OKLAHOMA STATE UNIVERSITY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Fall 2003
Midterm Exam #1

Do All Five Problems

Name : ______________________________

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Problem 1:

Let $\bar{y}(t)$ be the unit-step response of a linear time-invariant system. Show that the impulse response of the system equals $\frac{d\bar{y}(t)}{dt}$.

Problem 2:

a) For a moving-average (MA) model,

$$Y(z) = \left[b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}\right] U(z),$$

develop a simulation diagram in controllable canonical form.

b) For a autoregressive (AR) model,

$$Y(z)(1 + a_1 z^{-1} + \cdots + a_n z^{-n}) = U(z),$$

develop a simulation diagram in controllable canonical form.

c) Given a mixed or ARMA model,

$$\frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}},$$

show how to merge the simulation diagrams of the MA and AR models in a) and b) so as to obtain a minimal realization of the ARMA model.

Problem 3:
Show that a realization for the circuit shown below can be written as

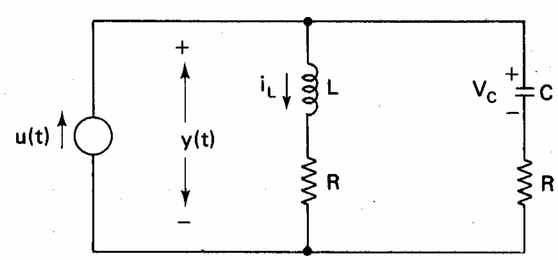
$$\dot{x}(t) = \begin{bmatrix} -2R/L & 1/L \\ -1/C & 0 \end{bmatrix} x(t) + \begin{bmatrix} R/L \\ 1/C \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} -R & 1 \end{bmatrix} x(t) + Ru(t)$$

if we choose $x_1(t) = i_L(t), x_2(t) = V_C(t)$. Show also that the transfer function is given by

$$H(s) = \frac{Y(s)}{U(s)} = \frac{Rs^2 + [(1/C) + (R^2/L)]s + (R/LC)}{s^2 + (2R/L)s + (1/LC)}.$$

Note that when $R^2 = L/C$, the transfer function is a constant, $H(s) = R$, for all values of s. This is known as a *constant-resistance network.*



Problem 4:
Let

$$H(z) = \begin{bmatrix} \dfrac{2 + z^{-1} - z^{-2}}{z^{2} - 1} & \dfrac{z^{-2}}{1 + 2z^{-1} - 3z^{-2}} \\ \dfrac{z^{-1} + 2z^{-2}}{1 - z^{-2}} & \dfrac{3 + z^{-2}}{1 + z^{-1} + 2z^{-2}} \end{bmatrix}$$

be a transfer function matrix. Find a minimal realization (i.e., simulation diagram and state space representation) for the discrete-time system, $H(z)$.

Problem 5:

Find the A, B, C, and D matrices for the composite system using two subsystems $\{A_i, B_i, C_i, D_i\}$, i=1, 2, connected in negative feedback, with $\{\mathrm{A}_1, \mathrm{B}_1, \mathrm{C}_1, \mathrm{D}_1\}$ in the forward loop and $\{\mathrm{A}_2, \mathrm{B}_2, \mathrm{C}_2, \mathrm{D}_2\}$ in the feedback loop.