Middle School Geometry
Session 4

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	Cover It Up	132	7.8, 8.7	Cover It Up	Pricing and coverage info from a paint store, yard/meter stick
	Architect's Square	135	6.17, 7.8, 8.9	Memoranda 1, 2, 3	Unit cubes (60/group), Isometric dot paper, supplies for making brochures

Topic: Circles

Description: Participants will explore relationships between the elements of a circle, including applied problems such as the relationship of circumference of a tennis ball relative to its diameter. Then they solve a problem that relates radius to area of a circle. They will use a dynamic geometry software program and a graphing calculator to approximate pi. They will use paper folding to investigate tangents to circles and inscribed angles.

Related SOL: 6.12, 7.8, 8.10

Activity: Circumference vs. Diameter

Format: Whole class demonstration with small group and whole class discussion

Objectives: Participants will measure the circumference of a tennis ball and compare it to the diameter of the ball.

Related SOL: 6.12

Materials: A can of tennis balls and a piece of string

Time Required: 15 - 20 minutes

Directions:

1) Bring a tennis ball can to class. Ask participants which is greater,: the height of the can or the circumference of the can? Have each participant write down his/her answer supported with reasons. Encourage small group discussion before having a whole class discussion.

2) Discuss participant conclusions with the whole class.

3) Wrap a string around a tennis ball and compare this to the height of the can. Discuss results.

Activity: Cake Problem

Format: Small group

Objectives: Participants will solve a real world problem using knowledge of area of circles.

Related SOL: 6.12

Materials: Cake Problem Activity Sheet

Time Required: 15 - 20 minutes

Directions:

1) Present the cake problem to participants. Make sure everyone understands the problem. Put a 10-inch diameter circle on the overhead projector to model the cake and ask a volunteer to make an estimate of the placement of the cut that solves the problem.

2) Give out the handout and have participants work in small groups to solve it. Have participants write up their solution.

3) Ask for volunteers to share solutions. Discuss variations on solutions.

4) Return to the transparency of the cake and draw the correct solution. How close did the estimate come?

Cake Problem

You have a cake that is 10 inches in diameter. You expect 12 people to share it, so you cut it into 12 equal pieces. (See Figure A).

Before you get a chance to serve the cake, 12 more people arrive! So you decide to cut a concentric circle in the cake so that you will have 24 pieces. (See Figure B).

How far from the center of the cake should the circle cut be made so that all 24 people get the same amount of cake?

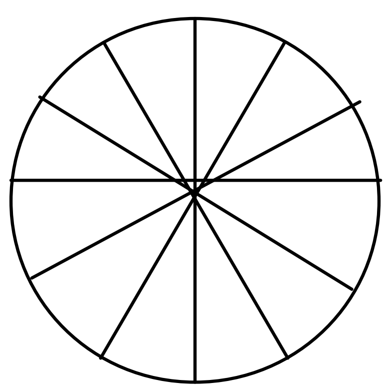


Figure A

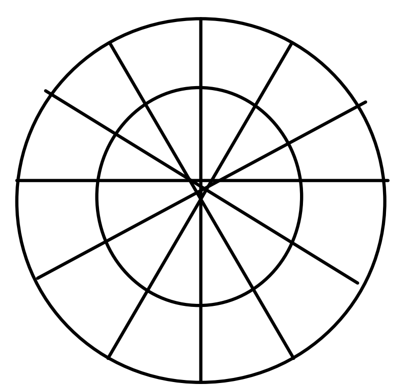


Figure B

Activity: Problem Solving With Circles

Format: Small group

Objectives: The participant will solve problems involving circumference of a circle and the volume of a cylinder.

Related SOL: 6.12, 7.8, 8.10

Materials: Calculators, Exploring Circumference and Perimeter, and Problem Solving Activity Sheets

Time Required: 45 minutes

Directions:

1) Refer to Exploring Circumference and Perimeter Activity Sheet and discuss the expectations with participants. Have them work in small groups to complete the problems. Discuss their solutions. (See the Teacher's Guide on the next page.)

2) Refer to the Problem Solving Activity Sheet and discuss it with participants. Let them work in groups to solve the problems. Then discuss the solutions with the whole class.

