

Chapter 4 -- Decimals

\$34.99 → decimal notation

ex. The cost of an object.

ex. The balance of your bank account

ex. The amount owed

ex. The tax on a purchase.

Just like Whole Numbers –

Place Value - 1.23456789

from the decimal point: ones, decimal (and), tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths,...see page 235

Write in words 3.456 → _____

ex. Write a check for \$38.43 → _____

ex. You buy two items that sell for \$85.90 and \$42.30 . You pay a total of \$13.22 in taxes. How much should you write a check for ? Write it.

Notice that we can write $8/4 =$ ____

$12/3 =$ _____

$24/6 =$ _____

We can write fractions as decimals

Sometimes they terminate some times they do not (nonterminating)

ex. $7/10 =$ ____

$3/8 =$ _____ but

$2/3 =$ _____

Other fractions:

$7/25 =$ _____

ex. $42/1000 =$ _____

$7/9 =$ _____

What place value is the digit 3 in the number 42.567321 ? → _____

Write as a decimal.

$3/10 = \underline{\hspace{2cm}}, \quad 93/100 = \underline{\hspace{2cm}}$

$58/1000 = \underline{\hspace{2cm}}$

Write as a fraction.

$0.4 = \underline{\hspace{2cm}}, \quad 0.24 = \underline{\hspace{2cm}}$

$0.004 = \underline{\hspace{2cm}}$

Write in words.

$0.023 = \underline{\hspace{4cm}}$

$23.05 = \underline{\hspace{4cm}}$

Order. Insert the correct symbol (< or >)

2.3

2.03

0.41

0.401

0.203

0.0343

Rounding: Round the number to the given place value

$8.345 \text{ tenths} \rightarrow \underline{\hspace{2cm}}$

$35.0256 \text{ thousandths} \rightarrow \underline{\hspace{2cm}}$

Addition and Subtraction of decimals

We still have the same rule of signs

ex. Add: $0.435 + 9.7 + 87.43$

ex. Subtract $43.82 - 9.876$

evaluate: $x - y$ if $x = -7.37$ and $y = -3.921$

Estimate: $53.29 - 0.89$

Multiplication of Decimals:

sum of the decimal places of the factors have to equal the decimal places of the product.

$3.2 \cdot 8 = \underline{\hspace{2cm}}$

$4.2 \cdot 1.2 = \underline{\hspace{2cm}}$

$-43.2 \times 2.01 = \underline{\hspace{2cm}}$

Special Case: When multiplying by powers of 10 such as 10, 100, 1000:

$\text{ex. } 10 \times 3.25 = \underline{\hspace{2cm}}$

$456.1 \times 100 = \underline{\hspace{2cm}}$

$56.02 \times 10^5 = \underline{\hspace{2cm}}$

Division: $8 \div 4 = 2$; 4 is the divisor and 2 is the quotient

Move the decimal point in the divisor to the right so that the divisor is a whole number. Move the decimal place in the dividend the same number of places. Place the decimal point directly above.

$\text{ex. } 256.04 \div 0.4 =$

$\text{ex. } 3.03505 \div 1.01$

Special Case: Division by powers of 10;

$48.03 \div 100 = \underline{\hspace{2cm}}$

$4.2105 \div 10 = \underline{\hspace{2cm}}$

$0.032 \div 1000 = \underline{\hspace{2cm}}$

We can write every fraction as a decimal if we interpret the fraction bar as a division. When we write a fraction as a decimal we get two types of decimal expression; terminating and nonterminating

$\text{ex. } 3/5 = \underline{\hspace{2cm}}$

$5/8 = \underline{\hspace{2cm}}$

$3/11 = \underline{\hspace{2cm}}$

$\text{ex. } 9/1000 = \underline{\hspace{2cm}}$

$\text{ex. } 2 \frac{5}{9} = \underline{\hspace{2cm}}$

We can also reverse the process and write a decimal expression as a fraction.

ex. $0.023 =$ _____

ex. $4.01 =$ _____ ex. $0.004 =$ _____

ex. $0.125 =$ _____

Estimate by rounding.

ex. $3.02 \cdot 49.1 =$ _____

ex. $456.1 \div 0.34 =$ _____

ex. $4.02 - 0.097 =$ _____

Evaluate if $x = 0.1$ and $y = -2.1$; $-x^2 =$ _____

$-xy =$ _____

Compare decimal expressions (and fractions) :

ex. $9/10$ 0.89

ex. $7/18$ 0.39

ex. $8/15$ 0.543

Other Examples:

ex. Your annual salary last year was \$52432. What was your monthly salary ? _____

ex. You travel 34.2 miles per gallon. If you have a car with a 12.8 gallon capacity, then what is the maximum number of miles that you can travel ?

ex. A year consists of 52 weeks. You get a biweekly salary of \$850. What is your yearly pay ?

ex. At the end of each of month you save a fixed amount into a savings account. At the end of $1\frac{1}{2}$ years you have \$ 2400. How much did you save each month ?

