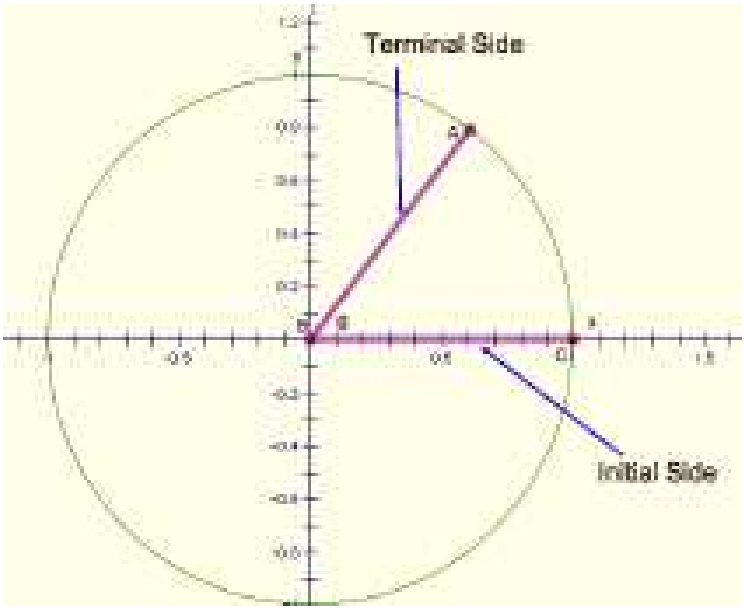


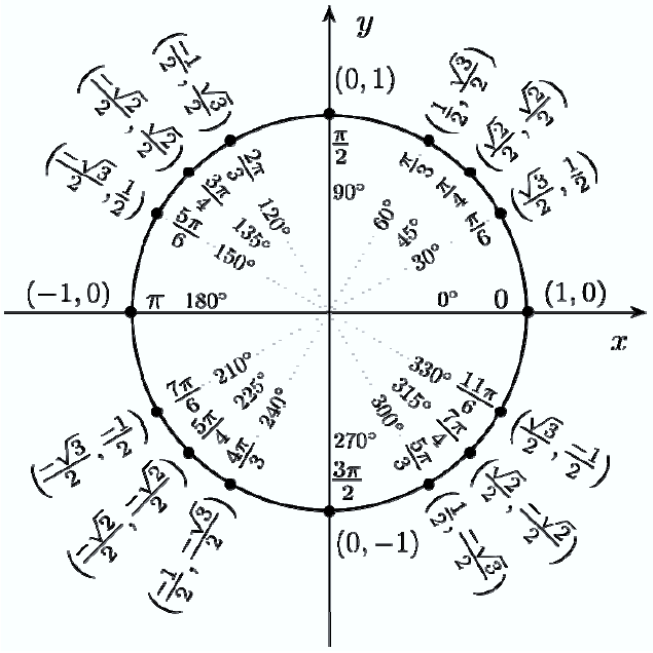
SUNY Oswego

School of Education

Teacher Work Sample

The Unit Circle

11th Grade Trigonometry



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Section I: Learning-Teaching Context

I will be teaching in Example High School. This is a large school consisting of grades 10-12. The total enrollment is 2,417 students. The average class size for mathematics is 23. The classes I will be teaching will be consistent with this. I think this is a good class size. I will take this into account when dividing the class into groups depending on available materials and the activity being done. This will also allow me to use the computer lab in my lessons.

I will be teaching in an 11th grade classroom which is called Math 11 and covers the last two semesters of the Math B curriculum. The school doesn't use a textbook for Math 11 but they use a series of packets put together by the teachers as a supplement. This high school does not have a textbook because they couldn't find a sufficient one for the curriculum. The packets can be a good resource but only if the students are in class and take notes. This lack of a textbook can be a burden for the students because since all notes are taken out of the packets. The packets are the only source for students and if the students miss the notes for any reason they have no alternative resource. The packets do have the advantage of being designed specifically for the curriculum so all standards are covered. For me this will be a challenge because I will not be using the packets so my notes and activities will need to be clear enough so the students will not need an alternative source.

The students are 92% white, 2% Asian, 1% Hispanic or Latino, 4% African American, and 1% American Indian. Because of this most of my examples will reflect this white dominated student body. As easy as it is to assume that the culturally relevant material should reflect the white students, I should also assume that these students haven't been exposed to a lot of diversity in their lives. Because of this I should take every opportunity to expose them to other cultures. I will do this with a lesson teaching about the origin of sine and how the ancient Arabs and Aryabhata used trigonometry.

Section I: Learning-Teaching Context

There is a high level (93%) of student stability The number of students eligible for free lunch is 11%. This is relatively low for a public school. This means that most of the students will have access to resources outside of school, like internet, books, etc. Virtually all of the students are English proficient with only 7 out of 2417 students being limited English proficient. In all likelihood a teacher at this school will not have too many encounters where language is an obstacle.

91% of students scored above 65% on the Math A regents. This means I can assume that most of the students know an acceptable amount of material from Math A. This is important because I am in a Math B classroom and this builds on many topics from Math A. The school is also in good standing in almost every category so it is realistic to have high expectations for the students as they have had good performance on standardized tests.

There are students in the classes I am observing with 504 plans. One is for a student with autism. This student needs to have limited distractions, extended time, verbal encouragement, on task reminders, and directions repeated. To limit distractions I will need to select this student's seat carefully. It will need to be away from distracting students, maybe in the front of the room and away from windows. I will also need to allow this student to go out of the room during tests and quizzes for extended time. There is also a student in the room who is a diabetic which might require him to have snacks or drinks. A learning disabled student is also in one of these classes. This student has slow hand writing. Some things I will need to do for this student include giving a copy of my notes to this student or having a note taker copy the notes for this student. This student will also need extended time on tests and quizzes.

The technology I have to work with will be useful. There is a computer lab in the library which I will use to do an activity involving the unit circle. I would like to use Geometers

Section I: Learning-Teaching Context

Sketchpad but the lab doesn't have it. The activity can be modified to work with web applets which will work just as well. Every math classroom has an ELMO. Most of my lessons will take advantage of the ELMO. I can use it to do warm ups, to go over homework, and to give notes. Each math classroom also has a set of calculators that the students are able to use. This means I can design activities where I ask the students to use calculators.

This high school uses block scheduling which can be a blessing and a curse. Block scheduling will allow me to do extended projects to that can be more interesting for students. Unfortunately I can not do extended projects everyday, and lecturing for 80 minutes can be boring for students. As long as I design my lessons to include a variety of activities the block scheduling will be a great tool for when I want to do in depth projects and experiments with students.

This high school also has a useful tool for students called "math help". This is held in the cafeteria and is available to students every period. At "math help" a student can go during a free period and get help from a math teacher on any work they bring. Because of "math help" students will have no excuse for not finishing assignments because they don't understand it.

Section II: Learning Goals & Objectives

Content Goals

Express and apply the six trigonometric functions as ratios of the sides of a right triangle (Standard A2.A.55)

Sketch the unit circle and represent angles in standard position (Standard A2.A.60)

Find the value of trigonometric functions, if given a point on the terminal side of angle θ (Standard A2.A.62)

Sketch and use the reference angle for angles in standard position (Standard A2.A.57)

Define radian measure (Standard A2.M.1)

Convert between radian and degree measures (Standard A2.M.2)

Sketch and recognize one cycle of a function of the form $y = A \sin Bx$ or $y = A \cos Bx$ (Standard A2.A.70)

Determine amplitude, period, frequency, and phase shift, given the graph or equation of a periodic function (Standard A2.A.69)

Know the exact and approximate values of the sine, cosine and tangent of 0º, 30º, 45º, 60º, 90º, 180º, and 270º angles (Standard A2.A.56)

Process Goals

Understand and make connections among multiple representations of the same mathematical idea (Standard A2.CN.1)

Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs and diagrams (Standard A2.CM.2)

Observe and explain patterns to formulate generalizations and conjectures (Standard A2.PS.3)

Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts (Standard A2.R.1)

Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion (Standard A2.RP.2)

Attitude/Disposition Goal

Develop an appreciation for the historical development of mathematics (Standard A2.CN.8)

Assessment Narrative

As evidence of student achievement I would collect two assessment pieces. These are the unit test and the Aryabhata homework in which the students are researching a nonwestern mathematician.

The unit test will be a culmination of the unit and will assess all of the content goals. These content goals are A2.A.56, A2.A.69, A2.A.70, A2.M.2, A2.M.1, A2.A.57, A2.A.62, A2.A.60, and A2.A.55. Some process standards will also be evident on the unit test. These will be shown through short answer questions where the students will have to create representations of mathematical concepts through sketching the unit circle and angles in standard position. These process goals are A2.R.1 and A2.CM.2. The selection of problems I will be analyzing will be such that all of these goals are assessed. I will be analyzing both multiple choice and short answer problems. I will also take a selection of 5-10 multiple choice questions from the unit test to serve as a pretest.

The class I will be collecting a sample from will be a regents level 11th grade math B class with 23 students. The diversity aspect I will be analyzing will be gender. There is a stereotype that males are better than females at math and I wonder if there are any trends either way. I will be looking at these trends as a whole or as they pertain to a certain type of problem.

The second form of assessment will be a homework that I assign in my social justice lesson. This lesson teaches about Aryabhata and the history of sine. The social justice goal for this lesson is for students to recognize a western bias in the curriculum and to gain an

appreciation for nonwestern mathematics. These goals and the attitude/disposition goal A2.CN.8 can all be assessed through the homework that I assign. The homework will ask he students to research a nonwestern contributor to mathematics and write a short paragraph describing who their contributor was and what they contributed to mathematics. I will by analyzing the diversity aspect of race. More specifically minorities vs. Whites students. I feel like this is appropriate given the assignment. I will test to see if a certain race is more accepting to nonwestern mathematics than others.

Achievement Goal	Assessments & Performance Criteria	Rationale	Adaptations
Content Goals: Express and apply the six trigonometric functions as ratios of the sides of a right triangle (Standard A2.A.55) Sketch the unit circle and represent angles in standard position (Standard A2.A.60) Find the value of trigonometric functions, if given a point on the terminal side of angle θ (Standard A2.A.62) Sketch and use the reference angle for angles in standard position (Standard A2.A.57) Define radian measure (Standard A2.M.1) Convert between radian and degree measures (Standard A2.M.2) Sketch and recognize one cycle of a function of the form y = A sin Bx or y = A cos Bx (Standard A2.A.70) Determine amplitude, period, frequency, and phase shift, given the graph or equation of a periodic function (Standard A2.A.69) Know the exact and approximate values of the sine, cosine and tangent of 0º, 30º, 45º, 60º, 90º, 180º, and 270º angles (Standard A2.A.56)	The content goals will be evaluated with a variety of methods. I will be constantly asking students questions while taking notes or doing activities to assess how they comprehend new material. Students will be assessed in during group work in class. This will be done by the students either explaining their work to the class, writing their work on the board or ELMO, or through collecting the group work. Students will be given homework each night to practice the new material covered in class. There will be a test at the end of my unit that will assess all of the content goals.	The reason for asking questions while taking notes and doing activities involving new material is to gage how fast to teach and to make sure the students understand something fully before moving on. Group work will be assessed to make sure the students are staying on task. When the work is assessed through sharing with the class this allows the other students to learn the information from another perspective. The homework is assigned to give students practice with the new material and to make sure that all students are learning the new material. The test is a final way to make sure that the students understand the important points of the unit and the content goals. The unit test will test the same thing but will cover more material not covered in my unit	Adaptations for autistic students will be to provide extended time for quizzes and tests, provide verbal encouragement, repeat directions, and give on task reminders. For the learning disabled students a note taker will write notes for the student of the student will be given a copy of the teacher's notes.