Exploring Circumference and Perimeter Teacher Guide

Using the Activity:
In this activity, participants can use the calculator to find the lengths of three different ramps in-line skaters might use. Participants will use the pi, square, and square-root keys on the calculator. The teacher may want to review the formula for finding the circumference (perimeter) of a circle. Discuss how to find the lengths of various arcs of the circle. To find the length of the third ramp, participants will need to use the Pythagorean Theorem.

Another important extension to this activity is finding the steepness of the various ramps. Participants can use this data to determine the level of difficulty of each ramp.

Answers:
First ramp: 51.41592654 ft
The length of the arc AB is one quarter the circumference of the circle.

Second ramp: 40.943951 ft
The length of the arc is one-sixth the circumference of the circle.

Third ramp: 41.540659 ft
The length of the ramp x is found by using the Pythagorean theorem.

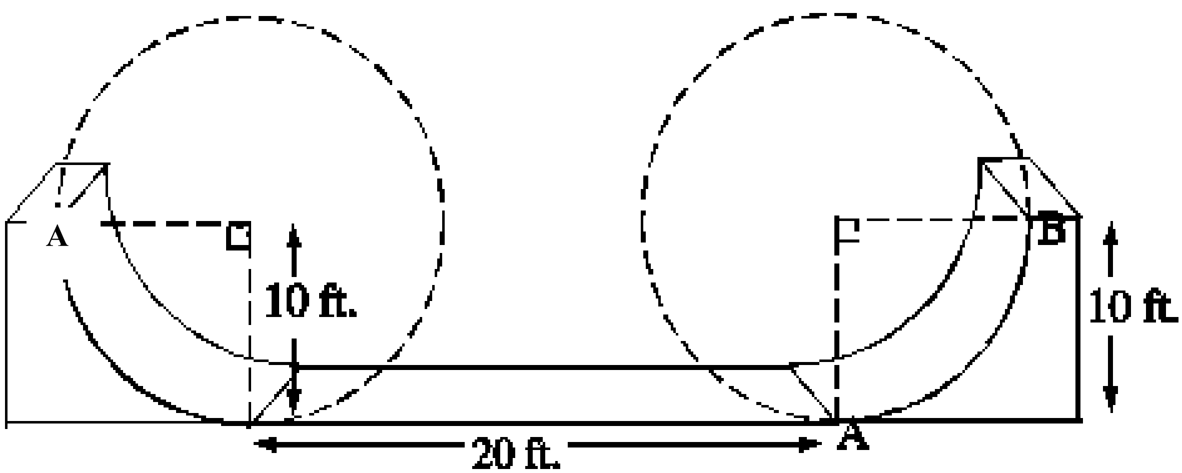
Discussion Questions:
1) What makes one ramp better than another?
2.)Which ramp is safest? Why?
3.)Which construction is more challenging? Why?

Thinking Cap:
D = 2L + (2)(1/2)(Pi)d = 2L + (Pi)d

(Adapted from lesson by Ann Mele, Assistant Principal, Offsite Educational Services; NY Public Schools; NY, NY found at http://pegasus.cc.ucf.edu/~ucfcasio/activities/line.htm.)

Exploring Circumference and Perimeter

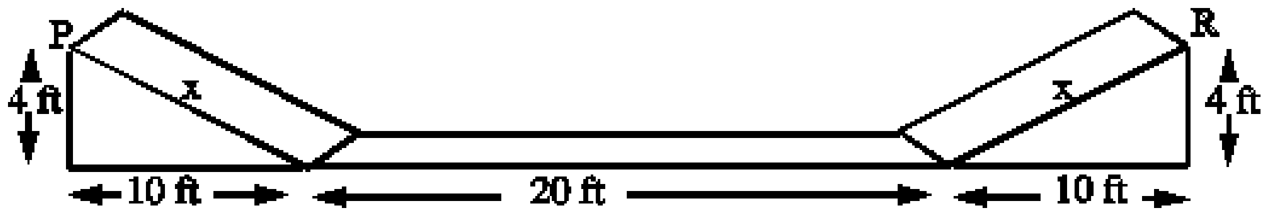
In-line skating has become a popular city sport. The parks department is thinking of constructing ramps in some of the local playgrounds. A "half-pipe" ramp is formed by two-quarter circle ramps each 10 feet high with a flat space of 20 feet between the centers of the circles from which the two-quarter circle ramps are formed.