Equations:

As before we have two basic types of operations that can be used to solve equations.

ex. $-4v = 7$ (write as a decimal)

ex. $-2.02 + 1.5x = -4.12$

ex. $-7.1 - x = 4.01$

$1.02 - x = 0.1 \rightarrow x = \underline{\hspace{2cm}}$

ex. $6.6 - 2.2x = 0.44 \rightarrow x = \underline{\hspace{2cm}}$

$x / -0.4 = 8 \rightarrow x = \underline{\hspace{2cm}}$

Square roots and perfect squares

Perfect Squares: 1, 4, 9, 16, 25, 36, 49, ...

Def. A square root of a positive number x is a number whose square is x .

ex. 4 is square root of $\underline{\hspace{2cm}}$ since $4^2 = \underline{\hspace{2cm}}$

ex. -8 is the square root of $\underline{\hspace{2cm}}$ since $(-8)^2 = \underline{\hspace{2cm}}$

Every positive number has

two square roots; one positive and one negative. We use the symbol $\sqrt{\quad}$ to represent the positive square root.

Notation: \sqrt{x}

$\sqrt{\quad}$ is called the radical, x is called the radicand

Each of these numbers has two square roots:

4 and 25 since $(-2)^2 = 4$, $(2)^2 = 4$; $(-5)^2 = (5)^2 = 25$

The radical only refers to the positive square root. $\sqrt{49} = 7$,

if we want the negative value we write $-\sqrt{49} = -7$

More Examples:

Find all of the square roots of

64 \rightarrow _____ , _____

121 \rightarrow _____ , _____

400 \rightarrow _____ , _____

Find

$\sqrt{81} =$ _____ ,

$\sqrt{169} =$ _____ ,

$-\sqrt{144} =$ _____

What is

$\sqrt{49} - \sqrt{16} =$ _____

$\sqrt{25} + \sqrt{4} =$ _____

$5 \cdot \sqrt{36} =$ _____

$\sqrt{25} \cdot \sqrt{16} =$ _____

$\sqrt{\frac{9}{16}} =$ _____

Not all whole numbers are perfect squares – what do we do when we try to find the square root of such a whole number ?

$$\sqrt{26} = \underline{\hspace{2cm}}$$

$$\sqrt{53} = \underline{\hspace{2cm}}$$

What two consecutive whole numbers does $\sqrt{37}$ lie between ?

$$\underline{\hspace{2cm}} < \sqrt{37} < \underline{\hspace{2cm}}$$

We can give approximations with a calculator or an algorithm – we also have exact values.

Property of radicals

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$. We can use this property to simplify radicals -

Find (exact value)

$$\sqrt{20} = \underline{\hspace{2cm}}$$

$$\sqrt{50} = \underline{\hspace{2cm}}$$

$$\sqrt{18} = \underline{\hspace{2cm}}$$

$$\sqrt{80} = \underline{\hspace{2cm}}$$

Real Numbers

Now that we have introduced radicals, we are ready to complete the set that we call the set of real numbers.

Sets of Numbers

$\{1, 2, 3, \dots\} \rightarrow$ _____

$\{0, 1, 2, 3, \dots\} \rightarrow$ _____

$\{\dots -3, -2, -1, 0, 1, 2, 3, \dots\} \rightarrow$ _____

A rational number is a number that can be written as a fraction a/b , where both a and b are integers
Every rational number has a decimal expression that is terminating or repeating.

$$1/5 = 0.2, \quad 3/8 = 0.375, \quad 5/9 = 0.5555\dots = \overline{.5}$$

$$0.231456231456\dots = \overline{0.231456}$$

An irrational number is a number whose decimal representation never terminates or repeats
-- can not be written as a fraction of integers

$$\pi = 3.1415926\dots, \quad \sqrt{2} = \text{_____}, \quad 0.312311223111222\dots, \quad 0.2010010001\dots,$$

The set of rational numbers and the irrational numbers when combined create the set of real numbers

