A COMPARISON OF LOW PERFORMING STUDENTS' ACHIEVEMENTS IN FACTORING CUBIC POLYNOMIALS USING THREE DIFFERENT STRATEGIES

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ABSTRACT

In this study, repeated measures design was employed to compare low performing students' achievements in factoring cubic polynomials using three strategies. Twenty-five low-performing Grade 12 students from a secondary school in Limpopo province took part in the study. Data was collected using achievement test and was analysed using repeated measures analysis of variance. Findings indicated significant differences in students' mean scores due to the strategy used. On average, students achieved better scores with the synthetic division strategy than with long division and equating coefficients strategies. The study recommends that students should be offered opportunities to try out a variety of mathematical solution strategies rather than confine them to only strategies in their prescribed textbooks.

KEYWORDS

Cubic Polynomials, equating coefficients, long division, low performing students, Synthetic division

1. INTRODUCTION

Cubic polynomials are functions of the form $y = Ax^3 + Bx^2 + Cx + D$, where the highest exponent on the variable is three, A, B, C and D are known coefficients and $A \neq 0$. Factoring cubic polynomials helps to determine the zeros (solutions) of the function. However, the procedure can be more tedious and difficult for students of low-mathematical ability especially in cases where strategies such as grouping and/or factoring the greatest common factor (GCF) are inapplicable.

Questions that require students to factorise cubic functions are common in the South African Grade 12 (senior certificate) Mathematics Examination. Our experiences show that many students find it difficult to factor cubic polynomials. Discussions with mathematics teachers and analyses of examiners' reports confirm that many South African Grade 12 students have difficulties in factoring cubic polynomials. The problem could be as a result of the way teachers expose the students to mathematical idea. Due to limited mathematical knowledge, many teachers tend to stick to teaching only what is in the prescribed textbooks, rushing through topics to cover the syllabus and avoiding the topics that they are not competent to handle (Cai et al, 2005).

In many classrooms, mathematics teaching and learning is confined to strategies that are in the prescribed textbooks and students who do not understand the solution are regarded as beyond redemption (Elmore, 2002). Yet, the growing demand for scientists and engineers in South Africa demands renaissance of mathematics teaching (McCrocklin & Stern, 2006).

Research on best practices of teaching some important mathematical aspect can help improve students' achievement in mathematics. Naroth (2010) asserts that the role of the mathematics teacher is to create an environment in which students explore multiple strategies of solving mathematical problems. Such exposures will likely help the students understand how and why certain strategies work (Naroth, 2010). Donovan and Bransford (2005) report that this serves as a scaffold to help students move from their own conceptual understanding to more abstract approaches of doing mathematics which involve their own reasoning and strategy development. However, some teachers do argue that exposing students to multiple strategies and

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heuristics will confuse the students (Naroth, 2010). The findings of this study could be drawn upon to assess such views.

A number of articles bemoan the poor achievement of South African students in matriculation examinations and international benchmark studies (for example, Howie, 2004; Mji & Makgato, 2006; Pourdavood et al, 2009). However, little is known about how teachers may address the plight of low-performing mathematics students.

This study seeks to compare low performing students' achievements in factoring cubic polynomials using three strategies and hopes to make a contribution towards addressing the plight of low-performing mathematics students in this mathematical aspect.

1.1 Strategies of Factoring Cubic Polynomials

The three strategies of factoring cubic polynomials students may use in cases where grouping and factoring the greatest common factor is inapplicable are equating coefficients, long division and synthetic division.