1) Find the distance a skater travels from the top of one ramp to the top of the other. (Hint: What is the length of AB?)

2) Another launch ramp is formed by 2 arcs each with a central angle of 60 degrees and a radius of 10 ft. Find the length from the top of one ramp to the top of the other. (Hint: What fractional part of the circle is each arc?)

3. A third ramp is a straight ramp 4 ft high and 10 ft long with a flat space of 20 ft. Find the length of the ramps from point P to point R. (Hint: Use the Pythagorean Theorem)



Thinking Cap

A school track is formed by 2 straight segments joined by 2 half circles. Each segment is L long and each half circle diameter is D in length. Write a formula for finding the distance, D, around the track.

Problem Solving

SODA STRAWS

How many straws full of pineapple juice can be taken from a 46 fl. oz. can of juice that is filled to the top?

diameter of the can __________

height of the can __________

diameter of straw __________

length of straw __________

Explain how you determined your answers.

DUCT TAPE

How many rolls of duct tape would it take to create a one-mile strip of tape? (You may not unroll the duct tape or read its length from the packaging). You may experiment with one 6-inch piece of tape.

1) Imagine tape as concentric circles...1st method

2) Find volume of tape...2nd method

(Adapted from lesson by Bob Garvey, Louisville Collegiate School, Louisville, KY)

Topic: Pythagorean Theorem

Description: Participants will engage in experiences that allow them to verify the Pythagorean Theorem and its converse. They are guided through several variations of proofs of the Theorem.

Related SOL: 8.10

Activity: Geoboard Exploration of the Pythagorean Relationship

Format: Whole class and small groups

Objectives: The participant will explore the Pythagorean relationship by constructing right triangles on a geoboard and verifying the Pythagorean Theorem.

Related SOL: 8.10

Materials: 11-pin geoboards or dot paper, Geoboard Exploration of Right Triangles Activity Sheet

Time Required: 30 minutes

Directions:

1) On a transparent geoboard on the overhead projector, construct a right triangle in which one leg is horizontal and the other is vertical. Ask a participant to construct a square on each leg and then on the hypotenuse of the triangle. Ask participants to find the area of each square. It may be difficult for some participants to recognize a way to find the area of the square on the hypotenuse, so you may need to assist them.

2) Refer to the Geoboard Exploration of Right Triangles Activity Sheet and have them fill in the data from the example that was done by the whole class. Then have small groups work to find several other examples and record them in the chart.

3) Using the Activity Sheet, debrief the examples with the whole class:

   - What patterns do you see? (If participants have not seen it before, you may tell them that this is the Pythagorean relationship that will be stated later as a theorem.)
   - Can you state the relationship in words? In symbols?
   - Do you think this is always true?
   - If you label the sides of the triangle, can you write a statement of what you think is true?
   - Does this procedure provide a proof that the relationship is always true?

Geoboard Exploration of Right Triangles

Make a right triangle on a large geoboard or dot paper. Construct a square on each side of the triangle. Label the shortest side a, the middle side b and the longest side (hypotenuse) c.

Complete the Table

Length of side a	Length of side b	Length of side c	Area of square on side a	Area of square on side b	Area of square on side c	$a^2 + b^2$

Activity: Egyptian Rope Stretching

Format: Large Group

Objectives: The participant will study an artifact of ancient Egyptian culture and will see how it was used as an application of the Pythagorean Theorem.

Related SOL: 8.10

Materials: A rope that has 13 knots tied at equal intervals

Time Required: 30-40 minutes

Directions:

1) Show participants the rope that has 13 knots tied at equal intervals. Tell them that a picture of a rope like this was found on inscriptions in tombs of ancient Egyptian kings. Ask participants to work in groups to figure out what the purpose of the rope might have been.

2) Ask participants to suggest ideas for the way the Egyptians used the rope. If participants come up with the idea that it was a used to make a template for determining right angles, let them demonstrate. If they do not, you should have two participants help you demonstrate. Have a participant hold knot #1 and knot #13 together. Have another hold a different knot. You should hold knot # 4. All three of you should stretch the rope and have the class observe the resulting shape. Make sure they understand that the only shape that can be formed when you pull is a right triangle.