Process Goals: Understand and make connections among multiple representations of the same mathematical idea (Standard A2.CN.1) Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs and diagrams (Standard A2.CM.2) Observe and explain patterns to formulate generalizations and conjectures (Standard A2.PS.3) Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts (Standard A2.R.1) Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion (Standard A2.RP.2)	Students will use technology through the graphing calculator and through an activity using Geometers Sketchpad. Students will be assessed on this by having to answer questions using the graphing calculator on homework and class work. The Geometers Sketchpad activity will include a worksheet that will be collected. Students will be able to demonstrate the relationship between the unit circle and the sine and cosine curves. They will do this by verbally explaining this in class.	The students will be taught how to recognize the graph of sine and cosine functions using period frequency and amplitude. The students will learn how to know how to graph these on the calculator because this will be a useful tool for saving time as well as increasing the number of ways the graph can be easily manipulated. Students will need to demonstrate how to make the connection between the sine and cosine curves and the unit circle because it makes them easier to remember as well as giving each one meaning by relating it to something else.	All of the above adaptations will be used as well as having the students discuss in groups the relationships between the unit circle and the sine and cosine curves. Discussing in small groups will allow the students to be able to hear the perspectives of students. Hearing the relationships discussed in a different way will help the students better understand.

Attitude/Disposition Goal: Develop an appreciation for the historical development of mathematics(Standard A2.CN.8)	This Attitude/ Disposition goal will be assessed through a homework assignment. The students will be asked to research a nonwestern contributor to mathematics. This will come after a lesson on Aryabhata and the history of sine. Students will also do an activity in class in which they will calculate sine the same way Aryabhata did.	Students will gain an appreciation for the way Aryabhata calculated sine by doing it themselves. The homework is a good way for students to gain an appreciation by researching what interests them the most.	The modifications will be the same including verbal encouragement, on task reminders, and extended time for the trig table activity. Also for the student with a learning disability I will provide a set of class notes for the Aryabhata notes.

Overview

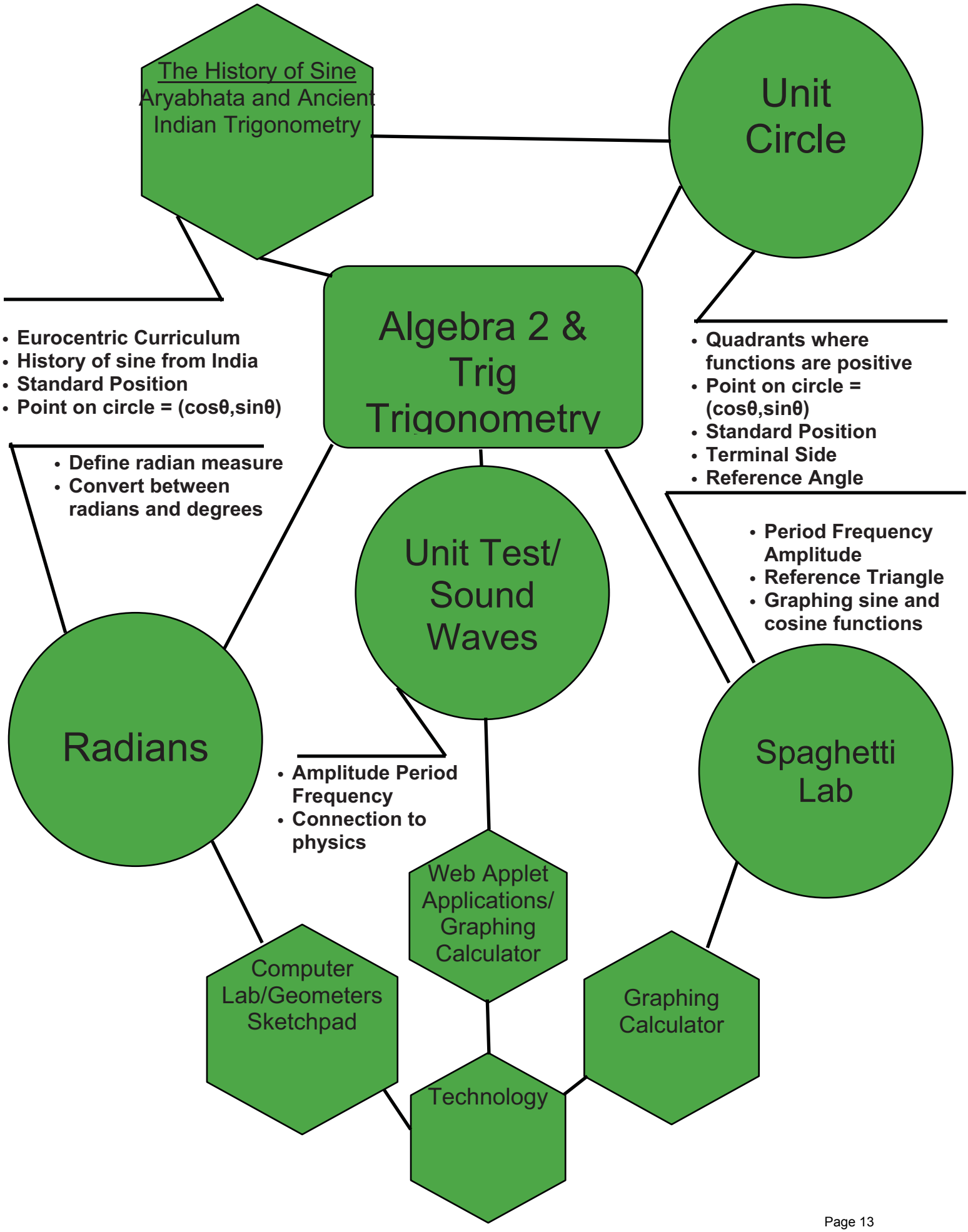
My teacher work sample will be a unit on 11th grade trigonometry. The unit will start by introducing the unit circle and the graph of the sine and cosine functions. I will do it this way because I want to emphasize the relationship between the two. I will then use the unit circle to let the students make conjectures and derive formulas related to the unit circle using Geometers Sketchpad. The unit will also cover expressing trigonometric functions as sides of a right triangle. As well as solving trigonometric equations for all angles from 0º to 360º using reference angles.

Rationale

I chose the topic of 11th grade trigonometry. Trigonometry is a good extension of the students math foundation and it can be used as a means of applying previously learned topics. The NCTM states, "Students should have opportunities to learn increasingly more sophisticated mathematical ideas as they progress through the grades." Trigonometry can be used across many fields in mathematics. A large portion of geometry involves using trigonometry. The NCTM geometry standards say that students should, "use trigonometric relationships to determine lengths and angle measures" (NCTM 2008). I plan on using sketchpad to show how these trigonometric relationships are interconnected, specifically how the unit circle relates to the sine and cosine functions and how the unit circle can be used to demonstrate trigonometric formulas. Students will also be able to use sketchpad to experiment with the unit circle to ultimately make conjectures on trigonometric identities and formulas. "Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction" (NCTM, 2008).

The NYS Core Curriculum in mathematics includes:

- A2.A.60 Sketch the unit circle and represent angles in standard position
- A2.A.68 Solve trigonometric equations for all values of the variable from 0º to 360º
- A2.A.55 Express and apply the six trigonometric functions as ratios of the sides of a right triangle


The History of Sine
Aryabhata and Ancient Indian Trigonometry
Unit Circle
Algebra 2 & Trig Trigonometry
• Eurocentric Curriculum
• History of sine from India
• Standard Position
• Point on circle = (cosθ,sinθ)
• Quadrants where functions are positive
• Point on circle = (cosθ,sinθ)
• Standard Position
• Terminal Side
• Reference Angle
• Define radian measure
• Convert between radians and degrees
• Period Frequency Amplitude
• Reference Triangle
• Graphing sine and cosine functions
Unit Test/ Sound Waves
Radians
Spaghetti Lab
• Amplitude Period Frequency
• Connection to physics
Web Applet Applications/ Graphing Calculator
Computer Lab/Geometers Sketchpad
Graphing Calculator
Technology

Dear Family,

We will soon be moving into this year's trigonometry unit. This unit is a great opportunity for students to use technology, to learn about an underappreciated ancient culture, and to make connections to other subjects as well as everyday life.

The unit will start with some review of basic trigonometry ratios as sides of a right triangle. This together with the Pythagorean theorem allow students to find missing pieces of right triangles. This skill is far reaching across all areas of mathematics and in life.

We will then move into the unit circle. The unit circle is an intricate part of trigonometry and a powerful representation of trigonometry functions. It is important that students recognize the connections between the trig functions and the unit circle. We will be doing a lab using spaghetti to demonstrate this relationship.

In order to promote social justice I will be teaching about the origins of sine. Sine was first used by Aryabhata, an ancient Indian mathematician. Aryabhata used a chord in the unit circle to define sine. This is similar to what we will be doing with the unit circle. Our class will take some time to investigate how Aryabhatas definition of sine applies to our lesson on the unit circle.

Technology will be used in this unit. The students will get the opportunity to use a program called Geometers Sketchpad. This is a program that lets students construct representations of geometric shapes. They can take measurements and manipulate these shapes to recognize relationships and make conjectures about these representations. They will be using this program to define radian measure. The students will do this by discovering the relationship between the circumference of a circle and its radius.

In an attempt to make the unit meaningful for students the students will see a demonstration using a web application using sound waves to introduce graphing sine and cosine functions. The students will look at a sine wave and listen to the sound it makes to determine how frequency and amplitude effect the sound a wave makes.

Trigonometry is an important topic which can be applied across many fields of mathematics. It is vital for the students to recognize how all the pieces of trigonometry fit together to promote a higher understanding of trigonometry and mathematics. It is my goal to not just teach a set of rules and procedures but to teach for understanding.

Sincerely

Student Teacher

Unit Circle

Name:	Student Teacher	Section Time:	Wednesday 4:30	#:11	

Mathematical Task: To describe a point on a unit circle as a function of an angle in standard position. To discover which quadrant trig values of an angle are positive and negative.

Grade Level, approximate number of students, types of students: Grade 11, 24 students, Regents level students.

Materials and Sources: PowerPoint, DIN, pairs check activity

NYS Mathematics Performance Indicator(s):

Content Strand:

- Sketch the unit circle and represent angles in standard position (Standard A2.A.60)
- Sketch and use the reference angle for angles in standard position (Standard A2.A.57)
- Find the value of trigonometric functions, if given a point on the terminal side of angle θ (Standard A2.A.62)
-

Process Indicators:

- Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts (Standard A2.R.1)
- Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs and diagrams (Standard A2.CM.2)
- Understand and make connections among multiple representations of the same mathematical idea (Standard A2.CN.1)

Performance Objectives:

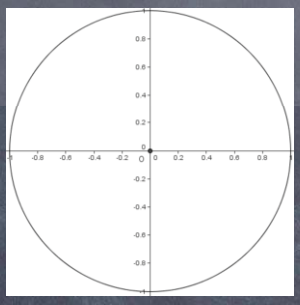
The students will demonstrate the ability to draw angles in standard position, write the sine and cosine of a point on a unit circle given its angle from the positive x-axis, and tell which functions are positive and negative in which quadrants by doing practice problems and explaining how they answered these practice problems when asked by the teacher.

Safety Precautions: Students will not wonder around the room during the pairs check activity.

