

# Penguin induced $B \rightarrow \eta$ transition form factor in light cone QCD

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## Abstract

We calculate the penguin form factor for the  $B \rightarrow \eta \ell^+ \ell^-$  decay. This form factor is calculated in light cone QCD sum rules, including contributions from wave functions up to twist-4, as well as mass corrections of the light  $\eta$  meson.

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# 1 Introduction

Rare B meson decays, induced by flavor changing neutral current (FCNC)  $b \rightarrow s(d)$  transition, provide potentially the stringiest testing ground for the Standard Model (SM) at loop level. These decays are also very suitable looking for new physics beyond the SM. Among all decays of B mesons, the semileptonic decays receive special attention, since their study offer one of the most efficient ways in determination of the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements. From experimental side, there scheduled an impressive program for study of both inclusive and exclusive B–decays in B factories, BaBar and Belle, as well as LHC–b machines. CLEO Collaboration [1] has measured the branching ratios of  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B \rightarrow \rho^- \ell^+ \nu$  decays, from which it is obtained that  $|V_{ub}| = (3.25 \pm 0.14_{-0.29}^{+0.21} \pm 0.55) \times 10^{-3}$ . In extraction of  $|V_{ub}|$  from  $B \rightarrow \pi(\rho)\ell\nu$  decay, main theoretical uncertainties come from  $B \rightarrow \pi(\rho)$  transition form factors. For an accurate calculation of the CKM matrix elements, hadronic form factors need to be determined more reliably.

It should be noted that the decay modes of  $B \rightarrow K\ell^+\ell^-$  ( $\ell = e, \mu$ ) has recently been observed with  $\mathcal{B}(B \rightarrow K\ell^+\ell^-) = (0.75_{-0.21}^{+0.25} \pm 0.09) \times 10^{-6}$  [2] and  $(0.78_{-0.20-0.18}^{+0.24+0.11}) \times 10^{-6}$  [3, 4]. At BaBar, an excess of events over background with  $2.8\sigma$  has been observed for the  $B \rightarrow K^*\ell^+\ell^-$  decay with  $\mathcal{B}(B \rightarrow K^*\ell^+\ell^-) = (1.68_{-0.58}^{0.68} \pm 0.28) \times 10^{-6}$  [4].

In this work we calculate the penguin form factor of the  $B \rightarrow \eta\ell^+\ell^-$  decay in light cone QCD sum rules. The form factors induced by the vector current in  $B \rightarrow \eta\ell\nu$  decay has already been calculated in light cone QCD sum rules in [5]. It should be mentioned here that  $B \rightarrow \eta$  form factors are related to the  $B \rightarrow \pi$  form factors through SU(3) symmetry, which are calculated in light cone QCD sum rules in [6]. A detailed description of the light cone QCD sum rule and its applications can be found in [7, 8].

Interest to  $B \rightarrow \eta\ell^+\ell^-$  and  $B \rightarrow \eta'\ell^+\ell^-$  has its grounds in the fact that they can give information about  $\eta$ – $\eta'$  mixing angle [9, 10]. Soon B factories will provide much more data and therefore a more reliable determination of the transition form factors and as a result a more precise determination of  $|V_{ub}|$  will be possible. The extraction of  $|V_{ub}|$  from the  $B \rightarrow \eta(\eta')\ell^+\ell^-$  decay would present an efficient and complementary alternative to its determination from  $B \rightarrow \pi(\rho)\ell^+\ell^-$  decay.

The present work is organized as follows. In section 2, we calculate the sum rule for the penguin form factor of the  $B \rightarrow \eta\ell^+\ell^-$  decay. Section 3 is devoted to the numerical analysis and the conclusion.