3) Have participants conjecture about how the Egyptians might have used this rope. (To build right angled corners on pyramids? To redraw the boundaries of the fields after the spring flooding of the Nile?)

4) Ask participants what other numbers of knots in a rope might be used to serve the same purpose of forming a right triangle. For example:
   - Could you obtain the right triangle result with a rope that has 20 equally spaced knots? Try it.
   - Could you do it with a rope with 31 knots (30 spaces)? Why or why not?

Topic: Solid Geometry

Description: Participants will explore three-dimensional shapes by sorting and classifying them, determining what they are by touch, building them, and taking them apart.

Related SOL: 6.17, 7.8, 8.7, 8.9

Activity: Polyhedron Sort

Format: Small Group/Large Group

Objectives: Participants sort polyhedra in three different ways and compare their sort to what they predict their participants would do. They then relate the sorts to the van Hiele levels of geometric understanding of polyhedra.

Related SOL: 6.17

Materials: One set of geometric solids per group of 4-6

Time Required: Approximately 10 minutes

Directions:

1) Divide the participants into small groups. Pass out the sets of geometric solids, one set per 4-6 participants. Have them sort the solids into groups that belong together, sketching the pieces they put together and the criteria they used to sort. Have them sort two or three times, recording each sort.

2) Ask the participants for some of their ways of sorting. Have them compare their ways with those of other groups.

Activity: What's My Shape? Ask Me About It.

Format: Large Group

Objectives: Participants will use logical reasoning to determine which polyhedron is in the box after asking questions and receiving information about it.

Related SOL: 6.17

Materials: One set of geometric solids, box or bag

Time Required: Approximately 10 minutes

Directions:

1) The teacher says to the participants, "This box (or bag) contains a polyhedron." Shake it so the participants can hear. "I'd like you to ask me some questions with yes or no answers to figure out what is in the box."

2) Questions and answers continue until the participants can figure out what shape is in the box.

Activity: What's My Shape? Touch Me.

Format: Large Group

Objectives: Participants will determine which polyhedron is in the bag by touch alone.

Related SOL: 6.17

Materials: One set of geometric solids, a paper bag

Time Required: Approximately 10 minutes

Directions:

1) The teacher says to the participants, "This bag contains a polyhedron." Shake it so the participants can hear. "One of you at a time may put your hand into the bag and touch the solid. Try to figure out what is in the bag."

2) The participants take turns touching the solid in the bag without looking and try to figure out what the solid is.

Activity: Creating Nets

Format: Small Group /Large Group

Objectives: Participants will determine the shapes that form various polyhedra and analyze how they fit together to form the polyhedra.

Related SOL: 6.17

Materials: Cardboard cereal boxes, canisters, milk cartons, etc., emptied and cleaned, scissors, Creating Nets Activity Sheet

