Cost |
| 0 | 0 | $50 \times(50 \times 0.2+100 \times 0.1)=1,000$ | 1000 |
| 50 | $50 \times 10=500$ | $50 \times(0.1 \times 50)=250$ | 750 |
| 100 | $100 \times 10=1,000$ |  | 0 |

The safety stock that minimizes total incremental cost is 50 sets. The reorder point then becomes 100 sets +50 sets or, 150 sets.
12.31
$\left.\begin{array}{lrrr}\hline \begin{array}{l}\text { Safety } \\ \text { Stock }\end{array} & \begin{array}{c}\text { Additional } \\ \text { Carrying Cost }\end{array} & 0 & \text { Stockout Cost }\end{array} \begin{array}{c}\text { Total } \\ \text { Cost }\end{array}\right]$

The BB-1 set should therefore have a safety stock of 30 units; ROP $=90$ units.
12.32 Only demand is variable in this problem so Equation (12-15) applies

$$
=\sqrt{(6 \times 625)+(40,000 \times 4)}=\sqrt{3,750+160,000}
$$

(a) ROP $=$ (average daily demand $\times$ lead time in days $)$

$$
=\sqrt{163,750} \cong 405
$$

$$
\begin{aligned}
& +\mathrm{Z} \sigma_{\mathrm{dLT}} \\
= & (1,000 \times 2)+(2.05)\left(\sigma_{\mathrm{d}}\right)(\sqrt{\text { lead time }}) \\
= & 2,000+2.05(100) \sqrt{2} \\
= & 2,000+290=2,290 \text { towels }
\end{aligned}
$$

$$
\text { So ROP }=1,200+(1.28)(405) \cong 1,200+518=1,718 \text { cigars }
$$

12.35 Fixed-period model.

$$
Q=\text { Target }- \text { onhand }- \text { orders not received }
$$

$$
=40-5-18=17 \text { poles. }
$$

(b) Safety stock $=290$ towels
12.33 Only lead time is variable in this problem, so Equation (12-16) is used.

$$
\begin{aligned}
Z= & 1.88 \text { for } 97 \% \text { service level } \\
\mathrm{ROP}= & (\text { daily demand } \times \text { average lead time in days }) \\
& +Z \times \text { daily demand } \times \sigma_{\mathrm{LT}} \\
\mathrm{ROP}= & (12,500 \times 4)+(1.88)(12,500)(1) \\
= & 50,000+23,500=73,500 \text { pages }
\end{aligned}
$$

12.34 Both lead time and demand are variables, so Equation (12-17) applies, in weeks. $Z=1.28$ for $90 \%$ service.

$$
\begin{aligned}
& \mathrm{ROP}=(200 \times 6)+1.28 \sigma_{\mathrm{dLT}} \\
& \text { where } \sigma_{\mathrm{dLT}}=\sqrt{\left(6 \times 25^{2}\right)+\left(200^{2} \times 2^{2}\right)}
\end{aligned}
$$

| Holding Cost | Ordering Cost |
| :---: | ---: |
| $\$ 2,000$ | 1,500 |
| 600 | 500 |
| 750 | 800 |
| 280 | 30,000 |
| 12,800 | 500 |
| 800 | 1,000 |
| 300 | $\$ 34,300$ |
| $\$ 17,530$ |  |

Note: Items of New Product Development, advertising, and research are not part of holding or ordering cost.

$$
\begin{gathered}
\text { Cost per order }=\frac{\$ 34,300}{200}=\$ 171.50 \\
\text { Holding cost per unit }=\frac{\$ 17,530}{10,000}=\$ 1.753 \\
\text { Therefore, EOQ }=\sqrt{\frac{(2)(1000)(171.5)}{1.753}}=442.34 \text { units. }
\end{gathered}
$$

12.37 Annual demand, $D=8,000$

Daily production rate, $p=200$
Set-up cost, $S=120$
Holding cost, $H=50$
Production quantity, $Q=400$
(a) daily demand, $d=D / 250=8,000 / 250=32$
(b) number of days in production run $=Q / p=400 / 200=2$
(c) number of production runs per year $=D / Q=8,000 / 400=20$ annual set-up cost $=20(\$ 120)=\$ 2,400$
(d) maximum inventory level $=Q(1-d / p)$

$$
=400(1-32 / 200)=336
$$

average inventory $=$ maximum $/ 2=336 / 2=168$
(e) total holding cost + total set-up cost $=(168) 50+20(120)$

