RIGA TECHNICAL UNIVERSITY

Vladislav Yevstignejev

Application of the Complete Bifurcation Groups Method for Analysis of Strongly Nonlinear Oscillators and Vibro-Impact Systems

Abstract of the Dissertation

Riga 2008

RIGA TECHNICAL UNIVERSITY
Transport and Theoretical Engineering Faculty
Institute of Mechanics

Vladislav Yevstignejev

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Abstract of the Dissertation

Scientific Supervisor
Dr. habil. sc. ing., professor
M. ZAKRZHEVSKY

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DISSERTATION
SUBMITTED TO RIGA TECHNICAL UNIVERSITY
FOR OBTAINING DOCTOR'S DEGREE

The dissertation submitted for obtaining a Doctor's Degree in Engineering Sciences will be defended in public at the Institute of Mechanics of Riga Technical University.

OFFICIAL OPPONENTS:

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CERTIFICATION

I hereby certify that I am the author of the present dissertation, which is submitted for consideration at Riga Technical University for obtaining a Doctor's Degree in Engineering Sciences. The dissertation has not been submitted for obtaining a scientific degree at any other university.

Vladislav Yevstignejev

Data

The dissertation is written in the Russian language and comprises an introduction, 7 chapters, conclusion, bibliography (156 sources) and three appendixes, all altogether 146 pages.

General characterization of the thesis

The present dissertation paper “Application of the Complete Bifurcation Groups Method for Analysis of Strongly Nonlinear Oscillators and Vibro-Impact Systems” concentrates to application of the new method of complete bifurcation groups, which is worked out in a group of professor M.V.Zakzhevsky and which is (on our point of view) a perspective method for problems of nonlinear dynamics, for global analysis of strongly nonlinear oscillators and vibro-impact systems. There is shown in the dissertation paper that application of method of complete bifurcation groups allows to implement a global bifurcation analysis of models of strongly nonlinear typical oscillators and vibro-impact systems, and find new bifurcation groups with unknown before periodical regular and chaotic regimes and new nonlinear effects, what is shown on typical piece-wise linear and smooth nonlinear systems with one or two degrees-of-freedom, as well as there is shown an opportunity to use new results in vibroengineering tasks. The thesis consists of introduction, seven chapters, conclusion, bibliography (156 sources) and three appendixes.

Urgency of the thesis

Design of machines, mechanisms and appliances requires modern knowledges in the region of nonlinear dynamics and nonlinear oscillation theory. For today there are worked out a lot of typical nonlinear dynamic models, which are being broadly used for analysis and synthesis of various technical systems and technological processes. They are used, for example, for calculations of vehicles (air, auto, railway) vibrations, of vibro-insulation appliances, oscillation of solid systems, various mechanisms and appliances with impact interaction, gears, electromechanical and electric appliances and many other systems.

However, in the present time, there is a situation when many important regimes, even in typical nonlinear dynamic systems, remains unnoticed while modeling by traditional analytic or numeral methods, despite modern opportunities of application of high-speed computers.

This conclusion is right even for simple nonlinear models, which are, for example, Duffing equation, piece-wise bilinear and trilinear models, system with one-sided or two-sided impact interactions etc.

In the present thesis there are analyzed the reasons for imperfection of traditional methods of nonlinear dynamics and laid out the basis of new alternative approach, which got the name “complete bifurcation groups method” and presented new qualitative research results, which are got by using the method mentioned above.

Basic statements of complete bifurcation groups method for nonlinear dynamic systems analysis presents scientific and practical interest, as existing methods don’t allow to find systematically all existing regimes in a system, and that, in its turn, keeps back from practical application of nonlinear effects, existing in such systems. This circumstance defines the urgency of the present dissertation paper.

Objective of dissertation paper

An objective of the dissertation paper is to show that using of the complete bifurcation groups method allows to implement global bifurcation analysis of strongly nonlinear oscillators and vibro-impact systems and to find unknown before forced regular and chaotic oscillations on examples of simple typical piece-wise linear and smooth systems with one or two degrees-of-freedom and to show an opportunity to use new (found regimes) results in tasks of vibroengineering.

