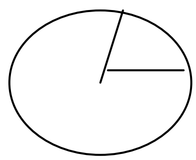
Math 1303 – Part II

We have discussed two ways of measuring angles; ***degrees and radians***

The opening of one of 360 equal central angles of a circle is what we chose to represent 1 degree

We defined a radian as the measure of an angle created when a central angle intercepts an arc of length r, the radius of the circle,



1 revolution = 360^o and also 1 revolution = 2π radians
In turn we get some of the basic angle relationships:

$\pi/2$ radians = 90^o , $\pi/6 = 30^o$, $\pi/4 = 45^o$ $\pi/3 = 60^o$, $\pi = 180^o$

as well as negative measures → $-\pi/6 = -$ .... and so forth..

Notice: we left off the word radians – although it is implied, it should be included.

We can convert from degrees to radians and from radians to degrees by using the fact that

2π radians = 360^o ➔ π radians = 180^o and 1 = 180^o / π radians or 1 = π radians / 180^o

Convert from degrees to radians ➔

-90^o = _______ radians 135^o = _______ radians

-200^o = _______ radians 1^o = _______ radians.

Convert from radians to degrees ➔

$2/3\ \pi$ radians = _________ $1/5\ \pi$ radians = ___________

$3/2\ \pi$ radians = ________ 1 radian = ______________

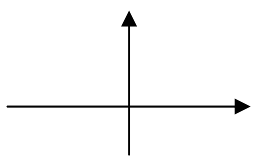
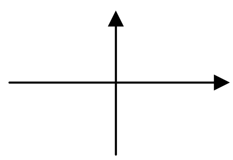
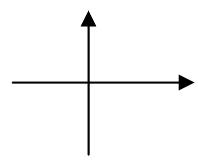
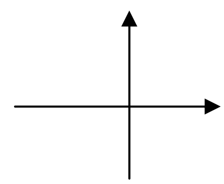
We can now find trigonometric functions of radian measures:

Find

$\sin \pi / 4 =$ _______ $\cos \pi/2 =$ ______ $\cot \pi/3 =$ _______

Def. Let θ be a standard position angle. Let θ_r represent the acute angle created by the terminal side of θ and the x-axis. *We call θ_r the related angle of θ* (your author calls it, the reference angle).

Note: it may be of help if we "drop" a vertical segment from the terminal side of θ to the x-axis and construct a triangle of reference.

Example

Find a positive angle θ so that $| \theta | < \leq 360^o$ and $\theta_r = 40^o$ and θ is located in each of the four quadrants

__________ __________ __________ ____________

Def. Let p and q be any two real numbers. We say p and q are numerically equal provided $|p| = |q|$.

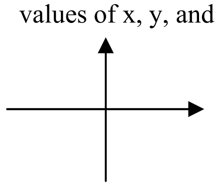
example: ¼ and - 0.25 are numerically equal What about the square roots of 7 ?

Property:

Any trig. function of a given angle θ will be numerically equal to the same trig. function of an angle whose related angle is equal to the related angle of θ.

For example: think of the acute angle θ and its supplementary angle ($180^o - \theta$) – Why can we say supplementary ?

When you draw these angles in standard position(especially the triangle of reference) you notice that the same values of x, y, and r are being used except for x. Similar idea if angle is in other quadrants.



Find

$\sin 135^o$ = ___________

angle of reference: $135^0{}_r$ =

$\cos(-120^o)$ = ______________

angle of reference: $-120^o{}_r$

$\tan 315^o$ = _________

angle of reference $315^o{}_r$ = _____

$\csc(390^o)$ = _______________

angle of reference: $390^o{}_r$ = _____

More Examples:

1.

2.

3.

4.

5.

Arc Lengths

From our definition of radians, we see that if

θ represents a central angle of a given circle of radius r measured in radians and
s is the length of the intercepted arc of θ, then

$$\underline{s = r\,\theta}$$

Pf. (sort of)
If θ represents 1 radian $\rightarrow$ s = 1(r) = r
If θ represents 2 radians $\rightarrow$ s = 2(r) = 2r
......
If θ is some known radian measure $\rightarrow$ s = θ r = r θ

NOTE: <u>θ must be in radian measure</u>

Examples:

1. A rope of length 10 feet is allowed to swing in a circle. What is the length of an arc that is created in a swing of 225^{o} ?

2. **A pendulum** swings back and forth. From beginning to end(on the opposite side) it swings through an arc of length 4 feet. If the pendulum is of length 2 feet, then what is the angle that it swings through ?
Find the answer to the nearest degree.

3.

4.

Linear and Angular Velocity

You took a small wheel from a toy and rotated the wheel on the ground until the beginning point had touched the pavement five additional times (five revolutions). It took you two seconds for that to happen.

You took a large tire(tractor tire) and did the same thing - five revolutions in two seconds.

Both of these wheels rotated at the same rate per time unit→ 5 revolutions per second = 10 π radians / 2 seconds
This is called the angular velocity - # of times the object rotated (in radians) per time unit

Both wheels moved a certain amount during the movement. Are both distances the same ?

This kind of motion is called **linear velocity**: distance per time unit

Formula:

Take our arc length formula and expand on it.

Write $s = r\,\theta$ but what happens when we put time into the equation – divide by the time unit

$s/t = r\theta / t$ ➔

replace s/t with v and replace θ/t with ω → we get $v = r\,\omega$

linear velocity ➔ ________________ angular velocity ➔ __________

ex. How fast is the 12 inch hour hand of a clock moving at the tip of the hand ? _______________

What is the angular velocity of the clock hand ?

ex. A car is moving at 60 mph. How fast are the 15-inche tires rotating in 1 minute ?

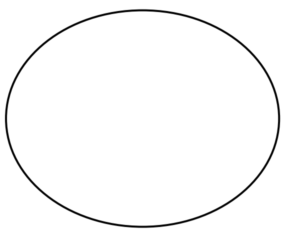
What is the angular velocity of a point on the earth

a) that is located at the north pole → ______________________

b) at the equator

What about linear velocity ?

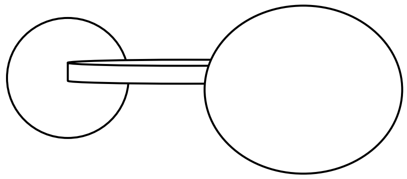
Latitude of a point on the surface of the earth.



