1. **(9 points)** *Calculate the following limits, or demonstrate that they do not exist:*

   (a) **(3 points)** $\lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4}$.

   Plugging in the value $x = 4$ gives $\frac{4^2-4\cdot 4}{4^2-3\cdot 4-4} = \frac{0}{0}$, suggesting that $(x-4)$ can be factored out of the numerator and denominator, and indeed:

$$\lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4} = \lim_{x\to 4} \frac{x(x-4)}{(x+1)(x-4)} = \lim_{x\to 4} \frac{x}{x+1}$$

   and now direct evaluation gives $\frac{4}{4+1} = \frac{4}{5}$.

   (b) **(3 points)** $\lim_{t\to 2} \frac{t^4-2}{2t^2-3t+2}$.

   Plugging in the value $t = 2$ gives $\frac{2^4-2}{2\cdot 2^2-3\cdot 2+2} = \frac{14}{4} = \frac{7}{2}$.

   (c) **(3 points)** $\lim_{x\to +\infty} \frac{1-x^2}{x^3-x+1}$.

   We can look at the dominant term in the numerator and denominator, and conclude that for $x$ of very large magnitude, $\frac{1-x^2}{x^3-x+1} \approx \frac{-x^2}{x^3}$, so that

$$\lim_{x\to +\infty} \frac{1-x^2}{x^3-x+1} = \lim_{x\to +\infty} \frac{-x^2}{x^3} = \lim_{x\to +\infty} \frac{-1}{x} = 0$$

   or, alternatively, we could divide the numerator and denominator by $x^3$ to get:

$$\lim_{x\to +\infty} \frac{1-x^2}{x^3-x+1} = \lim_{x\to +\infty} \frac{\frac{1-x^2}{x^3}}{\frac{x^3-x+1}{x^3}} = \lim_{x\to +\infty} \frac{\frac{1}{x^3}-\frac{1}{x}}{1-\frac{1}{x^2}+\frac{1}{x^3}} = \frac{0-0}{1-0+0} = 0.$$

2. **(3 points)** *Find a value $a$ so that the function* $f(x) = \begin{cases} 2^x & \text{if } x < 3 \\ ax^2 & \text{if } x \geq 3 \end{cases}$ *is continuous everywhere.*

   The expressions $2^x$ and $ax^2$ are themselves both continuous on their domain, so the only potential problem is that the piecewise function may not be continuous at the point where it transitions between these two behaviors; thus we need simply to ensure continuity at the point $x = 3$:

$$\begin{aligned} \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) &= f(3) \\ 2^3 = a\cdot 3^2 &= a\cdot 3^2 \\ 8 &= 9a \\ \frac{8}{9} &= a \end{aligned}$$

3. **(4 points)** *Prove by epsilon-delta methods that* $\lim_{t\to 6} 8-3t = -10$.

   The statement $\lim_{t\to 6} 8-3t = -10$ is an assertion that, for every value $\epsilon > 0$, a value $\delta$ can be furnished such that, if $0 < |t-6| < \delta$, then $|8-3t-(-10)| < \epsilon$. We may justify this assertion by explicity determining how $\delta$ is calculated from $\epsilon$ to make this inference true.

$$\begin{aligned}
|8 - 3t - (-10)| &< \epsilon \\
|18 - 3t| &< \epsilon \\
|-3(t-6)| &< \epsilon \\
|-3| \cdot |t-6| &< \epsilon \\
3|t-6| &< \epsilon \\
|t-6| &< \frac{\epsilon}{3}
\end{aligned}$$

so we may declare that the choice of $\delta$ equal to $\frac{\epsilon}{3}$ is sufficient to meet whatever challenge we are given.

4. **(4 points)** *Using the difference quotient, find the slope of the tangent line to the graph of* $f(x) = 2x^2 - 3x + 1$ *at the point* $(2, 3)$.

We will need to find the slope of this line. Since the tangent line touches the graph at the $x$-value of 2, the slope will be given specifically by $f'(2)$, which we calculate using the difference-quotient definition of the derivative:

$$\begin{aligned}
f'(2) &= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} \\
&= \lim_{h \to 0} \frac{[2(2+h)^2 - 3(2+h) + 1] - (2 \cdot 2^2 - 3 \cdot 2 + 1)}{h} \\
&= \lim_{h \to 0} \frac{(2 \cdot 2^2 + 8h + h^2 - 3 \cdot 2 - 3h + 1(-(2 \cdot 2^2 - 3 \cdot 2 + 1)}{h} \\
&= \lim_{h \to 0} \frac{8h + h^2 - 3h}{h} = \lim_{h \to 0} 8 + h - 3 = 5
\end{aligned}$$

so $f'(2) = 5$, and thus we want a line of slope 5 through the point $(2, 3)$. Using point-slope form, this line has equation

$$(y - 3) = 5(x - 2)$$

or, in slope-intercept form:

$$y = 5x - 7$$