We can plot these numbers on a number line ---

ex. Plot $-5, 7, 3.5, \sqrt{3}$ _____

ex. We can also write real numbers on the number line with inequalities –

a) Write down all of the real numbers less than 2

b) Write down all of the real numbers greater than or equal to -1

c) write down all the numbers between -2 and 3

ex. Which of the values $-4, 0, 2, 3/2$, make the inequality $x \geq 2$ true? _____

ex. Which of the values $2, 2.1, 0.21$ make the inequality $x > 2.01$ true? _____

Properties of Real Numbers :

1. $a + b = b + a \rightarrow$ _____ 2. $ab = ba \rightarrow$ _____

3. $(a + b) + c = a + (b + c) \rightarrow$ _____ 4. $a(bc) = a(bc) \rightarrow$ _____

5. $a(b + c) = ab + ac \rightarrow$ _____

6. Addition Property of 0 \rightarrow _____, we call 0 the additive identity

7. Multiplication Property of 0 \rightarrow _____

8. Multiplication Property of 1 \rightarrow _____, we call 1 the multiplicative identity

9. Inverse Property of addition \rightarrow _____, a and $(-a)$ are called additive inverses of each other (we also call them) _____ of each other

10. Inverse Property of Multiplication \rightarrow If $a \neq 0$, then _____, a and $1/a$ are called multiplicative inverses of each other.

Find

$6(3x) = \underline{\hspace{2cm}} \text{ Why?}$

$(3x) \cdot 5 = \underline{\hspace{2cm}} \text{ Why?}$

$(2x)(5x) = \underline{\hspace{2cm}} \text{ Why?}$

$1 \cdot c = \underline{\hspace{1cm}} \quad -1 \cdot c = \underline{\hspace{1cm}}$

$(-3) \cdot (-c) = \underline{\hspace{2cm}}$

$-3t + 8 + 3t = \underline{\hspace{2cm}}$

$2(3x + 4) = \underline{\hspace{2cm}}$

$-2(3x + 5) = \underline{\hspace{2cm}}$

$-4(2x + 4) = \underline{\hspace{2cm}}$

$-4(3x - 4) = \underline{\hspace{2cm}}$

$2(x - 3a + 2c) = \underline{\hspace{2cm}}$

$-(x + 2y) = -1 \cdot (x + 2y) = \underline{\hspace{2cm}}$

Terms, coefficients, variable terms, variable expressions, combining similar terms,

How many terms? what are they? $3x^3 + 3x - 4 \rightarrow \underline{\hspace{2cm}}$

How many are constant terms?

Same Questions for the polynomial $-2x^2 + 2y - 5 \rightarrow \underline{\hspace{2cm}}$

What are the coefficient terms of $x^2 + x + 1 \rightarrow$ _____

of $-2x^2 - x + 3 \rightarrow$ _____

Similar Terms (Like Terms) :

Combine terms. (Alternate form of the distributive law)

Instead of thinking of the distributive law in the form: $a(b + c) = ab + ac$,

let us think of it as $ab + ac = a(b + c)$

$$8x + 13a = \underline{\hspace{2cm}} \quad 12c - 16c = \underline{\hspace{2cm}} \quad 6c - 6c = \underline{\hspace{2cm}}$$

$$2x - 4y + 3x = \underline{\hspace{2cm}} \quad 3x + y + 4x - 4y = \underline{\hspace{2cm}}$$

$$2x + 3(x + 4) = \underline{\hspace{2cm}} \quad 4 + 2(3x - 1) = \underline{\hspace{2cm}}$$

$$2c - 2[2 - 3(2c - 3)] = \underline{\hspace{2cm}}$$

Polynomials :

Def. A monomial is a number, a variable, or a product of numbers and variables. Exponents on variables must be nonnegative integers.

Def. A polynomial is a variable expression in which the terms are monomials (a sum of monomials)

A polynomial with one term is called a _____
with two terms → _____
with three terms → _____

Addition of polynomials. Combine similar terms (like terms)

ex. $(3x^2 + 2x - 3) + (5x^2 - 4x - 12) =$ _____

ex. $(2xy + x) + (5xy + y) =$ _____

Subtraction: We perform subtraction operations as before - write as an addition problem and use addition rules.

ex. $(2x - 3xy) - (4x - 2y) =$ _____

ex. $(2x + y - 3) - (3x - 2y - 4) =$ _____

Degree of a monomial:

$x^3 \rightarrow \underline{\hspace{2cm}}$

$x^{10} \rightarrow \underline{\hspace{2cm}}$

$5x^6 \rightarrow \underline{\hspace{2cm}}$

$2x^2y \rightarrow \underline{\hspace{2cm}}$

$12x^4y^8 \rightarrow \underline{\hspace{2cm}}$

Degree of a polynomial:

$1 + x + x^{10} \rightarrow \underline{\hspace{2cm}}$

$2 + 3y - 4x^{20} \rightarrow \underline{\hspace{2cm}}$

$2xy + 3 \rightarrow \underline{\hspace{2cm}}$

$2^5 + x + x^3 \rightarrow \underline{\hspace{2cm}}$

$4x^3y^2 + 3y^4 \rightarrow \underline{\hspace{2cm}}$

Multiplication of Polynomials:

Rules of exponents:

1. If m and n are positive integers (natural numbers), then

$$x^m \cdot x^n = x^{m+n}$$

ex. Find $a^4 \cdot a^8 = \underline{\hspace{2cm}}$

$4^3 \cdot 4^{10} = \underline{\hspace{2cm}}$

$(-2)^4 (-2)^3 = \underline{\hspace{2cm}}$

What about $x^3 \cdot y^5 = \underline{\hspace{2cm}}$ or

$(4x^3)(3x^2) = \underline{\hspace{2cm}}$

$(2a^3b)(a^2b^5) = \underline{\hspace{2cm}}$

2. If m and n are positive integers, then $(x^m)^n = \underline{\hspace{2cm}}$

ex. Find $(2^3)^4 = \underline{\hspace{2cm}}$

$(x^3)^2 = \underline{\hspace{2cm}}$

3. If m , n , and p are positive integers, then

$(x^m y^n)^p = \underline{\hspace{2cm}}$

ex $(2x^3)^4 = \underline{\hspace{2cm}}$

$(x^4 y^2)^5 = \underline{\hspace{2cm}}$

These types of rules help us multiply monomials together but what about the product of polynomials ?

Products of powers of 10.

What is $123.45 \cdot 100$? = _____

Find $3.01 \cdot 1000$ _____

Quotients of powers of 10 .

What is $123.45 \div 10^2$? = _____

Find $0.056 \div 0.01$ = _____

Convert to a decimal –

$9/16$ = _____

$7/4$ = _____

$5/3$ = _____

$3 \frac{2}{9}$ = _____

Exponents – Just like we worked with natural numbers, whole numbers, integers, we can work with fractions.

ex. $(\frac{2}{3})^2$ = _____

ex. $(2 \frac{1}{5})^2$ = _____

ex. $(2 \frac{1}{2})^3 \cdot (1 \frac{1}{2})^3$ = _____

HW: page 322 → 1, 5, 10, 15, 20, 25, ...95
Math 130A – Week 6 day 2, October 3, 2001 - Quiz #11

0.1 Homework – 20, 15, 10, 0 → _____

0.2 { 0, 1, 2, 3, ... } is called the set of _____

0.3 The set of rational numbers consists of _____ and proper fractions or mixed numbers.
Each of these numbers can be written as a fraction and have a terminating decimal expression or have a repeating part if it is nonterminating

1. Write as a fraction 0.019 → _____

2. Write as a fraction in simplest (reduced) form; 0.02 → _____

3. Write in decimal form

a) $\frac{3}{1000} =$ _____

b) $\frac{3}{8} =$ _____

4. Write in words; 103.016 → _____

5. Round to the nearest thousandth 31.00167 → _____

6. Perform the given operation (add, subtract, multiply, divide)

a) $3.1 + (-1.01) =$ _____

b) $2.01 \cdot 1.1 =$ _____

c) $1.02 - 4.1 =$ _____

d) $3.08 \div 0.4 =$ _____

7. Evaluate. $x^2 - y$ if $x = -1.1$ and $y = 0.1$

8. Find the solution of each of the following equations.

a) $0.2x = 9.8$

b) $4.2 - 2x = 6.8$

More HW problems on page 267-272

Square roots , radicals $\sqrt{\quad}$, Product Property of Square roots

Name _____ Math 130A – Quiz #12, October 5, 2001

0.1 Homework 20, 15, 10, 0 → _____

1. Give me an example of an improper fraction . _____

2. Write $6/5$ in decimal form . _____

3. Solve for x. $0.5 - 0.2x = 1.03$

4. $2 \cdot 0.4 =$ _____

5. $4 + 3.05 =$ _____

6. $10.1 - 8.01 =$ _____

7. A pen sells for \$1.50 after taxes. You pay with \$20 and get \$6.50 back in change – how many pens did you buy if all your money went towards the purchase of pens.

