High energy behavior of quark elastic form factors within the Wilson integral approach: perturbative and nonperturbative contributions*

Igor O. Cherednikov†
Joint Institute for Nuclear Research
RU-141980 BLTP JINR, Dubna, Russia
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The Wilson contour integral approach is applied to resum the soft gluon radiative correctins to the quark form factors in the Sudakov regime. The one-loop order results for the quark-photon (color singlet form factor) and quark-gluon (color non-singlet form factor) vertices are presented. The explicit expressions for the vacuum averaged contour integrals in g^2 accuracy are derived for an arbitrary gauge field. The corresponding one-loop cusp anomalous dimensions are found in the case of perturbative gluon field in arbitrary covariant gauge. It is shown that the gauge dependence drops out from the leading high energy behavior.

In this report, the brief summary of the recent study (within the framework of the world-line contour integrals) of the perturbative and nonperturbative contributions to the soft gluon radiative effects for the quark form factors at large transferred momenta is given.

In sector of the strong interactions of the Standard Model, the elastic form factors of quarks are the most elementary entities exhibiting the large resumed logarithmic corrections at large transferred momentum. These quantities enter into the quark-photon and quark-gluon vertices in the calculations of various QCD processes at the partonic level, and are under active investigation nowadays [1–4].

Here we apply the powerful Wilson contour integral techniques [5–7] to perform the Sudakov resummation of soft gluon radiative corrections to the *quark-vector boson* vertices with large transferred momentum. The world-line formulation of a quantum theory is actively developed not only due to the wide range of applications in the perturbative QFT, but also from the point of view of the various string theories (for a recent review, see [8] and Refs. therein). The attractive feature of this approach is that it does not refer directly to the standard perturbative techniques allowing one to avoid the explicit diagrammatic calculations which use to be very involved in non-Abelian gauge theories. Therefore, this method is equally suitable for the perturbative as well as nonperturbative calculations [9]. In this report, we demonstrate how the Wilson integrals formalism can be applied for arbitrary gauge fields in n-dimensional space-time, in a particular case of the on-shell quark form factor.

Among a number of independent form factors entering the *quark-vector boson* vertex, the only one contains IR singularities and is not power-suppressed in the Sudakov kinematics (see, *e.g.*, [10, 11] and Refs. therein). This form factor can be defined via the amplitude of the elastic quark scattering in an external gauge field—the quark on a mass shell comes from infinity, emits the hard vector boson at the origin, and goes away to infinity:

$$u_i(p)\,[\mathcal{M}_\mu]^a_{ij}\,v_j(p') = F\left[(p-p')^2;\xi\right]\,\bar{u}_i(p)t^a_{ij}\gamma_\mu v_j(p') \quad , \tag{1}$$

where p, p' are the momenta of the in-coming and out-going quarks, and ξ is the covariant gauge-fixing parameter. In this formulae, we write down the vector boson's and quark's color indices $a, (i, j)$ for generality, assuming that the external vector boson is a colored gluon [11]. The case of a photon is trivially restored by means of the replacement of t^a_{ij} with unity matrix. The Sudakov kinematics is determined by the small masses of the quarks and large squared transferred momentum:

$$m^2 = p^2 = p'^2 \quad , \quad (pp') = m^2\cosh\chi \ , \ (pp') >> m^2 \tag{2}$$

Following the Refs. [6, 7], we express the IR sensitive contribution of the resumed soft gluon radiative corrections to the form factor as the vacuum average of two path-ordered exponentials

$$t^a_{ij}\ F\left[Q^2;\xi\right] = \left\langle 0\middle|\mathcal{T}\left\{W_{ii'}t^a_{i'j'}W_{j'j}\right\}\middle|0\right\rangle \ , \tag{3}$$

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†Electronic address: igorch@thsun1.jinr.ru,igor.cherednikov@jinr.ru

where

$$W_{ii'} = \mathcal{P} \exp \left[ig \ t^{\alpha} v_{\mu} \int_{-\infty}^{0} d\sigma \ A^{\alpha}_{\mu}(v\sigma) \right] \Bigg|_{ii'} \ , \ W_{j'j} = \mathcal{P} \exp \left[ig \ t^{\beta} v'_{\mu} \int_{0}^{\infty} d\sigma \ A^{\beta}_{\mu}(v'\sigma) \right] \Bigg|_{j'j} \ . \tag{4}$$