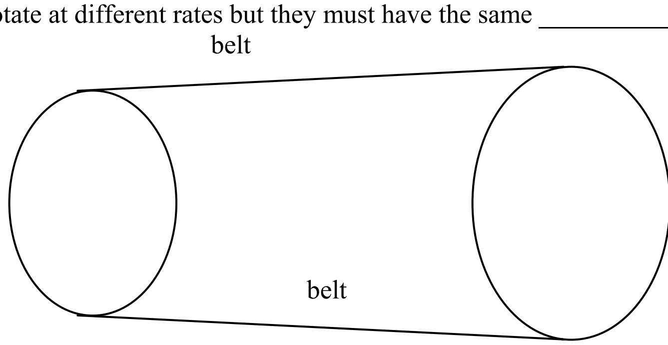
ex. What about the linear and angular velocity of a point on the earth at a latitude of 60^{o} N .

Plus:

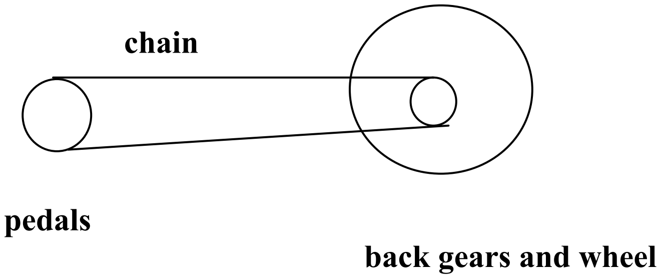
1) Wheel connected by a rod – What is their relationship in terms of linear and angular velocity ?
(wheels, gears must move together and rotate at the same time)



2) Two wheels connected by a belt (road, chain,…) --- What is their relationship ?
They may rotate at different rates but they must have the same ______________



3) Bicycle example:



Back to s=rθ

Note:

If θ is small with r large ($\theta << r$), then the intercepted chord and arc are essentially the same.

Find the apparent diameter (θ) of the moon as seen from the earth . Distance to the earth 384,000 km with diameter 3,470 km.

At what distance must a 6 ft person stand from you, so that his apparent diameter is the same as the moon?

If a 40 foot-tall tree appears to be three inches tall to you, then how far are you from the tree ?

More Examples:

Two planes take off from an airport. a) 1st one is flying at 255 mph on a course of 135^{o} b) second one at 225^{o} at 275 mph. How far apart are they after two hours and what is the bearing of the second plane from the first one ? Assume no wind currents.

A 10 lb. Object is on an incline of 15^{o} . Find the force against the plane and the force needed to keep the object from sliding.

We have talked about linear and angular velocities: $v = r\omega$

a) 45/161 A lawnmower has a blade that extends 1 foot from its center. The tip of the blade is traveling at the speed of sound; 1,100 ft per 3second. Through how many rpm is the blade turning.

b) 53/162 A woman rides a bicycle for 1 hour and travels 16 km. Find the angular velocity of the wheel if the radius is 30 cm.

Area of a sector: $A = \frac{1}{2} r^2 \theta$ (Why ?)

Construction:

Area of a circle: πr^2

Area of ½ a circle: $(\pi r^2) / 2$, Area of 1/3 a circle: $(\pi r^2)/3$,....

Think of the sector in terms of the angle θ.
We can write $\theta / 2\pi$ to represent the corresponding ratio of the central angle to the whole angle. The areas will be in the same ration –

Area of the sector / Area of the whole circle = $\theta / 2\pi$ $\rightarrow$ Area of sector = A = $(\pi r^2) \cdot \theta / 2\pi = \dfrac{\theta r^2}{2}$

$$\mathbf{A = \tfrac{1}{2} r^2 \theta}$$

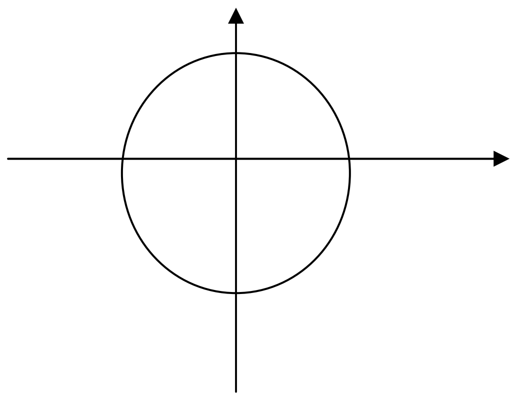
Circular Functions:

We found that a trigonometric function can be defined on an angle θ independent of the point used (as long as the point is on the terminal side of θ. Why not use points that lie one unit away from the center.

Begin with a unit circle (radius = 1) centered at the origin. Let θ be an angle in standard position with P(x,y) a point on the terminal side of θ . Notice how the six trig. functions can be defined in terms of points on this circle --- r =1 with x and y having varying values.

$\sin\theta =$
$\cos\theta =$
$\tan\theta =$

$\csc\theta =$
$\sec\theta =$
$\tan\theta =$



We have three conclusions:

1. Given any point P(x,y) on the unit circle - we can write P in terms of trig. functions.

P(x,y) = ___________

Notice the trig. functions and the arc length formula:

$\sin\theta = y$, $\cos\theta = x$, and $s = r\theta$

2. There is a relation between the angle θ measured in radians and s the length of the intercepted arc.

If P(x,y) is a point on the terminal side of θ (standard position) and $r = 1$, then

s = rθ ➔ _____________ What does that say about θ and s ?

Here, θ represents an angle, while s represents a length (a real number ?)

3. We can work with real numbers instead of angles
So instead of finding $\sin\theta$ we can find $\sin s$ - where s represents length , a real number. Same idea with the other trigonometric functions.
Why would this be useful ?

Every real number can be thought of as angle (radian measure) . We can find trig. functions of these angles.

Find sin 1. ____________ Find sin 3. _____________

Find sin $\pi/2$ = ___________ (Does $\pi/2$ represent an angle in radian measure or just a length – a real number ?)

Also notice that the coordinates of any point on the unit circle can now be labeled in terms of the sine and cosine functions (angle must be in standard position).

Find x and y if $\theta = 45^o$. _______________ If $\theta = 210^o$ ➔ ____________

This works for any circle centered at the origin.

ex. Write (-2, 2) in terms of trig. functions of some angle θ.

We can now define the trig. functions in terms of real values; not just angles measured in degrees.

$f(x) = \sin x$, $f(x) = \cos(x)$ $f(x) = \tan x$

Which means we can find points of the form (x,y) that satisfy the functions. ➔ we can graph them.

ex. $f(x) = 2x^2 - 1$

Def. Let f(x) represent a function.

1) We say f is ***even*** provided $f(-x) = f(x)$

2) We say f is ***odd*** provided $f(-x) = -f(x)$

3) otherwise , we say f is ***neither odd nor even.***

ex. $f(x) = x^2 + 2$ ➔ $f(x) = 4x^3 - 2x$ ➔ ________

What about $f(x) = 2^x - 2^{-x}$? ➔ ____________

What about any of the six trig. functions ?

