Extraction of the static magnetic form factor and the structure function of the neutron from inclusive scattering data on light nuclei

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Abstract

We show that quasi-elastic inclusive electron scattering data on light nuclei for medium Q^2 furnish information on $G_M^n(Q^2)$, whereas the deep- inelastic region for large Q^2, provides the Structure Function $F_2^n(x,Q^2)$. Common to the two extractions is the possibility to de-convolute medium effects, which is most accurately done for light targets. Results are independent of the target.

Introduction. Most neutron observables can only indirectly be extracted from experiments on a nuclear medium, in which the n is embedded. We discuss below the neutron static magnetic form factor and its Structure Function (SF).

Consider the reduced cross section for inclusive scattering of unpolarized electron of energy E from non-oriented targets A over een angle θ

$$\frac{A^{-1}d^2\sigma_{eA}(E;\theta,\nu)/d\Omega\, d\nu}{\sigma_M(E;\theta,\nu)} = \Big[\frac{2xM}{Q^2}F_2^A(x,Q^2) + \frac{2}{M}\tan^2(\theta/2)F_1^A(x,Q^2)\Big] \tag{1}$$

$F_k^A(x,Q^2)$ are two nuclear structure functions (SF), functions of $Q^2=\boldsymbol{q}^2-\nu^2$ ($\nu,\boldsymbol{q}$ are the energy-momentum transfer) and the Bjorken variable $x=Q^2/2M\nu$, with range $0\le x\le A$

(M is the nucleon mass). Of crucial importance is a relation between the SF of nuclei and of nucleons. For instance (for $Z = N$) [1]

$$F_k^A(x, Q^2) = \int_x^A \frac{dz}{z^{2-k}} [f^{PN,A}(z, Q^2) \Big[F_k^p\Big(\frac{x}{z}, Q^2\Big) + F_k^n\Big(\frac{x}{z}, Q^2\Big)\Big] \Big/ 2 \qquad (2)$$

The two SF are related by $f^{PN,A}$, the SF of a nucleus, composed of point-nucleons. A standard calculation of F_k^A thus requires data on F_k^p, an assumed form for F_k^n and in addition, a computed, unphysical $f^{PN,A}$.

We separate F_k^N in NE ($\Gamma^* + N \to N$) and NI parts ($\gamma^* + N \to$ hadrons, partons), leading to the corresponding components $F_k^{A,NE}$ [2] ($\eta = Q^2/4M^2$)

$$F_1^{A,NE}(x) = \frac{f^{PN,A}(x)}{4} G_d^2 [(\alpha_p \mu_p)^2 + (\alpha_n \mu_n)^2] \qquad (3a)$$

$$F_2^{A,NE}(x) = \frac{x f^{PN,A}(x) G_d^2}{2(1+\eta)} \Big[(\alpha_p \gamma)^2 + \Big(\frac{\mu_n \eta}{1 + 5.6\eta}\Big)^2 + \eta[(\alpha_p \mu_p)^2 + (\alpha_n \mu_n)^2]\Big], \qquad (3b)$$

where reference to Q^2 has been dropped. Instead of the actual static electromagnetic form factors $G_{M,E}^N(Q^2)$, we use in Eq. (3) their deviations from the standard dipole form [3–5].

$$\alpha_N \equiv G_M^N / \mu_N G_d \qquad ; N = p, n \qquad (4a)$$

$$\gamma \equiv \frac{\mu_p G_E^p}{G_M^p} = \frac{G_E^p}{\alpha_p G_d} \qquad (4b)$$

$$\gamma = 1 + \theta(Q^2 - 0.3) \approx \qquad [1 - 0.14(Q^2 - 0.3)] \; ; Q^2 \lesssim 5.5 \qquad (4c)$$

For G_E^n we use the Galster parametrization [6]. Nuclear NI components completely dominate cross sections on the inelastic side $x \lesssim 1$ of the QEP, while for $x \gtrsim 1$ NE>NI. Those regions will be treated separately.

Quasi-elastic region $x \lesssim 1$: G_M^n. Consider first the x, Q^2 dependence of $F_k^{A,NE}(x, Q^2)$. The latter is primarily due to the form factors in Eqs. (3), which decrease with growing Q^2. The x-dependence resides in $f^{PN,A}(x, Q^2)$, which sharply decreases with growing $|1 - x|$ away from the QEP at $x \approx 1$. From the above one concludes that $\ln[\sigma^{A,NE}/A]$ grows with increasing ν (decreasing x for fixed Q^2), while in general for $A \geq 12$ there is a mere break in the slope in the QE region $|1 - x| \ll 1$ for $A \geq 12$ (Fig. 1a) [7].

The unusual structure of the lightest nuclei, causes $f^{PN,A}(x,Q^2)$ to be narrow and sharply peaked. With no interference of NI, the above change in slope may develop into a QE peak, as observed for D [8] and ^{4}He [9] (Fig. 1b). For the same targets one can compute with great precision ground states [10] and non-diagonal target density matrices in the expression for $f^{PN,A}$ [11,12].