8. 6 % of the students in Math 130A are known to drop out of school at the end of the semester. If there are 240 students enrolled during the semester, then how many will return next semester ?

(HINT: use the fact that $6\% = 0.06$)

Name _____ Math 130A – October 2, 2002 – week 6 – day 2

1. What do you call the set of real numbers that can be written as fraction (have terminating decimal expressions or repeat)?

2. Find the square roots of 36 . _____

3. Simplify the following radicals.

a) $\sqrt{49} =$ _____

b) $\sqrt{100} =$ _____

c) $\sqrt{1600} =$ _____

d) $\sqrt{16} \cdot \sqrt{25} =$ _____

e) $4\sqrt{36} =$ _____

4. Complete the formula $\rightarrow \sqrt{a b} =$ _____

5. Write the following in simplest form by using the formula in #4.

a) $\sqrt{8} =$ _____

b) $\sqrt{32} =$ _____

6. Use a number line to plot $x > 4 \rightarrow$ _____

7. Use a number line to plot $x \leq -2$

8. List two consecutive integers that $\sqrt{32}$ lies between? $\underline{\hspace{2cm}} < \sqrt{32} < \underline{\hspace{2cm}}$

Math 130A – Week 7, day 2, October 10, 2001 - Quiz

1. Simplify

a) $4(3x) = \underline{\hspace{2cm}}$

b) $-2(3x) = \underline{\hspace{2cm}}$

c) $-\frac{5}{3}(2a) = \underline{\hspace{2cm}}$

d) $(4x)(5x) = \underline{\hspace{2cm}}$

e) $-4(-\frac{1}{4}a) = \underline{\hspace{2cm}}$

f) $-4h + 3h = \underline{\hspace{2cm}}$

g) $2(h + 5) = \underline{\hspace{2cm}}$

h) $4 + (-2) - 2y = \underline{\hspace{2cm}}$

2. List the terms of

3. What are the coefficients of

4. Simplify by combining like (similar) terms

HW page 322

Math 130A – Week 5, September 24, 2001

HW: page 211 – 1, 6, 11, 16, 21, 26, 31, 36, 41, 3, 8, 13, 18, 23, 28, 33, 38,
page 199 1, 6, 11, 16, ...166

Exponents and Complex Fractions - section 3.6 page 213-222.

Name _____ Math 130A, Quiz #9 , September 26, 2001

1. Did you do all/most of your homework ? _____ yes/no

2. Homework points 20, 15, 10 _____

3. Find the sum of $3\frac{1}{4}$ and $2\frac{3}{8}$. _____

4. What is the total of $(-3\frac{2}{5})$ and $(+2\frac{1}{4})$? _____

5. Find the difference; $4\frac{1}{2} - 2\frac{3}{4} =$ _____

6. Write as an improper fraction; $2\frac{3}{5} =$ _____

7. Find $4 + 6\frac{3}{11} =$ _____

8. What is the product of 4 and $\frac{3}{8}$? _____

9. Find $8 \cdot 2\frac{1}{2} =$ _____

10. Is 2 a solution of $\frac{2}{3} = \frac{x}{3}$? _____

Name _____ Math 130A, Quiz 10, September 28, 2001

1. Which of these represents an improper fraction ?

$15/17$, $2\frac{1}{2}$, $27/8$, $12/16$, or none of these → _____

2. Multiply

$$3\frac{1}{5} \cdot 3\frac{3}{4} = \underline{\hspace{2cm}}$$

3. What is the product of 4 and $2\frac{1}{8}$? _____

4. Find the solution of each of the following equations.

a) $x/3 = 2\frac{1}{3}$ → $x = \underline{\hspace{2cm}}$

b) $3x/5 = -4$ → $x = \underline{\hspace{2cm}}$

c) $2/3 - \frac{3}{4}x = \frac{1}{4}$ → $x = \underline{\hspace{2cm}}$

5. Let $x = 4\frac{1}{4}$ and $y = -2\frac{3}{8}$. Find

a) $x - y = \underline{\hspace{2cm}}$

b) $x + 2y = \underline{\hspace{2cm}}$

6. $2/3$ of a household's budget goes towards essentials. If the \$2400 is budgeted for this month, how much of it goes towards the household's budget.

7. Find $-4\frac{3}{8} - (-2\frac{3}{4})$

September 28, 2001