

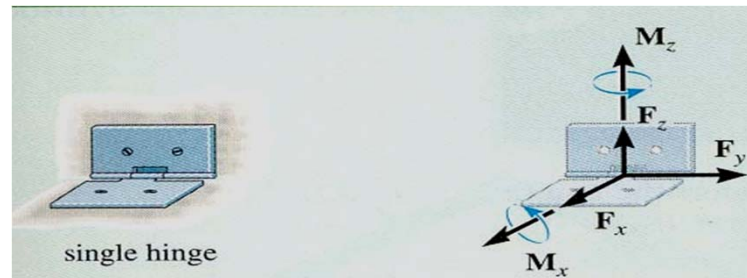
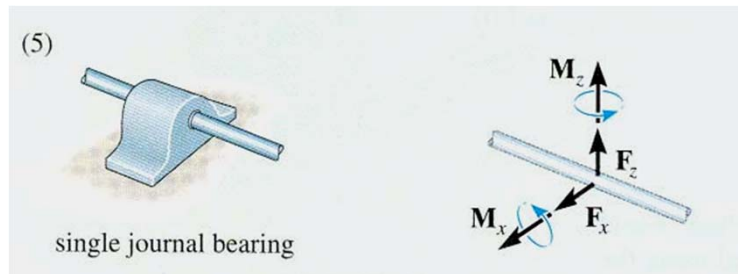
Equilibrium in Three Dimension

5.5 FREE-BODY DIAGRAMS

The first step in solving 3D equilibrium problems is to draw a free-body diagram of the body:

Support Reactions should be studied

SUPPORT REACTIONS IN 3-D (Table 5-2)



A few examples are shown above. Other support reactions are given in your text book (Table 5-2).

As a general rule, if a support prevents translation of a body in a given direction, then a reaction force acting in the opposite direction is developed on the body. Similarly, if rotation is prevented, a couple moment is exerted on the body by the support.

5.6 EQUATIONS OF EQUILIBRIUM

As stated earlier, when a body is in equilibrium, the net force and the net moment equal zero, i.e., $\sum \mathbf{F} = 0$ and $\sum \mathbf{M}_O = 0$.

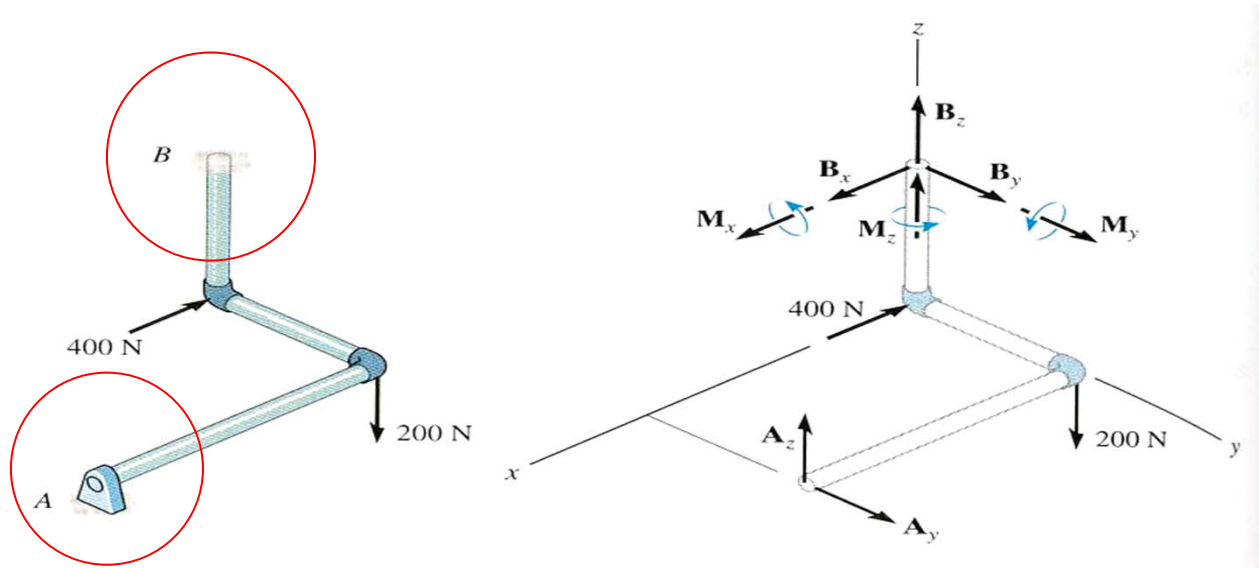
These two vector equations can be written as six scalar equations of equilibrium (EofE). These are

