Linear Algebra (Math 2890) Practice Problems 1

Topics for Midterm I:1.1-1.5, 1.7, 1.8

1. Show that $A = \begin{bmatrix} 1 & 2 & -2 & 0 & 7 \\ -2 & -3 & 1 & -1 & -5 \\ -3 & -4 & 0 & -2 & -3 \\ 3 & 6 & -6 & 5 & 1 \end{bmatrix}$ is row equivalent to $\begin{bmatrix} 1 & 0 & 4 & 0 & -3 \\ 0 & 1 & -3 & 0 & 5 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

2. Determine if the following systems are consistent and if so give all solutions in parametric vector form.
   (a)

$$\begin{array}{rrl} x_1 & -2x_2 & = 3 \\ 2x_1 & -7x_2 & = 0 \\ -5x_1 & +8x_2 & = 5 \end{array}$$

   (b)

$$\begin{array}{llllll} x_1 & +2x_2 & -3x_3 & +x_4 & = 1 \\ -x_1 & -2x_2 & +4x_3 & -x_4 & = 6 \\ -2x_1 & -4x_2 & +7x_3 & -x_4 & = 1 \end{array}$$

3. Let $A = \begin{bmatrix} 1 & 3 & -4 & 7 \\ 2 & 6 & 5 & 1 \\ 3 & 9 & 4 & 5 \end{bmatrix}$.

   (a) Find all the solutions of the non-homogeneous system $Ax = b$, and write them in parametric form, where $b = \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$.

   (b) Find all the solutions of the homogeneous system $Ax = 0$, and write them in parametric form.

   (c) Are the columns of the matrix A linearly independent? Write down a linear relation between the columns of A if they are dependent.

4. Let $S = Span\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ -3 \end{bmatrix} \right\}$.

(a) Find all the vectors $u = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ such that the u is in S. Write these u in parametric form. Justify your answer.

(b) Is $v = \begin{bmatrix} -1 \\ 3 \\ -2 \\ 1 \end{bmatrix}$ in S.

(c) Is $w = \begin{bmatrix} 1 \\ 3 \\ -2 \\ 1 \end{bmatrix}$ in S.

5. Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & b \\ 3 & 5 & a & 1 \end{array}\right]$$

(a) For what values of a will the system have a unique solution? What is the solution?(your answer may involve a and b)

(b) For what values of a and b will the system have infinitely many solutions?

(c) For what values of a and b will the system be inconsistent?

6. Determine if the columns of the matrix form a linearly independent set. Justify your answer.

$$\begin{bmatrix} -2 & 1 \\ 4 & -2 \\ 0 & 0 \\ -6 & 3 \end{bmatrix}, \begin{bmatrix} -2 & 1 \\ 4 & -2 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 2 \\ 1 & -1 & -1 \\ 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -4 & -3 & 1 & 5 & 1 \\ 2 & -1 & 4 & -1 & 2 \\ 1 & 2 & 3 & 6 & -3 \\ 5 & 4 & 6 & -3 & 2 \end{bmatrix}.$$

7. $M = \begin{bmatrix} 1 & 1 & 2 \\ 1 & a+1 & 3 \\ 1 & a & a+1 \end{bmatrix}$.

   (a) Describe the values of a so that the column vectors of M are linearly independent.

   (b) Describe the values of a so that the column vectors of M are linearly dependent.

8. Let $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Suppose $T : R^3 \mapsto R^2$ is a linear transformation such that $T(e_1 + e_2) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $T(e_1 - e_2) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $T(e_1 + e_2 + e_3) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. What is $T(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix})$?