$f(x) = \sin x$? ____________ $f(x) = \cos x$? ________ $f(x) = \tan x$? _______

May help if you think in terms of related angles and the sign of the functions in the four quadrants.

Other examples: (Odd, Even, or Neither)

$y = |x|$ $y = x^2$ $y = -2$ $x = 4$

Now that we have our trig functions in terms of radians or real numbers – can we graph them ?

Recall: in terms of degrees every angle of the form $\theta \neq 360^{o} + \theta$, however the trig function of each will be equal (assuming the function is defined at these angles). Why ?

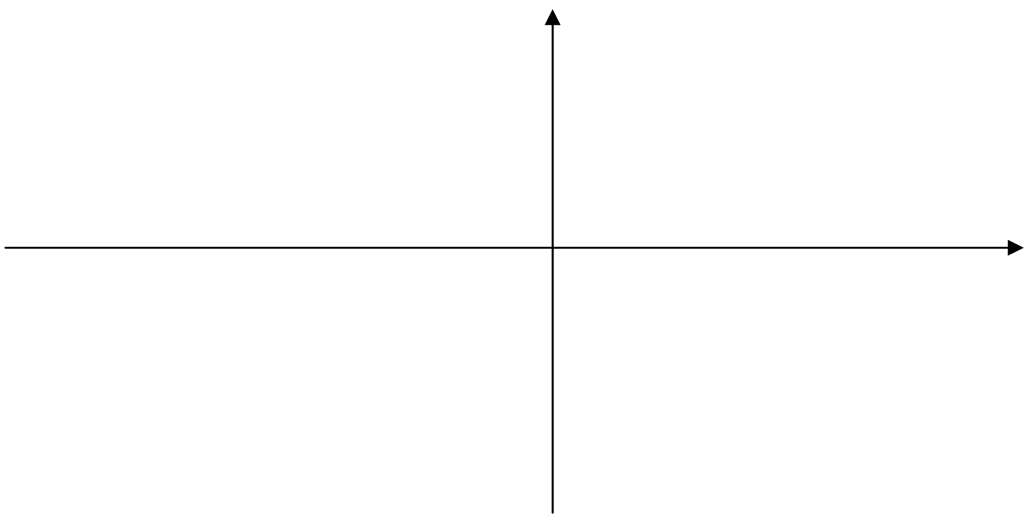
y = sin x (f(x) = sin x) :

Table method:

x

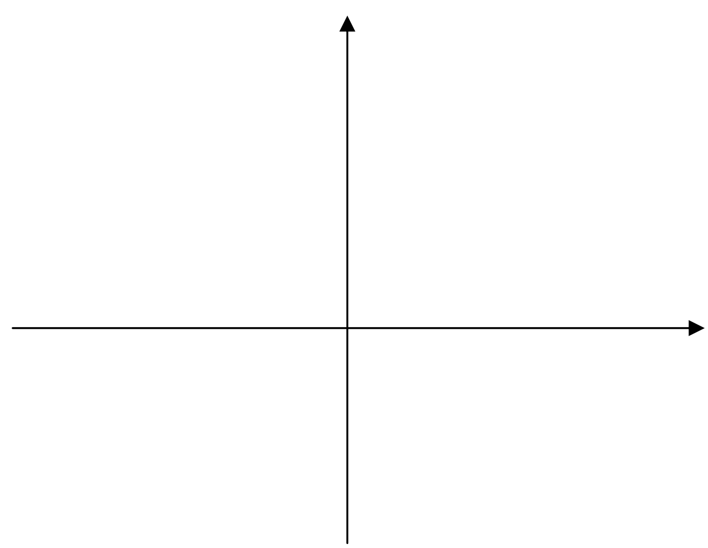
x_d

y

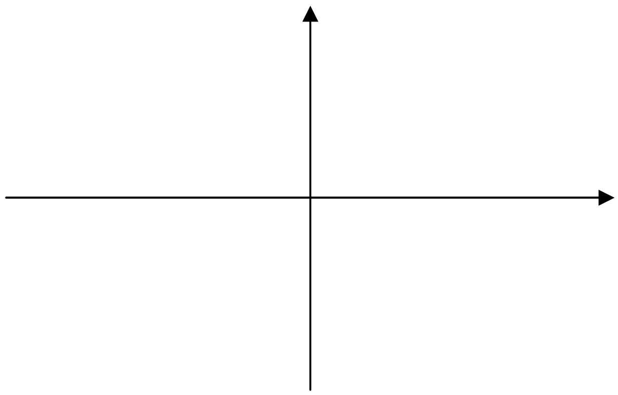


Other Curves:

f(x) = cos x

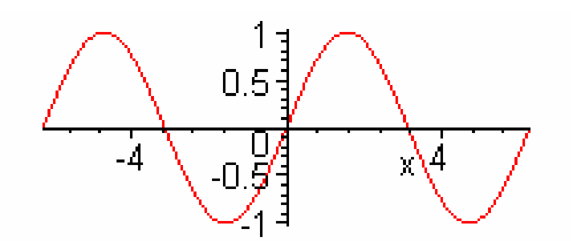


f(x) = tan x

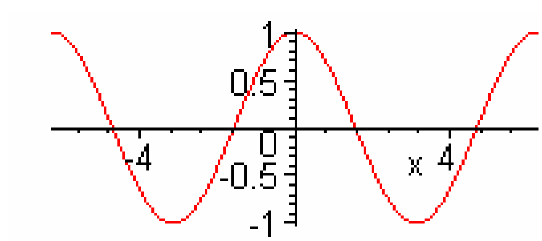


Here are the graphs we should have obtained

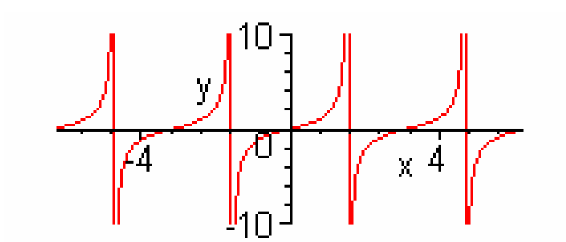
f(x) = sin x



f(x) = cos x

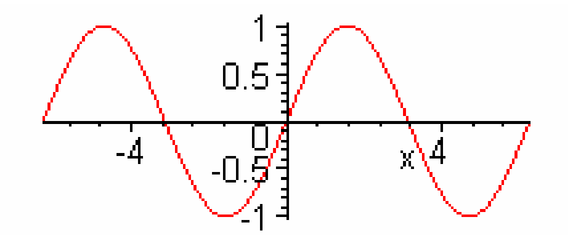


f(x) = tan x

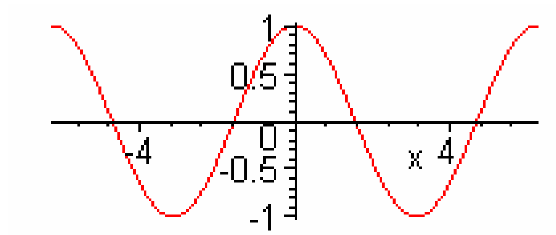


What about the graphs of the reciprocal functions ? We use original trig. functions to get graph of reciprocal functions.