$$\sum F_X = \sum F_Y = \sum F_Z = 0$$

$$\sum M_X = \sum M_Y = \sum M_Z = 0$$

The moment equations can be determined about any point. Usually, choosing the point where the maximum number of unknown forces are present simplifies the solution. Those forces do not appear in the moment equation since they pass through the point. Thus, they do not appear in the equation.

5.7 CONSTRAINTS FOR A RIGID BODY

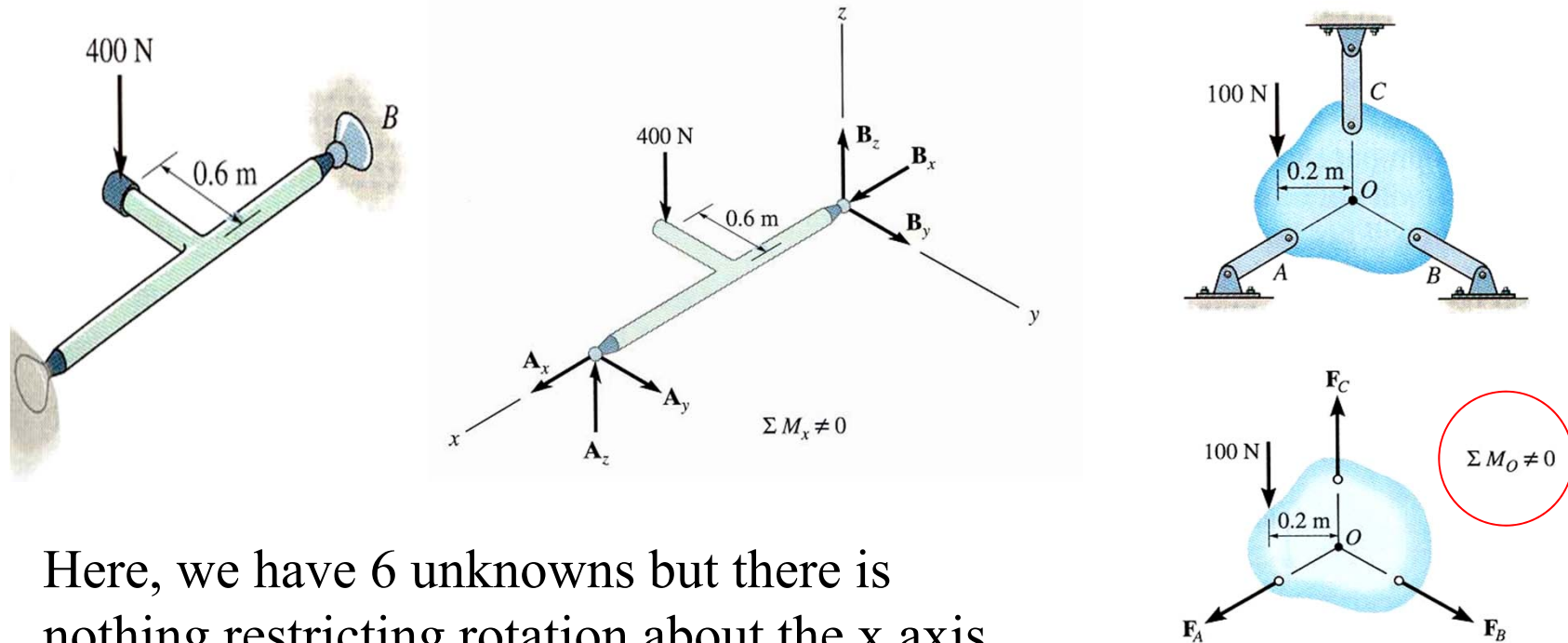


Redundant Constraints: When a body has more supports than necessary to hold it in equilibrium, it becomes statically indeterminate.