1.1.1 Equating Coefficients

Let us consider the function: $f(x) = x^3 - 6x^2 + 11x - 6$. To factorise this cubic function, we first find one factor by inspection. Now $(x+c)$ can only be a factor of $f(x)$ if c is a factor of the constant term (- 6). Hence the only possible factors of f(x) are: $x \pm 1; x \pm 2; x \pm 3; x \pm 6$. Let $f(x) = x^3 - 6x^2 + 11x - 6$. Then, by trial and error, $f(1) = 0$. Thus, by the factor theorem of algebra, $(x-1)$ is a factor of f(x).The strategy of equating coefficients is then set out as follows:

$$\begin{aligned} x^3 - 6x^2 + 11x - 6 &= (x - 1)(Ax^2 + Bx + C) \\ &= Ax^3 + Bx^2 + Cx - Ax^2 - Bx - C \\ \therefore x^3 - 6x^2 + 11x - 6 &= Ax^3 + (B - A)x^2 + (C - B)x - C \end{aligned}$$

Equating coefficients:

$A = 1$ ------ (i)
$B - A = - 6$ ------ (ii)
$C - B = 11$ ------ (iii)
From (i) and (ii) $B = - 5$
$C = 6$ (Equating constants)

$\therefore x^3 - 6x^2 + 11x - 6 = (x - 1)(x^2 - 5x + 6) = (x - 1)(x - 3)(x - 2)$

Students' success in using the strategy depends on their ability to simplify brackets and group like terms. Understanding the meaning of the term *coefficient* is equally important and the strategy also requires students to formulate and solve linear equations.

1.1.2 Long Division

Consider $f(x) = x^3 - 6x^2 + 11x - 6$. Then, $f(1) = 0$. Thus, by the factor theorem of algebra, $(x-1)$ is a factor of f(x). Having identified one factor by inspection (as explained above), the long division procedure is then set out as shown below:

$$\begin{array}{rl} & x^2 - 5x + 6 \\ (x-1) & \overline{)\,x^3 - 6x^2 + 11x - 6} \\ & \underline{-(x^3 - x^2)} \\ & -5x^2 + 11x \\ & \underline{-(-5x^2 + 5x)} \\ & 6x - 6 \\ & \underline{-(6x - 6)} \\ & 0 \end{array}$$

$$\therefore x^3 - 6x^2 + 11x - 6 = (x - 1)(x^2 - 5x + 6) = (x - 1)(x - 3)(x - 2)$$

Here, knowledge of laws of exponents, particularly those relating to multiplication and division is a prerequisite. Students should also be able to subtract directed numbers and work with brackets.

1.1.3 Synthetic Division

Let $f(x) = x^3 - 6x^2 + 11x - 6$, then $f(1) = 0$ which implies that $x = 1$ is a root of the cubic polynomial. The procedure for synthetic division is then set out as shown below:

$$\begin{array}{c|cccc} 1 & 1 & -6 & 1 & -6 \\ & \downarrow & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \\ & a & b & c & \end{array}$$

$$\therefore\ x^3 - 6x^2 + 11x - 6 = (x - 1)(x^2 - 5x + 6) = (x - 1)(x - 3)(x - 2)$$

This strategy requires students to understand the synthetic division algorithm, which involves multiplying and adding integers repeatedly.

1.2 Objectives of the Study

This study sought to compare the three strategies of factoring cubic polynomials presented above. The objectives were to first test whether there are any significant differences in students' achievement scores as a result of the strategies used to factor cubic polynomials and secondly, to determine which strategies are better understood and preferred by low-performing students.

1.3 Theoretical Framework

This study was largely influenced by some aspects of Bruner's cognitive theory and Van de Walle's constructivist theory of mathematics education. According to Bruner (1960), any mathematical idea can be taught in a simple form for any student to understand as long it is adapted to the student's intellectual capacity and experience. Van de Walle (2004) asserts that all students can learn all the mathematics we want them to learn provided we offer them opportunities to do so. Based on these two learning perspectives, the researchers conceived that even low-performing Grade 12 students are capable of learning any mathematical aspect we want them to learn provided they are offered opportunities to explore different strategies of solving mathematics problems. As students solve mathematical problems using different strategies, they are likely to arrive at a strategy they understand better which they can easily employ to solve such problems in future.