## 2 Light cone QCD sum rules for the penguin form factors in $B \rightarrow \eta$ transition

The penguin form factor of the  $B_d \rightarrow \eta$  transition is defined as

$$\langle \eta(p) | \bar{d}\sigma_{\mu\nu}q^\nu(1 + \gamma_5)b | B(p_B) \rangle = 2i[p_\mu q^2 - q_\mu(pq)] \frac{f_T}{m_B + m_\eta}. \quad (1)$$

The starting point for the calculation of the form factor  $f_T$  in Eq. (1) is the following correlator function:

$$\Pi_\mu(p, q) = i \int d^4x e^{iqx} \langle \eta(p) | \mathcal{T} \{ \bar{q}\sigma_{\mu\nu}q^\nu(1 + \gamma_5)b(x)\bar{b}(0)i(1 - \gamma_5)q \} | 0 \rangle$$

$$= i\Pi^T[p_\mu q^2 - (pq)q_\mu] , \quad (2)$$

which is calculated in an expansion around the light cone  $x^2 = 0$ . The main reason for choosing the chiral  $\bar{b}i(1 - \gamma_5)q$  current instead of the  $\bar{b}i\gamma_5q$  current which has been used in the calculation of the  $B \rightarrow \pi$  form factor [8], is because twist-3 wave functions do not contribute for this choice, which are the main inputs of the light cone QCD sum rules and which bring about the main uncertainty to the results [11].

Following the general idea QCD sum rules to obtain the penguin form factor is by matching the representation of the correlator function in hadronic and quark-gluon languages. Let us first consider the hadronic representation of the correlator function. By inserting a complete set of states with the same quantum numbers of the B meson between the currents in the correlator, and singling out the pole term of the lowest pseudoscalar B meson, we get

$$\begin{aligned} \Pi_\mu(p, q) &= \frac{\langle \eta | \bar{q}\sigma_{\mu\nu}q^\nu(1 + \gamma_5) | B \rangle \langle B | \bar{b}i(1 - \gamma_5)q | 0 \rangle}{m_B^2 - (p + q)^2} \\ &+ \sum_h \frac{\langle \eta | \bar{q}\sigma_{\mu\nu}q^\nu(1 + \gamma_5) | h \rangle \langle h | \bar{b}i(1 - \gamma_5)q | 0 \rangle}{m_h^2 - (p + q)^2} , \\ &= i\Pi^T[p_\mu q^2 - (pq)q_\mu] , \end{aligned} \quad (3)$$

where the sum in Eq. (3) describes the contributions of the higher states and continuum. For the invariant amplitude  $\Pi^T$  one can write a general dispersion relation in the B meson momentum squared  $(p + q)^2$  as

$$\Pi^T(q^2, (p + q)^2) = \int ds \frac{\rho(s)}{s - (p + q)^2} . \quad (4)$$

The spectral density corresponding to (3), is

$$\rho(s) = 2 \frac{f_T^\eta(q^2)}{m_B + m_\eta} \frac{m_B^2 f_B}{m_b} \delta(s - m_B^2) + \rho^h(s) , \quad (5)$$

where we have used the definition

$$\langle B | \bar{b}i\gamma_5q | 0 \rangle = \frac{m_B^2 f_B}{m_b} .$$

The first term in Eq. (5) represents the ground state B meson contribution and  $\rho^h(s)$  corresponds to the spectral density of the higher resonances and the continuum. The spectral density  $\rho^h(s)$  can be approximated by invoking the quark-hadron duality ansatz

$$\rho^h(s) = \rho^{QCD}(s - s_0) , \quad (6)$$

where  $s_0$  is the continuum threshold. As a result, the hadronic representation of the invariant amplitude  $\Pi^T$  takes the following form

$$\Pi^T = 2 \frac{f_T^\eta(q^2) m_B^2 f_B}{(m_B + m_\eta) m_b [m_B^2 - (p + q)^2]} + \int_{s_0}^{\infty} ds \frac{\rho^{QCD}(s)}{s - (p + q)^2} + \text{subtractions} . \quad (7)$$