A problem that is statically indeterminate has more unknowns than equations of equilibrium.

Are statically indeterminate structures used in practice? Why or why not?

IMPROPER CONSTRAINTS



Here, we have 6 unknowns but there is nothing restricting rotation about the x axis.

In some cases, there may be as many unknown reactions as there are equations of equilibrium. However, if the supports are not properly constrained, the body may become unstable for some loading cases.

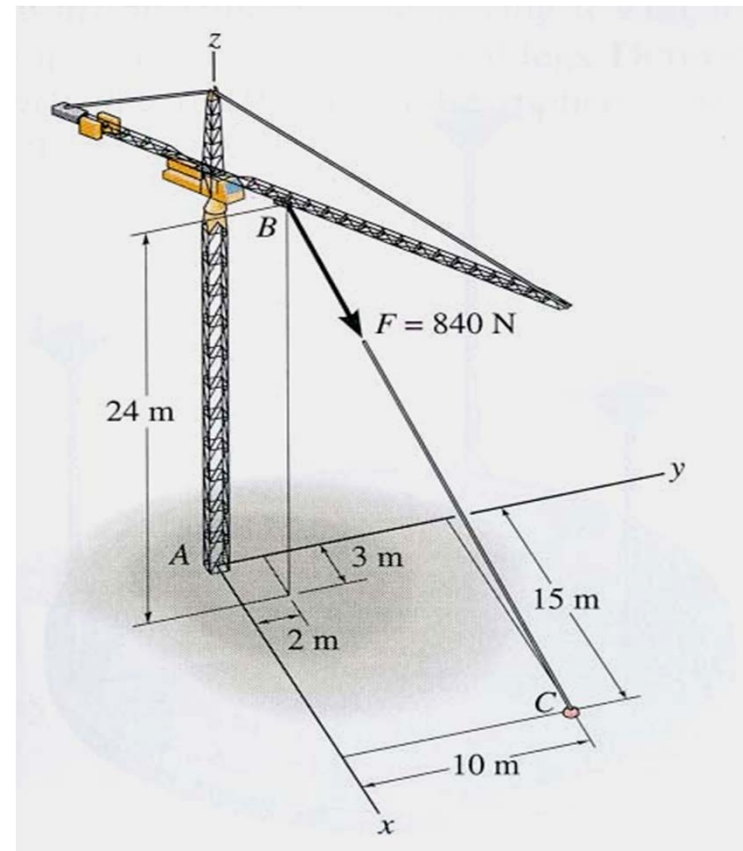
EXAMPLE

Given: The cable of the tower crane is subjected to 840 N force. A fixed base at A supports the crane.