2. RESEARCH DESIGN

In this study, the repeated measures research design (Shuttleworth, 2009) was employed. This research design uses the same participants for each treatment condition and involves each participant being tested under all levels of the independent variable (Shuttleworth, 2009). The researchers adopted the repeated-measuresresearch design because it allows statistical inference to be made with fewer participants and enables researchers to monitor the effect of each treatment upon participants easily. According to Minke (1997), the primary strengths of the repeated measures design are that it makes an experiment more efficient, maintains low variability and keeps the validity of the results higher with small number of participants.

2.1 Sample

A purposive sample of twenty-five low-performing Grade 12 students from a secondary school in the Capricorn District in Limpopo province took in the study. Low performing students are students that persistently scored below pass mark in mathematics examinations for three years before this study. The school and the students were used because they consented to participate in the study. According to Tabachnick and Fidell (2006), the minimum sample size for detecting treatment effect(s) in a repeated-measures design is 10 + the number of dependent variables (3 in this case). Hence, the recommended minimum sample size was satisfied.

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2.2 Instruments

A cognitive test was used to collect data to measure students' achievement in factoring cubic polynomials. The test items were generated based on the concept and depth of knowledge specified in the National Curriculum Statement, Mathematics Grades 10-12 (Department of Education [DoE], 2008). The test was made up of essay type questions designed to allow the students show their understanding of the three strategies of factoring cubic polynomials. The appropriateness of the test items was evaluated by six mathematics teachers who had at least five years of mathematics teaching experience. After the evaluation process, the test was pilot-tested on a sample of ten students (from another school) in order to detect and correct any errors and ambiguities in the instrument before the main study was conducted. The final instrument was a ten-item instrument.

2.2.1 Reliability and Validity of the Instrument

The reliability of the achievement test was established by calculating Kuder-Richardson (KR 20) reliability estimate, using data from the pilot study. From the Kuder-Richardson 20 calculations, a reliability value of 0.71 was obtained meaning that the instrument was reliable (Gay et al, 2011).

The test's content validity was established through expert judgement. The experts were one Mathematics subject advisor, one Head of Mathematics Department and four mathematics teachers who had experience in teaching Grade 12. They independently judged if the test items reflected the content domain of the study. Based on their judgements, the content validity ratio (CVR) of each item was calculated using $CVR_i = \frac{\left[n_e - \left(\frac{N}{2}\right)\right]}{\left(\frac{N}{2}\right)}$ where CVR_i is the content validity ratio for the i^{th} item; n_e is the number of judges rating the item as reflected the content domain of the study and N is the total number of judges (Lawshe, 1975). The mean of the test items' CVR_i was computed in order to find the content validity index (CVI) of the test. A CVI value of $+1.00$ was obtained which implies that there was complete agreement among the judges that the test items reflected the content domain of the study (Wynd et al, 2003).

2.3 Procedures

After the students were exposed to the three strategies of factoring cubic polynomials, the test was administered to assess individual students' ability to use each of the three strategies. Students wrote the test three times, using a different strategy each time. The duration of the test was one hour and it was marked out of fifty.

2.4 Research Hypothesis

The research hypotheses were:

$\mathrm{H_0}: \bar{\mathrm{x}}_1 = \bar{\mathrm{x}}_2 = \bar{\mathrm{x}}_3$ (There is no difference between the mean scores of the students using the three strategies. That is the mean scores of the students using the three strategies are equal)

$\mathrm{H_A}$: At least one mean ($\bar{\mathrm{x}}_\mathrm{i}$) is different from the others.

3. FINDINGS

Table 1 shows the descriptive statistics of the students' scores using each of the three strategies. The result shows that the mean percentage scores of the students for the three strategies were 54.32% for equating coefficients, 31.76% for long division and 67.68% for synthetic division. Although, these results seem to suggest that synthetic was better than the other two strategies, other statistical tests had to be conducted to determine if the differences in the mean scores were statistically significant. Hence, the repeated-measures ANOVA F-test was applied to the data.

Table 1. Descriptive statistics of the students' scores.

