

## Lesson Plans

### Day 1 $S_{(a,b)} = (a x, by)$ Size Change

Example:  $S_{(2,3)} = (2x, 3y)$

#### Materials:

- coordinate graph paper
- individual coordinate white boards

#### STEPS:

- Exercise in small groups (3 or 4), graphing various figures (triangle, quadrilaterals, ...) and the corresponding images under various size changes in the form of  $S_{(a,b)} = (ax, by)$  on graph paper or on white boards.
- Group should look for similarities and differences that occur between the pre-image and image. Hint: look at lengths, perimeter, area, angles, congruence, and parallelism.
- Groups should look for any patterns and form a hypothesis. The group should test these hypotheses with additional examples. Distance and slope formulas may be used. Show all work.
- Sketchpad may be used to test these additional examples.
- Group should summarize their finding and share with the class.
- Class should summarize findings.

### Day 2 $S_{(a,b,c)} = (ax, by, cz)$

Example:  $S_{(2,3,4)} = (2x, 3y, 4z)$

#### Materials:

- isometric dot paper
- various 3-D models (created now or in a previous lesson)

Example:

<u>Cubes</u>	<u>Boxes</u>	<u>Cylinders</u>
1x1x1	1x2x3	radius=1, height=2
2x2x2	2x2x4	r=2, h=2
3x3x3	3x6x6	r=2, h=8
4x4x4	4x2x4	r=1, h=4
5x5x5	5x10x5	r=3, h=4
6x6x6	3x3x6	r=3, h=2

\*Other figures may also be used.

STEPS:

- A. Exercise in small groups, drawing 3-D models and their images under various size changes in the form  $S_{(a,b,c)} = (ax, by, cz)$  on 3-D coordinate graph paper and isometric dot paper.
- B. Group should look for similarities and differences that occur between the pre-image and the image. Hint: length, perimeter, angles, area, congruence, parallelism.
- C. Groups should look for any patterns and form a hypothesis. The group should test these hypotheses with additional examples. Distance and slope formulas may be used. Show all work.
- D. Students should use the 3-D models to assist them in analyzing the size changes and in making and testing the hypothesis.
- E. Group should summarize their finding and share with the class.
- F. Class should summarize findings.

Day 3  $S_k(x,y) = (kx, ky)$

Example:  $S_3(x,y) = (3x, 3y)$

Materials:

- coordinate graph paper or white boards
- sketch pad

STEPS:

- A. Individually students should graph a triangle or other polygon. Apply  $S_3$  to the pre-image.
- B. Individuals should look for similarities and differences that occur between the pre-image and the image. Back-up findings through work shown. Hint: length, perimeter, angles, area, congruence, parallelism.
- C. Individuals should look for any patterns and form a hypothesis. The individual should test these hypotheses with additional examples. Distance and slope formulas may be used. Show all work.
- D. Students should move into groups to discuss and share findings.
- E. Groups should use sketchpad to test their findings. Groups should look at other scale changes and compare length, width, area, corresponding angles, and parallelism.
- F. Groups should explore on sketchpad what happens when  $k > 1$ ,  $k < 1$ ,  $k = 0$ ,  $k < -1$ ,  $k > -1$ , etc.
- G. Groups will share and discuss finding with class.

## Day 4 $S_k$ without graph

### Materials:

- template
- poster board
- cartoon character

### STEPS:

- A. Teacher lesson on
1. Locating centers of size changes given to similar figures
  2. Finding images of size changes without using ordered pairs
  3. Using proportions to find various lengths.
- B. Students will work in groups to produce one large poster of a cartoon character by applying a size change without using ordered pairs. Students will also use the method demonstrated in class and show all work.

## Day 5 Proportions and Similar Figures

### Materials:

- Posters from previous lesson
- Template
- Meter stick
- worksheet

### STEPS:

- A. Groups will present posters from the previous day to the class.
- B. Groups will analyze and measure other group's posters to find the K factor (ratio of similitude) that was used to produce the image.
- C. Using a worksheet, groups will locate centers of size change, compare length, areas and their proportions of various similar figures to come up with relationships between them. They should test their findings by analyzing the other posters.
- D. The groups will share their findings with the class.

## Day 6 $S_k$ Fundamental Theorem of Similarity

Under  $S_k$ ,  $k$  is the ratio of corresponding lengths of Image over Pre-image.  
 $k^2$  is the ratio of corresponding areas of Image over Pre-image.  
 $k^3$  is the ratio of corresponding volumes of Image over Pre-image.