Part of Lesson	**Teacher and Student Actions**	**Mathematical Questions.** These can repeat for different parts of the lesson if appropriate. Include both lower order and higher order questions for each lesson.
Launch/Anticipatory Set/Before State prior knowledge needed, then explain how you will access prior knowledge and draw the students into this particular lesson. Also explain how you will launch the "problem" the students will solve during the explore. Some problems need more extensive launches than others.	**Teacher Actions:** The teacher will have the DIN ready on the projector as the students come in the room. The DIN will be quick and easy as serve as a simple review of trig ratios. When class starts the teacher walk around and help the students with the DIN. When the students finish the DIN the teacher will go over it on the ELMO. The teacher will go over any questions the students have on the homework. **Student Actions:** The students will come into the classroom and immediately start working on the DIN. The students will then ask any questions they have on the DIN while the teacher is going over it.	What tricks do you know to remember trig ratios? (SOHCAHTOA) What is the Pythagorean theorem? Can tangent be represented in terms of sine and cosine?
Explore/During Describe what the teacher will do and how the students will be engaged in the lesson/activity.	**Teacher Actions:** The teacher will go through some notes using Powerpoint. Some new topics will be discussed. There will be times during the notes where the teacher will stop while the students do some practice problems. While the students are working on practice problems the teacher will walk around and ask questions and help students. The teacher will then go over the problems while asking students to explain how they did them. The teacher will then resume with the notes. **Student Actions:** The students will take notes given by the teacher. When the teacher gives practice problems the students will try them and ask questions if they need to.	When drawing an angle in standard position where is the initial side? Where is the terminal side? How do you represent a point on the unit circle using trig functions? Which trig functions are positive in quadrant I, II, III, and IV?

Summary/Closure/After Describe how you will summarize the lesson. This is a most important part of a problem solving lesson where the students share their strategies. The teacher should facilitate the discussion and the students should be actively engaged.	**Teacher Actions:** When the class finishes going through the notes the teacher will pass out a pairs check activity. The students will do this with a partner. While the students are working the teacher will walk around and help students who have questions. What the students don't finish on the pairs check in class will be assigned for the students to work on by themselves for homework. **Student Actions:** The students will work on the pairs check activity with a partner. The students should work together on a set of problems until they get the same answer. If the students don't finish the worksheet they will complete it for homework.	Can two angles have the same reference angle? Can two functions be positive in the same quadrant? How do you draw a reference angle?
Assessment Clearly describe how the teacher will know if the students met the objective. Assessment should be embedded within the activities. The assessment may include a separate independent component.	One form of assessment will come from talking to the students during the pairs check activity and through asking the students questions during the notes. A second form will come from the pairs check activity done in class and for homework.	
Modification for Advanced Learners	Advanced learners will be asked to sketch the tangent line of a given angle on the unit circle.	
Modifications for two types of disabilities (state the name of the disability here). You may include a modification for ELL.	The learning disabled student will be given a copy of the Powerpoint so he doesn't fall behind trying to copy the notes. The autistic student will be placed in the front of the room away from the windows and other distractions during notes to eliminate distractions	

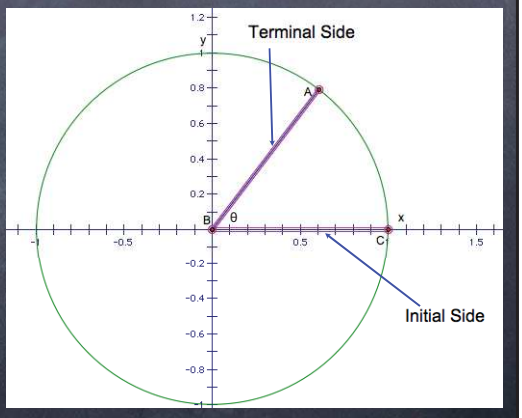
Unit Circle



- The unit circle is a circle centered at the origin with a radius of 1

Standard Position/Terminal Side

- Standard Position - An angle with its vertex at the origin and one side, the initial side, on the positive x axis.
- Terminal Side - The side of the angle which is not on the x-axis. “The ending side”



Draw the following angles in standard position on the unit circle. What quadrant is each angle in?

1) 30

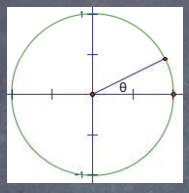
2)300

3)120

4)90

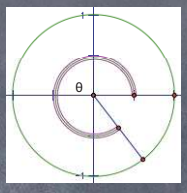
Draw the following angles in standard position on the unit circle. What quadrant is each angle in?

1) 30



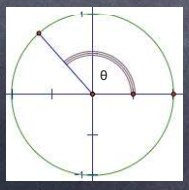
Quadrant I

2)300



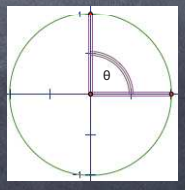
Quadrant IV

3)120



Quadrant II

4)90

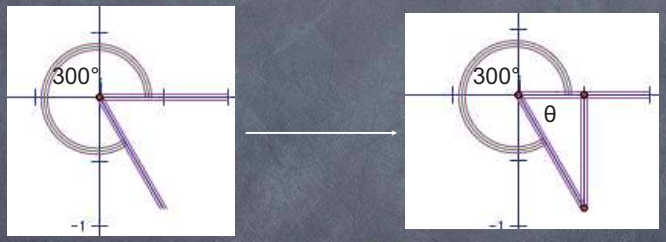


θ is not in a quadrant

Reference Angle

- The reference angle is an acute angle that touches the x-axis.
- To create a reference angle you should:
  - Sketch the angle in standard position
  - Create a right triangle with the x-axis
  - Find the reference angle by adding or subtracting from the x-axis

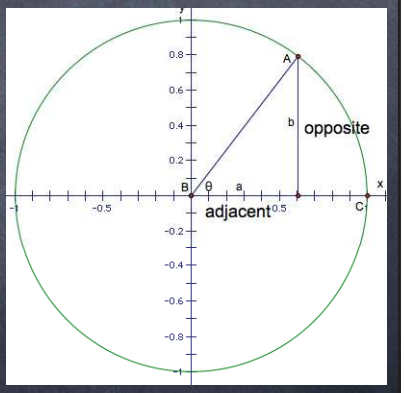
What is the reference angle for 300°?



θ = 360 - 300 = 60

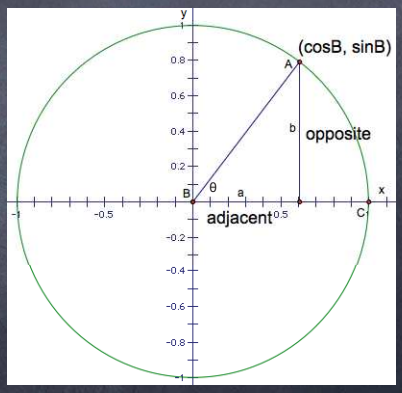
Trig Ratios

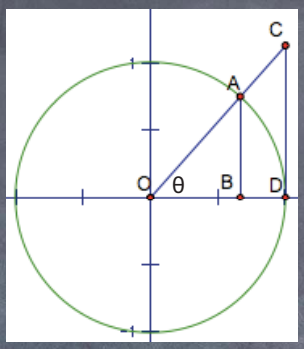
- We know cosθ = adj/hyp and sinθ = opp/hyp
- hyp = 1 because the hypotenuse is the radius of a unit circle
- opp = side b
- adj = side a
- therefore cosθ=adj/hyp = b/1 = b and sinθ = opp/hyp = a/1 = a



What does this tell us?

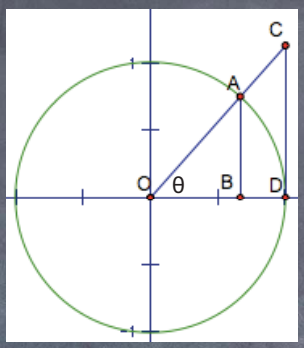
- We know in a unit circle cosθ = a and sinθ = b
- we also know a is the x-coordinate of point A and b is the y-coordinate of point A
- therefore in a unit circle the coordinates of point A can be represented as (cosθ, sinθ)





Which side represents sinθ?

Which side represents cosθ?



Which side represents sinθ?

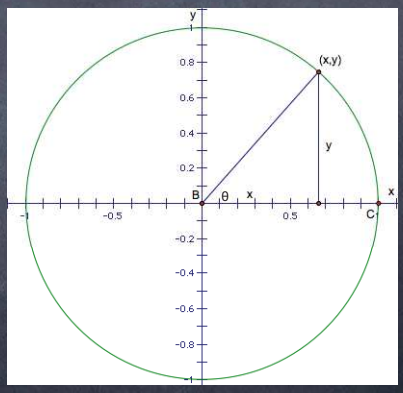
BA

Which side represents cosθ?

OB

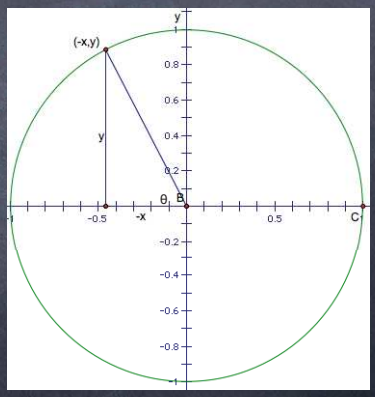
Quadrant I

- cosθ = adj/hyp = x/1 = x
- sinθ = opp/hyp = y/1 = y
- tanθ = opp/adj = y/x
- cosθ is positive
- sinθ is positive
- tanθ is positive



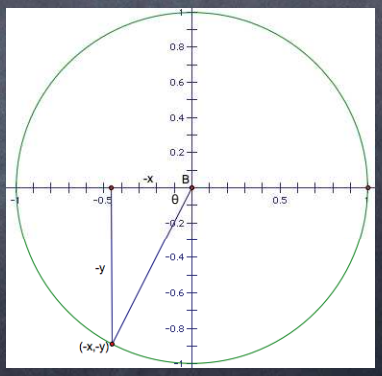
Quadrant II

- cosθ = adj/hyp = -x/1 = -x
- sinθ = opp/hyp = y/1 = y
- tanθ = opp/adj = y/-x = -y/x
- cosθ is negative
- sinθ is positive
- tanθ is negative



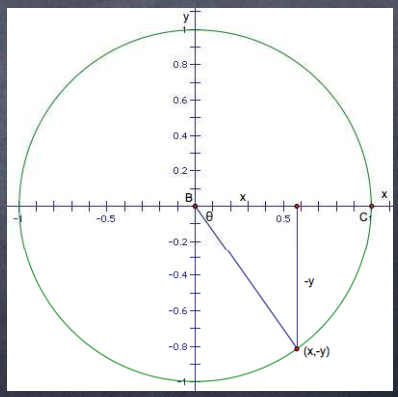
Quadrant III

- cosθ = adj/hyp = -x/1 = -x
- sinθ = opp/hyp = -y/1 = -y
- tanθ = opp/adj = -y/-x = y/x
- cosθ is negative
- sinθ is negative
- tanθ is positive



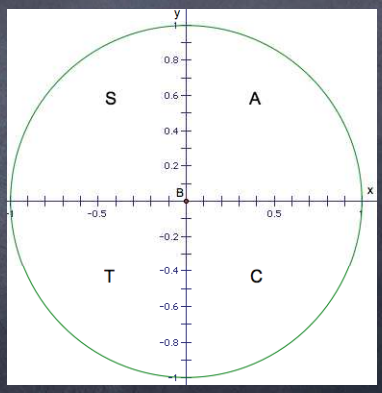
Quadrant IV

- cosθ = adj/hyp = x/1 = x
- sinθ = opp/hyp = -y/1 = -y
- tanθ = opp/adj = -y/x
- cosθ is positive
- sinθ is negative
- tanθ is negative



All Students Take Calculus

- This diagram can be used to remember which functions are positive in each quadrant
- Starting in the 1st quadrant the letters ASTC are placed in a quadrant
- A - all (all)
- S - sine (students)
- T - tangent (take)
- C - cosine (calculus)



Find the quadrant each angle lies in.

1)$\sin\theta > 0$ and $\cos\theta < 0$

3)$\sin\theta > 0$ and $\tan\theta > 0$

2)$\tan\theta > 0$ and $\sin\theta < 0$

4)$\sin\theta < 0$ and $\cos\theta > 0$

Find the quadrant each angle lies in.