In order to obtain the sum rule for  $f_T^\eta(q^2)$ , we proceed to calculate of the correlator function from QCD side. This can be done by using the light cone OPE method. For this purpose, we work in the large space-like momentum regions  $(p+q)^2 - m_b^2 \ll 0$  for the  $b\bar{q}$  channel and  $q^2 \ll m_b^2 - \mathcal{O}(\text{few } GeV^2)$  for the momentum transfer, which correspond to the small light cone distance  $x \approx 0$  and are required by the validity of the OPE. After contracting  $b$  quark field, we get

$$\Pi_\mu(p, q) = i \int d^4x e^{iqx} \langle \eta(p) | \bar{q}(x) \sigma_{\mu\nu} q^\nu (1 + \gamma_5) \mathcal{S}^b(x, 0) i(1 - \gamma_5) q | 0 \rangle, \quad (8)$$

where  $\mathcal{S}^b(x, 0)$  is the full quark propagator. In presence of the background gluon field, its explicit expression can be written as

$$\begin{aligned} \langle 0 | \mathcal{T} \{ b(x) \bar{b}(x) \} | 0 \rangle &= i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{\not{k} + m_b}{k^2 - m_b^2} \\ &- ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 du \left[ \frac{1}{2} \frac{\not{k} + m_b}{(k^2 - m_b^2)^2} G^{\alpha\beta}(ux) \sigma_{\alpha\beta} - \frac{1}{k^2 - m_b^2} ux_\alpha G^{\alpha\beta}(ux) \gamma_\beta \right], \end{aligned} \quad (9)$$

where the first term on the right hand side corresponds to the free quark propagator,  $G^{\alpha\beta}$  is the gluonic field strength and  $g_s$  is the strong coupling constant. We see from Eqs. (8) and (9) that, in order to calculate the theoretical part of the correlator, the matrix elements of the nonlocal operators between  $\eta$  meson and vacuum states are needed.

Here we would like to remark that in the following calculation  $\eta$ - $\eta'$  mixing will be neglected, since in octet-singlet basis this angle is about  $\theta \approx 10^\circ$  [12]. Hence, in the above-mentioned basis, the interpolating current for  $\eta$  meson is chosen as the SU(3) octet axial-vector current

$$J_\mu = \frac{1}{\sqrt{6}} (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d - \bar{s} \gamma_\mu \gamma_5 s). \quad (10)$$

In order to simplify the notation we will use  $\bar{q}\Gamma q$  to denote

$$J_\mu = \frac{1}{\sqrt{6}} (\bar{u} \Gamma_\mu u + \bar{d} \Gamma_\mu d - \bar{s} \Gamma_\mu s),$$

and introduce  $F_\eta = f_\eta/\sqrt{6}$ . Here,  $f_\eta$  is the leptonic decay constant of  $\eta$  meson and is to be determined from the relation

$$\langle 0 | \bar{q} \gamma_\mu \gamma_5 q | \eta^{(\rho)} \rangle = i f_\eta p_\mu. \quad (11)$$

It is easy to see from Eqs. (8) and (9) that the terms containing even number of Dirac matrices do not give any contribution. Remaining matrix elements can be parametrized in terms of  $\eta$  meson functions up to twist-4 defined as

$$\begin{aligned} \langle \eta(p) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle &= -i f_\eta p_\mu \int_0^1 du e^{iupx} \left[ \varphi_\eta(u) + \frac{1}{16} m_\eta^2 x^2 A(u) \right] \\ &- \frac{i}{2} f_\eta m_\eta^2 \frac{x_\mu}{px} \int_0^1 du e^{iupx} B(u), \end{aligned} \quad (12)$$

$$\begin{aligned} \langle \eta(p) | \bar{q}(x) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(ux) q(0) | 0 \rangle &= f_\eta m_\eta^2 \left[ p_\beta \left( g_{\alpha\mu} - \frac{x_\alpha p_\mu}{px} \right) - p_\alpha \left( g_{\beta\mu} - \frac{x_\beta p_\mu}{px} \right) \right] \\ &\times \int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) e^{ipx(\alpha_1 + u\alpha_3)} + f_\eta m_\eta^2 \frac{p_\mu}{px} (p_\alpha x_\beta - p_\beta x_\alpha) \int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) e^{ipx(\alpha_1 + u\alpha_3)} \end{aligned} \quad (13)$$