Under the above circumstances one tends to ascribe the total cross sections on the elastic side $x \gtrsim 1$ to NE. With $G^p_{E,M}$ known and small G^n_E, this enables the extraction of G^n_M from NE. Tests for the above allocations are: i) Around $x \lesssim 1$, $\sigma^A/\sigma_M \propto f(x,Q^2)$, i.e. of a bell shape in $1-x$. ii) $G^n_M(Q^2)$ should be independent of the value of the individual x from which the one extracts G^n_M. iii) Idem for the chosen target.

Our analysis comprises older D data, where separation into transverse and longitudinal SF, with the former $\mathcal{R}_T \propto [G^p_M]^2 + [G^n_M]^2$ [13]. Although direct and simple, it requires high-quality data in order to allow an accurate Rosenbluth separation and to obtain a precise G^n_M. Table I summarizes all our findings for $\alpha_n(Q^2)$ while Fig. 2 shows all $\alpha_n(Q^2)$, extracted thus far. Our values follow the trend of previously measured values and adds points for intermittent Q^2. Hardly any target dependence has been detected.

<u>The deep-inelastic region, $x \ll 1$: extraction of $F^n_2(x,Q^2)$</u>. That region is dominated by NI. We focus on $F^n_2(x,Q^2)$, commonly estimated from the 'primitive' ansatz $F^n_2 \approx 2F^D_2 - F^p_2$, which is only reliably for $x \lesssim 0.3$. Instead of a vehicle to compute F^A_k, we now consider Eq. (2) in the inverse sense: Can one, with data on σ^A, Eq. (1), known F^p_2 and computed $f^{PN,A}$ extract $F^n_2(x,Q^2)$?

Virtually all previous methods addressed a D target (e.g. [14]). We outline and apply a method [19], which with sufficient kinematics available [7,8], is applicable to all targets.(see Refs. [15,16] for treatments of isobar pairs). Again a test is an outcome, independent of A. As to F^A_2, in order to separate it from F^A_1, one needs in addition to cross sections, an assumption on $R^{-1}(x,Q^2)+1 \propto 2xF^A_1(x,Q^2)/F^A_2(x,Q^2)$. Alternatively, one may for every data point determine a relative deviation of theory and data, and ascribe it in equal measure to the two SF. The procedure produces quasi-data for $F^{A;qd}_2$.

All modern data thus far [7,8] appear to yield F_2^A in disjoint x, Q^2 regions, whereas the inversion of Eq. (2) requires data over a large x-range for the same Q^2. Even with careful binning and/or interpolation, we could only construct a single set for $Q^2 \approx 3.5$ GeV^2 , $x \gtrsim 0.55$, which x-range misses a crucial part of the DI region. Fortunately, one can use the fact, that, independent on Q^2, $F_2^p(x, Q^2) \approx 0.32$ for $x \approx 0.16$. Eq. (2) then proves the same for $F_2^A(x, Q^2)$, permitting extrapolation into the vital DI region.

We have used several inversion methods, all based on a parametrization

$$F_2^n(x, Q^2) = F_2^n(x, Q^2; d_k) = C(x, Q^2; d_k) F_2^p(x, Q^2)$$
$$C(x, Q^2; d_k) = \sum_{k \geq 0} d_k(Q^2)(1-x)^k, \tag{5}$$

with mildly constrained parameters. First we take $C(0) = 1$, ensuring a finite outcome for the Gottfried sumrule $S_G(Q^2) = \int_0^1 \frac{dx}{x}[F_2^p(x, Q^2) - F_2^n(x, Q^2)]$. Next we exploit the above $'$primitive$'$ ansatz for, say, $x = 0.2$. For the simplest choice $k_{max} = 2$ only one parameter is left, e.g. $d_0 = C(1)$. It moreover proved useful to parametrize F_2^p as follows

$$F_2^p(x, Q^2) = x^{-a^2} \sum_{m \geq 1} c_m (1-x)^m ; x \geq 0.02 \tag{6a}$$
$$= \quad 0.42 \qquad\qquad ; x \leq 0.02 \tag{6b}$$

In the region $0.02 \lesssim x \lesssim 0.9$, the above practically coincides with the standard parametrization [17]. Fig. 3 shows our results for C, F_2^n for fixed $Q^2 = 3.5$ GeV^2 and given F_2^p. The band in C reflects results from several inversion methods and from different targets D,C,Fe. The value of C at the elastic point $x = 1$ has been the subject of several estimates with results, marked by small horizontal lines. All those, as well as our C, assumed smooth, i.e. resonance-averaged behaviour of F_2^N (cf. lower part of Fig. 3).