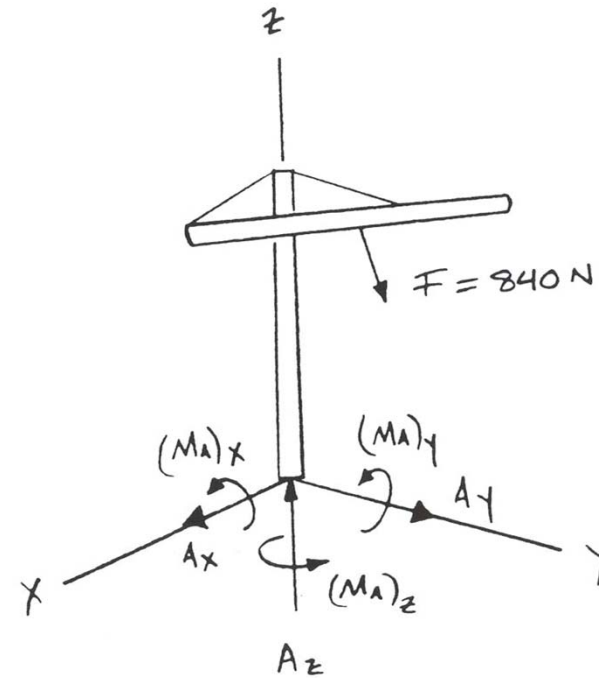
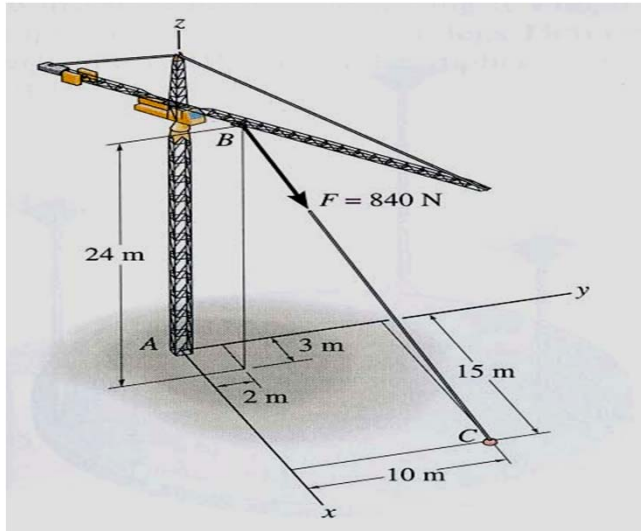
Find: Reactions at the fixed base A.

Plan:

- Establish the x, y and z axes.
- Draw a FBD of the crane.
- Write the forces using Cartesian vector notation.
- Apply the equations of equilibrium (vector version) to solve for the unknown forces.



EXAMPLE (continued)



$$\mathbf{r}_{BC} = \{12 \mathbf{i} + 8 \mathbf{j} - 24 \mathbf{k}\} \text{ m}$$

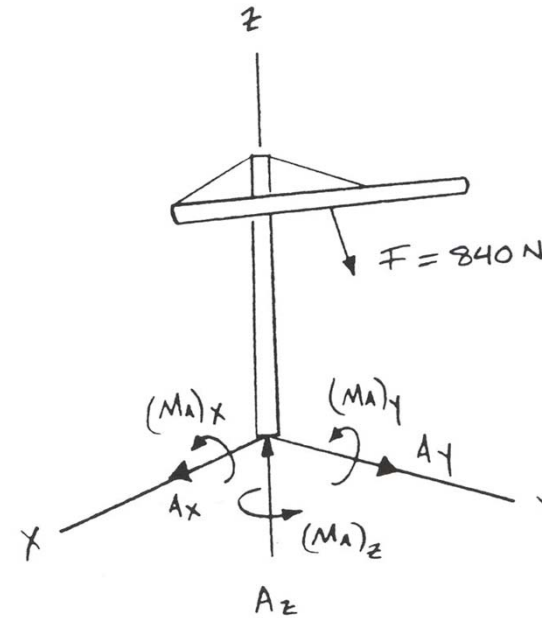
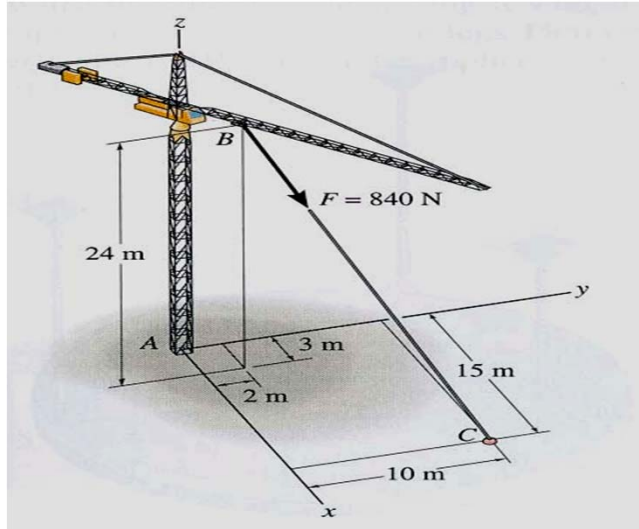
$$\mathbf{F} = F [\mathbf{u}_{BC}] \text{ N}$$