$$\begin{aligned} \langle \eta(p) | \bar{q}(x) g_s \tilde{G}_{\alpha\beta}(ux) \gamma_\mu q(0) | 0 \rangle &= i f_\eta m_\eta^2 \left[ p_\beta \left( g_{\alpha\mu} - \frac{x_\alpha p_\mu}{px} \right) - p_\alpha \left( g_{\beta\mu} - \frac{x_\beta p_\mu}{px} \right) \right] \\ &\times \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{ipx(\alpha_1 + u\alpha_3)} + i f_\eta m_\eta^2 \frac{p_\mu}{px} (p_\alpha x_\beta - p_\beta x_\alpha) \int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) e^{ipx(\alpha_1 + u\alpha_3)}, \end{aligned} \quad (14)$$

where

$$\tilde{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}, \text{ and } \mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3).$$

In Eqs. (12)–(14), the function  $\varphi_\eta(u)$  is the leading twist–2,  $A(u)$ ,  $\varphi_\parallel(\alpha_i)$ ,  $\varphi_\perp(\alpha_i)$ ,  $\tilde{\varphi}_\parallel(\alpha_i)$ , and  $\tilde{\varphi}_\perp(\alpha_i)$  are all twist–4 wave functions. Inserting Eqs. (12)–(14) and Eq. (9) into Eq. (8) and completing integration over the variables  $x$  and  $k$ , we get for the invariant structure

$$\begin{aligned} \Pi^T &= 2F_\eta \int_0^1 \frac{du}{m_b^2 - (q + pu)^2} \left\{ \varphi_\eta(u) - \frac{1}{2} m_b^2 m_\eta^2 \frac{A(u)}{[m_b^2 - (q + pu)^2]^2} \right\} \\ &- 4F_\eta m_\eta^2 \int_0^1 duu \int \mathcal{D}\alpha_i \frac{\varphi_\parallel(\alpha_i) - 2\tilde{\varphi}_\perp(\alpha_i)}{\{m_b^2 - [q + p(\alpha_1 + u\alpha_3)]^2\}^2} \\ &+ 2F_\eta m_\eta^2 \int du \int \mathcal{D}\alpha_i \frac{2\varphi_\perp(\alpha_i) - \varphi_\parallel(\alpha_i) + 2\tilde{\varphi}_\perp(\alpha_i) - \tilde{\varphi}_\parallel(\alpha_i)}{\{m_b^2 - [q + p(\alpha_1 + u\alpha_3)]^2\}^2}. \end{aligned} \quad (15)$$

The next and the last step in obtaining the sum rule for penguin form factor is to carry out the Borel transformation with respect to the variable  $(p + q)^2$  which enhances the ground state contribution and suppresses contributions of the higher states and the continuum. Finally, matching this result with the corresponding invariant amplitude that is calculated in hadronic and quark languages, we get the sum rule. Subtraction of the continuum contribution is performed by using quark–hadron duality (more about subtraction of continuum and higher state contributions in light cone QCD can be found in [14, 15]). Performing Borel transformation in Eq. (15), we get for the theoretical part

$$\begin{aligned} (\Pi^T)^B &= F_\eta \left\{ 2 \int_\delta^1 \frac{du}{u} \varphi_\eta(u) e^{-s(u)/M^2} - \frac{m_b^2 m_\eta^2}{2} \int_\delta^1 \frac{du}{u^3} \frac{A(u)}{M^4} e^{-s(u)/M^2} \right. \\ &- 4m_\eta^2 \int duu \int \mathcal{D}\alpha_i \frac{\varphi_\parallel(\alpha_i) - 2\tilde{\varphi}_\perp(\alpha_i)}{M^2 k^2} \theta(k - \delta) e^{-s(k)/M^2} \\ &\left. + 2m_\eta^2 \int du \int \mathcal{D}\alpha_i \frac{2\varphi_\perp(\alpha_i) - \varphi_\parallel(\alpha_i) + 2\tilde{\varphi}_\perp(\alpha_i) - \tilde{\varphi}_\parallel(\alpha_i)}{M^2 k^2} \theta(k - \delta) e^{-s(k)/M^2} \right\}, \end{aligned} \quad (16)$$