The quark trajectories can be parameterized as:

$$\boldsymbol{In}: \ x_{\mu} = v_{\mu} \ \tau \ \ , \ \ \tau \in [-\infty, 0] \ \ , \ \ v_{\mu} = p_{\mu}/m \ , \ \boldsymbol{Out}: \ y_{\nu} = v'_{\nu} \ \sigma \ \ , \ \ \sigma \in [0, +\infty] \ \ , \ \ v'_{\nu} = p'_{\nu}/m \ . \tag{5}$$

For calculations to the $O(g^2)$ accuracy, it is convenient to present the gauge field propagator $D_{\mu\nu}(z)$:

$$\left\langle 0 \middle| \mathcal{T} A^{\alpha}_{\mu}(x) A^{\beta}_{\nu}(y) \middle| 0 \right\rangle = \mathcal{D}^{\alpha\beta}_{\mu\nu}(x-y) = \delta^{\alpha\beta} D_{\mu\nu}(x-y) \tag{6}$$

in the form [12]:

$$D_{\mu\nu}(z) = g_{\mu\nu} \partial_{\rho} \partial^{\rho} D_1(z^2) - \partial_{\mu} \partial_{\nu} D_2(z^2) \ . \tag{7}$$

First, we present the vacuum averaged Wilson integral Eq. (3) in n-dimensional space-time, for an arbitrary gauge field which can be of any origin, for instance, it may be nonperturbative. The leading order $\sim g^2$ terms stem from the expressions:

$$W^{(1)}_{LO} = - \frac{g^2}{2} \ t^a_{ij} \ C_F \ v_{\mu} v_{\mu'} \ \int_0^{\infty} d\sigma \int_0^{\infty} d\sigma' \ D_{\mu\mu'} \left[v(\sigma - \sigma') \right] \ , \tag{8}$$

$$W^{(2)}_{LO} = - \frac{g^2}{2} \ t^a_{ij} \ C_F \ v'_{\mu} v'_{\mu'} \ \int_{-\infty}^{0} d\sigma \int_{-\infty}^{0} d\sigma' \ D_{\mu\mu'} \left[v'(\sigma - \sigma') \right] \ , \tag{9}$$

and

$$W^{(12)}_{LO} = - \frac{g^2}{2} \ G_F \ t^a_{ij} \ v_{\mu} v'_{\nu} \ \int_0^{\infty} d\tau \int_0^{\infty} d\sigma \ D_{\mu\nu}(v\tau + v'\sigma) \ . \tag{10}$$

The general result in n dimensions and arbitrary covariant gauge, with the gauge field two-point correlator expressed as Eq. (7), reads:

$$W^{(1)} = W^{(2)} = - t^a_{ij} \ \frac{g^2}{2} C_F \ \left[(n-2) D_1(-b_{\perp}^2) + 2 b_{\perp}^2 D'_1(-b_{\perp}^2) + D_2(-b_{\perp}^2) \right] \ , \tag{11}$$

$$W^{(12)}(\chi) = t^a_{ij} \ g^2 G_F \left[\chi \coth\chi \left((n-2) D_1(-b_{\perp}^2) + 2 b_{\perp}^2 D'_1(-b_{\perp}^2) \right) + D_2(-b_{\perp}^2) \right] , \tag{12}$$

up to $O(g^4)$ order terms. Here $C_F = \left(N_c^2 - 1\right)/2N_c$ is the quadratic Casimir operator in the fundamental representation, and G_F is the color factor which is

$$G_F^S = C_F \tag{13}$$

for the external photon field (color singlet form factor) and

$$G_F^{NS} = C_F - \frac{C_A}{2} \ , \ C_A = N_c \ , \tag{14}$$

for the gluon probe (color non-singlet form factor). The space-like transversal vector $\vec{b}_{\perp}$ is introduced in order to regulate the UV divergence.

Performing the standard renormalization procedure within the $\overline{MS}$ scheme, described in detail in Refs. [6, 13, 14], one finds for the *perturbative gluon field* (with λ^2 being the IR cutoff)

$$W^{(1)}_{LO}(\alpha_s, \mu^2/\lambda^2; \xi) = W^{(2)}_{LO}(\alpha_s, \mu^2/\lambda^2; \xi) = t^a_{ij} \frac{\alpha_s}{4\pi} C_F \left(1 - \frac{\xi}{2} \right) \ \ln \frac{\mu^2}{\lambda^2} \ , \tag{15}$$

and

$$W^{(12)}_{LO}(\alpha_s, \chi, \mu^2/\lambda^2; \xi) = -t^a_{ij} \frac{\alpha_s}{2\pi} G_F \left[\chi \coth\chi - \frac{\xi}{2} \right] \ln \frac{\mu^2}{\lambda^2} \ . \tag{16}$$