Time Required: Approximately 10 minutes

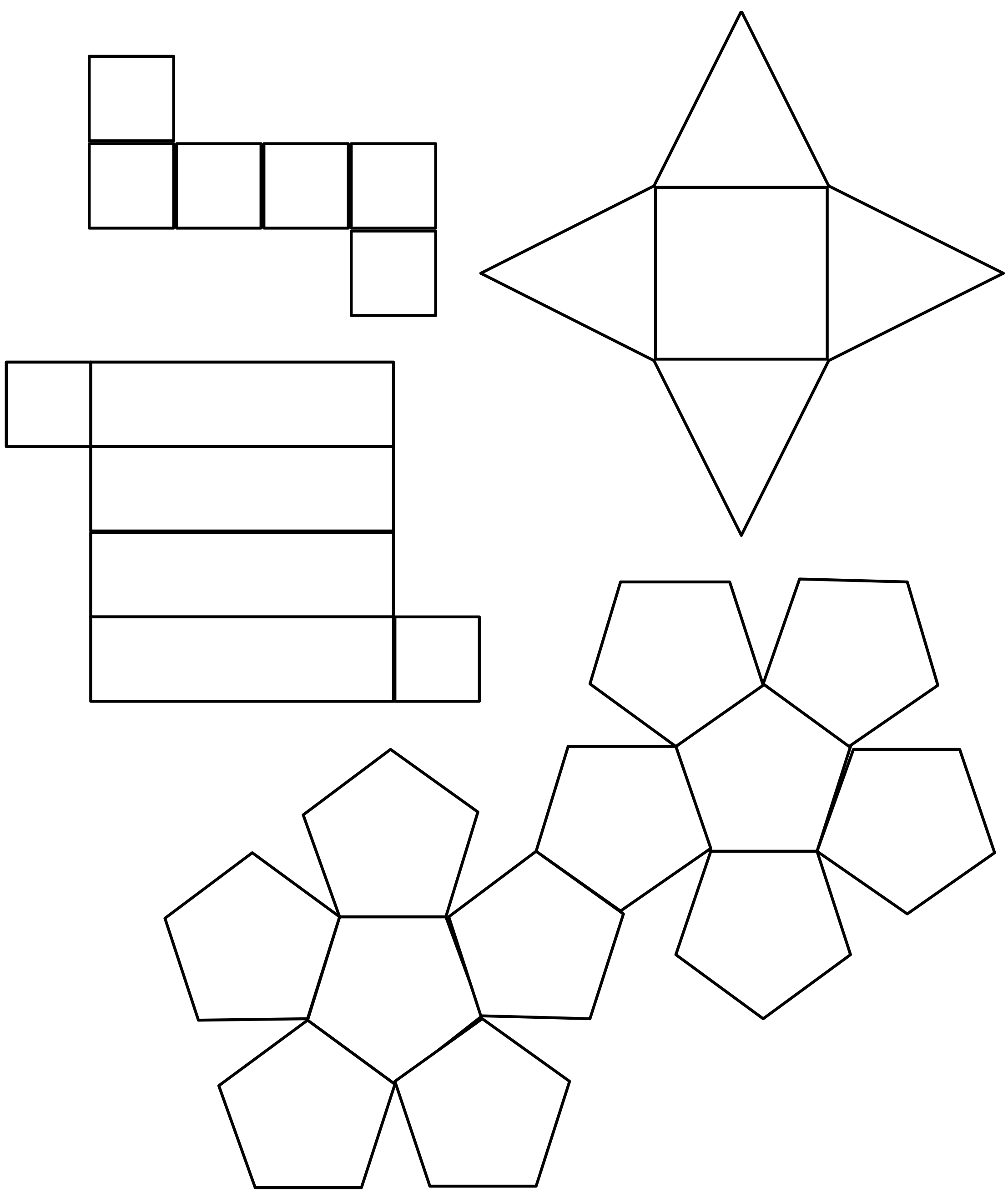
Directions:

1) The teacher or class collects a variety of cardboard containers such as cereal boxes. The teacher (or the participants) carefully cut apart a container along its seams, in such a way that the container can be flattened out, but each piece is connected. This shape is called the **net**. Some nets appear on the following page.

2) The class should identify each shape formed.

3) The class should see how many different nets they can find for the same container.

Creating Nets



Activity: Building Polyhedra

Format: Small Group / Large Group

Objectives: Participants will determine the shapes that form various polyhedra and analyze how they fit together to form the polyhedra.

Related SOL: 6.17

Materials: Scissors, tape or glue, geometric solids, and handout made by tracing the sides of various geometric solids; or a commercial 3-dimensional building kit

Time Required: Approximately 20 minutes

Directions:

1) Have each participant choose a geometric solid from the set. Have the participant make a handout by tracing each of the faces to form its net. Repeat for 2 or 3 more solids.

2) Pass out scissors, tape or glue, and multiple copies of handout, or a commercial 3-dimensional building kit.

3) Working in small groups, the participants should predict which solid the net will make. The teacher may show several solids, one of which is the one that the net will form.

4) After the predictions are made, the participants should cut out the net and tape it together.

5) The participants should compare their solids to their predictions and to the original solids.

Activity: 6-Flexagon

Format: Individual

Objectives: Participants will construct a six-flexagon (hexaflexagon) and explore what faces are visible as the structure is flexed.