Student	Scores		
	Equating coefficients (Strategy 1)	Long division (Strategy 2)	Synthetic division (Strategy 3)
1	26	30	42
2	30	2	54
3	22	4	54
4	28	6	32
5	32	4	80
6	84	78	84
7	48	18	46
8	24	6	66
9	66	22	92
10	78	62	92
11	72	28	82
12	82	28	84
13	62	18	58
14	34	34	88
15	32	62	94
16	66	20	52
17	90	78	92
18	54	14	88
19	76	16	28
20	88	54	80
21	26	14	30
22	70	66	74
23	68	56	50
24	38	34	68
25	64	40	82
Mean ($\bar{x}$)	54.32	31.76	67.68
SD	23.03	23.91	21.41

A one-way repeated-measures analysis of variance was performed on the data to evaluate the study hypotheses.

3.1 Results of Repeated-Measures ANOVA

3.1.1 Sphericity

Sphericity is the condition where the variances of the differences between all combinations of the repeated-measures levels are equal. Violation of this assumption causes the repeated-measures ANOVA test to increase Type I error rate (Laerd, 2012). The SPSS computed significance value for the ANOVA test would be too low and thus we risk rejecting the null hypothesis when actually we should not.

Table 2. Mauchly's Test of Sphericity.

Mauchly's Test of Sphericity[a]							
Measure: MEASURE_1							
Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	**Sig.**	Epsilon[b]		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Strategy	.959	.974	2	**.615***	.960	1.000	.500

Mauchly's Test of Sphericity indicates that the assumption of sphericity has not been violated ($\chi^2(2) = .974, \mathrm{p} = .615$) , which is non-significant. Hence, there is no need to adjust the degrees of freedom of the repeated-measures ANOVA F-Test and we report the results in the row labelled 'Sphericity Assumed' in Table 3.

3.1.2 ANOVA F-test

Table 3 shows the main results of the repeated-measures ANOVA F-test. The results in the row labelled 'Sphericity Assumed' indicate a statistically significant main effect of the independent variable (solution strategy) on the dependent variable (students' test scores) ($F(2, 48) = 32.066$, $p = .000$). Therefore, the null hypothesis that the average scores for the three strategies are the same is rejected and we conclude that at least one mean ($\bar{x}_i$) is different.

Table 3. ANOVA F-test.

Tests of Within-Subjects Effects						
Measure: MEASURE_1						
Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Strategy	Sphericity Assumed	16480.747	**2**	8240.373	**32.066**	**.000***
	Greenhouse-Geisser	16480.747	1.920	8581.923	32.066	.000
	Huynh-Feldt	16480.747	2.000	8240.373	32.066	.000
	Lower-bound	16480.747	1.000	16480.747	32.066	.000
Error (strategy)	Sphericity Assumed	12335.253	**48**	256.984		
	Greenhouse-Geisser	12335.253	46.090	267.636		
	Huynh-Feldt	12335.253	48.000	256.984		
	Lower-bound	12335.253	24.000	513.969		

Since a statistically significant result was found, the Bonferroni post hoc analysis was conducted to compare the mean scores for the three strategies in order to determine exactly where the differences lied.

Bonferroni post hoc analysis

The Bonferroni pair wise comparison table (Table 4) provides a comparison of the mean scores for all paired combinations of the levels of the repeated factor (solution strategy).

Table 4. Bonferroni pair wise comparisons.

Pair wise Comparisons						
Measure: MEASURE_1						
(I) Strategy	(J) Strategy	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
1	2	22.56	4.135	**.000***	11.917	33.203
	3	-13.36	4.933	**.037***	-26.054	-.666
2	1	-22.56	4.135	**.000***	-33.203	-11.917
	3	-35.92	4.500	**.000***	-47.500	-24.340
3	1	13.36	4.933	**.037***	.666	26.054
	2	35.92	4.500	**.000***	24.340	47.500

*. The mean difference is significant at the .05 level.
b. Adjustment for multiple comparisons: Bonferroni.