$$
=\$ 8,400+\$ 2,400
$$

$$
=\$ 10,800
$$

(f) $Q=\sqrt{\frac{2 D S}{H\left(1-\frac{d}{p}\right)}}=\sqrt{\frac{2(8000) 120}{50\left(1-\frac{32}{200}\right)}}=213.81$
total holding cost + total set-up cost $=4,490+4,490=\$ 8,980$ Savings $=\$ 10,800-\$ 8,980=\$ 1,820$
12.38 (a) $d=75 \mathrm{lbs} /$ day 200 days per year $D=15,000 \mathrm{lbs} /$ year $H=\$ 3 / \mathrm{lb}$./year $S=\$ 16 /$ order

$$
Q=400 \mathrm{lbs} \text { of beans }
$$

(b) Total annual holding cost $=\frac{Q}{2} H=(200)(\$ 3)=\$ 600$
(c) Total annual order cost $=\frac{D}{Q} S=(37.5)(16)=\$ 600$
(d) $\mathrm{LT}=4$ days with $\sigma=15$ Stockout risk $=1 \%$ $\mathrm{Z}=2.33$
ROP $=$ Lead time demand + SS
where $\mathrm{SS}=(Z)\left(\sigma_{\mathrm{dLT}}\right)$ and lead time demand $=(\mathrm{d})(\mathrm{LT})$

$$
\sigma_{\mathrm{dLT}}=(\sqrt{L T})(15)=(\sqrt{4})(15)=30
$$

$\mathrm{ROP}=369.99 \quad$ where $\mathrm{ROP}=(\mathrm{d})(\mathrm{LT})+\mathrm{SS}$
(e) $\mathrm{SS}=69.99$ from part (d)
(f) Annual holding cost $=\$ 209.37$
(g) $2 \%$ stock out level $\Rightarrow \mathrm{Z}=2.054$
$\mathrm{SS}=(Z)\left(\sigma_{\mathrm{dLT}}\right)=61.61$
The lower we make our target service level, the less S.S. we need.

## INTERNET HOMEWORK PROBLEMS

Problems 12.39-12.51 are found on our companion web site at www.prenhall.com/heizer.
12.39

| SKU | Annual <br> Demand | Cost (\$) | Demand $\times$ Cost | Classification |
| :---: | :---: | :---: | :---: | :---: |
| A | 100 | 300 | 30,000 | A |
| B | 75 | 100 | 7,500 | B |
| C | 50 | 50 | 2,500 | C |
| D | 200 | 100 | 20,000 | A |
| E | 150 | 75 | 11,250 | B |

Obviously, with so few items, the breakdowns into A, B, and C cannot follow the guidelines exactly.
12.40

| Item | Annual Demand | Cost (\$) | $\begin{aligned} & \text { Demand } \times \\ & \text { Cost } \end{aligned}$ | Classification |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| E102 | 800 | 4.00 | 3,200 | C |  |
| D23 | 1,200 | 8.00 | 9,600 | A | 27\% |
| D27 | 700 | 3.00 | 2,100 | C |  |
| R02 | 1,000 | 2.00 | 2,000 | C |  |
| R19 | 200 | 8.00 | 1,600 | C |  |
| S107 | 500 | 6.00 | 3,000 | C |  |
| S123 | 1,200 | 1.00 | 1,200 | C |  |
| U11 | 800 | 7.00 | 5,600 | B | 16\% |
| U23 | 1,500 | 1.00 | 1,500 | C | 33\% |
| V75 | 1,500 | 4.00 | 6,000 | B | 17\% |

12.41 $E O Q=\sqrt{\frac{2(1000) 62.50}{0.50}}=500$ units
$12.42 \quad 300=\sqrt{\frac{2(8,000) 45}{H}} \Rightarrow 90,000=\frac{720,000}{H}$

$$
H=\frac{720,000}{90,000}=\$ 8
$$

12.43 (a) Economic Order Quantity:
$Q=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2 \times 1,500 \times 150}{45}}=100$ units
where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost
(b) Holding cost $=\frac{Q H}{2}=\frac{100 \times 45}{2}=\$ 2,250.00$
(c) Order cost $=\frac{D S}{Q}=\frac{1500 \times 150}{100}=\$ 2,250.00$
(d) Reorder point:

Reorder point $=$ demand during lead time

$$
\begin{aligned}
& =\frac{1,500}{300} \text { units } / \text { day } \times 6 \text { days } \\
& =30 \text { units }
\end{aligned}
$$

12.44 Reorder point $=$ demand during lead time

$$
\begin{aligned}
& =500 \text { units } / \text { day } \times 14 \text { days } \\
& =7,000 \text { units }
\end{aligned}
$$