1) $\sin\theta > 0$ and $\cos\theta < 0$

II

2) $\tan\theta > 0$ and $\sin\theta < 0$

III

3) $\sin\theta > 0$ and $\tan\theta > 0$

I

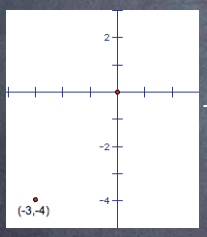
4) $\sin\theta < 0$ and $\cos\theta > 0$

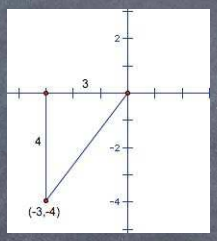
IV

Finding a trig value given a point on the terminal side of an angle

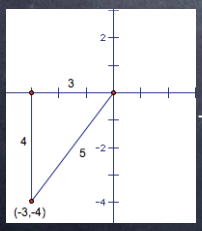
1. Plot the point
2. Create a right triangle to the x-axis
3. Use the pythagorean theorem to find any missing sides
4. Use the proper trig ratio to find the value for the desired trig function
5. Determine if the trig function is positive or negative depending on the quadrant

If the terminal side of an angle, θ, in standard position passes through the point (3,4), find cosθ.

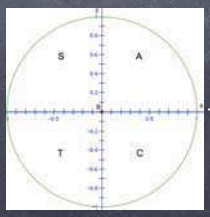




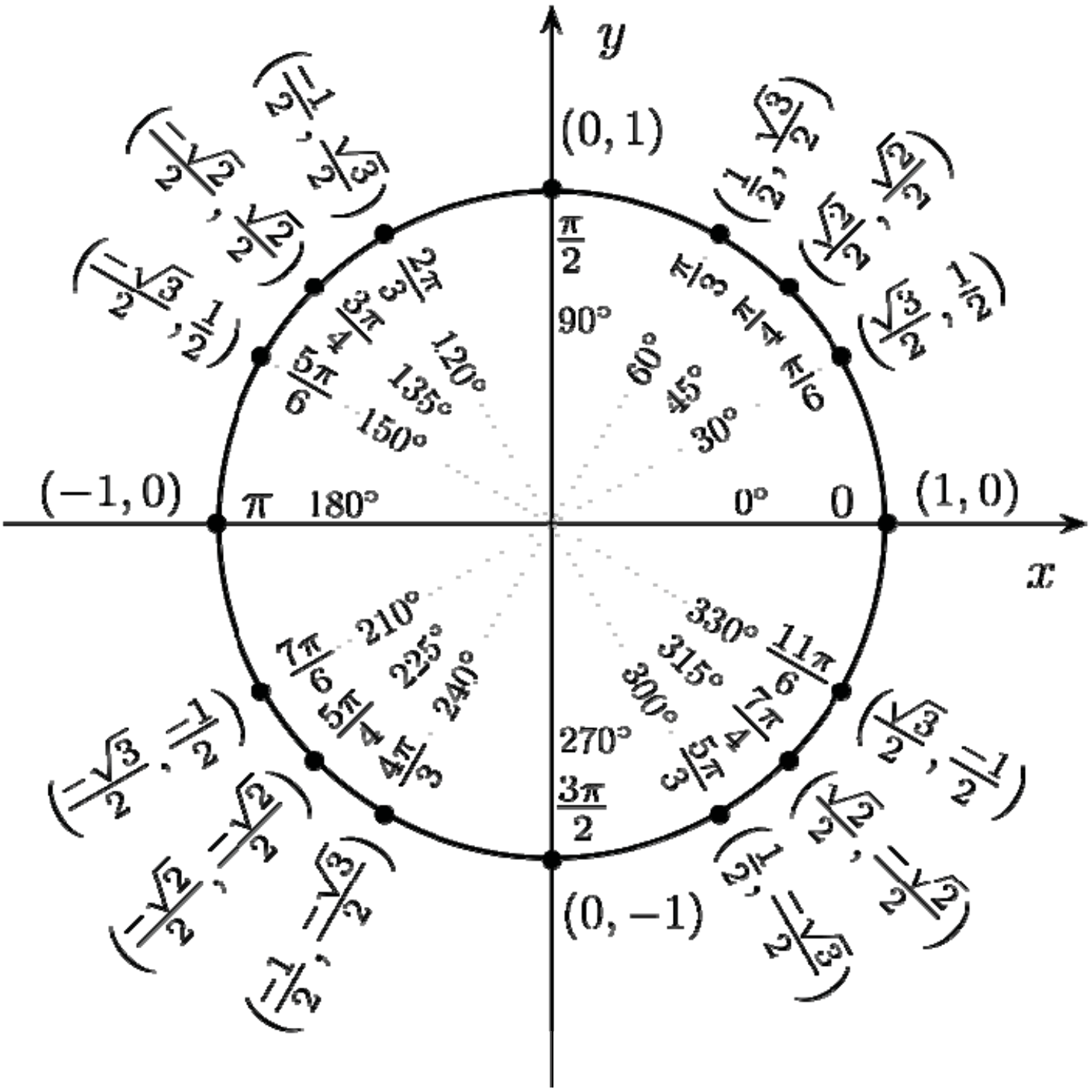
leg^2+leg^2 = hyp^2
3^2+4^2 = hyp^2
9+16 = hyp^2
25 = hyp^2
5 = hyp



cosθ = adj/hyp
cosθ = 3/5



cosθ = -3/5


y
x
(0, 1)
(1, 0)
(−1, 0)
(0, −1)
π/2
90°
π/3
60°
π/4
45°
π/6
30°
0°
0
π
180°
2π/3
120°
3π/4
135°
5π/6
150°
7π/6
210°
5π/4
225°
4π/3
240°
3π/2
270°
5π/3
300°
7π/4
315°
11π/6
330°
(1/2, √3/2)
(√2/2, √2/2)
(√3/2, 1/2)
(−1/2, √3/2)
(−√2/2, √2/2)
(−√3/2, 1/2)
(−√3/2, −1/2)
(−√2/2, −√2/2)
(−1/2, −√3/2)
(1/2, −√3/2)
(√2/2, −√2/2)
(√3/2, −1/2)

Name_____
Block_____
Unit Circle Pairs Check

Directions: Each pair of questions has the same answer. You will work on this activity with a partner. You should work on one set of problems with your partner until you both get the same answer before moving on to the next set. You are responsible for answering all problems. What you don't finish in class you will finish for homework.

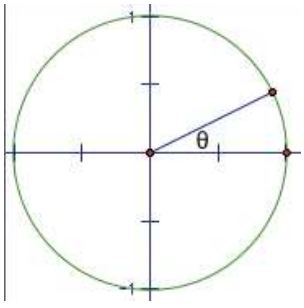
1a. What quadrant is θ in if $\sin\theta > 0$ and $\cos\theta < 0$?	1b. What quadrant is θ in if $\tan\theta < 0$ and $\cos\theta < 0$
2a. What is the reference angle for 200°?	2b. What is the reference angle for 340°?

3a. If the terminal side of an angle, θ, in standard position passes through the point (6,-8), find cosθ.	3b. If the terminal side of an angle, θ, in standard position passes through the point (-4,3) find sinθ.

4a. If cosθ = □ in quadrant IV, then find sin θ.	4b. If tanθ = ¾ in quadrant III then find cosθ.
5a. Let the point (-√2/2,√2/2) be on the terminal side of an angle θ in standard position. If the distance from the point to the origin is one unit find sinθ.	5b. Let tanθ = -1 in quadrant IV. Find cosθ

6a. If $\sin\theta = -3/5$ and $\cos\theta < 0$ find $\tan\theta$	6b. If $\sin\theta = \sqrt{7}/4$ and $\tan > 0$ find $\cos\theta$

Extension: A tangent line is one that intersects a circle at only one point. The tangent function can be thought of this way in a geometric sense. Given the following angle drawn in standard position draw the tangent line for this angle. Hint: you need to extend one of the lines.



Radians with Sketchpad

Name:	Student Teacher	Section Time:	Wednesday 4:30	#:11	

Mathematical Task: To define radian measure and develop a way to convert between degrees and radians.

Grade Level, approximate number of students, types of students: Grade 11, 23 students, Regents level.

Materials and Sources: Computer lab, a projector, handouts for activity and homework

NYS Mathematics Performance Indicator(s):

State the Content Strand:

- A2.M.1 - Define radian measure
- A2.M.2 - Convert between radian and degree measures

Process Indicators:

- A2.R.1 - Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts
- A2.RP.2 - Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion
- A2.CM.2 - Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs and diagrams

Performance Objectives:
The students will demonstrate the ability to define radian measure and convert between radians and degrees by completing the sketchpad activity and the homework.

Safety Precautions: Students will treat all equipment in the computer lab properly.

Part of Lesson	**Teacher and Student Actions**	**Mathematical Questions.** These can repeat for different parts of the lesson if appropriate. Include both lower order and higher order questions for each lesson.
Launch/Anticipatory Set/Before State prior knowledge needed, then explain how you will access prior knowledge and draw the students into this particular lesson. Also explain how you will launch the "problem" the students will solve during the explore. Some problems need more extensive launches than others.	**Teacher Actions:** Class will start with the DIN. The DIN will have some questions about the relationship between the central angle of a circle and its intercepted arc because the students will need to know this information for the activity. After the students have time to work on the DIN the teacher will go over it on the ELMO. The teacher will then go over any questions the students have on the homework from the night before. When the class is finished going over the homework the teacher will hand out a short quiz. After the quiz the teacher will lead the students to the computer lab for a sketchpad activity. **Student Actions:** The students will enter the room and begin work on the DIN. After the teacher goes over the DIN the students will ask questions about the homework. When the class is finished going over the homework the students take a quiz. After the quiz the students will quietly walk together to the computer lab.	What is the relationship between the central angle of a circle and its intercepted arc? How many degrees are in a circle? What is the definition of radius?

Explore/During Describe what the teacher will do and how the students will be engaged in the lesson/activity.	**Teacher Actions:** Once the class is at the computer lab the teacher will briefly explain the activity and pass out the directions. The teacher will go thought the first page on the projector while the students follow along. The teacher will stop periodically and walk around to make sure the students are keeping up. When the class is finished going through the first page the teacher will tell the students to begin work on the second page by themselves. The teacher will walk around while the students are working on the second page to help students who are struggling and to ask questions to make sure the students are making the right connections and conjectures. **Student Actions:** The students will follow along on their own computer as the teacher does the first page. The students will work on the second page by themselves. They are welcome to ask a classmate for help. If students are having trouble making the constructions the teacher might ask a student who has done it to go to the computer with the projector and do that construction for the class. If a student finishes early they can help another student who hasn't finished yet.	What is the formula for finding the radian measure of a central angle of a circle? Can you explain to the class how to make the last construction? Why is it necessary to divide the arc length in step 7 by the radius?
Summary/Closure/After Describe how you will summarize the lesson. This is a most important part of a problem solving lesson where the students share their strategies. The teacher should facilitate the discussion and the students should be actively engaged.	**Teacher Actions:** When the students have finished the activity the teacher will ask the students to get into small groups and discuss the answers they got and what their final conjecture was. After a couple of minutes the class someone from each group will tell the class what their rule was for converting between radians and degrees. The teacher will then write and explain on the board that to convert from degrees to radians you multiply the degree measure by $\pi/180$, and to convert from radians to degrees you multiply the radian measure by $180/\pi$. Also the teacher will go over the formula $\theta = s/r$ and explain where each of the variables can be found on a circle. When class is about to end the teacher will pass out the homework. **Student Actions:** The students will share, in a small group the answers they got from the activity and come up with a rule to share with the class. When the class finishes sharing their rules the students will write down the rules the teacher goes over on the board to make sure they came up with the right conjectures.	What is the rule for converting between radians and degrees? Did anyone have a different rule that their group members? Why can we multiple by $\pi/180$ without changing the value?

Assessment Clearly describe how the teacher will know if the students met the objective. Assessment should be embedded within the activities. The assessment may include a separate independent component.	Throughout the activity the teacher will be walking around and engaging in discussion with the students to see if they are making the right connections. Also the teacher will assess what the students got from the activity when they share the rules they came up with at the end of the class. The homework will be using the conjectures they made during class and will be another form of assessment.
Modification for Advanced Learners	Advanced learners will be asked to find another way to determine how many times the radius will fit around the edge of the circle using the formula for circumference of a circle.
Modifications for two types of disabilities (state the name of the disability here). You may include a modification for ELL.	The teacher will chose the seat for the student with autism carefully to reduce distractions. This student will also need frequent encouragement and on task reminders while working on the computer because this activity provides many opportunities to get distracted. The learning disabled student will be given extra time to complete the assignment and will be given a sheet with the important formulas and conversions.