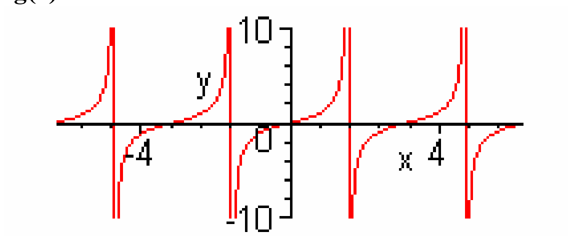
f(x) = sin x → g(x) = csc x



f(x) = cos x → g(x) = sec x



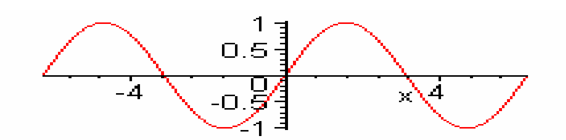
f(x) = tan x → g(x) = cot x



Now that we have seen the graph of each trigonometric functions we can state other definitions.

ex. When is sin x = 1 ➔ x = ______________________________

graph of y = sin x



For what values is tan x = -1 ➔ x = ______________________

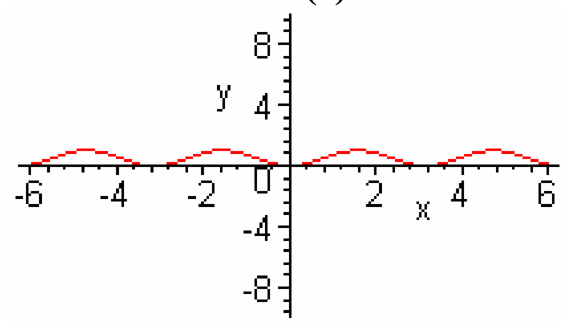
graph of y = tan x

Def. Let f(x) represent any function. We say f has <u>period p</u> provided

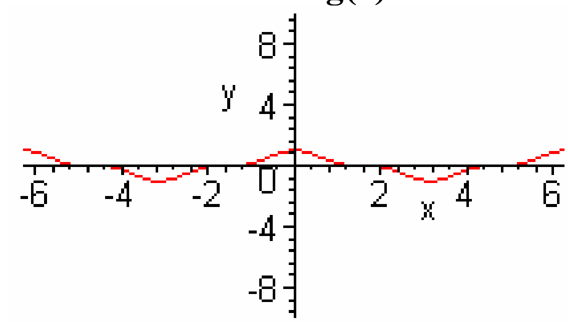
a) $f(x + p) = f(x)$ for all x and b) p is the smallest such value.

Example: Look at these two graphs and see if you can guess their period

f(x)



g(x)



$f(x) = \sin x$. Using the graph we can guess what p is equal to. ➔ p = _____________

What about the other five trig. functions ?

period of

cos x ➔ ____________ tan x ➔ ______________

sec x ➔ ____________ cot x ➔ _______________

csc x ➔ _____________

Other functions:

Consider $f(x) = \sin^3 x$. What is its period ? ➔ _______________

What about $f(x) = \cos^2 x$? ➔ ______________

Def. Let f(x) be a given function with $m \le f(x) \le M$ (m is the smallest value, M is the largest value)

We say f has amplitude $A = \frac{1}{2}(M - m)$

What are the amplitude of each of the following six trig. function ?

f(x) = :

sin x ➔ __________ cos x ➔ _____________ tan x ➔ __________

Def. (Domain) Given a function y = f(x). The domain represents the set of all values of x that x can assume.

(Range): The range of the function y = f(x) is the set of all values that y can assume.

Find the domain and the range of all six trigonometric functions.

Function	Domain	Range
f(x) = sin x		
f(x) = cos x		
f(x) = tan x		
f(x) = sec x		
f(x) = csc x		
f(x) = cot x		

More on amplitude and period of a function.

Consider functions of the form y = a sinx, y = a cos x.
What is the amplitude in each case ?

ex. y = 2sin x ➔ _____________ y = ¼ cos x ➔ ______________

Consider functions of the form $y = \sin bx$, $y = \cos bx$, or $y = \tan bx$.

What is the period in each case ?

ex. $y = \sin 2x$ ➔ ________________ $y = \cos x/2$ ➔ ________________

ex. $y = \tan 4x$ ➔ ________________

Conclusion:

In general, ***Amplitude*** ***Period***

given $y = a \sin bx$

➔

or $y = a \cos bx$

➔

or $y = a \tan bx$

Phase –shift and other graphs

The graph of $y = A \sin (Bx + C)$

We already know the effects of A and B but how does C affect the graph ?

Write

$y = A \sin (Bx + C) = A \sin [B (x + C/ B)]$ and consider the graphs represented by

$y = A \sin x$, $y = \sin Bx$, $y = \sin Bx$, and $y = A \sin [Bx + C]$

Graph:

$y = \sin 2x$

$y = \cos \pi x/2$

$y = 2 \sin (x + \pi/4)$

$y = - 4 \cos 2x$

$y = \sin (2x - \pi/4)$

<u>Domain and Range of Trig. Functions.</u>

function	**Domain**	**Range**
y = sin x		
y = cos x		
y = tan x		
y = cot x		
y = sec x		
y = csc x		

<u>Domain and Range:</u>

of some basic equations:

$y = |x|$

$y = x^2 + 2$

$y = 2^x$

$f(x) = \dfrac{x}{x+2}$

$g(x) = \dfrac{3}{\sin x + 2}$

$y = [\,x\,]$

Domain and range of other trig. functions.

1.

2.

3.

4.

<u>Inverse Trig. Relations and Functions</u>

We have solved equations of the form $\sin\theta = 1/2$ and found the values of θ that make this statement true.

Review:

Relation: a correspondence between two sets.
The domain of a relation: set of all first coordinates
The range : set of all second coordinates

Function: a relation in which every first element is matched with exactly 1 element from the second set.

ex.

Let $f(x) = 2x + 4$ be given – rewrite in the form $y = 2x + 4$.

What happens when we interchange the x and the y variables.
Solve for y and label this new relation (function ?)

$g(x) =$ ____________

Look at the relationship between f(x) and g(x). If (p, q) is a point on f, give me a point on g.

