Complex: Numbers and Basic Arithmetic, Algebra

NAME:

1. Draw the complex plane C and label the real axis and the imaginary axis on your graph paper.

2. Plot the complex number z = 3 + 2i in the complex plane C. The real part of z is denoted by $Re(z)$ and the imaginary part of z is denoted by $Im(z)$. What does $Re(z) =$? What does $Im(z) =$? Notice the similarity with plotting an ordered pair $(3, 2)$ in the xy plane.

3. Plot w = 4 - 3i.

4. Calculate the sum z + w =

Calculate the difference z - w =

Plot these vectors.

5. Plot the complex numbers $z = 2$ and $z = 3i$.

6. Complex multiplication behaves like regular multiplication of factors, but using the special relation which defines i:

$$i = \sqrt{-1} \quad \text{so} \quad i^2 = -1.$$

Multiply: z w = (3 + 2i)(4 - 3i) =

Is there some geometric interpretation of complex multiplication?

7. Division is done by making use of the complex conjugate. If w = 4-3i then the complex conjugate of w, denoted by $\bar{w}$ is $\bar{w} = 4 + 3i$ (one changes the sign on the imaginary part of w.). To solve z/w we multiply by the complex conjugate:

$$\frac{z}{w} = \frac{z}{w}\frac{\bar{w}}{\bar{w}}$$

For z and w given above, determine

$$\frac{z}{w} =$$

Polar Coordinates: A complex number z can be given by polar coordinates (r, θ) so that $z = r\cos\theta + ir\sin\theta$.

Euler's Formula makes use of polar coordinates:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

so that we can write a complex number using the complex exponential function:

$$z = r(\cos\theta + i\sin\theta) = r\cos\theta + ir\sin\theta = re^{i\theta}$$

8. For $z = e^{i(\pi/4)}$, rewrite z in the form $z = u + iv$. Simplify your answer using your trig values.

$z =$

For this z value, give the polar coordinates, $r =$ and $\theta =$. Plot this complex number.

9. Write $z = e^{i\pi}$ in the form $z = u + iv$ and then plot it: $z =$

10. Use the normal rules of exponents rewrite $z = e^{3+i(\pi/4)}$ in the form $z = re^{\imath\theta}$ and then plot z on the complex plane. What is the value of r and of θ?

Also rewrite $z = e^{3+i(\pi/4)}$ in the form $z = u + \imath v$. What do u and v equal?

11. Rewrite the point $z = \frac{3}{2} + i\frac{3\sqrt{3}}{2}$ in the form $z = re^{i\theta}$. You will first need to determine r and θ. Note that if $z = u + \imath v$ then $r = \sqrt{u^2 + v^2} = (z\bar{z})^{1/2}$. Also $\theta = \arctan(y/x)$.

12. Write z = 3 + 2i and w = 4 - 3i each in the form $re^{i\theta}$. Using these exponential formulas, calculate (z w) in exponential form. Just use the regular rules of exponents. Give $r(zw)$ and $\theta(zw)$.