Related SOL: 7.8, 8.7

Materials: Strips of sturdy paper about 2" wide, model constructed by the teacher, Flexagon Fun Activity Sheet

Time Required: Approximately 30 minutes

Directions:

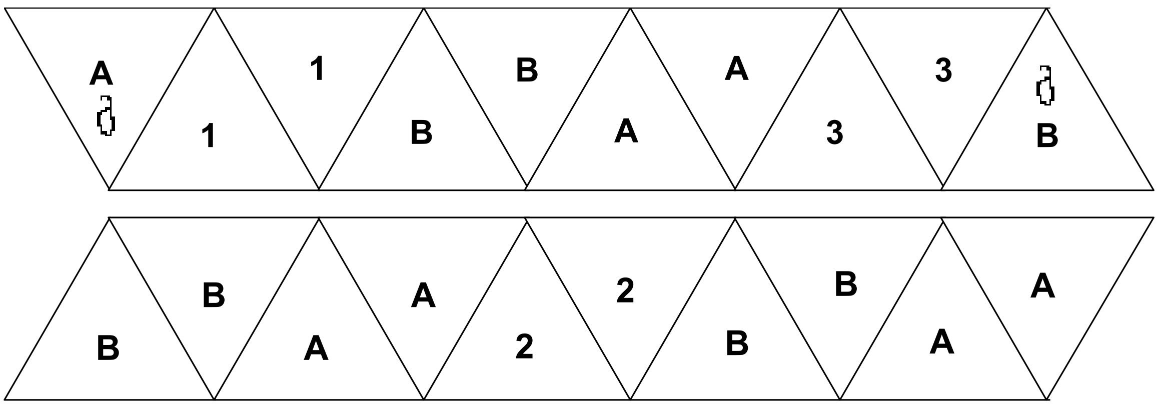
1) Review the terms mountain fold (the fold is at the top with the paper coming down) and valley fold (the fold is at the bottom with the paper coming up) so that all participants are familiar with the terms.

2) Have participants create a flexagon using the pattern and instructions provided on the activity sheet.

3) Participants should explore which faces of the flexagon are visible at which times. How can they use this information to make a game of the flexagon?

Flexagon Fun

1) Using the pattern below, mark your strip of paper as indicated.

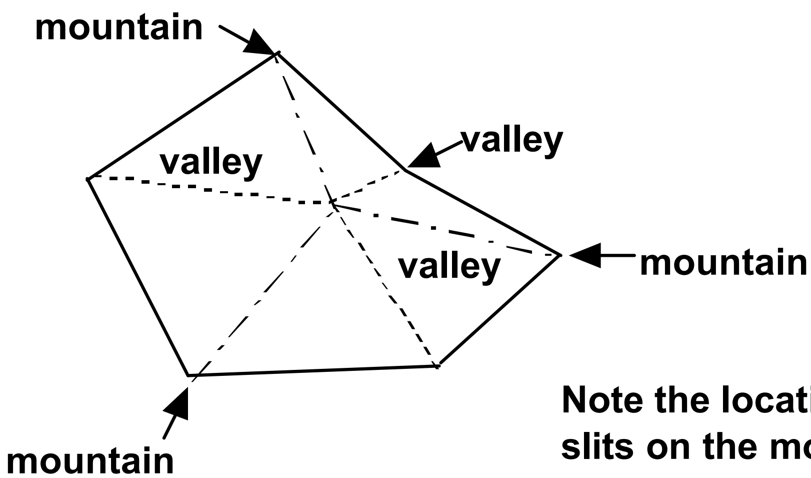


2) Crease all fold lines (dotted) in both directions so that you will be able to flex your construction easily.

3) Fold in order (so that the numbers aren't visible) triangle 1 onto triangle 1, triangle 2 onto triangle 2, triangle 3 onto triangle 3, etc., until you fold a triangle with a teardrop on top of the other.

4) Glue or clip the last two (teardrop) triangles.

5) Gently flex your model by making mountain and valley folds as shown below. Which faces can you see at any one time? What patterns do you notice between the pattern above and the flexagon you've created?



Note the location of the slits on the mountain folds.

Activity: Cover It Up

Format: Individual or Small Group

Objectives: Participants will calculate the surface area of the classroom. Participants will use this calculation in figuring out materials needed and cost for a classroom-painting job.

Related SOL: 7.8, 8.7

Materials: Cover It Up Activity Sheet, pricing and coverage information from the paint store, measurement tools

Time Required: Approximately 2-3 class periods

Directions:

1) In the first class period, participants will calculate the surface area of the classroom. The problem can be made more or less complex by including windows and doors (or not) and by painting the ceiling and/or floor or not. The activity sheet assumes that walls and ceiling are painted and that windows and doors are subtracted from the wall surface area. It will need to be modified if other assumptions are made.