where

$$\begin{aligned} s(u) &= \frac{m_b^2 - q^2 \bar{u} + m_\eta^2 u \bar{u}}{u}, \quad s(k) = s(u \rightarrow k), \\ k &= \alpha_1 + u\alpha_3, \quad \bar{u} = 1 - u, \quad \bar{k} = 1 - k, \\ \delta &= \frac{m_\eta^2 + q^2 - s_0 + \sqrt{(m_\eta^2 + q^2 - s_0)^2 + 4m_\eta^2(m_b^2 - q^2)}}{2m_\eta^2}. \end{aligned}$$

In the same manner, performing Borel transformation in Eq. (7) and equating it to Eq. (16), we finally get the following sum rule for the penguin form factor

$$f_T^\eta(q^2) = \frac{(m_B + m_\eta)m_b}{2m_B^2 f_B} e^{m_B^2/M^2} (\Pi^T)^B . \quad (17)$$

### 3 Numerical analysis

In this section we present the result of our numerical calculations on penguin form factor  $f_T^\eta(q^2)$ . It follows from Eqs. (16) and (17) that the main input parameters of the sum rule (17) are the  $\eta$  meson wave functions. The explicit expressions of the wave functions  $\varphi_\eta(u)$ ,  $A(u)$ ,  $\varphi_\parallel(\alpha_i)$ ,  $\varphi_\perp(\alpha_i)$ ,  $\tilde{\varphi}_\parallel(\alpha_i)$  and  $\tilde{\varphi}_\perp(\alpha_i)$  are all given in [13]. The other necessary input parameter of the sum rule is the leptonic decay constant  $F_\eta$ . As has already been noted, we will  $\eta$ - $\eta'$  mixing. Furthermore, since  $\eta$  meson is an isoscalar, we have

$$F_\eta^d = F_\eta^u \equiv F_\eta = \frac{f_\eta}{\sqrt{6}} ,$$

where for the leptonic decay constant  $\eta$  meson, we quote the result of a recent analysis which predicts  $f_\eta = 159 \text{ MeV}$  [16]. Moreover, the leptonic decay constant B meson is chosen to have the value  $f_B = 160 \text{ MeV}$  [14, 17].

Having all these input parameters at hand, we proceed carrying out numerical calculations. First of all, since  $M^2$  is an auxiliary Borel parameter, we must find a region of  $M^2$  where a physically measurable quantity be practically independent of it. The lower bound of  $M^2$  is determined by the fact that nonperturbative terms must be subdominant. The upper limit of  $M^2$  is determined by the condition that the higher states and continuum contributions are less than, for example, 30% of the total result. Our numerical analysis shows that both conditions are satisfied in the region  $8 \text{ GeV}^2 \leq M^2 \leq 16 \text{ GeV}^2$ . Moreover, it should be emphasized that light cone QCD sum rule predictions are reliable in the region of momentum transfer square, i.e.,  $q^2 \leq m_b^2 - 2m_b\Lambda$ , where  $\Lambda$  is a typical hadronic scale having the value  $\Lambda \simeq 0.5 \text{ GeV}$ , which yields  $q^2 \lesssim 18 \text{ GeV}^2$ .

In Fig. (1) we present the dependence of the form factor  $f_T^\eta(q^2)$  on the Borel parameter  $M^2$  at different values of momentum transfer square,  $q^2 = 0 \text{ GeV}^2$ ,  $q^2 = 5 \text{ GeV}^2$  and  $q^2 = 10 \text{ GeV}^2$ , at two different choices of the continuum threshold  $s_0 = 35 \text{ GeV}^2$  and  $s_0 = 40 \text{ GeV}^2$ . We observe from this figure that,  $f_T^\eta$  seems to be practically independent of the Borel parameter  $M^2$ , as  $M^2$  varies in the region  $8 \text{ GeV}^2 \leq M^2 \leq 16 \text{ GeV}^2$ .