Notation: Let $f(x) = 2x - 3$ and $g(x) = 4 - x$

We can define $f + g$: **$f - g$:**

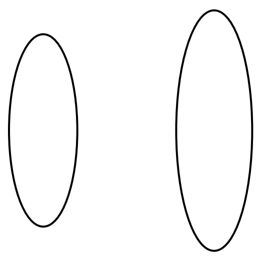
fg: **f/g:**

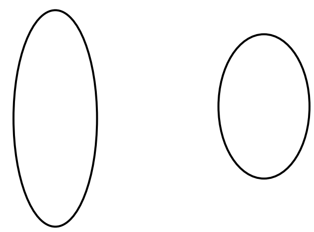
Begin with a number: say 4→ double it and add 3 --after you finish, take the resulting value and square it

In terms of functions:

Let f(x) = ______________ and g(x) = __________

Is there one function that does both functions at the "same" time – in the same equation ?





f(x): double it and add three **g(x) : square it**

Notation: We write g o f (x) and call it a composition function. **g o f (x) = g (f (x)).**

ex. $f(x) = 2x + 3$ **and** $g(x) = x^2$

find f + g: $f(x) + g(x) = (f + g)(x) =$ ____________

f /g: $\dfrac{f(x)}{g(x)} =$ ____________

g o f: $g \circ f(x) = g(f(x)) =$ _______________

ex. Given $f(x) = 3x - 4$ **and** $g(x) = \dfrac{x+4}{3}$,

find $g(f(2)) =$ ________ **What about** $f(g(5)) =$ ______________

In general find $g(f(t))$. ____________

Now we can talk about inverse trig. relations as well as inverse trig. functions.

W.L.O.G:

given $f(x) = \sin x$ write down the inverse relation →

Is this a function or is it just a relation ?

Similarly we have the inverse relations for the other five trig. functions.

$f(x) = \cos x$ →

$f(x) = \tan x$ →

$f(x) = \cot x$ →

$f(x) = \sec x$ →

$f(x) = \csc x$ →

What do their graphs look like ?

We want to be able to talk about inverse trig. functions – so let's redefine the inverse relations so that we make them functions.

Function	**Domain**	**Range**	**Inverse Relation**	**Inverse Function**
f(x) = sin x				
f(x) = cos x				
f(x) = tan x	**all real numbers except π/2, - π/2 and coterminal angles**			
f(x) = csc x	**all real numbers except 0, π and all coterminal angles**			
f(x) = sec x	**all real numbers except π/2, 3π/2 and all coterminal angles**			
f(x) = cot x	**all real numbers except 0, π and all coterminal angles**			

Problems:

Find each of the following values –

1) $\sin \pi/3 =$ _________

2) $\arcsin 1 =$ ____________

3) $\arccos \frac{1}{2} =$ ________

4) $\mathrm{Tan}^{-1}(-1) =$ __________

5) $\operatorname{arcsec} \frac{\sqrt{3}}{2} =$ ________

6)

7) $\sin(\arcsin(-1/2)) =$ __________

8) $\sin^{-1}(\sin \pi/7) =$ ___________

9) $\cos(\arccos \pi/2) =$ ____________

10) $\arccos(\cos 4\pi/3) =$ ___________

Also,
Provide a sketch of $y = 2 + 4\cos 2x$

Identities on page 243-245

1)

2)

3)

4)

True or False

______________ $\sin(A+B) = \sin(A) + \sin B$

Identities

Prove the following identities: (page 253 : 1 – 35)

a)

b)

c)

One more type of graph

f(x) = 2 + sin x **g(x) = cos x - 4**

More on identities:
Basic Rules

1) **Work each side independent from each other**

2) **Use rules of algebra; common denominators, reduce, factor, …**

3) **Begin with the most complex side**

4) **When in doubt , change to sines and cosines**

Examples:

36/254

40/254

Other Important Identities:

1) sum and difference identities **2) double angle identities** **3) half-angle identities**

Sum and Difference

True or False. sin (A + B) = sin A + sin B ________________

Construct a circle centered at the origin with angles A and B as drawn below.

Use the distance formula to come up with the following derivation.

If $\cos(A + B) =$ ___________________________________, then what will $\cos(A - B) = ?$

$\cos(A - B) =$ ________________________________

What about $\sin(A + B)$? and $\sin(A - B)$?

We need to see the relationship between $\sin(90^o - A)$ and $\cos\theta$. Is there any

Notice that $\cos(90^o - A) =$

So, $\sin(90^o - A) = \cos(90^o - (90^o - A))$

Now,
We want $\sin(A + B) = \cos[90^o - (A + B)] =$

We end up with the following four identities
Identities:
cos(A + B) = ________________________________

cos (A – B) = ______________________________________

sin (A + B) = ________________________________

sin(A – B) = _______________________________________

examples: Write as a single trig. function of a single angle and whenever possible simplify to a numeric value.

Write each of the following as a single trig. function of a single angle.

$\cos 3\theta \cos 2\theta$ - $\sin 3\theta$ $\cos 2\theta$ = ___________________________

$\sin \theta \cos \theta/2$ - $\cos \theta \sin \theta/2$ = _____________________________

$\sin 2\theta \sin \theta$ + $\cos 2\theta \cos \theta$ = ______________________________

examples:

Find $\cos 255^{o}$. ____________________ **find $\sin(-165^{o}) =$** _____________________

1)

2)

3)

Other identities:

Double angle Identities

Look at $\sin 2A$. We can write $\sin 2A = \sin(A + A)$. Now what ?

What about $\cos 2A$?

We did not look at an identity for the tangent function of the sum/difference of two angles.
There is one, as well as a double angle identity

$\tan(A + B) =$ ____________________________ **$\tan 2A =$ _______________**

Half-angle Identities:

Use the double angle identity cos 2A = ______________________ and the Pythagorean identity ➔ ________________

To create two new identities:

sin A = **cos A =**

Both of these can also be written as

sin A/2 = **cos A /2 =**

examples:

1) **write as a single trig. function of a single angle.**

a)

b)

c)

d)

2) graph each of the following

a) **b)**

3) Let sin A = -3/5 with A not in quadrant III and sin B = 12/13 with B not in quadrant I. Find

a) sin 2A = ____________ **b) cos 2A = __________________**

c) sin (A + B) = _______________________

d) cos (A + B) = ______________________

e) What quadrant is A + B in ? ___________________ Why ?

f) sin A / 2 = ___________________

g) cos A / 2 = ____________________

h) tan 2A = ____________________

4) Find sin A if sin (A + B) = 3/5 with A + B not in Quadrant II and cos B = -3/ 5, B not in quadrant II.

one possible solution:

use $\sin(A + B) = 3/5$ and $\cos(A + B) = ?$ $\rightarrow$ this will create a system of two equations in "2 variables" which can be solved.