$$= 840 [12 \mathbf{i} + 8 \mathbf{j} - 24 \mathbf{k}] / (12^2 + 8^2 + (-24^2))^{1/2}$$

$$= \{360 \mathbf{i} + 240 \mathbf{j} - 720 \mathbf{k}\} \text{ N}$$

$$\mathbf{F}_A = \{A_X \mathbf{i} + A_Y \mathbf{j} + A_Z \mathbf{k}\} \text{ N}$$

EXAMPLE

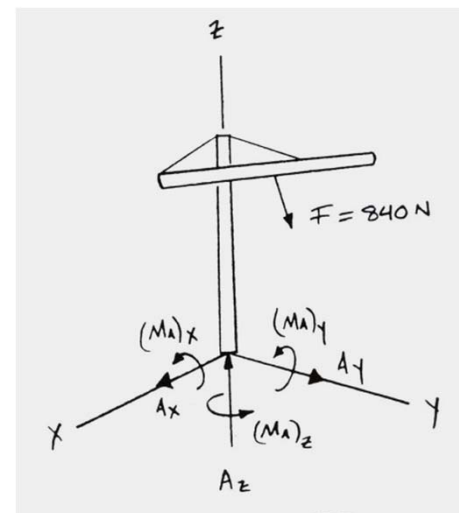
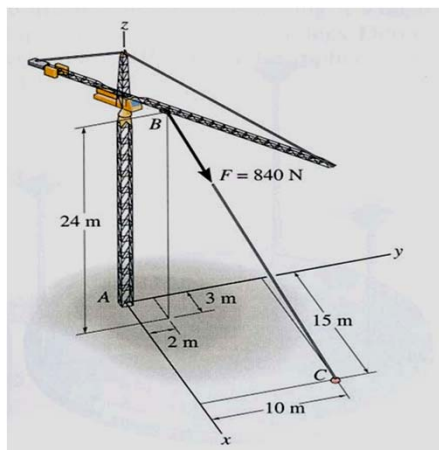


From E-of-E we get, $\mathbf{F} + \mathbf{F}_A = \mathbf{0}$

$$\{(360 + A_X) \mathbf{i} + (240 + A_Y) \mathbf{j} + (-720 + A_Z) \mathbf{k}\} = \mathbf{0}$$

Solving each component equation yields $\underline{A_X = -360 \text{ N}}$,
 $\underline{A_Y = -240 \text{ N}}$, and $\underline{A_Z = 720 \text{ N}}$.

EXAMPLE



Sum the moments acting at point A.

$$\begin{aligned}
 \sum \mathbf{M} &= \mathbf{M}_A + \mathbf{r}_{AC} \times \mathbf{F} = 0 \\
 &= M_{AX} \mathbf{i} + M_{AY} \mathbf{j} + M_{AZ} \mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 15 & 10 & 0 \\ 360 & 240 & -720 \end{vmatrix} = 0 \\
 &= M_{AX} \mathbf{i} + M_{AY} \mathbf{j} + M_{AZ} \mathbf{k} - 7200 \mathbf{i} + 10800 \mathbf{j} = 0 \\
 &\underline{M_{AX} = 7200 \text{ N} \cdot \text{m}}, \underline{M_{AY} = -10800 \text{ N} \cdot \text{m}}, \text{ and } \underline{M_{AZ} = 0}
 \end{aligned}$$

Note: For simpler problems, one can directly use three scalar moment equations, $\sum M_x = \sum M_y = \sum M_z = 0$

Question

1. The rod AB is supported using two cables at B and a ball-and-socket joint at A. How many unknown support reactions exist in this problem?
 - 1) 5 force and 1 moment reaction
 - 2) 5 force reactions
 - 3) 3 force and 3 moment reactions
 - 4) 4 force and 2 moment reactions