12.45 (a) Economic Order Quantity:

$$
Q=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2 \times 5,000 \times 30}{50}}=77.46 \text { or } 78 \text { units }
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost
(b) Average inventory $=\frac{78}{2}=39$ units
(c) Number of orders per year $=\frac{\text { Demand }}{E O Q}=\frac{5,000}{78}$

$$
=64.1 \text { or } 64 \text { orders }
$$

(d) Assuming 250 business days per year, the optimal number of business days between orders is given by:
Optimal number of days $=\frac{250}{64}=3.91$ days
(e) Total cost $=$ order cost + holding cost

$$
\begin{aligned}
& =\frac{D S}{Q}+\frac{Q H}{2}=\frac{5,000 \times 30}{78}+\frac{78 \times 50}{2} \\
& =1,923.02+1,950=\$ 3,873.08
\end{aligned}
$$

Note: Order and carrying costs are not equal due to rounding of the EOQ to a whole number. If an EOQ of 77.46 is used, the order and carrying costs calculate to $\$ 1,936.49$ for a total cost of \$3,872.98.
(f) Reorder point:

Reorder point $=$ demand during lead time

$$
=\frac{5,000 \text { units }}{250 \text { days }} \times 10 \text { days }=200 \text { units }
$$

This is not to say that we reorder when there are 200 units on hand (as there never are). The ROP indicates that orders are placed several cycles prior to their actual demand.
12.46 (a) Total cost $=$ order cost + holding cost $=\frac{D S}{Q}+\frac{Q H}{2}$

For $Q=50$ :
$\frac{600 \times 60}{50}+\frac{50 \times 20}{2}=720+500=\$ 1,220$
(b) Economic Order Quantity:
$Q=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2 \times 600 \times 60}{20}}=60$ units
where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost
For $Q=60$ :

$$
\frac{600 \times 60}{60}+\frac{60 \times 20}{2}=600+600=\$ 1,200
$$

(c) Reorder point:

Reorder point $=$ demand during lead time

$$
=\frac{600 \text { units }}{250 \text { days }} \times 10 \text { days }=24 \text { units }
$$

12.47 Economic Order Quantity, noninstantaneous delivery:

$$
Q=\sqrt{\frac{2 D S}{H\left(1-\frac{d}{p}\right)}}=\sqrt{\frac{2 \times 8,000 \times 100}{0.80\left(1-\frac{40}{150}\right)}}=1,651.4 \text { or } 1,651 \text { units }
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost, $d=$ daily demand rate, $p=$ daily production rate
12.48 Economic Order Quantity:

$$
Q=\sqrt{\frac{2 D S}{H}}
$$

where: $D=$ annual demand, $D=$ setup or order cost, $H=$ holding cost, $p=$ price/unit
(a) Economic Order Quantity, standard price:

$$
Q=\sqrt{\frac{2 \times 2,000 \times 10}{1}}=200 \text { units }
$$

Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
& =\frac{D S}{Q}+\frac{Q H}{2}+P D \\
& =\frac{2,000 \times 10}{200}+\frac{200 \times 1}{2}+(2,000 \times 1) \\
& =100+100+2,000 \\
& =\$ 2,200
\end{aligned}
$$

(b) Quantity Discount:

Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
= & \frac{D S}{Q}+\frac{Q H}{2}+P D \\
= & \frac{2,000 \times 10}{2,000}+\frac{2,000 \times 1}{2} \\
& +(2,000 \times 0.75) \\
= & 10+1,000+1,500=\$ 2,510
\end{aligned}
$$

Note: No, EOQ with 200 units and a total cost of $\$ 2,200$ is better.
12.49 Under present price of $\$ 7.00$ per unit, Economic Order Quantity:

$$
\begin{aligned}
& Q=\sqrt{\frac{2 D S}{H}} \\
& Q=\sqrt{\frac{2 \times 6,000 \times 20}{0.15 \times 7}}=478.1 \text { or } 478 \text { units }
\end{aligned}
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost, $p=$ price/unit
Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
& =\frac{D S}{Q}+\frac{Q H}{2}+P D \\
& =\frac{6,000 \times 20}{478}+\frac{478 \times 0.15 \times 7}{2}+(7 \times 6,000) \\
& =251.05+250.95+42,000 \\
& =\$ 42,502.00
\end{aligned}
$$

Note: Order and carrying costs are not equal due to rounding of the EOQ to a whole number. Under the quantity discount price of $\$ 6.65$ per unit:
Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
& =\frac{D S}{Q}+\frac{Q H}{2}+P D \\
& =\frac{6,000 \times 20}{3,000}+\frac{3,000 \times 0.15 \times 6.65}{2}+(6,000 \times 6.65) \\
& =40.00+1,496.25+39,900=\$ 41,436.25
\end{aligned}
$$