### Materials:

- Similar 3-D models from Day 1.
- Template

### STEPS:

- A. In Small groups, students will measure length and calculate areas and volumes of similar 3-D models that were used in Day 1.
- B. The students will work in small groups to try to find patterns that lead to the following conclusions: In any  $S_k$ ,  $k$  is the ratio of lengths of Image/Pre-image  
 $k^2$  is the ratio of areas of Image/Pre-image  
and  $k^3$  is the ratio of volumes of Image/Pre-image
- C. Students will share and discuss finding with the class.

## Days 7 & 8 $S_k$ word problems dealing with real world situations.

### Materials:

- Worksheets

### Example problems:

- Two similar quadrilaterals have areas of 6 and 54 square inches. If the smaller quadrilateral has a perimeter of 8 inches, what is the perimeter of the larger?
- A toy truck, which is 17 inches long can hold 3 lbs of sand. Its real version can hold 12000 lbs. How long is the real truck?
- A 6-inch single topping pizza at Sue's Pizza Palace costs \$3.99. What should a 15-inch single topping pizza cost? (Assuming same thickness for the pizzas)

Answers to the above are: 24, 269.9 inches (22 feet 6 inches), \$24.94

### STEPS:

- A. The students will individually work on the worksheets.
- B. Small groups will be formed and students will compare and defend answers.
- C. Small groups will volunteer to present their solution to the class, with class discussion to follow.

## Day 9 Lewis and Clark dugout canoe size change problem

### Materials:

- Ken Burn's Lewis and Clark videos from PBS(optional)
- Model dugout canoe(optional)
- Lewis and Clark "peace medal" Jefferson nickels(optional)
- Worksheet

### STEPS:

A. Present background necessary for lesson.

Near the mouth of the Marias river, two cottonwood dugout canoes were built. The canoes were of lengths 25 feet and 33 feet. They were also approximately 3 feet wide. The canoes and all of their gear had to be portaged several miles around the Great falls of the Missouri river.

B. Size change data.

Model canoe's length 10 inches

Model canoe's weight 90.6 grams or 0.19988625 lbs.

Model canoe's load it could carry 84.4 grams or 0.1862075

Model canoe's width 2 inches

C. Students should make predictions as to the weight of the 25 foot and the 33 foot canoes. These should be recorded on the board.

Students should work individually to calculate the weight of the two canoes and how much of a load they could carry. How does this compare to your prediction? Does your answer make sense? Also Compare the weights of the 25 foot and 33 foot canoes. Does the weight seem reasonable?

D. In small groups, compare and contrast answers. Make a decision and present best answer to the class. Discussion will follow once all groups have put up their answers.

E. Sample answers:

Using similarity and the lengths,

Weight of the 25 foot canoe would be 5396.9 lbs.

Weight of the 33 foot canoe would be 12,412.8 lbs.

This seems a little high. Why?

F. Sample answers:

Using  $S_{(a,b)} = (ax, bx)$

Where **a** is the ratio of length, and **b** is the ratio of width.

Weight of the 25 foot canoe would be 3238.2 lbs if height ratio was 30 and  
1942.9 lbs if height ratio was 18.

Weight of the 33 foot canoe would be 5642.2 lbs if height ratio was 39.6 and  
2584.6 lbs if height ratio was 18

## Lewis and Clark Dugout Canoe Size Change Problem

The lengths of the two canoes were 25 feet and 33 feet long.  
The widths were three feet.

Predict the weight of the canoes.

25-foot canoe weighs \_\_\_\_\_ lbs.

33-foot canoe weighs \_\_\_\_\_ lbs.

The model is 10 inches long.

It is two inches wide.

The model weighs 90.9 grams (0.19988625 lbs)

The maximum load it can carry is 84.4 grams (0.1862075 lbs)

### A. Calculations for the 25-foot canoe

1. Weight = \_\_\_\_\_

2. Load = \_\_\_\_\_

### B. Calculations for the 33-foot canoe

1. Weight = \_\_\_\_\_

2. Load = \_\_\_\_\_

### C. Individual Reflection Questions

1. How close were your predictions to your calculated weights? Explain.

2. Do your calculated weights seem reasonable? Explain

3. In comparing the weights of the 25 foot to the 33-foot canoe, do the weights seem reasonable? Explain.

**D. Group Questions**

1. Record the weights of the canoes of each group member.

Name \_\_\_\_\_ 25-foot canoe wt. \_\_\_\_\_ 33- foot canoe wt. \_\_\_\_\_

2. Did everyone agree on the weights?

3. If not, after discussion, does everyone agree on the weight?

4. How can you convince me your groups answer is correct?

## DAY 10 Lewis and Clark lead canister size change performance package part 1

### Materials:

- Template
- Compass
- Scissor
- Tape
- Part 1 area and volume calculation sheet

### STEPS:

A. Present background necessary for lesson.

Problem: How to keep gun powder dry

Solution: Cylindrical lead canister made out of  $\frac{3}{8}$  inch lead sheets, which contained the gunpowder. No dimension of the canister was recorded in the journals of the expedition. After numerous hours of research and calculation by your math teacher, the following calculation may be used for this project.

