Date:

## NUMB3RS Activity: Social Security Numbers

Charlie and Amita are trying to track down someone based on a Social Security Number. Charlie explains that numbers are assigned according to a system. "The first three digits of a social are assigned by the zip code of the applicant. And the next two break down numbers from the same area into smaller, easier-to-manage blocks." Amita says, "Even if we focus on the numbers generated in southern California, that still leaves us with a lot of data." Charlie adds, "Each hospital is assigned a batch of Social Security numbers based on birth rate projections."

A Social Security number has the form XXX-XX-XXXX. The first three digits of the number are called the "area number." As Charlie mentions, these are assigned by zip code, with lesser numbers for the East Coast and greater numbers for the West Coast. As the population has grown, some exceptions have arisen, but a given "area number" will refer to a certain area of the country.

To understand some mathematical ideas behind Social Security numbers, let's look at the following definitions:

- 1. Multiplication principle: Suppose that a task is composed of two consecutive operations. If the first operation can be performed in m ways and for each of those, the second operation can be performed in n ways, then the complete task can be performed in  $m \cdot n$  ways.
- 2. Generalized multiplication principle: Suppose that a task consists of *t* operations performed consecutively. Suppose that the first operation can be performed in  $m_1$  ways; for each of these, the second operation can be performed in  $m_2$  ways; for each of these, the third operation can be done in  $m_3$  ways; and so forth. Then the complete task can be performed in  $m_1 \cdot m_2 \cdot m_3 \cdot \ldots \cdot m_t$  ways.

Complete the following:

- 1. If Lile's wardrobe consists of 3 pairs of winter pants and 10 sweaters, how many different outfits can Lile make?
- 2. A combination lock has a code that consists of 3 digits. Each digit can be any number from 0 to 9, but the combination cannot be 000. How many combinations are possible?
- 3. If you are asked to select an integer from 236 to 578, including those two values, how many choices do you have?

4. If each of the digits of a Social Security number can be any digit from 0 to 9, how many different Social Security numbers are possible?

Not all possible Social Security numbers are valid. One restriction is that no block of numbers can be all zeros (the area number can never be 000, the next two cannot be 00, and the last set of four digits cannot be 0000, but zero can be used otherwise).

- 5. a. What is the lowest possible Social Security number?
  - b. With the "no block of zeros" restriction, how many different Social Security numbers are possible?

In 1936, when numbers were first assigned, the area numbers were either from 001 to 585 or from 700 to 729 (the first set were assigned by the area of the country and the second set were assigned to railroad workers). The "no block of zeros rule following question 4 still applies.

6. In 1936, how many Social Security numbers were available?

During the episode, Amita and Charlie are searching a database for a Social Security number from California. They know the possible area numbers, but, as Amita comments "that still leaves us with a lot of data." The area numbers for California range from 545 to 573 and from 602 to 626.

7. How many possible Social Security numbers are there for people applying from California?

The two digits after the area number are called the group number. They are assigned in the following order:

- a.) odd numbers from 01 to 09
- b.) even numbers from 10 to 98
- c.) even numbers from 02 to 08
- d.) odd numbers from 11 to 99
- 8. The highest group number assigned for each Social Security number area is published monthly. If 26 is the highest group number assigned to area 862, explain why the Social Security number 862-06-2775 is not yet a valid number.