Therefore, the new policy, with a total cost of $\$ 41,436.25$, is preferable.
12.50 Economic Order Quantity:

$$
Q=\sqrt{\frac{2 D S}{H}}
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost, $P=$ price/unit
(a) Order quantity 9 sheets or less, unit price $=\$ 18.00$
$Q=\sqrt{\frac{2 \times 100 \times 45}{0.20 \times 18}}=50$ units
Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
= & \frac{D S}{Q}+\frac{Q H}{2}+P D \\
= & \frac{100 \times 45}{50}+\frac{50 \times 0.20 \times 18}{2}+(18 \times 100) \\
= & 90+90+1,800 \\
= & \$ 1,980 \text { (see note at end of problem } \\
& \text { regarding actual price) }
\end{aligned}
$$

(b) Order quantity 10 to 50 sheets: unit price $=\$ 17.50$
$Q=\sqrt{\frac{2 \times 100 \times 45}{0.20 \times 17.50}}=50.7$ units or 51 units
Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
= & \frac{D S}{Q}+\frac{Q H}{2}+P D \\
= & \frac{100 \times 45}{51}+\frac{51 \times 0.20 \times 17.50}{2} \\
& +(17.50 \times 100) \\
= & 88.23+89.25+1,750.00=1,927.48
\end{aligned}
$$

Note: Order and carrying costs are not equal due to rounding the EOQ to a whole number. See note at end of problem regarding price.
(c) Order quantity more than 50 sheets: unit price $=$ \$17.25

$$
Q=\sqrt{\frac{2 \times 100 \times 45}{0.20 \times 17.25}}=51.1 \text { units or } 51 \text { units }
$$

Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
= & \frac{D S}{Q}+\frac{Q H}{2}+P D \\
= & \frac{100 \times 45}{51}+\frac{51 \times 0.20 \times 17.25}{2} \\
& +(17.25 \times 100) \\
= & 88.24+87.98+1,725.00=\$ 1,901.22
\end{aligned}
$$

Therefore, order 51 units.
Note: Order and carrying costs are not equal due to rounding of the EOQ to a whole number.

Important Note: Students will likely complete all three sets of calculations, including the calculations of total costs. They should be prompted to realize that calculations of total cost under (a) and (b) are actually inappropriate because the original assumptions as to lot size would not be satisfied by the calculated EOQs.
12.51 $Z=1.28$ for $90 \%$ service level Safety stock $=(1.28)(15)=19.2$ or 19 Reorder point $=36+19=55$ TVs

## Case Studies

## ZHOU BICYCLE COMPANY

1. Inventory plan for Zhou Bicycle Company. The forecasted demand is summarized in the following table.

| Jan | 8 | July | 39 |
| :--- | ---: | :--- | ---: |
| Feb | 15 | Aug | 24 |
| Mar | 31 | Sept | 16 |
| April | 59 | Oct | 15 |
| May | 97 | Nov | 28 |
| June | 60 | Dec | $\frac{47}{}$ |
|  |  | Total | 439 |

Average demand per month $=439 / 12=36.58$ bicycles. The standard deviation of the monthly demand $=25.67$ bicycles. The inventory plan is based on the following costs and values.

| Order cost | $=\$ 65 /$ order |
| :--- | :--- |
| Cost per bicycle | $=\$ 102.00$ |
| Holding cost | $=(102.00) \times(1 \%) \times 12$ per year per bicycle |
|  | $=\$ 12.24$ per year per bicycle |
| Service level | $=95 \%$, with corresponding $Z$ value of 1.645 |
| Lead time | $=1$ month $(4$ weeks $)$ |
| Total demand/year | $=439$ units of bicycles |

The solution below uses the simple EOQ model with reorder point and safety stock. It ignores the seasonal nature of the demand. The fluctuation in demand is dealt with by the safety stock based on the variation of demand over the planning horizon.

Economic order quantity $\left(Q^{*}\right)$ is given by:

$$
Q=\sqrt{\frac{2 \times(\text { Total demand }) \times(\text { Ordering cost })}{\text { Holding cost }}}
$$

where the total demand and the holding cost are calculated on the same time unit (monthly, yearly, etc.). Thus,

$$
Q=\sqrt{\frac{2 \times 439 \times 65}{12.24}} \approx 68 \text { units of bicycles }
$$

2. The reorder point is calculated by the following relation:

Reorder point $(\mathrm{ROP})=$ average demand during the lead time $(\mu)$ $+z \times$ (standard deviation of the demand during the lead time $(\sigma)$ )
Therefore, $($ ROP $)=36.58+1.645(25.67) \approx 79$ bicycles.
Safety stock ( $s s$ ) is given by $s s=z \sigma=1.645(25.67) \approx 42$ bicycles. Inventory cost is calculated as follows:

Total annual
inventory cost $=$ Annual holding cost + Annual ordering cost

$$
\begin{aligned}
& =\frac{1}{2} Q^{*}(\text { Holding cost })+s s(\text { Holding cost }) \\
& +\frac{\text { Total Demand }}{Q^{*}}(\text { Ordering cost }) \\
& =\$ 416.16+\$ 514.08+\$ 419.63=\$ 1,349.87
\end{aligned}
$$

(rounded to integer values)
This case can be made more interesting by asking the students to trace the inventory behavior with the above plan (assuming that the forecast figures are accurate and ignoring the forecast errors) and to see the amount of total stockout, if any. The students then can calculate the lost profit due to stockout and add it to the total cost.
3. A plot of the nature of the demand clearly shows that it is not a level demand over the planning horizon. An EOQ for the entire year, therefore, may not be appropriate. The students should try to segment the planning horizon in a way so that the demand is more evenly distributed and come up with an inventory plan for each of these segments (e.g., quarterly inventory planning). The challenge is then to manage the transition from one planning period to the next. Again, a plot of the inventory behavior may be of help to the students.


## 2 STURDIVANT SOUND SYSTEMS

1. Compute the optimal order quantity. First, determine the cost under the present policy:

Number of orders/year $=52$ weeks $\div 4$ weeks $=13$ orders
Average order size $=5,000 / 13=384.6$ or 385 units
Total cost $=$ order cost + holding cost + purchase cost
Purchase cost $=5000$ units $\times 60 /$ unit $=300,000$
Order cost $=\$ 20 /$ order $\times 13$ orders $=260$

$$
\text { Carrying cost }=\frac{384.6 \text { units/order } \times \$ 6 / \text { unit }}{2 \text { Total cost }}=\frac{1154}{\$ 301,414}
$$

Next, develop an Economic Order Quantity, and determine the total costs:

$$
Q=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2 \times 5,000 \times 20}{6}}=182.5 \text { or } 183 \text { units }
$$

where: $D=$ annual demand, $S=$ setup or order cost, $H=$ holding cost.
2. Determine the appropriate reorder point (in units).

Reorder point $=$ demand during lead time $=20 \times 5=100$
3. Compute the cost savings that the company will realize if it implements the optimal inventory procurement decision.

Total cost $=$ order cost + holding cost + purchase cost

$$
\begin{aligned}
& =\frac{D S}{Q}+\frac{Q H}{2}+P D \\
& =\frac{5,000 \times 20}{183}+\frac{183 \times 6}{2}+(5,000 \times 60) \\
& =546.45+549.00+300,000.00 \\
& =\$ 301,095.45
\end{aligned}
$$

Note: Order and carrying costs are not equal due to rounding of the EOQ to a whole number.

The cost savings under the EOQ ordering policy would then be:
Cost under present policy: $\$ 301,414.00$
Cost under EOQ policy: $\quad \frac{301,095.45}{\$ 318.55}$
which is a very small savings.
4. The typical costs associated with procurement of materials include costs of preparing requisitions, writing purchase orders, receiving merchandise, inspecting goods, storage, updating inventory records, and so forth. These costs are usually fixed, regardless of the size of the order. A large order may require more processing time (in inspection, for example), but the increase in procurement costs is typically minimal. As lot size increases, the number of orders decreases (assuming a constant requirement level). Consequently, procurement costs typically decrease with an increase in lot size.

## Video Case Study

## INVENTORY CONTROL AT WHEELED COACH

The 7 minute video, filmed specifically for this text, is available from Prentice Hall and designed to supplement this case. A 2 minute edited version of the video also appears on the student CD in the text.

1. Wheeled Coach implements $A B C$ analysis by identifying the annual use of those high dollar items and classifying them as A. They represent some $15 \%$ of the total inventory items, but $70-80 \%$ of the total cost. B items are those items that are of medium value that represent $30 \%$ of the items and $15-25 \%$ of the value. The low dollar items are class C items, which represents 5\% of the annual dollar volume, but about $55 \%$ of the total items.
2. The inventory control manager at Wheeled Coach would want to not only have ABC analysis but implement tight physical control of the stockroom. He would also implement a cycle counting system, and ensure that issues require engineering change notices
for those items not initially included on the bill of material. To the extent feasible, stockrooms would be consolidated.
3. The inventory control manager would implement these changes through effective leadership, hiring and training of cycle counters, and effective training and education of all staff, from engineering through cycle counters, so that each understands the entire system and the importance of maintaining accurate inventory. We would also want to be assured that all concerned employees understand the importance of accurate inventory records, tight inventory control, and locked stockrooms. Management would have to exhibit the proper leadership and support of the entire system, including accurate bills of material, rapid issuing of ECN's, training budgets, etc.