Having this window for  $M^2$ , we next study the dependence  $f_T^\eta(q^2)$  on  $q^2$ , at three fixed values of the Borel parameter  $M^2 = 8 \text{ GeV}^2$ ,  $M^2 = 12 \text{ GeV}^2$  and  $M^2 = 16 \text{ GeV}^2$ , picked obviously from the above-mentioned working region of  $M^2$ , again at two fixed values of the continuum threshold,  $s_0 = 35 \text{ GeV}^2$  and  $s_0 = 40 \text{ GeV}^2$ , as before. Depicted in Fig. (2) is the dependence of the form factor on the momentum transfer  $q^2$ , which clearly demonstrates that  $f_\eta^T(0) = 0.16 \pm 0.03$ . As we have noted earlier, the prediction by the light cone QCD sum rule is not reliable in the region  $q^2 \geq 18 \text{ GeV}^2$ . In order to extend the present result to whole physical region, we look for some convenient parametrization of the form factor in such a way that in the region  $4m_\ell^2 \leq q^2 \leq 18 \text{ GeV}^2$  this parametrization coincides with

the light cone QCD sum rule prediction. The best parametrization of  $f_T^\eta$  with respect to  $q^2$  can be written in terms of three parameters in the following way

$$f_T^\eta(q^2) = \frac{f_T^\eta(0)}{1 - a_F \frac{q^2}{m_B^2} + b_F \left(\frac{q^2}{m_B^2}\right)^2}. \quad (18)$$

For the values of these parameters for the penguin form factor we obtain  $a_F = 1.08$  and  $b_F = 0.09$ , where the quoted errors can be attributed to the variation in  $s_0$  and  $M^2$ . As has already mentioned earlier, the form factor for the  $B \rightarrow \eta$  transition can be related to the corresponding  $B \rightarrow \pi$  transition form factor through SU(3) symmetry. For example, the value  $f_T^\eta(q^2 = 0) = 0.17$  is obtained using SU(3) symmetry seems to be in quite a good agreement with our prediction of  $f_T^\eta(q^2 = 0)$ . In conclusion, we have calculated the penguin form factor for the  $B \rightarrow \eta \ell^+ \ell^-$  decay in light cone QCD sum rule method, including contributions of wave functions up to twist-4 and mass correction of the  $\eta$  meson.

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## Figure captions

**Fig. (1)** The dependence of the form factor  $f_T^\eta$  on the Borel parameter  $M^2$  at  $q^2 = 0 \text{ GeV}^2$ ,  $5 \text{ GeV}^2$ , and  $10 \text{ GeV}^2$ , at fixed values of the momentum threshold  $s_0 = 35 \text{ GeV}^2$  and  $s_0 = 40 \text{ GeV}^2$ .

**Fig. (2)** The dependence of the form factor  $f_T^\eta$  on the momentum transfer  $q^2$  at  $M^2 = 8 \text{ GeV}^2$ ,  $12 \text{ GeV}^2$ , and  $16 \text{ GeV}^2$ , at fixed values of the momentum threshold  $s_0 = 35 \text{ GeV}^2$  and  $s_0 = 40 \text{ GeV}^2$ .