Name_____
Block_____
Sketchpad Activity

1) Construct a circle using the create circle button. Label the center of the circle as O

2) Construct two points on the circle label them A and B. Connect each of these points with the center of the circle making two radii.

3) Measure the radius of the circle by selecting segment OA and clicking on length under the measure menu. Record this value below

Radius = _____

4) Measure the arc length of arc AB by selecting points A and B and the edge of the circle. Then click on arc length under the measure menu.

Arc AB = _____

5) A radian is a unit of angle measurement defined for a central angle of a circle by comparing the the length of the intercepted arc of a circle with the radius. This relationship can be represented by this formula **θ = s/r** where θ is a central angle, s is the arc length and r is the radius

Using the calculate tool in the measure menu, calculate the arc length divided by the radius. This will be the radian measure of angle AOB record this value below.

Angle AOB = _____radians

6) Now measure this same angle in degrees by selecting the points AOB in that order then selecting angle from the measure menu. Record the value below.

Angle AOB =_____˚

7) Using the formula θ = s/r, what is the angle measure in radians for an angle whose arc length is equal to its radius?

_____ radians

8) Change angle AOB until the radius is equal to the arc length. This should give you the degree measure for one radian. Record this below.

1 radian = _____degrees

This means that if the central angle of a circle is equal to 57.3˚ then the intercepted arc of this circle is equal to the radius. Using this fact you will, on your own, measure how many times the radius can be wrapped around the circle.

1) Start by constructing a circle with center O and radius OA

2) To create an angle of 1 radian or 57.3˚ we will rotate point A around the center of the circle by 57.3˚. To do this select points O and A and then select rotate under the transform menu. Label this new point B

3) Measure the length of arc AB. This should be equal to the radius because of the relationship **θ = s/r** mentioned earlier.

4) Rotate this new point around the center of the circle by 57.3˚. Repeat this all the way around the circle. The length between each point is one length of the radius. Knowing this, how many full lengths fit around the circle.

Record this value_____

5) The radius doesn't fit around the edge of the circle evenly so we will have to find how much of the last radius it takes to reach the original point. This value will be the length of the arc between the last rotated point and the original point divided by the length of the radius. Measure the length of the arc between the last rotated point and the original point.

Record this value _____

6)Now use the calculate tool to divide the arc length you just measured by the length of the radius. This will tell you how many times the radius fits between these two points. It should be less than one time.

Record this value_____

7)This value plus how many full times the radius fits around the edge of the circle will give you exactly how many times the radius fits around the circle.

Record this value_____

8)What is this number in terms of π?

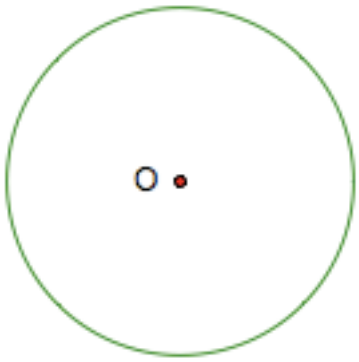
Circumference = _____π radians

9) Knowing that there are 360˚ in a circle and also knowing how many π radians are in a circle, how many degrees are in π radians?

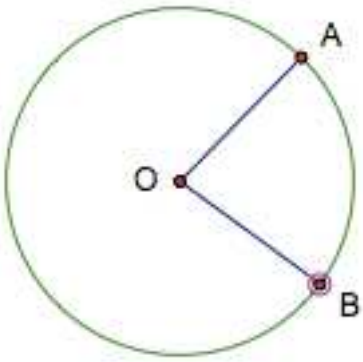
Record this value_____

10) Use this relationship to develop a rule for converting between radians and degrees.

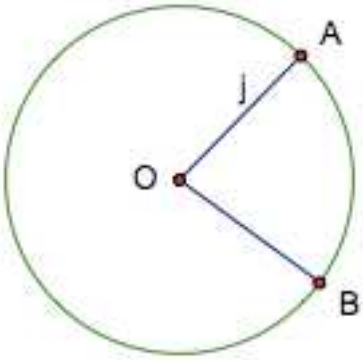
1)



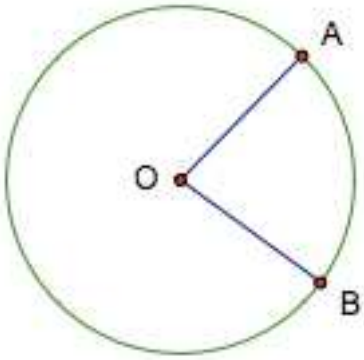
2)



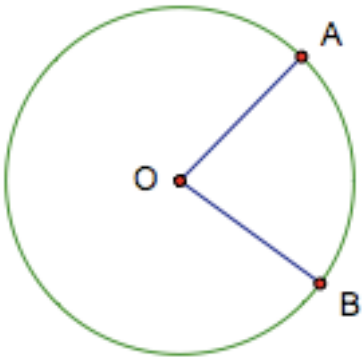
3) radius = 2.29 cm



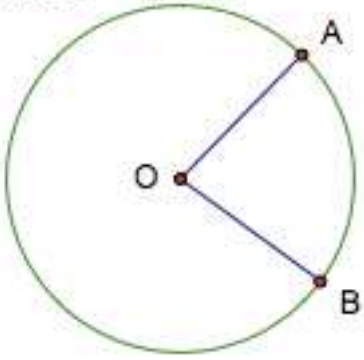
4) Arc AB = 3.28 cm
radius = 2.29 cm



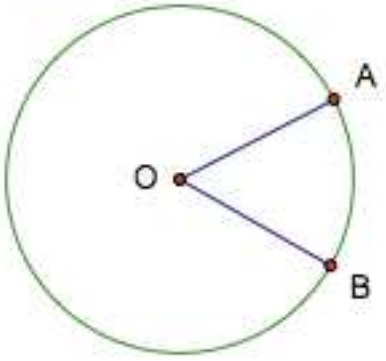
5) Angle AOB radians = 1.43
Arc AB = 3.28 cm
radius = 2.29 cm



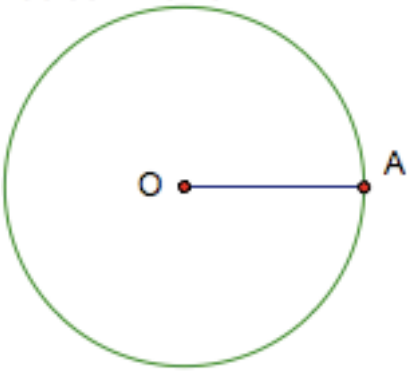
6) Angle AOB degrees = 81.98°
Angle AOB radians = 1.43
Arc AB = 3.28 cm
radius = 2.29 cm



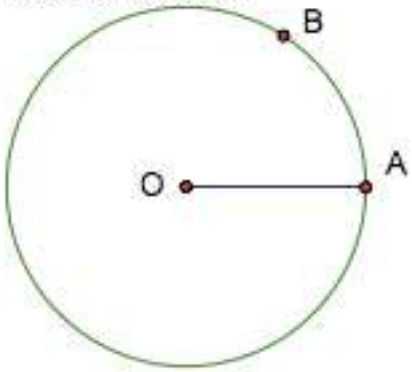
8) Angle AOB degrees = 57.30°
Angle AOB radians = 1.00
Arc AB = 2.29 cm
radius = 2.29 cm



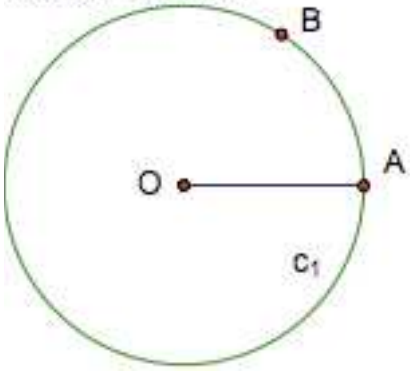
1) radius = 2.37 cm



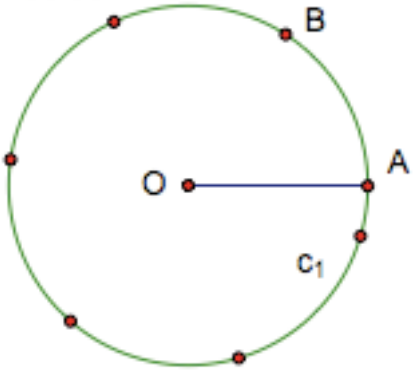
2) radius = 2.37 cm



3) Arc AB = 2.37 cm
radius = 2.37 cm

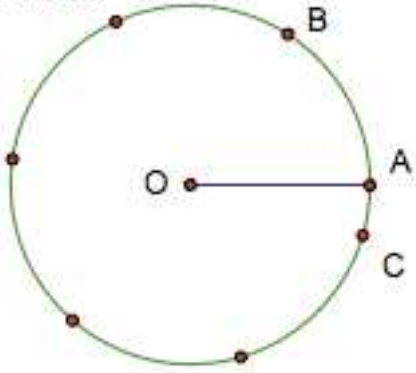


4) Arc AB = 2.37 cm
radius = 2.37 cm



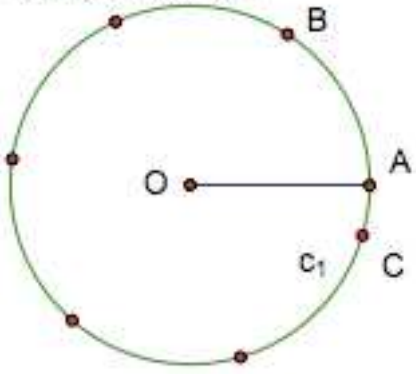
5)

Arc CA = 0.67 cm
Arc AB = 2.37 cm
radius = 2.37 cm



6) $\frac{\text{(Arc CA)}}{\text{radius}} = 0.28$

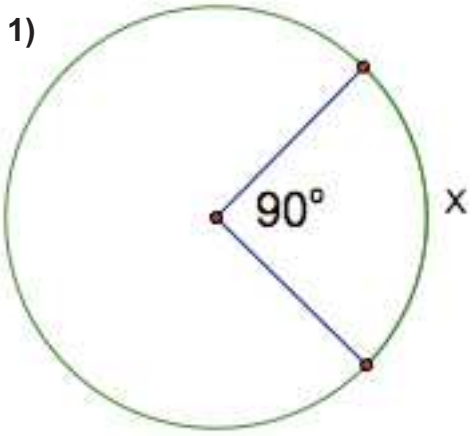
Arc CA = 0.67 cm
Arc AB = 2.37 cm
radius = 2.37 cm



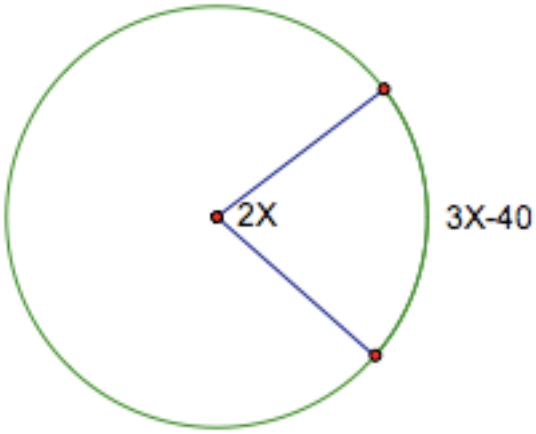
DIN

Solve for x

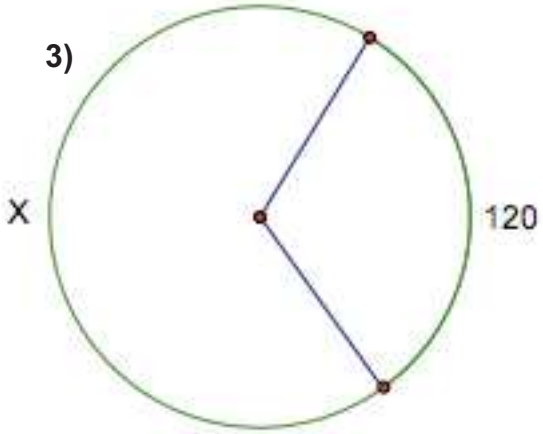
1)