## Internet Case Studies*

## 1 MAYO MEDICAL CENTER

1. The benefits of bar codes in hospitals are much the same as in any inventory application. These benefits include ease (low cost) of collecting inventory data and accuracy of inventory records. Such systems in turn contribute to systems with low inventory investment, but that have materials when they are needed.
2. A natural extension with the hospital suggests accurate charges to patient bills, reduced pilferage, and improved care through reduction of shortages.
3. A natural extension in the supply chain suggests more accurate inventory, which means orders placed at the correct time for the correct quantity. Accurate inventory records also support blanket ordering and quantity discounts.
4. EDI and Internet connections reduce costs for both purchaser and supplier as well as reducing communication delay.

## SOUTHWESTERN UNIVERSITY: F

Key Points: This case lets the student look at a simple inventory problem that can be discussed at several levels. By using a standard EOQ formula, the student gets a fast, easy solution that is close. However, the case lends itself to further discussion that can make the limitations of EOQ readily apparent.

1. Because this is a one-year demand, demand violates the EOQ assumption of constant demand. Therefore, the number of orders should not be prorated (as does the standard EOQ computation) nor are all orders at the EOQ optimum of 60,000 . The total cost and total profit will not be accurate if the theoretical solution is used.

Theoretical Solution: Maddux should order 60,000 per order from First Printing. The simple theoretical EOQ solution is $3 \frac{1}{3}$ orders of 60,000 each for a setup cost of $\$ 1,000$, and the total is $\$ 310,600$. The instructor can accept this as less than precise, but adequate. The solution is close because the total EOQ line is so flat (robust) around the optimum. Alternatively, the instructor can expand the discussion to the real application.

Excel OM software output (theoretical solution) is shown below.

|  | Data |  |  |  |
| :--- | ---: | :---: | ---: | ---: |
| Demand rate, D | 200,000 |  |  |  |
| Setup cost, S | 300 |  |  |  |
| Holding cost \%, I | 0.5 |  |  |  |
|  | Range 1 | Range 2 | Range 3 | Range 4 |
| Minimum quantity | 10,000 | 30,000 | 60,000 | 250,000 |
| Unit Price, $P$ | 1.62 | 1.53 | 1.44 | 1.26 |


|  | Results |  |  |  |
| :--- | :---: | ---: | ---: | ---: |
|  | Range 1 | Range 2 | Range 3 | Range 4 |
| Q* (Square root form) | $12,171.61$ | $12,524.48$ | $12,909.94$ | $13,801.31$ |
| Order quantity | $12,171.61$ | $30,000.00$ | $60,000.00$ | $250,000.00$ |
| Holding cost | $\$ 4,929.50$ | $\$ 11,475.00$ | $\$ 21,600.00$ | $\$ 78,750.00$ |
| Setup cost | $\$ 4,929.50$ | $\$ 2,000.00$ | $\$ 1,000.00$ | $\$ 240.00$ |
| Unit costs | $\$ 324,000.00$ | $\$ 306,000.00$ | $\$ 288,000.00$ | $\$ 252,000.00$ |
| Total cost | $\$ 333,859.01$ | $\$ 319,475.00$ | $\$ 310,600.00$ | $\$ 330,990.00$ |

Actual Solution. The demand is not constant. Maddux needs 200,000 programs this year. The programs will be different next year when he will also have a new forecasted demand, depending on how the team does this year. Maddux's real solution will be more like this one: Maddux should order programs from First Printing. He places 3 orders for 60,000 and 1 for 20,000 at an actual total cost of $\$ 308,800$.
$\left.\begin{array}{rl} & =(\$ 1.44 \times 200,000)=\$ 288,000 \\ \text { Theoretical unit cost } & (\$ 1.44 \times 3 \times 60,000)+(\$ 1.53 \\ & \times 20,000)=\$ 259,200+\$ 30,600 \\ = & \$ 289,600\end{array}\right)$
2. The insert ordering includes another set of issues. Although some students might use a standard Quantity Discount Model and suggest that the order quantity should be 60,000 units, purchased from First Printing, as shown in the Excel OM printout below, the real problem is somewhat different.