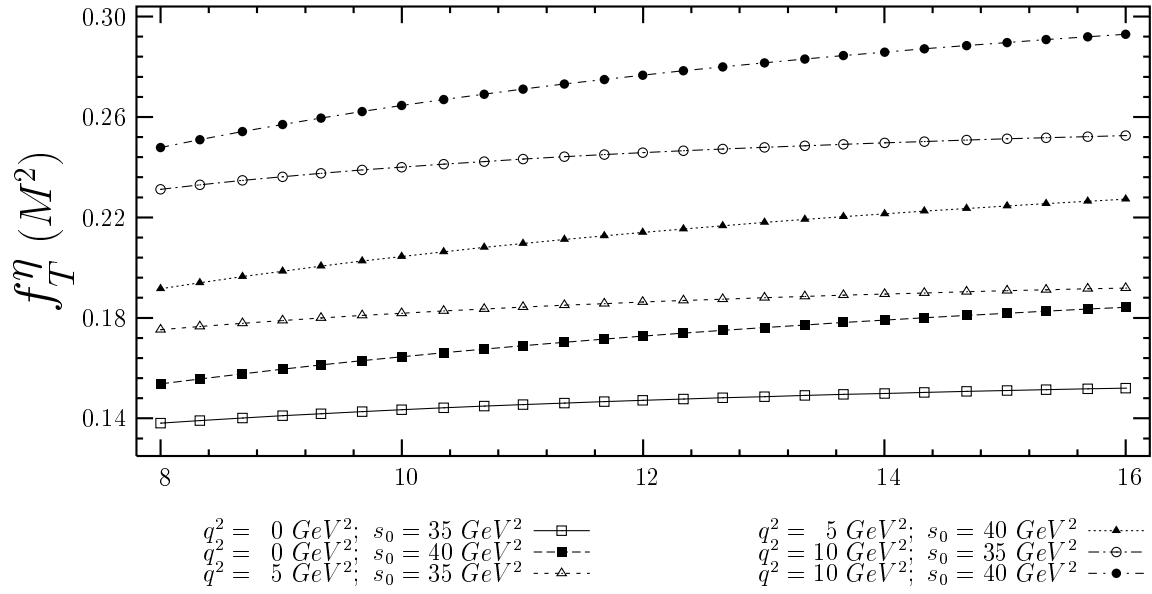


Figure 1:

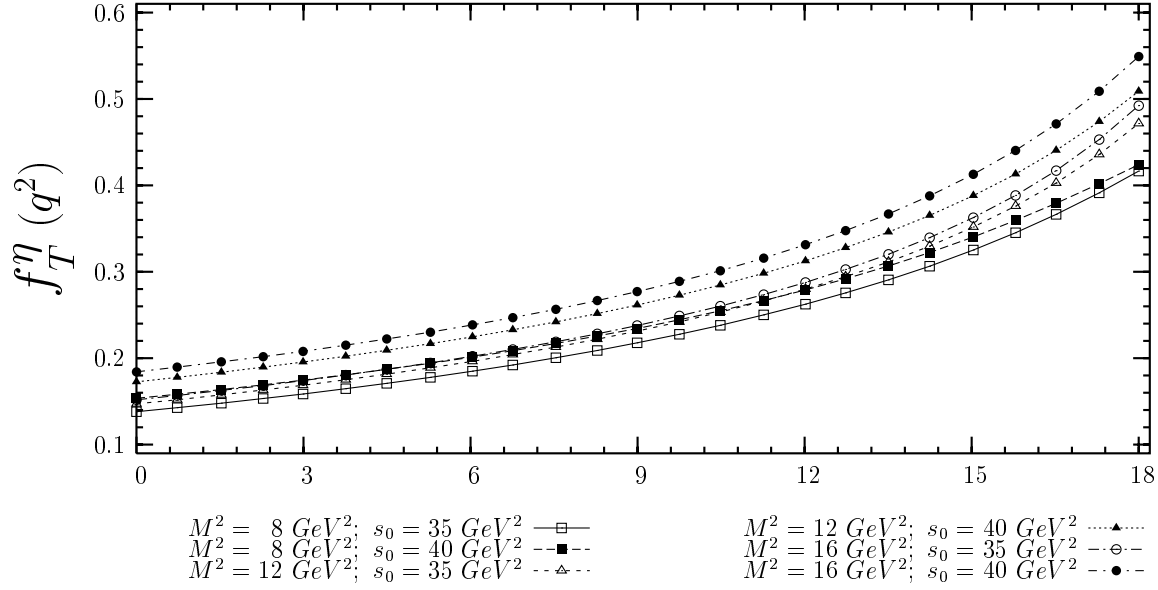
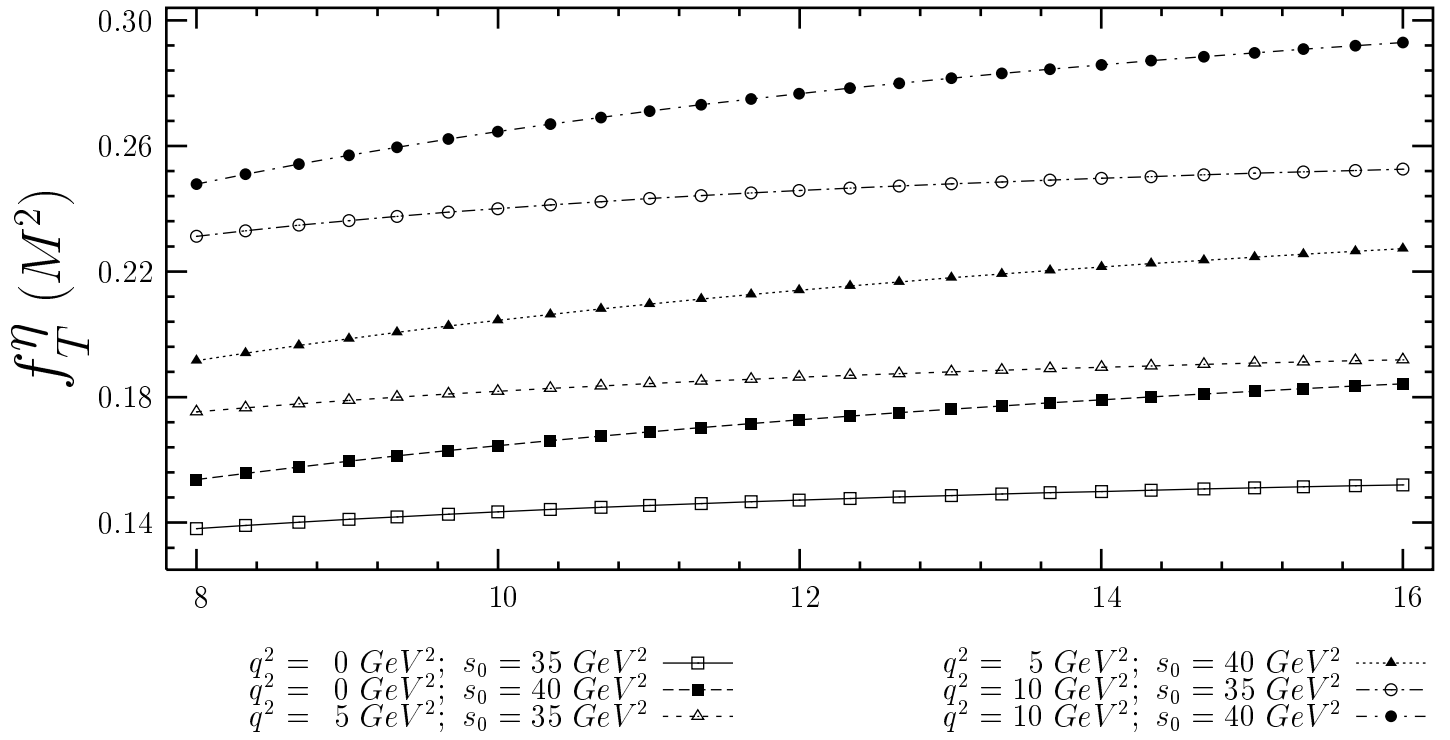
 $q^2 (GeV^2)$ 


Figure 2:

$$M^2 \text{ (GeV}^2\text{)}$$



$q^2 \text{ (GeV}^2\text{)}$