Week 11 – Math 1303 – April 1, 2002
More Examples on solving Equations –

1) Find ALL POSSIBLE solutions of the equations $\tan x = 0$

2) Solve for θ if $0 < \theta < 360^o$, $\sin^2\theta - \sin\theta - 1 = 0$

3) Solve for x if $0 < x < 2\pi$, $\sin 2\theta - \cos\theta = 0$

4) $\cos 2\theta = -1$, find all θ if $0 < \theta < 2\pi$

5) $\sqrt{3}$ sec = 2

6) If $\sin\theta = 2/3$ and θ is not in QI find

a) $\sec\theta$ = __________ **b) $\sin 2\theta$ = _______________**

b) $\cos 2\theta$ = ___________ **$\sin 2\theta$ = _____________**

7) csc 3θ = 2

8) cos θ - sin θ = 1

Another type of equation – parametric equations

A parametric equation is an equation in which both x and y depend on a third variable (a parameter). Sometimes these are useful and sometimes they are necessary.

ex. $x = 2t$ $\qquad\qquad$ $y = t^2$

ex. $x = \sin t$, $y = \cos t$

Math 1303 –
Notes 3

Review of Identities:

cos(A + B) = ______________________________

cos (A – B) = ____________________________________

sin (A + B) = ______________________________

sin(A – B) = _____________________________________

examples:

Find cos 255^{o} . __________________ find sin (- 165 ^{o}) = ___________________

Write each of the following as a single trig. function of a single angle.

cos 3θ cos 2θ - sin 3θ cos 2θ = _________________________

sin θ cos θ/2 - cos θ sin θ/2 = ___________________________

$\sin 2\theta \sin \theta + \cos 2\theta \cos \theta =$ ______________________________

Other identities:

Double angle Identities

Look at $\sin 2A$. We can write $\sin 2A = \sin(A + A)$. Now what ?

What about $\cos 2A$?

By definition of the tangent function we can write similar identities for

$\tan(A + B)$ and the $\tan 2A$.

$\tan(A + B) =$ ______________________________ $\tan 2A =$ _______________

Half-angle Identities –

Use the double angle identity $\cos 2A =$ ______________________ and the Pythagorean identity ➔ ________________

To create two new identities:

$\sin A =$ $\cos A =$

Both of these can also be written as

$\sin A/2 =$ $\cos A/2 =$

2) graph each of the following

a) $y = 4 \sin 3x \cos 3x$ b) $y = 4\cos^2 x/4 - 4 \sin^2 x/2$

c) $y = 4\sin x \cos 3x - 4\cos x \sin 3x$

3) Let $\sin A = -3/5$ with A not in quadrant III and $\sin B = 12/13$ with B not in quadrant I.

Find

a) $\sin 2A =$ ____________ b) $\cos 2B =$ __________________

c) $\sin(A + B) =$ ______________________ d) $\cos(A + B) =$ ______________________

e) What quadrant is $A + B$ in ? ___________________ Why ?

f) $\sin A / 2 =$ ___________________ g) $\cos A / 2 =$ ____________________

h) $\tan 2A =$ ____________________

4) Find $\sin A$ if $\sin(A + B) = 3/5$ with $A + B$ not in Quadrant II and $\cos B = -3/5$, B not in quadrant II.

HW: page 264: 29, 33, 35, 36, 43, 47, 51, page 272: 1, 5, 9, 17, 21, 23, 25, 29, 31, 33,

HW: 37, 39, 41, 43, 45, 47, 51, 57 page 278: 1, 5, 9, 15, 17, 19, 23, 33,

Equations

2) Find ALL POSSIBLE solutions of the equations $\tan x = 0$

2) Solve for θ if $0 < \theta < 360^o$, $\sin^2\theta - \sin\theta - 1 = 0$

3) Solve for x if $0 < x < 2\pi$, $\sin 2\theta - \cos\theta = 0$

4) $\cos 2\theta = -1$, find all θ if $0 < \theta < 2\pi$

5) $\sqrt{3}$ sec $= 2$

6) If $\sin\theta = 2/3$ and θ is not in QI find

a) $\sec \theta$ = __________ b) $\sin 2\theta$ = _______________

b) $\cos 2\theta$ = ___________ $\sin 2\theta$ = _____________

7) Solve.

$\csc 3\theta = 2$

9) $\cos \theta - \sin \theta = 1$

Another type of equation – parametric equations

A parametric equation is an equation in which both x and y depend on a third variable (a parameter). Sometimes these are useful and sometimes they are necessary.

ex. $x = 2t$ $y = t^2$

ex. $x = \sin t$, $y = \cos t$

Name ______________________________ Math 1303 - Short QZ

1. Find the domain of

a) $y = \cos x$ **b) $y = 3x / (x + 2)$**

2. Find the range of $y = \sec x$

3. Sketch the graph of

a) $y = x^2 + 4x$

b) $y = 4 \sin 4\pi x$

4. What is the phase – shift of $y = \sin(x - \pi/3)$ and in what direction is it shifted from the graph of $y = \sin x$?

5. Complete the following identities:

$\cos(A + B) =$ ____

$1 - \cot^2 A =$ _____________

6. True or False.

_________ $\sin(\pi/2 - x) = \cos x$ if $0 < x < \pi/2$

______ $\sec(4\pi + x) = \sec x$ for all x

Name ____________________________________ Math 1303 Quiz February 20, 2002 -- Short Quiz

1. Use $s = r\theta$ to find s when $r = 4$ inches, $\theta = 90^o$ ➔ ___________________

2. Use $s = r\theta$ to find θ in degrees, when $s = 2$ feet and $r = 4$ inches ➔ ______________

3. What is the angular velocity of the hour hand of a clock if the hands are 10 inches long ? __________

4. Identify each of the following functions as ; even, odd, or neither

a) $f(x) = 4x^3$ ➔ ________________

b) $f(x) = 3x^2$ ➔ __________________

5. The sum of two odd functions is ___________________

Name ____________________________ Math 1303 - Long Quiz - February 15, 2002

1. Change from radians to degrees

2π ➔ ___________________

$3\pi / 5$ ➔ ___________________

2. Change from degrees to radians.

-90^o ➔ ___________ 120^o ➔ ______________

3. Give exact answer to each of the following.

a) Find $\sin \pi/4$. ________________ b) $\cos \pi/6$ = _____________

4. Find a positive angle in each of QII, QIII, and QIV so that their absolute value is less than 360^o and they all have related angle $\theta_r = 40^o$

QII ➔ __________ **QIII ➔ _______** **QIV ➔ ____________**

5. An airplane is flying due north at 300 mph. A west wind causes to move off course by 30^o . What is

a) the speed of the wind ➔ _______________

b) the ground speed ➔ _____________

Name ______________________________ Math quiz, February 18, 2002 - Short Quiz.