Outside diameter 3"

Height of cylinder 6"

Thickness of lead sheets  $\frac{3}{8}$  inch

B Students working individually should create an accurate net for the lead cylinder using the given information from above.

C. Students working individually should create and put together a 3-D model from the net in part B.

D. Students working individually calculate the following by showing the formula and all work: area of the base, perimeter of the base, lateral area of the base, surface area, volume, volume of the lead, volume of the powder.



## **Day 11 Lewis and Clark lead canister size change performance package part 2**

### Materials:

- Data results from part 1
- Part 2 length, area and volume calculation sheet.

### STEPS:

A. Students should work individually to answer questions relating to size changes relating to the lead canister. Length, width and height variables will be changed by a constant value, volumes will be change by a constant value, surface area will be changed by a constant value.

B. In real life the thickness of the canister was not change if the canister was made with different dimensions. The sheets of lead were  $\frac{3}{8}$  of an inch thick. Students should work individually to answer questions relating to size changes relating to the lead canister.

## **Day 12 Lewis and Clark lead canister size change performance package part 3**

### Materials:

- Data results from part 1
- Data results from part 2

### STEPS:

A. Photo copy then correct individual student work.

B. Hand back un-corrected original student work. Place students in small groups to compare and contrast their answers. The groups should discuss and come up with what they believe the best answer is along with the reasons for making their decision.

C. Individual students should correct their own papers. They should also state where their mistakes were made.

D. Group should share their write their answers on the board, followed by class discussion.

PERFORMANCE PACKAGE TASK 1  
**SIZE CHANGES OF THE CORP OF DISCOVERY**

**Content Standard: V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT**

***Specific Statement(s) from the Standard:***

**B. Geometry**

**3. Know and use properties of two- and three-dimensional figures to solve real-world and mathematical problems such as: finding area, perimeter, volume and surface area; applying direct or indirect methods of measurement; the Pythagorean theorem and its converse; and properties of 45°-45°-90° and 30°-60°-90° triangles.**

**6. Use numeric, graphic and symbolic representations of transformations such as reflections, translations and change of scale in one, two and three dimensions to solve real-world and mathematical problems.**

**C. Measurement**

**Standard: Use the interconnectedness of geometry, algebra and measurement to explore realworld and mathematical problems.**

***Product(s):***

Lewis and Clark Lead Canister Project

Create a replica of a Lewis and Clark Lead canister

Calculate length, area and volume and apply the Fundamental Theorem of Similarity.

***Task Description:***

Students will:

- Draw a net to scale of a cylinder.
- Cut out and put together a 3-D models of a cylinder.
- Calculate the lengths, areas and volumes by writing the formulas and showing all work.
- Calculate the lengths, areas and volumes of similar canisters by using the fundamental theorem of similarity.

***Special Notes:***

This is a Lewis and Clark bicentennial project. A small historical view of the Lewis and Clark Expedition would be helpful.

This project could also be adapted to other geographical historic situations.

PERFORMANCE PACKAGE TASK 1  
**SIZE CHANGES OF THE CORP OF DISCOVERY**

**FEEDBACK CHECKLIST FOR TASK 1**

The purpose of the checklist is to provide feedback to the student about his/her work relative to the content standard. Have the standard available for reference.

Y=Yes

N=Needs Improvement

<u>Student</u>		<u>Teacher</u>
_____	Net of cylinder properly drawn to scale	_____
_____	Model of 3 D cylinder properly constructed	_____
_____	Used the correct numbers for the formulas	_____
_____	Accurately calculated lengths, areas and volumes by showing work	_____
_____	Accurately calculate corresponding areas and volumes of similar figures by using the fundamental theorem of similarity.***corresponding lengths given	_____
_____	Accurately calculate corresponding lengths and volumes of similar figures By using the fundamental theorem of similarity.***corresponding area given	_____
_____	Accurately calculate corresponding lengths and areas of similar figures by using the fundamental theorem of similarity.***corresponding volumes given	_____

**Overall Comments** (information about student progress, quality of the work, next steps for teacher and student, needed adjustments in the teaching and learning processes, and problems to be addressed):