The above is an undesired feature of averaging: the lowest inelastic threshold of $F_2^N(x, Q^2)$, occurs at a mass $M + m_\pi$, or equivalently, at $x_{thr}(Q^2) = [1 + 2Mm_\pi/Q^2]^{-1}$. In particular $x_{thr}(3.5) \approx 0.93$, which is marked in Fig. 3 by a vertical line. For $x_{th} < x < 1$, $F^N(x, Q^2)$ is strictly 0. In particular the mention prediction of C out to the elastic border, merely reflects the different approach to 0 of the p, n SF. As a consequence $C(x \to 1)$ is due to purely NE parts of F_2^N, and equals (cf. Eq. (3b))

$$\lim_{x \to 1} C(x, Q^2) = \Big[\frac{\mu_n \alpha_n(Q^2)}{\mu_p \alpha_p(Q^2)}\Big] \Big[1 + \frac{4M^2}{Q^2} \Big(\frac{\gamma(Q^2)}{\mu_p}\Big)^2\Big]^{-1}, \tag{7}$$

From Eqs. (4), (7) one then *computes*

$$C(x = 1, 3.5) \approx 0.61, \tag{8}$$

surprisingly close to the *extracted* value as the ratio of the two F_2^N, which tend to 0 in a different way for $x \to 1$. More extensive reports can be found in Refs. [18,19].

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Figure captions

Fig. 1a,b. Partial data and predictions for inclusive cross sections ($E = 4.045$ GeV, $\theta = 15^\circ, 23^\circ, 30^\circ$) on D,Fe.

Fig. 2. $\alpha_n = G_M^n/\mu_n G_d$ as function of Q^2. Shown are some previous representative results. Filled squares, diamonds, triangles and stars are our results.

Fig. 3. The ratio $C(x, 3.5) = F_2^n(x, 3.5)/F_2^p(x, 3.5)$ for $Q = 3.5$ GeV^2 from data on D, C, Fe. The drawn line corresponds to $C(1) = 0.54$ and the band represents the spread from averages over different targets and methods. The numbers on the right abscissa are standard quark model and QCD predictions for $C(1)$ with 0.61, the NE limit (7).

TABLES

TABLE I. Extraction of $\alpha_n(Q^2)$ from QE inclusive scattering data on D, ^{4}He. Columns give target, beam energy E, scattering angle θ, ranges of Bjorken x and Q^2, range of SF of target composed of point-nucleons and (between brackets) its maximal value. The last column gives $\alpha_n(Q^2)$ with deviations from average over the considered x-intervals.

target	E (in GeV)	θ	x	Q^2(in GeV^2)	$f^{PN,A}(x,Q^2)$	$\alpha_n(Q^2)$
^{4}He [9]	2.02	20°	1.125-0.848	0.444-0.430	0.97-1.49 (1.49)	0.988±0.055
-	3.595	16°	1.125-0.930	0.887-0.864	1.16-1.90 (1.90)	0.967±0.028
-	3.595	20°	1.095-0.925	1.295-1.250	1.44-2.16 (2.16)	0.988±0.018
D [8]	4.045	15°	1.131-0.953	0.988-0.972	1.31-3.65 (4.30)	1.039 ±0.020
-	4.045	23°	1.079-0.978	1.976-1.929	2.44-5.18 (5.18)	1.062 ±0.009
D [13]	5.507	15.2°	1.063-0.978	1.769-1.741	2.89-5.04 (5.31)	1.047 ±0.019
-	2.407	41.1°	1.081-0.957	1.803-1.721	2.37-4.89 ((5.32)	1.048 ±0.007
-	1.511	90.0°	1.059-0.977	1.812-1.728	3.21-4.79 (5.26)	1.057 ±0.009
$\mathcal{R}_T^{D,NE}$ [13]	3.809	20°	1.141-0.962	$< Q^2 >$=1.75	1.79-3.38 (5.31)	1.004±0.014 (1.052 [13])
D [13]	5.507	19.0°	1.104-1.000	2.561-2.501	1.69-5.65 (5.98)	1.030 ±0.016
-	2.837	45.0°	1.101-0.991	2.613-2.500	1.69-5.91 (5.94)	1.031 ±0.018
-	1.968	90.0°	1.064-0.984	2.608-2.474	3.06-5.71 (5.90)	1.078 ±0.027
$\mathcal{R}_T^{D,NE}$ [13]	5.016	20°	1.068-0.940	$< Q^2 >$=2.50	2.92-4.16 (5.94)	0.986 ±0.014 (1.014 [13])
$\mathcal{R}_T^{D,NE}$ [13]	5.016	20°	1.051-0.958	$< Q^2 >$=3.25	3.50-6.15 (6.43)	0.940±0.013 (0.967 [13])
$\mathcal{R}_T^{D,NE}$ [13]	5.016	20°	1.079-1.038	$< Q^2 >$=4.00	3.80-6.20 (6.50)	0.830±0.016 (0.923 [13])