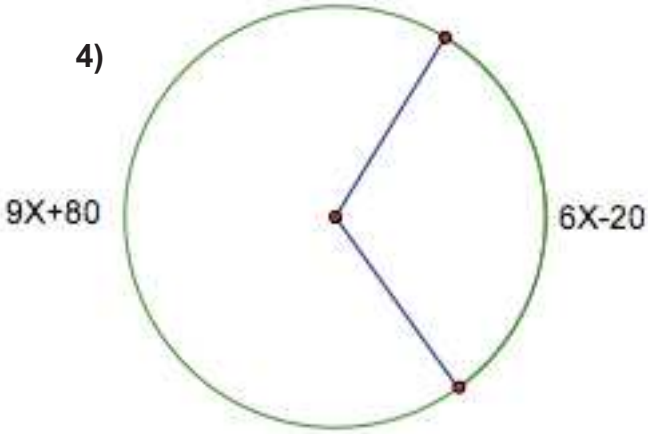
2)



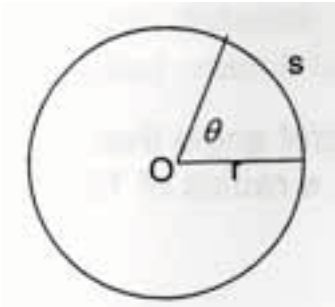
3)



4)



Name_____
Block_____
Radian Homework



degrees --> radians π/180
radians --> degrees 180/π

$$\theta = \frac{s}{r}$$

1) Find S if r = 5 and θ = 10 radians

2) Find θ if s = 18 and r = 3

3) Find r if θ = 3 radians and s = 15

4) How many degrees are in 3π/4 radians?

5) How many radians are in 120 degrees in terms of π?

6) In a circle, a central angle of 2 radians intercepts an arc of 6 inches. Find the length of the radius of the circle.

7)Find the positive radian measure of a central angle that intercepts an arc of 10 cm on a circle with a radius of 15 cm.

8)The bottom of a pendulum traces an arc of 3 feet in length when the pendulum swings though an angle of ½ a radian. Find the number of feet in the length of the pendulum.

9)The pendulum of a clock swings though an arc of 50 cm. Find the length of the pendulum.

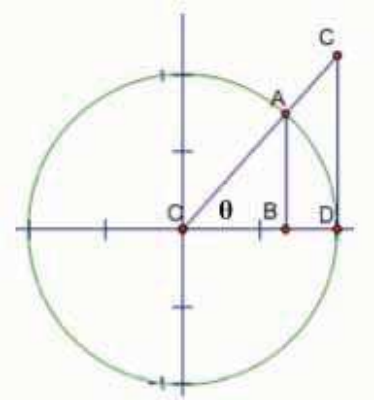
10) Circle O has a radius of 5 inches. The central angle of circle O intercepts an arc of length 10 inches. What is the measure of the central angle **to the nearest degree.**

11) Circle O has a radius of 10 inches. What is the length, in inches of the arc subtended by a central angle measuring 60 degrees

Name_____
Block_____
Trigonometry Quiz

1) Which segment represents sinθ?

a) OA
b) OB
c) BA
d) CD



2) What is the reference angle for 210°?

a) 30°
b) 50°
c) 60°
d) 150°

3) If the terminal side of an angle sketched in standard position passes through the point (-5,12) find the sine of that angle.

a) 12
b) -5/12
c) 12/13
d) 5/13

4) Let θ be an angle sketched in standard position. If $\sin\theta > 0$ and $\cos\theta < 0$ in what quadrant does the terminal side of θ lie?

a) I
b) II
c) III
d) IV

5) If $\tan\theta = -6/8$ and $\sin\theta > 0$ find $\cos\theta$

a) 3/5
b) 4/5
c) 3/4
d) -4/5

Spaghetti Lab

Name:	Student Teacher	Section Time:	Wednesday 4:30	#:11	

Mathematical Task:
Make conjectures relating graphs of trigonometric equations to the unit circle.

Grade Level, approximate number of students, types of students: Grade 11, 23 students, regents level

Materials and Sources: poster paper, unit circle, spaghetti, straight edge, markers, string, tape

NYS Mathematics Performance Indicator(s):

State the Content Strand:

- A2.A.56 – Know the exact and approximate values of the sine, cosine and tangent of 0º, 30º, 45º, 60º, 90º, 180º, and 270º angles
- A2.A.60 – Sketch the unit circle and represent angles in standard position
- A2.A.70 - Sketch and recognize one cycle of a function of the form $y = A \sin Bx$ or $y = A \cos Bx$
- A2.A.55 - Express and apply the six trigonometric functions as ratios of the sides of a right triangle
- A2.A.69 - Determine amplitude, period, frequency, and phase shift, given the graph or equation of a periodic function

Process Indicators:

- A2.PS.3 – Observe and explain patterns to formulate generalizations and conjectures
- A2.R.1 – Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts
- A2.CM.2 – Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs and diagrams
- A2.RP.2 – Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion

Performance Objectives:
The students will demonstrate the ability to represent trigonometric functions graphically and relate the result to the unit circle by successfully completing the steps of the lab and creating an accurate poster.

Safety Precautions:
Students will remain in their designated area during the activity. They will also follow all directions on appropriate use of supplies.

Part of Lesson	Teacher and Student Actions	**Mathematical Questions.** These can repeat for different parts of the lesson if appropriate. Include both lower order and higher order questions for each lesson.
Launch/Anticipatory Set/Before State prior knowledge needed, then explain how you will access prior knowledge and draw the students into this particular lesson. Also explain how you will launch the "problem" the students will solve during the explore. Some problems need more extensive launches than others.	**Teacher Actions:** The teacher will begin class by walking around and helping the students while they are working on the DIN. The teacher will then go over the DIN on the ELMO. The DIN will be about graphing angles in standard position because this will be important for the lesson. The teacher will then begin the spaghetti lab activity by telling the students that the sine and cosine functions can be graphed on an x/y plane and that they will discover this relationship in a lab. **Student Actions:** Students will begin work on the DIN as soon as they come in the room. They will ask for help if they are having trouble with the DIN.	Where are the terminal and initial sides of an angle when sketched in standard position? What axis do you draw the reference triangle to? Which side of the reference triangle represents sine? Cosine?

Explore/During Describe what the teacher will do and how the students will be engaged in the lesson/activity.	**Teacher Actions:** The teacher will describe the spaghetti lab directions in full before distributing materials. All questions will be clarified before students begin. Students will then be assigned to groups and asked to rearrange themselves in the room. Materials will then be distributed and students will be instructed to begin. As students work on the activity the teacher will interact with each group. Any questions will be addressed and questions will be fielded to ensure students understand the lesson. The teacher will also inform students of how much time remains in the block so they can budget their time accordingly. With about 10 minutes left, the teacher will instruct students to clean up their space and return all materials. **Student Actions:** Students will be attentive as directions are given. They will be asked to repeat the direction to ensure clarification. Students will then separate into assigned groups and begin work on the activity. They will follow all directions pertaining to the spaghetti lab and manage their time accordingly. Students will answer questions pertaining to the lab and demonstrate their understanding of the material. Students will raise their hand should any questions arise. When the students have finished their graphs they will clean up their workspace and return any materials.	Did you expect sinθ to be negative between π and 3π/4? Why? Can you draw a reference triangle for 90°? How can you use the unit circle to find the sine and cosine of 90° without drawing a reference triangle?
Summary/Closure/After Describe how you will summarize the lesson. This is a most important part of a problem solving lesson where the students share their strategies. The teacher should facilitate the discussion and the students should be actively engaged.	**Teacher Actions:** After the students have finished with their graphs and they have had an opportunity to work on the questions, the teacher will bring the class back together to discuss the questions on the back of the procedure sheet. The teacher will ask the students for the answers they got and will clarify any confusion to make sure the students have the right information. If there is any time left over the teacher will go over the directions for the homework. **Student Actions:** The students will share their answers they got to the questions on the back with the class and will write down any answers they didn't get on their own. The students will ask questions if they don't understand how to get an answer.	How is the radius of the circle related to the maximum value of the sine and cosine curve? Where does the sine graph equal the cosine graph? What type of transformation would make the cosine graph look like the sine graph?

Assessment Clearly describe how the teacher will know if the students met the objective. Assessment should be embedded within the activities. The assessment may include a separate independent component.	Students will be assessed while they are working on the activity, as well as by their finished product. Answers to questions fielded by the teacher throughout the activity will serve as formative assessment. The students' posters and written answers will serve as summative assessment. One last form of assessment will come from the homework handed out at the end of class.
Modification for Advanced Learners	Advanced learners will be prompted to repeat the activity for the sine curve if they finish the activity on the cosine curve with time remaining.
Modifications for two types of disabilities (state the name of the disability here). You may include a modification for ELL.	The learning disabled student will be given an answer key for the questions so he doesn't have to struggle to keep up with writing the answers while the class goes over them. The autistic student will be able to choose a specific job so he can focus on one task and not get distracted.

Spaghetti Lab

Materials: 2 pieces of poster paper, unit circle, spaghetti, straight edge, marker, string, tape

Procedure:

1. Stick both pieces of poster paper on the wall so that the width is longer than the height.
2. On each poster paper draw a set of coordinate axes with the y-axis near the left side of the paper and the x-axis through the middle of the paper.
3. Wrap the string around the unit circle with one end at 0°. Mark every point on the string that is given on the unit circle.
4. Place one end of the string at the origin of each set of axes and unwrap the string along the x-axis. Transfer the marks on the string to the x-axis including 0° and 360° and label the points accordingly.
5. Create the reference triangle to the first point, 30°, on the unit circle by breaking pieces of spaghetti. The horizontal side will be the cosine of the angle and the vertical side will be the sine of the angle.
6. Move the vertical piece to the sine graph and place it perpendicular to the x axis starting at the point labeled 30°. Tape the piece in place. Do the same for the horizontal side moving it to the cosine graph
7. Repeat steps 5 and 6 for every point on the unit circle. Be careful to place the spaghetti pieces below the x-axis when they represent a negative value.
8. After all the triangles have been constructed, the spaghetti has been transfered to the graphs, and the dots made, draw a smooth curve though all the dots.

Questions:

For y = sin x

- What are the coordinates of the ordered pairs for the x-intercepts, minimum and maximum points'?

- Compared to the radius, what is the height of the triangle at 30°? What is the value of sin 30°?

- Write a short paragraph explanation to tell a classmate about why sin 30˚ = sin 150˚.

For y = cos x

- What are the coordinates of the ordered pairs for the x-intercepts, minimum and maximum points'?

- Compared to the radius, what is the height of the triangle at 60°? What is the value of cos 60°?

- Write a short paragraph explanation to tell a classmate about why cos 60˚ = -cos 120˚.

Comparing sine and cosine graphs

- Approximately where would the sin x = cos x?

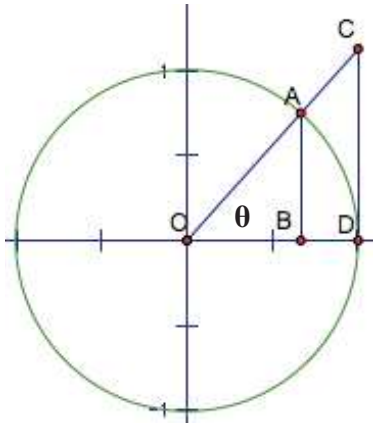
- Where is the spaghetti piece for 120˚ on the sine graph the same length as the piece of spaghetti on the cosine curve?