2) Once the participants know how much surface is to be covered, they should use the information from paint stores about coverage and cost to figure how much paint is needed.

3) Finally, participants should write a formal proposal for the job explaining what their cost estimate is and how they arrived at this figure. You may wish to require them to include a diagram.

Cover It Up

As an independent painting contractor, you have been asked to provide an estimate for the cost of painting your classroom. In order to prepare your estimate, you need to sketch the classroom, calculate the surface area, and figure the cost of paint. We suggest the following process for calculating your estimate.

1) Sketch a bird's eye view of the classroom in the space below. Be sure to mark which walls have windows and doors.

2) Measure each wall, window, and door and mark the measurements on your diagram.

3) List each wall, window, and door below and calculate its area.

Object	Length	Width	Area

Extend this table as needed.

4) Find the area of the ceiling. It must be painted as well.

5) Find the total area of the walls and ceiling by adding each area together.

6) Find the total area of windows and doors. Subtract this area from the total you found in step 5 to find the area you need to paint.

7) A paint can includes information about cost and coverage. You will want to put two coats of paint on the walls and ceiling. This means you will cover the area you found in step 6 twice.

8) Using the information about coverage and cost, find the number of cans of paint you need and the cost for that number of cans.

9) Finally, write a letter to your teacher submitting your bid. Your letter should be carefully written (neatly, with correct grammar and spelling) and should include the following information:

   How did you calculate the area to be painted?
   How many cans of paint are required?
   How much will the paint cost?

Activity: Architect's Square

Format: Small Group

Objectives: Participants will create all possible arrangements of four unit cubes. Participants will draw these structures on isometric paper. Participants will make accurate calculations about the surface area of these figures.

Related SOL: 6.17, 7.8, 8.9

Materials: Unit cubes (approximately 60 per group); isometric dot paper; Memoranda to Architects 1, 2, and 3; Architect's Square Activity Sheet, supplies for making brochures (optional)

Time Required: Approximately 5 class sessions

Directions:

1) Have the participants work in groups of 2-4. Instruct them to build all possible arrangements of four unit cubes. These will be houses, and the first memorandum sets the stage for the activity. Participants should record their figures on isometric dot paper. (2 days)

2) Once participants have found all 15 possible arrangements, they should use the information in Memorandum #2 to calculate costs for each house. (1-2 days)

3) Using their information, participants should next plan a brochure advertising their houses. They should think about the features of each house for an elderly couple, a single person, and a family with small children. Memorandum #3 contains this information.

Memorandum #1

TO: All Architects

RE: Housing Designs

We are beginning work on a new subdivision of modular homes. Each home will include four units. All units are shaped as cubes and adjacent units must touch over a full face. It is not acceptable to have faces overlap partially, nor is it acceptable for cubes to touch at edges only.

Your task is to figure out how many different houses we can design. A design is considered different from another if they cannot be superimposed without reflection. Use the cubes provided to you to design and record (on dot paper) all possible home designs.

Memorandum #2

TO: All Architects

RE: Housing Costs

Now that you have drawn all possible houses, you must figure the cost of each house. There are three factors that influence the cost of our houses. Each square unit of land has a fixed cost, each square unit of wall has a fixed cost, and each square unit of roof has a fixed cost. These costs are as follows:

Land	\$10,000 per square unit
Wall	\$ 5,000 per square unit
Roof	\$ 7,500 per square unit

Using your drawings, models you build, and these figures, complete a chart like the one below to price each home.

Design #	Units of Land	Land Cost	Units of Wall	Walls Cost	Units of Roof	Roof Cost	Total Cost

Memorandum #3

TO: All Architects

RE: Advertising

Now that your houses are designed and costs calculated, we need your help with a marketing plan. There are three groups we would like to target — elderly couples, families with small children, and single people. Look at your design and consider such factors as stairs to climb, space for people to spread out, and cost. Which home design would be best for each of these three groups?

Plan a brochure that includes designs for at least six different homes, including the three you selected above. For each home, include a sketch of the home on dot paper, the calculations for the cost of the home, and three to five selling points for that design. Your brochure should be colorful and easy to read.