1. What is larger 50^o or 1 radian ? ___________________

2. Find $\sin \pi$. _______________

3. Use the formula $s = r\theta$

a) What is the radius of a circle that has a 45^o central angle intercepting an arc of length $\pi/2$ inches ?

b) A pendulum swings through a four-inch arc. If the pendulum is of length 1 inch. Then what angle does it swing through in degrees?

c) A circle of radius 10 feet. A central angle $\pi/4$ intercepts an arc of what length ?

Name ____________________________________ Math 1303 – LOng quiz, Febr. 22, 20001

1. Give me a rough sketch of $y = \sin x$

2. Use $s = r\theta$

   a) What restrictions does θ have ? ____________

   b) If $s = 24$ inches and $\theta = 1$ radian, find r in terms of feet

3. Given $v = rw$

   a) What restrictions are placed on w ? __________________________

b) What is the value of v when r = 12 inches and w = $\pi/4$ radians/ min

4. What is the speed of a point on the edge of a circle that is rotating at 2 revolutions /sec and has radius 6 inches ?

Name ________________________ Math 1303 – Short Quiz – February 27, 2002

1. Find the period of each of the following trig. functions.

a) $f(x) = \sin x$ ➔ ________________ b) $y = \sec x$ ➔ ____________________

c) $g(x) = \cot x$ ➔ ______________

2. Find the amplitude of each of the following functions.

a) $f(x) = 4\sin x$ ➔ ____________ b) $h(x) = \frac{1}{2} \cos x$ ➔ ____________

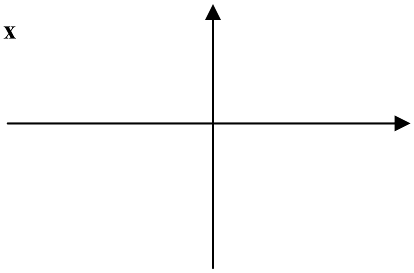
c) $y = 2 \tan x$ ➔ ______________

3. What is the period of $f(x) = \sin 4x$ ➔ _____________

What is the period of $y = \cos x/2$ ➔ _____________

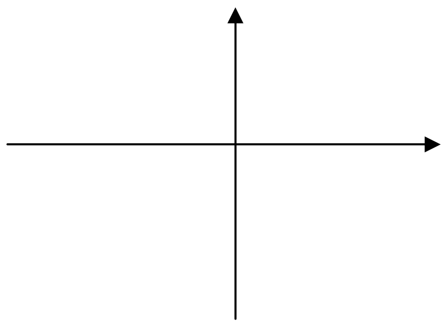
4. **Give a rough but accurate sketch (x and y-intercepts,…) of**

a) $y = \cos x$



b) $f(x) = \tan x$

5. **Which of the six trig. functions is best represents the following graph ?**



Name ____________________________ **Math 1303.010 – February 11, 2002 – Short Quiz**

1. **Write the definition of** $\sec\theta$ ➔ ___________________

2. **Complete each of the following identities ---**

a) $\sin^2\theta$ + ____________ = 1

b) 1 - ______________ = $-\cot^2\theta$

3. **An object is flying 300 feet above the ground it spots an object a 30° angle of depression – How far (along the ground) is the object on the ground from the flying object ?**

4. An airplane is flying at a bearing of S 80^o W at 200 mph it encounters a wind from the south causing the airplane to fly due west . What is the speed of the wind ?

Name _________________________________ Math 1303 – March 1, 2002 --- Long Quiz

1. Use: opp. , adj., and hyp. to define

sin θ = ________

2. Which if any of these functions is even ?

sin x, cos x, cot x NONE of these

3. What is the domain of y = sin x ? ______________________________

4. What is the range of f(x) = cos x ? ______________________________

5. Find $\sin x \bullet \csc x$ = ______________

6. What is the phase-shift of $y = 4 \tan(x - \pi/4)$? Indicate which direction. ____________________

7. Give me a rough but accurate sketch of

a) $y = \cos x/2$

b) $f(x) = 4 \sin x$

8. Use the given graph to sketch the graph of $y = \sin(2x + \pi/2)$

Go over the last three identities –

Review for Exam II:

Know:

1) Distance formula between points, plot point, rectangular coordinate system

2) Pythagorean Thm.

3) Similar, congruent

4) Definition of trig. functions in terms of x,y, and r as well as hyp., opp., and adjacent

5) Basic Definitions: complement, supplementary, cofunctions, reciprocals, coterminal, standard position, axis angle, quadrant angle

6) triangle of reference, related angle,

7) special angles: 30^o, 45^0, 60^o, axes angles

8) radians and degrees

9) $s = r\,\theta$, $v = r\,w$

10) Identities

11) graphs of functions, amplitude, period, phase-shift

12) domains of trig. functions, range of trig. functions

13) odd, even functions 14) fundamental identities 15) major items

HW: page 264: 29, 33, 35, 36, 43, 47, 51, page 272: 1, 5, 9, 17, 21, 23, 25, 29, 31, 33,

HW: 37, 39, 41, 43, 45, 47, 51, 57 page 278: 1, 5, 9, 15, 17, 19, 23, 33,

Week 10 finish off material and begin equations.

HW: page 95 ----26,
page 108 --- 22,
page 151 --- 18,
page 153 --- 57,
page 161 --- 49,
page 167 --- 51
page 297 --- 37, 51, 55, 59
page 304 – 5, 9, 15, 23,
page 309 1, 7, 19, 23, 29, 35, 47, 48

Name __________________________ Math 1303 - Short Quiz - March 20, 2002

1. Sketch the graph of $y = 2\sec x$ **2. Sketch the graph of $y = \cos 2x$**

3. **Complete the following identities:**

a) $\sin^2 A +$ _________ $= 1$ b) $1 + \tan^2 A =$ ______________

c) $\cos(A + B) =$ ______________________________________

d) $\sin(A - B) =$ ____________________________________

4. **Find all x's so that $0 < x < 2\pi$ if $\sin x = -\frac{1}{2}$. $x =$** __________________________

5. **Prove $\cos(90^\circ - \theta) = \sin\theta$**

6. **Write $\cos 4\theta \cos\theta + \sin 4\theta \sin\theta$ as a single trig. functions of one angle.** _______________

Name ____________________________ **Math 1303 - Long Quiz, April 1, 2002**

1. **What "famous holiday" is celebrated on April 1 ?** ___________________________