- What type of transformation would you do to make the cosine graph look like the sine graph?

DIN

1) Sketch 300° in standard position and find the reference angle.

2) Which side represents sinθ? Which side represents cosθ?



3) When θ is sketched in standard position what quadrant is the terminal side of θ in if $\sin\theta > 0$ and $\cos\theta < 0$?

4) When θ is sketched in standard position what quadrant is the terminal side of θ in if $\cos\theta > 0$ and $\tan\theta > 0$?

Name_____
Block_____
Spaghetti Lab Homework

Full Cycle - one complete pattern of the trig function without repeating
Period - the length of the interval over which the function completes one cycle
Amplitude - the distance from the middle of the trig function to the highest point
Frequency - the number of cycles completed between 0 and 2π

Directions - Graph each equation in your graphing calculator and find the period amplitude and frequency. Make sure you are in radian mode and set your window to look like this.
xmin = 0
xmax = 4π
xscl = π/2
ymin = -5
ymax = 5
yscl = 1

1) $y = 3 \cos \frac{1}{2} x$

2) $y = \frac{1}{2} \cos 2x$

3) $y = 2 \sin 4x$

4) $y = \sin \frac{1}{2} x$

5) $y = 3 \sin x$

6) Using the equations $y = a \sin bx$ and $y = a \cos bx$ and what you saw in the previous examples try to generalize what the constants a and b represent. Think amplitude period and frequency.

Unit Test Sound Wave Activity

Name:	Student Teacher	Section Time:	Wednesday 4:30	#:11	

Mathematical Task: To review all content needed for the exam and to extend these concepts to other content areas.

Grade Level, approximate number of students, types of students: Grade 11, 24 students, regents level students

Materials and Sources: Web applet, sound wave activity, DIN, unit test,

NYS Mathematics Performance Indicator(s):

State the Content Strand:

State 1-3 relevant Content Performance Indicators using bullets.

- Determine amplitude, period, frequency, and phase shift, given the graph or equation of a periodic function (Standard A2.A.69)

Process Indicators:

State 4-6 relevant Process Indicators illustrating a variety of processes using bullets.

- Understand and make connections among multiple representations of the same mathematical idea (Standard A2.CN.1)
- Observe and explain patterns to formulate generalizations and conjectures (Standard A2.PS.3)
- Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts (Standard A2.R.1)
- Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion (Standard A2.RP.2)

Performance Objectives:

Students will demonstrate the ability to apply their knowledge of the sine wave to make conjectures regarding the effect amplitude and frequency have on the sound a sound wave makes.

Safety Precautions: If the class goes to the computer lab the students will walk together as a class.

Part of Lesson	**Teacher and Student Actions**	**Mathematical Questions.** These can repeat for different parts of the lesson if appropriate. Include both lower order and higher order questions for each lesson.
Launch/Anticipatory Set/Before State prior knowledge needed, then explain how you will access prior knowledge and draw the students into this particular lesson. Also explain how you will launch the "problem" the students will solve during the explore. Some problems need more extensive launches than others.	**Teacher Actions:** When class starts the teacher will instruct the students to start the DIN. The DIN will serve as review and will ask the students to list everything they will need to know for the test. While the students are working the teacher will take attendance and check homework. After 5-10 minutes the teacher will make a list of things they need to know with the help of the students. The teacher will make sure the list is complete and will answer any questions the students have about these topics. The teacher will then go over any questions the students have on the homework making sure the students know how to graph trig functions on the calculator and find period frequency and amplitude using the calculator. **Student Actions:** Students will enter the room and begin work on the DIN. When instructed to by the teacher the students will tell what they got on their list to compile a class list. The students will then ask questions on any of the topics they need clarification on. Students will then ask questions they have on the homework.	What will you need to know for the exam? Are there any questions on specific topics for the exam? Does anyone have questions about graphing sine and cosine functions on the graphing calculator? What is period frequency and amplitude? What are a and b in y = a sin bx?

Explore/During Describe what the teacher will do and how the students will be engaged in the lesson/activity.	**Teacher Actions:** Depending on how much time is left the teacher will either take the class to the computer lab to do the sound wave activity or the class will do it together in the classroom. If it is done in the computer lab the students will follow the directions on the activity and do the questions on their own. Most likely they will be doing it in the classroom in which case the teacher will bring the website up on the projector. The teacher will go through the directions and discuss the questions with the class. The teacher will watch the clock to ensure that the students have at least 40 minutes to complete the exam. **Student Actions:** If the students go to the computer lab they will walk quietly as a class to the lab. They will then follow the directions and answer the questions. If the activity is done in the class the students will participate in discussion as the class goes through the activity together.	How does the amplitude affect the sound a sound wave makes? How does frequency affect the sound a sound wave makes? When you turn up the volume on your radio what are you doing to the sound waves that come out of that radio?
Summary/Closure/After Describe how you will summarize the lesson. This is a most important part of a problem solving lesson where the students share their strategies. The teacher should facilitate the discussion and the students should be actively engaged.	**Teacher Actions:** With at least 40 minutes remaining the teacher will pass out the exam. The teacher will tell the students that there is no talking during the exam and anyone suspected of cheating will receive a 0 on the exam. The teacher will sit behind the students during the exam so the students don't know if they are being watched or not. The teacher will give the students reminders of how much time they have left and will tell them if they finish early to check their answers. **Student Actions:** The students will take the test and budget their time as best they can. When the students are finished they will check their answers then sit quietly until the period ends.	
Assessment Clearly describe how the teacher will know if the students met the objective. Assessment should be embedded within the activities. The assessment may include a separate independent component.	Obviously the main assessment will be the unit test. This test will assess everything the students learned in this unit. The students will also be assessed by their work on the sound wave activity. The teacher will be asking the students questions while they are working on it as a form of assessment.	
Modification for Advanced Learners	Advanced learners will be asked how to change the equation of a sound wave instead of the graph to change to volume and pitch.	

Modifications for two types of disabilities (state the name of the disability here). You may include a modification for ELL.	The student with autism and the student with a learning disability will be sent out of the room during the exam for extended time. If the class goes to the computer lab the student with autism will be given a seat away from distractions and will be given on task reminders.

DIN

The DIN will be to list all of the things you will need to know for the exam. This is a list of all of the points I am looking for on the DIN.

1) SOHCAHTOA

2) Sketch the unit circle and represent angles in standard position - terminal side - initial side

3) point on unit circle = $(\cos\theta, \sin\theta)$

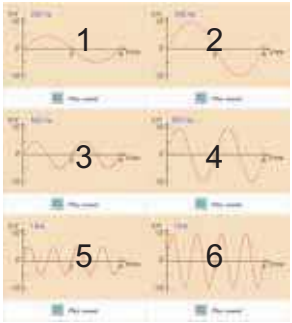
4) reference angle

5) converting between radians and degrees

6) amplitude frequency period

Name_____
Block_____
Sound Wave Activity

Directions: go to the website http://www.doctronics.co.uk/signals.htm#listening. At the top under navigation click on Listening to waves. Scroll down so you have 6 pictures of sine waves on the screen. You will be comparing the waves and the sounds these waves make. To hear the sound click on the  icon. For the purposes of this activity we will number the pictures as shown below.



1) What has changed graphically between graph 1 and graph 2? Think amplitude, period, and frequency. Tell what has changed and how it has changed.

2) Listen to the sound for graph 1 and graph 2. How is the sound different in 2 compared to 1?

3) What has changed graphically between graph 1 and graph 3? Think amplitude, period, and frequency. Tell what has changed and how it has changed.

4) Listen to the sound for graph 1 and graph 3. How is the sound different in 3 compared to 1?

5) How does the amplitude of a sound wave effect the sound a wave makes?

6) How does frequency of a sound wave effect the sound a wave makes?

Name_____
Block_____
Trigonometry Unit Test

Directions: The first 11 multiple choice questions are worth 2 points each and the last four short answer questions are worth 3 points each for a total of 34 points. There will be partial credit awarded on the short answer questions but not for the multiple choice.

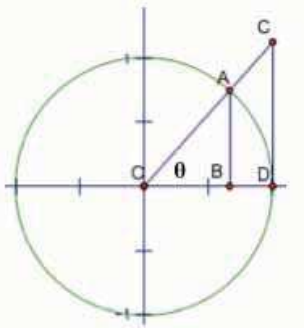
1) If θ is an angle in standard position with $\sin\theta > 0$ and $\tan\theta < 0$ in which quadrant is the terminal side of θ?
   a) I
   b) II
   c) III
   d) IV

2) What is the reference angle for 110˚?
   a) 20˚
   b) 70˚
   c) 30˚
   d) 60˚

3) If θ is an angle in standard position and its terminal side passes through point $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ on a unit circle, which of the following is equal to cosθ?
   a) $\frac{1}{2}$
   b) $\frac{\sqrt{3}}{2}$
   c) $-\frac{1}{2}$
   d) $\frac{\pi}{3}$

4) Which segment represents cosθ?
   a) OD
   b) OB
   c) BA
   d) CD

5) How many degrees are in $\frac{7\pi}{4}$ radians?
   a) 135
   b) 180
   c) 225
   d) 315



6) How many π radians are in 240 degrees?

a) $\frac{3\pi}{4}$
b) $\frac{4\pi}{3}$
c) $\frac{2\pi}{3}$
d) $\frac{\pi}{3}$

7) What is the positive radian measure of a central angle that intercepts an arc of 25 cm on a circle with a radius of 50 cm?
a) $\frac{1}{2}$ radians
b) 2 radians
c) $\frac{\pi}{2}$ radians
d) π radians

8) Which is the largest angle measure?
a) 105˚
b) $\frac{3\pi}{4}$ radians
c) 1.8 radians
d) $\frac{2\pi}{3}$ radians

9) What is the Amplitude of the graph of the equation y = -3 sin 2x?
a) 2
b) 3
c) -3
d) 2π

10) What is the period of the graph of the equation y = cos 2x?
a) 2
b) 1
c) 2π
d) π

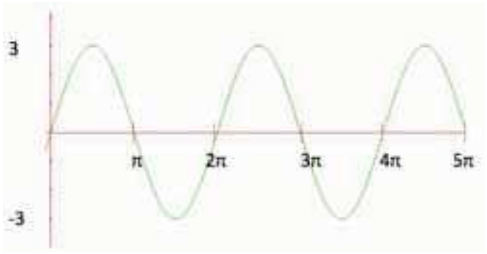
11) What is the frequency of the graph of the equation y = -2 sin 5x?
a) 2
b) 5
c) -2
d) π

12) If the terminal side of an angle θ in standard position passes through the point $(1, \sqrt{3})$, find sinθ.

13) If $\cos\theta = -\frac{5}{13}$ and $\tan\theta > 0$ find $\sin\theta$.

14) The length of a pendulum is 8 feet. When the pendulum swings through an angle of $\frac{3}{4}$ radians. Find the length of the arc, in feet, the bottom of a pendulum traces.

15) Find the amplitude period and frequency of the graph below.



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Section V: Analysis of Student Learning

The sample I collected was a quiz that followed the lesson I taught to one class. This lesson was not directly from my TWS but it contained similar material. The quiz had 5 questions and assessed the following two learning goals. A2.A.69 - Determine amplitude, period, frequency, and phase shift, given the graph or equation of a periodic function.

On the quiz, questions 1, 2, 3, 6, 7, 8, 9, and 10 all assess standard A2.A.69. 1, 2, 3, 8, 9, and 10 assess the students' ability to determine amplitude period and frequency given an equation. Questions 6 and 7 assess the student's ability to determine amplitude period and frequency given the graph. The students were able to find amplitude in both cases about the same with 21 and 22 answering correctly out of 23. But there was a big difference when they were asked to find period. Questions 3 and 9 asked the students to find period given an equation. 17 students answered question 3 correctly and 16 answered question 9 correctly. When given a graph in question 7 only 5 students were able to find the period. I think this is a result of the students not being able to recognize one cycle of the sine and cosine curves. As this test pertains to the process goals standard A2.CN.1 - Understand and make connections among multiple representations of the same mathematical idea was assessed in this quiz. Overall I don't think the students performed well on this objective because they were not able to reach the same conclusions given a graph of a sine and cosine curve that they were given an equation of a sine and cosine curve.