Here the UV-normalization point is taken to be $\mu^2 = 4\vec{b}^{-2}$. The total one-loop contribution to the form factor is given by the sum

$$F_{LO}\left[Q^2; \xi\right] = W_0 + 2W^{(1)}_{LO}\left(\alpha_s, \frac{\mu^2}{\lambda^2}\right) + W^{(12)}_{LO}\left(\alpha_s, \frac{\mu^2}{\lambda^2}, \chi\right) + O(\alpha_s^2) \ . \tag{17}$$

The high-energy asymptotic behavior of the form factor is determined by the (in general, gauge-dependent) cusp anomalous dimension which is derived from the renormalization of the Wilson integral (3) [6]:

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} + \delta(\alpha_s, \xi) \xi \frac{\partial}{\partial \xi}\right) \ln F(\chi) = -\frac{1}{2} \Gamma_{cusp}\left[\alpha_s(\mu^2); \chi\right] \ . \tag{18}$$

This anomalous dimension reads in our case:

$$\Gamma_{cusp}\left[\alpha_s(\mu^2); \chi\right] = \frac{\alpha_s}{\pi} \left[G_F \chi \coth\chi + \frac{C_F - G_F}{2} \xi - C_F \right] + O(\alpha_s^2) \ . \tag{19}$$

It is easy to see that for the color singlet case, the gauge dependent term is cancelled, as it should be for the gauge invariant quantity.

Taking into account that the cusp anomalous dimension is linear in $\ln Q^2$ at large χ [6]:

$$\Gamma_{cusp}(C_\chi; \alpha_s) = \ln q^2 \ \Gamma_{cusp}(\alpha_s) + O(\ln^0 q^2) \ , \tag{20}$$

one obtains the leading (double-logarithmic) behavior of the quark form factor from the corresponding evolution equation:

$$F\left[Q^2\right] = \exp\left(-G_F \int_{\lambda^2}^{Q^2} \frac{d\mu}{2\mu} \ \ln \frac{Q^2}{\mu} \ \frac{\alpha_s(\mu)}{\pi} + \text{NLO terms} \right) F\left[\alpha_s(\lambda^2)\right] =$$

$$= \exp\left[-\frac{2G_F}{\beta_0} \ln q^2 \ln\ln q^2 + O(\ln q^2) + \text{NLO terms} \right] F\left[\alpha_s(\lambda^2)\right] \ , \tag{21}$$

where $q^2 = Q^2/\Lambda^2_{QCD}$ is dimensionless variable. In any case, the dependence from the gauge-fixing parameter ξ drops out of the leading logarithmic expression for Γ_{cusp}, and yields no influence on the main asymptotics. Thus we find that the cusp anomalous dimension of non-singlet quark form factor is negative:

$$\Gamma^{NS}_{cusp}\left[\alpha_s(\mu^2); \chi\right] =$$

$$= \frac{\alpha_s}{\pi} \left[\left(C_F - \frac{C_A}{2} \right) \chi \coth\chi + \frac{C_A}{4} \xi - C_F \right] + O(\alpha_s^2) = -\frac{\alpha_s}{\pi} \left[\frac{1}{2N_c} \chi \coth\chi - \frac{N_c}{4} \xi - \frac{N_c^2 - 1}{2N_c} \ , \right] \tag{22}$$

that corresponds to the enhancement of the resumed Sudakov logarithms at large Q^2, while in the singlet case it is positive:

$$\Gamma^{S}_{cusp}\left[\alpha_s(\mu^2); \chi\right] =$$

$$= \frac{\alpha_s}{\pi} \left[C_F \chi \coth\chi + (C_F - C_F) \xi - C_F \right] + O(\alpha_s^2) = \frac{\alpha_s}{\pi} \frac{N_c^2 - 1}{2N_c} \left[\chi \coth\chi - 1 \right] + O(\alpha_s^2) \ , \tag{23}$$

that yields the well known Sudakov suppression.

The formulas (11, 12) can be applied directly to compute the leading contributions to the Wilson vacuum average for an arbitrary $SU(N_c)$ gauge field in any covariant gauge, in n-dimensional space-time. For instance, the nonperturbative instanton-induced corrections (within the framework of the Instanton Liquid Model (ILM) of QCD vacuum [15]), as well as IR-renormalon effects, are evaluated within this approach in Refs. [12].

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