| Data |  |  |  |  |
| :--- | ---: | :---: | :---: | ---: |
| Demand rate, $D$ | 200,000 |  |  |  |
| Setup cost, $S$ | 300 |  |  |  |
| Holding cost \%, I | 0.05 | Range 1 | Range 2 | Range 3 |
|  | Range 4 |  |  |  |
| Minimum quantity | 10,000 | 30,000 | 60,000 | 250,000 |
| Unit price, $P$ | 0.81 | 0.765 | 0.72 | 0.63 |

Therefore, Maddux should order 40,000 inserts from First Printing for each game at a cost of $\$ 32,430$ per game, and $5 \times 32,430$ $(5$ games $)=\$ 162,150$ per season.

$$
\text { Unit cost }=\$ 0.765 \times 40,000=\$ 30,600
$$

Ordering cost $=5$ orders must be placed @ 40,000 inserts; 5 setups cost $\$ 1,500$ @ $\$ 300$ each.
Holding cost $=5 \%$ of $\$ 0.765 \times(40,000 / 2)=\$ 1,530$ (assume average inventory is 20,000 ).
Per game insert cost $=(\$ 0.765 \times 40,000)+(\$ 300)$

$$
\begin{aligned}
& +(5 \% \text { of } \$ 0.765 \times 40,000 / 2) \\
= & \$ 30,600+\$ 300+\$ 1,530=\$ 32,430
\end{aligned}
$$

Per season insert cost $=\$ 32,430 \times 5$ games $=\$ 162,150$
3. Total cost for the season is: Programs $=\$ 308,800$

$$
\text { Inserts }=\$ 198,750
$$

Total cost for season $=\$ 507,550$
4. Maddux might do several things to improve his supply chain.

- Ask the potential vendors if there is an additional discount if he buys programs and inserts from the same vendor.
- Ask if he can have the same discount schedule if he places a blanket order for all 200,000, but ask for releases on a per game basis.
- He may also be able to save money if he can reduce his trips to Ft. Worth by combining pickups of programs and inserts.
- He might also prevail upon the vendors to hold the programs and inserts at the printing plant until just before the game, reducing his holding cost.


## 3 PROFESSIONAL VIDEO MANAGEMENT

1. To determine the reorder points for the two suppliers, daily demand for the videotape systems must be determined. Each video system requires two videotape systems that are connected to it, thus the demand for the videotape units is equal to two times the number of complete systems.

The demand for the complete video system appears to be relatively constant and stable. The monthly demand for the past few months can be averaged, and this value can be used for the

|  | Resuls |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Range 1 | Range 2 | Range 3 | Range 4 |
| $Q^{*}$ (Square root form) | $54,433.1$ | $56,011.2$ | $57,735.0$ | $61,721.3$ |
| Order quantity | $54,433.1$ | $56,011.2$ | 60,000 | 250,000 |
| Holding cost | $\$ 1,102.27$ | $\$ 1,071.21$ | $\$ 1,080.00$ | $\$ 3,937.50$ |
| Setup cost | $\$ 1,102.27$ | $\$ 1,071.21$ | $\$ 1,000.00$ | $\$ 300.00$ |
| Unit costs | $\$ 162,000.00$ | $\$ 153,000.00$ | $\$ 144,000.00$ | $\$ 126,000.00$ |
| Total cost | $\$ 164,204.54$ | $\$ 155,142.43$ | $\$ 146,080.00$ | $\$ 130,237.50$ |

Maddux needs 40,000 inserts for each game and must order them on a per game basis. Inserts for each game are unique, as statistics and lineup for each team changes as the season progresses. If 60,000 people are going to attend the game, then 40,000 inserts are required ( 2 of 3 people or $2 / 3$ of 60,000 ). Therefore, the quantity discount issue, although it should be evaluated, takes second place to the necessity of ordering 40,000 inserts for each game.
average monthly demand. The average monthly sales is equal to $(7,970+8,070+7,950+8,010) / 4=8,000$. Therefore, the average monthly demand of the videotape systems is 16,000 units, because two tape units are required for every complete system. Annual demand is 192,000 units $(192,000=12 \times 16,000)$.

We will assume that there are 20 working days per month ( 5 working days per week). Making this assumption, we can determine the average daily sales to be equal to the average monthly sales divided by 20 . In other words, the daily sales are equal to 800 units per day $(800=16,000 / 20)$.

To determine the reorder point for Toshiki, we must know the lead time. For Toshiki, it takes 3 months between the time an order is placed and when the order is actually received. In other words, the lead time is 3 months. Again, assuming 20 working days per month, the lead time for Toshiki is 60 days $(60=20 \times 3)$. In order to determine the reorder point, we multiply the demand, expressed as units per day, times the lead time in days. For Toshiki, the reorder point is equal to 48,000 units $(48,000=800 \times 60)$. The reorder point will be greater than the EOQ (see question 2 for EOQ calculations), thus the lead time will likely be more important for ordering more inventory.