From the significance values of each pair wise comparison, we found:

The mean difference between the equating coefficients strategy ($\bar{x} = 54.32, \mathrm{s.d} = 23.03$) and the long division strategy ($\bar{x} = 31.76, \mathrm{s.d} = 23.91$) is statistically significant. The mean difference (22.56) had a probability ($p = .000$) which is less than alpha .05. The difference between the two means would be considered a substantial difference. Hence, the null hypothesis that these two means were equal was rejected and we therefore conclude that the students had better test scores on factoring cubic polynomials using the strategy of equating coefficients than using the long division strategy.

The mean difference (-13.36) between equating coefficients strategy ($\bar{x} = 54.32, \mathrm{s.d} = 23.03$) and synthetic division strategy ($\bar{x} = 67.68$, $\mathrm{s.d} = 21.41$) had a probability ($p = .037$) which is less than alpha $(.05)$. This implies that the difference is also statistically significant. Therefore, the null hypothesis that these two means were equal was rejected and we conclude that the synthetic division strategy yielded better test results for the students than the strategy of equating coefficients.

The mean difference between the long division strategy ($\bar{x} = 31.76$, $\mathrm{s.d} = 23.91$) and the synthetic division strategy ($\bar{x} = 67.68, \mathrm{s.d} = 21.41$) had a probability($p = .000$). This is less than alpha $(.05)$, meaning that it is statistically significant. The difference (-35.92) would be considered a substantial difference. Hence, the null hypothesis that these two means were equal was rejected and we concluded that synthetic division had better test scores than long division.

Since the students' mean score using synthetic division strategy (67.68%) was better than their mean scores using long division (31.76%) and the of equating coefficients (54.32%) strategies, we concluded that the strategy of synthetic division made the low-performing students to achieve better test scores in factoring cubic polynomials.

3.1.3 Confidence Intervals of the Means

Table 5 shows the 95% confidence intervals of the mean percentage scores of each of the three strategies.

Table 5. Confidence intervals of the means.

Estimates				
Measure: MEASURE_1				
Strategy	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1	54.32	4.607	44.81	63.83
2	31.76	4.781	21.89	41.63
3	67.68	4.283	58.84	76.52

The result shows that if one takes repeated samples of low-performing Grade 12 students from the population and subject them to the same conditions, one would be 95% confident that their mean score would lie between 44.81 and 63.83 percent with the strategy of equating coefficients, 21.89 and 41.63 percent with long division strategy and between 58.84 and 76.52 percent with synthetic division strategy.

3.2 Discussion

Firstly, the study sought to test whether there were significant differences in low performing students' test scores due to using three different strategies to factor cubic polynomials. Secondly, the study sought to see which strategy was preferred and better understood by the participants. Results from repeated-measures ANOVA indicated that indeed there were statistically significant differences in students' scores due to the effect of using different strategies to factor cubic polynomials. Post hoc analysis showed that students scored better with synthetic division strategy than using long division and the strategy of equating coefficients. An important practical implication of the findings of the study is that it may take several attempts to see positive results in students' achievement but we should not give up. If one strategy does not work, we should try another. The findings also debunk the perception that exposing students to multiple solution strategies serves to confuse the students. Instead, making different solution strategies available to students could help many students to achieve better in mathematics. The findings are consistent with previous assertions by Bruner (1960), Van de Walle (2004), Donovan and Bransford (2005) and Naroth (2010).

3.3 Recommendations and Conclusion

The findings of this study point to the need for teachers to expose students to different strategies for solving not only cubic polynomials but also handling other mathematical aspects. The strategies for solving mathematical problems should not be limited to the strategies in the textbooks. This will enable teachers to offer students opportunities to explore techniques of dealing with mathematical problems and understand how and why certain strategies work (Naroth, 2010).

We also recommend that teachers be given opportunities to attend professional development workshops that are conducted by experts in the field to enable them learn and develop expertise on how to teach the problematic mathematical aspects such as factoring cubic polynomials.

A similar study with a large randomised sample of students could provide more definitive evidence to strengthen the findings of this study. Future research should extend this study to other mathematical aspects and Grades to see if similar results are obtainable. Such studies could contribute towards improving students' achievement and the quality of mathematics teaching in South African secondary schools.

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