The diversity characteristic I chose to analyze was gender. It is important to understand the learning of the students in these different subgroups to ensure that I am practicing teaching strategies that are relevant to both groups. If one group scores higher than the other it might be the case that I am using examples and assignments that promote learning in only one of these groups. As the results show, this was not necessarily the case. By simply comparing mean scores

between each subgroup (male/female), I cannot make a statistical inference. However, the scores appear close in value, and I do not feel there is a great difference in achievement between males and females in this particular sample. Watching these mean scores over time, and with a larger sample size might allow me to make more conclusions. The average score for males was a 77 and the average score for females was a 72. This is evidence that both groups are achieving at a relatively average level. The one question that gave the students trouble was question 7. This, as mentioned before, asked students to find the period given the graph. On question 7, 4 out of five students who answered it correctly were male. Because this is such a small sample of only five students it's hard to say that this result is significant but it is good to note that maybe the females are having more trouble making the connections given a graph.

PRE/POST TEST SCORE ANALYSIS: Student Characteristics

DIRECTIONS: Enter the name of up to 30 students who took BOTH tests below. This information will appear on other worksheets automatically.
TIP -- Put the names in alphabetical order if desired BEFORE you enter grades on the next sheet.
OPTIONAL -- Describe what each code below means, then enter the appropriate "characteristic code" for each student on the list.
Characteristic codes must refer to ONE mutually exclusive characteristic (e.g., for SES, 1=Eligible for free lunch,
2=Eligible for reduced lunch, 3=Not eligible for free or reduced lunch). Leave unused codes blank.

1=	male	3=	
2=	female	4=	

#	Student Name (Last, First)	Char Code
1	KE	2
2	ES	2
3	GW	1
4	LP	2
5	AW	2
6	RL	1
7	ZF	1
8	JD	1
9	JH	2
10	MM	2
11	CB	2
12	JC	1
13	BL	2
14	JB	1
15	ML	1
16	HB	2
17	JI	1
18	SP	1
19	TB	2
20	KS	2
21	KH	1
22	CC	2
23	CD	2
24		
25		
26		
27		
28		
29		
30		

PRE/POST TEST SCORE ANALYSIS: Post-Test Data Entry

DIRECTIONS: For each student, enter a "1" for any question that was answered CORRECTLY on the post-test.

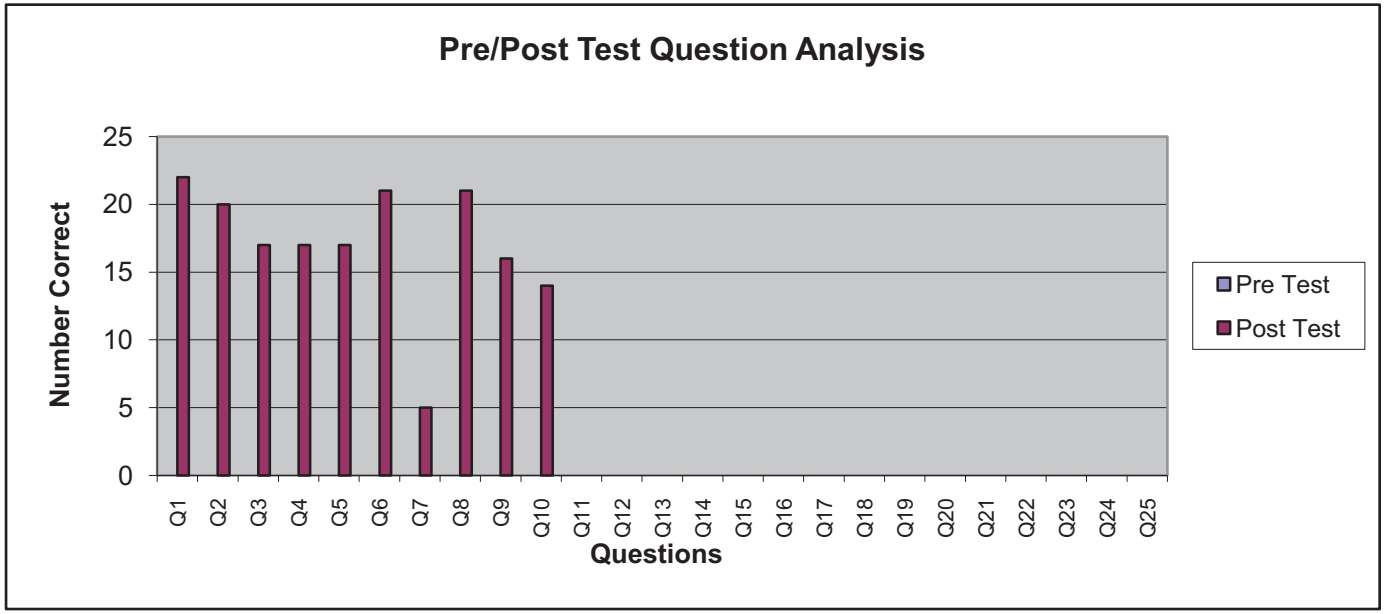
Total number of questions on the test: Q= 10

Total number of students who took the test: N= 0

		Question																									Post-Test	
#	Student Name	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Total	%
1	KE	1	1	1	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	70%
2	ES	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	80%
3	GW	1	1	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	60%
4	LP	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	60%
5	AW	1	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	40%
6	RL	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	90%
7	ZF	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	100%
8	JD	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	70%
9	JH	1	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	90%
10	MM	1	1	1	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	60%
11	CB	1	1	1	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	70%
12	JC	1	1	1	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	80%
13	BL	0	0	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	40%
14	JB	1	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	90%
15	ML	1	1	0	1	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	70%
16	HB	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	100%
17	JI	1	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	40%
18	SP	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	100%
19	TB	1	1	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	70%
20	KS	1	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	90%
21	KH	1	1	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	70%
22	CC	1	1	1	1	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	70%
23	CD	1	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	90%
24		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0%
25		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0%
26		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0%
27		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0%
28		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0%
29		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0%
30		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0%
	Sum Qs and Means	22	20	17	17	17	21	5	21	16	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0%

PRE/POST Question SUMMARY ANALYSIS

	Question																								
	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	Q22	Q23	Q24	Q25
Pre Test	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Post Test	22	20	17	17	17	21	5	21	16	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0



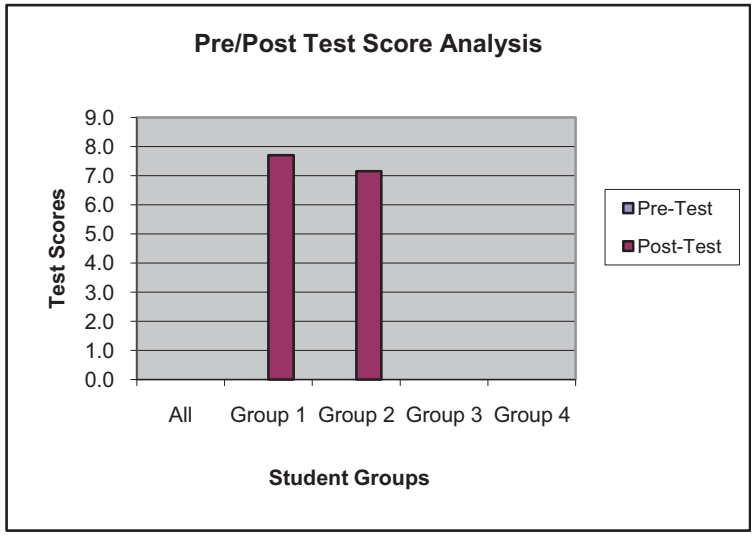
PRE/POST TEST SCORE SUMMARY ANALYSIS

Student Group Characteristics:
1= male
2= female
3=
4=

Total number of questions on the test: Q= 10
Total number of students who took the test: N= 0

#	DO NOT REPORT Student Names	Char Code	Pre-Test Total	Pre-Test %	Post-Test Total	Post-Test %	Pre vs Post Test Gain/Loss	Pre vs Post Test %
1		2	0	0%	7	70%	7	70%
2		2	0	0%	8	80%	8	80%
3		1	0	0%	6	60%	6	60%
4		2	0	0%	6	60%	6	60%
5		2	0	0%	4	40%	4	40%
6		1	0	0%	9	90%	9	90%
7		1	0	0%	10	100%	10	100%
8		1	0	0%	7	70%	7	70%
9		2	0	0%	9	90%	9	90%
10		2	0	0%	6	60%	6	60%
11		2	0	0%	7	70%	7	70%
12		1	0	0%	8	80%	8	80%
13		2	0	0%	4	40%	4	40%
14		1	0	0%	9	90%	9	90%
15		1	0	0%	7	70%	7	70%
16		2	0	0%	10	100%	10	100%
17		1	0	0%	4	40%	4	40%
18		1	0	0%	10	100%	10	100%
19		2	0	0%	7	70%	7	70%
20		2	0	0%	9	90%	9	90%
21		1	0	0%	7	70%	7	70%
22		2	0	0%	7	70%	7	70%
23		2	0	0%	9	90%	9	90%
24		0	0	0%	0	0%	0	0%
25		0	0	0%	0	0%	0	0%
26		0	0	0%	0	0%	0	0%
27		0	0	0%	0	0%	0	0%
28		0	0	0%	0	0%	0	0%
29		0	0	0%	0	0%	0	0%
30		0	0	0%	0	0%	0	0%
	Mean Scores		0	0%	0	0%	0	0%

	Mean Scores All	Group 1	Group 2	Group 3	Group 4
Pre-Test	0.0	0.0	0.0		
Post-Test	0.0	7.7	7.2		
Difference	0.0	7.7	7.2		
Total Number	23	10	13	0	0



Section VI: Evaluation and Reflection

The lesson I taught at my practicum was not from my teacher work sample. I taught a lesson about determining amplitude period and frequency from both a graph and an equation. I also taught vertical shifts of the sine and cosine curve. I did this by going through the class packet of notes. I taught the lesson to two classes. During the first class I didn't get through as much material as I wanted to. I didn't get all the way through teaching vertical shifts but I did feel confident the students understood what I did get through with determining amplitude period and frequency from both a graph and an equation. This was reflected in the quiz the students took the next week. This is the student work sample I collected. The students performed well when given an equation. The only problem on the quiz was that the students had some trouble determining period given a graph. I think the problem was they were confusing period and frequency. A lot of the students would find the period and then take 2π/period to reach their answer.

Another thing I noticed during the first class was that when I was going through some examples with the class the students were getting bored. This was because I went through too many examples. This boredom lead to some students talking and I had to stop and refocus them. I should have been able to tell when the students learned the material and just moved on.

The second time I taught the lesson I was more conscious of my time management. I was able to better assess when the students understood the new material and I skipped some of the last examples. This allowed me to get through all of the material I planned on originally.

I think what this reveals about my teaching is that I need to work on my time management and my assessment of student understanding while teaching. I need to be able to assess when the students understand new material and then move on so they don't get bored and disengage. This has been something I have worked on all semester and will continue to work on.

Section VI: Evaluation and Reflection

My host teacher would let me go over the DINs with the class and I would often go too slowly. I have learned to assess what the students have picked up on and what to move more quickly through.

If I were to teach this lesson differently I would try to have the students do more practice problems on their own. The whole lesson was me lecturing. This is a very boring and sometimes ineffective practice. There were times where I would ask the students to answer parts of problems while I was doing the problem on the ELMO but this is not the same as the students actually doing the problems on their own. I would have at least liked to take a break from the notes and give the students a set of problem to try for themselves.