For Kony, the reorder point can be computed in the same manner. Assuming again that there are 5 working days per week, we can compute the lead time in days. For Kony, it takes 2 weeks between the time an order is placed and when it is received. Therefore, the lead time in days is equal to 10 days $(10=2 \times 5)$. With the lead time expressed in days, we can compute the reorder point for Kony. This is done by multiplying the lead time in days times the daily demand. Therefore, the reorder point for Kony is 8,000 $(8,000=800 \times 10)$.
2. To make a decision concerning which supplier to use, total inventory cost must be considered for both Toshiki and Kony. Both companies have quantity discounts. Because there are two suppliers, we had to make two separate quantity discount computer runs. The first run was for Toshiki. The second run was for Kony. Toshiki had the lowest total cost of $\$ 40,950,895$. The EOQ for the minimum cost inventory policy was 20,001 . Kony had a cost of \$42,406,569.
3. Each alternative that Steve is considering would have a direct impact on the quantity discount model and the results. The first strategy is to sell the components separately. If this is done, the demand for videotape systems could change drastically. In addition to selling the videotape units along with the complete system, additional tape units could be demanded. An increase in demand could change the outcome of the quantity discount model. The second strategy would also have an impact on the results of the analysis. If other videotape systems can be used as well, there will be fewer videotape systems ordered when obtaining the complete system. At this time, exactly two videotape systems are sold with every complete system. Implementing the second strategy would cause this ratio to drop below two. Again, this will change the annual demand figures.

## WESTERN RANCHMAN OUTFITTERS (WRO)

The EOQ for a yearly demand of 2,000 , order cost of $\$ 10.00$, and holding cost of $0.12(10.05)=\$ 1.206$ is

$$
E O Q=\sqrt{\frac{2(10)(2,000)}{1.206}}=182.12
$$

The solution recommends 2,000/182 $=11$ orders to be submitted per year; WRO orders monthly. The EOQ is about 182 pairs, as compared to 167 ordered monthly. The annual cost difference is minimal.

There is one remaining problem that the model doesn't solve, but which Mr. Randell has. That is the problem of the unreliability of the supplier. By ordering one extra time ( 12 orders per year instead of 11) and by ordering extra quantities judiciously, Mr. Randell has managed to keep WRO almost totally supplied with the requisite number of Levis 501. Further, because the actual solution is so close to the model solution, and because we have seen that the EOQ is a robust model, Mr. Veta can feel that he is keeping his inventory goals close to the minimum while still meeting his goal of avoiding stockouts.

The conclusion is that the model has been shown to be practically valid with minor adjustments that compensate for the unreliability of the manufacturer.

This case differs from most in that the EOQ is just a starting point for discussion. Students must then develop their own approach and reasoning for why the current policy is acceptable or unacceptable.

## LAPLACE POWER AND LIGHT CO.

The optimal order quantity is given by:

$$
\begin{aligned}
& Q^{*}=\sqrt{\frac{2 D S}{H}}=\sqrt{\frac{2(499.5) \times 50}{41.4}} \\
& Q^{*}=34.74 \text { thousand feet }
\end{aligned}
$$

The reorder point is given by:

$$
\begin{aligned}
\text { ROP } & =\text { Daily demand } \times \text { Lead time } \\
& =\left(\frac{499.5}{260}\right)(60)
\end{aligned}
$$

$$
\mathrm{ROP}=115.27 \text { thousand feet }
$$

Currently, the company is committed to take $1 / 12$ th of its annual need every month. Therefore, each month the storeroom issues a purchase requisition for 41,625 feet of cable.

$$
\begin{aligned}
\text { Present TC } & =\left(\frac{499.5}{41.625}\right)(50)+\left(\frac{41.625}{2}\right)(41.4)+(499.5)(414) \\
& =600+861.62+206,793 \\
& =\$ 208,254.62 \\
\text { Optimum TC } & =\left(\frac{499.5}{34.74}\right)(50)+\left(\frac{34.74}{2}\right)(41.4)+(499.5)(414) \\
& =718.91+719.12+206,793 \\
& =\$ 208,231.03 \\
\text { Savings } & =\text { Present } \mathrm{TC}-\text { Optimum TC }=\$ 23.59
\end{aligned}
$$

Ordering costs are assumed to be a linear function because no matter how large an order is or how many orders are sent in, the cost to order any material is $\$ 50$ per order.

The student should recognize that it is doubtful the firm will or should alter any current ordering policy for a savings of only \$23.