2. **Sketch a rough but accurate graph of $f(x) = \tan x$.**

3. **Find all of the values of x so that $\sin x = 1$ and x is in degrees.**

4. **Find all values or r so that $\cos r = -1$ and $0 \le r < 2\pi$**

5. **Complete the identities.**

   a) **$\sin(A + B) =$ ____________________________**

   b) **$\sin 2A =$ ____________________________**

   c) **$\cos A/2 =$ ____________________________**

Math 1303 – Week 11, day 3

Name ____________________________ Short –Quiz April 3, 2002

1. **Identify the following graph.**

2. **Find all of the solutions of the equation $\sin\theta = 1$**

3. **Find the solutions of the equations for values of x so that $0^o \le x < 360^o$**

   a) **$\sin^2 x - \cos x = 1$**

b) $\cos^2 x - \sin^2 x = 0$

4. Prove.

$\sin^2 \theta + \sin 2\theta = (\sin \theta + 2\cos\theta) / \csc \theta$

Week 11 - Day 3 – April 5, 2002
Test III Review –

Old Material:

1) Distance formula between points, plot point, rectangular coordinate system
2) Pythagorean Thm. 3) Similar, congruent
4) Definition of trig. functions in terms of x,y, and r as well as hyp., opp., and adjacent
5) Basic Definitions: complement, supplementary, cofunctions, reciprocals, coterminal, standard position, axis angle, quadrant angle
6) triangle of reference, related angle,
7) special angles: 30^o, 45^0, 60^o, axes angles --- may need to use without a calculator
8) radians and degrees
9) $s = r\,\theta$, $v = r\,w$

10) Identities

11) graphs of functions, amplitude, period, phase-shift

12) domains of trig. functions, range of trig. functions

13) odd, even functions 14) fundamental identities 15) major items

14) Solving equations ---

15) All the new identities: cos (A + B) = …., double angle, half-angle

Review Session: Sunday Evening - @ 6:00 – 7:00 PM

Math 1303 - Test III –

1. Define csc θ:________________ **2. Find the reciprocal function of cos θ = ______**

3. True or False:

a) All pairs of complementary angles are acute.

4. IF sin θ = 3/8 with θ not in quadrant I, then find

sec θ = ______ sin 2θ = _________ cos 2 θ = _____________ sin θ/2 = _________

cos θ/2 = __________

5. IF sin A = - 3/5 and cos B = -12/13 with A and B not in Quadrants II or IV

find sin (A + B) = ______________ cos (A + B) = ____________

What quadrant is A + B in ? __________________

6. Find all of the solutions of the equation

tan θ = - 1 ➔ θ = ____

7. Find the solution of each of the following equations. Assume that θ lies between 0^o and 360^0 Round to the nearest degree whenever necessary.

a) $\sin^4 \theta - \cos^4 \theta = 0$

b) $\sec^2 \theta - 4\sec \theta + 3 = 0$

c) $\sin^3 \theta - \cos^3 \theta = 0$

d) $\sin \theta + \cos \theta = 0$

8. Identify as Functions or just Relations

9. Find the domain of

a) $f(x) = \cos x$ ➔ _____________ $g(x) = \tan x$ ➔ ____________________

10. What is the range of the following

a) $f(x) = \sin x$ ➔ _______________ $g(x) = x + 2$ ➔ __________________

11. If $f(x) = x^2 + 2x + 1$, then find $f(2)$ = _____________

12. Word Problems and Identities –

Inverse relations:

Let $f(x) = 2x + 4$ be given – rewrite in the form $y = 2x + 4$.
What happens when we interchange the x and the y variables.

Solve for y and label this new relation (function ?) $g(x)$ = ____________

Look at the relationship between $f(x)$ and $g(x)$. If (p, q) is a point on f, give me a point on g.

Notation: Let $f(x) = 2x - 3$ and $g(x) = 4 - x$
We can define

$f + g$:

$f - g$:

fg:

f/g:

Begin with a number: say 4→ double it and add 3 --after you finish, take the resulting value and square it

In terms of functions:

Let f(x) = _______________ and g(x) = __________
(double your number and add 3) (square the value)

Is there one function that does both functions at the "same" time – in the same equation ?

Notation: We write g o f (x) and call it a composition function.
g o f (x) = g (f (x)).

ex. from above.

ex. let f(x) = 3x – 2 g(x) = x + 5

Find g o f: Find f o g :

find g o f (2) = __________ f o g (- 1) = __________

Certain functions provide special results:

ex. f(x) = 2x – 3 g(x) = (x + 3) / 2

Find f o g (1) = _______ f o g (- 2) = ____________

This is the idea of inverse relations. Look at the example at the previous page at the beginning of our discussion of inverse relations. This is one way of creating inverse relations.

Look at trig. functions and how they relate to their inverse relations.
We have solved equations of the form sin θ = 2/3 and found the values of θ that make this statement true.
Change the notation of the statement above.

Function	Domain	Range	
f(x) = sin x			
f(x) = cos x			

$f(x) = \tan x$	all real numbers except $\pi/2$, $-\pi/2$ and coterminal angles		
$f(x) = \csc x$	all real numbers except 0, π and all coterminal angles		
$f(x) = \sec x$	all real numbers except $\pi/2$, $3\pi/2$ and all coterminal angles		
$f(x) = \cot x$	all real numbers except 0, π and all coterminal angles		

Define the inverse relation of following trig. functions. What is the domain and range of each.

$y = \sin x$ ➔ ________________ D: R:

$y = \cos x$ ➔ ________________ D: R:

$y = \tan x$ ➔ ________________ D: R:

Change the range of the three relations so that they represent functions. The domain is still the same.

$y = \arcsin x = \sin^{-1} x$ D: R:

$y = \arccos x = \cos^{-1} x$ D: R:

$y = \arctan x = \tan^{-1} x$ D R:

Graphs:

$y = \arcsin x$ $y=\arccos x$ $y=\arctan x$

Problems: Simplify each of the following.

1) $\sin^{-1} \frac{1}{2} = x$ ➔ ____________

2) $\sin^{-1} (- 1) = x$ ➔ __________

3) arctan 3 ➔ ______________

4) $\text{arc cos } \pi = x$ ➔ _____________

Since All of the above represent angles , we could go back and talk about the topics discussed earlier in terms of these inverse trig. functions.

5) $\sin (A) = ?$ If $A = \sin^{-1} 4/7$ ➔ ____________________

6) $\cos (\text{arc cos } 7/9) =$ _______________________

7) $\sin (\tan^{-1} 3/8) =$ ________________________

We can work with identities.

8) $\sin (\sin^{-1} 3/5 + \cos^{-1} 12/13) =$ ________________________

9) $\sin (2 \arcsin 5/6) =$ _____________________

10) $\cos (2 \sin^{-1} (- 5/8)) =$ _____________________________

We can also solve equations.

11)

12)