Scientific novelty and main results

In the present dissertation paper there are obtained the following main new qualitative results:

1. Shown that using of the complete bifurcation groups method and on its base worked out complex approach for qualitative global analysis of forced oscillations in strongly nonlinear oscillators and vibro-impact systems allows to implement full qualitative topological analysis of various bifurcation groups and to find new periodic and chaotic regimes.
2. Using of the complete bifurcation groups method for investigation of forced oscillations on the example of simple oscillation system with bilinear elastic characteristics allowed to find new bifurcation groups with complex protuberances, with rare periodical and chaotic regimes.
3. There is studied an interaction between various existing bifurcation groups (1T and sub-harmonic nT) on the example of oscillation system with bilinear elastic characteristic. Obviously for the first time, there are built bifurcation maps of existing various regimes on a plane of two parameters (frequency of excitation – coefficient of dissipation). There is shown that existing subgroups with unstable periodic infinitum (UPI) in a main bifurcation group always leads to chaotic behavior of a system: chaotic attractor and transient chaos.
4. On the base of complete bifurcation groups method there are built typical bifurcation diagrams for vibro-impact systems with one-sided impact. Comparison of results of implemented bifurcation analysis of dynamics of stiff and soft vibro-impact systems has shown that using of hypothesis of momentary impact leads to qualitative mistakes even at high values of stiffness of stop.
5. At the using of complete bifurcation groups method for system with two degrees-of-freedom there is found new nonlinear effect. Particularly for system with three equilibrium positions there is found unknown before bifurcation group of sub-harmonic regimes – "desert" isle, which consists just of unstable periodical regimes. On this unstable isle at parameters changing there born rare stable periodical and chaotic regimes.
6. From analysis of various oscillators and vibro-impact systems with bifurcation groups with rare attractors of tip type, obviously for the first time, is obtained that there are always its own chaotic attractors in those groups.

7. An opportunity of practical using of new research results in tasks of nonlinear vibroengineering: vibromoving, vibromixing, vibropolishing, vibrowelding etc. is shown. There is analyzed the influence of electro-drive at the using of phenomenon of multiplicity.

Thereby in the thesis there is shown that using of the complete bifurcation groups method allows to implement global bifurcation analysis of strongly nonlinear oscillators and vibro-impact systems, to find new nonlinear effects and bifurcation groups, unknown before periodic and chaotic regimes, what is shown on typical piese-wise linear and smooth systems with one or two degrees-of-freedom and shown an opportunity of using new results in tasks of vibroengineering.

Practical value of the thesis

The obtained results about the using of the complete bifurcation groups method for analysis of strongly nonlinear oscillators and vibro-impact systems have, on our point of view, huge practical value.

First of all, those results allows engineer to understand the nature of undesirable stable periodic or chaotic regimes, which appears at definite parameters and to explain existence and birth of chaotic regimes and rare attractors.

Secondly, the results of the dissertation paper may be used in tasks of vibroengineering, because particularly chaotic oscillators allows to increase effectiveness of various processes at processing of materials: cutting, milling, polishing, etc; to implement complicated movement regimes (vibro-transformer of movement) at minimal expenses, for example, in vibro-stands for checking of fatigue resistance of constructions (aviation, appliances construction); to decrease dissipation in a system, etc.

Furthermore, obtained results (typical bifurcation diagrams, bifurcation maps) are used in studying process at studying a course "Nonlinear dynamics. Introduction" in Riga Technical university and for student's research works.

Reliability of results

Reliability of results is ensured by using of modern methods for analysis of strongly nonlinear systems and worked out systematic approach for investigation of forced oscillations in nonlinear dynamic systems, by comparing of exact analytical and numerical methods of calculations of piece-wise linear systems, by coincidence of new research results obtained by minimum three methods.

Publications

On the theme of dissertation paper there are 15 scientific publications.

Approbation of thesis

Different parts of dissertation paper was reported on International Student Conference "Nonlinear Dynamics, Chaos, Catastrophes and Control" (Riga, 2001); on XIII International

Symposium “Dynamics of vibro-impact (strongly nonlinear) systems” (Moscow, 2001); on II world congress of Latvian scientists (2001); on International Scientific Conferences of RTU (Riga, 2001, 2002, 2008); on International Conference “Vibroengineering - 2001” (Kaunas, 2001); on International Scientific School “Nonlinear Dynamics, Chaos, Catastrophes and Control” (Jurmala-Riga, 2002); on XIV International Symposium “Dynamics of vibro-impact (strongly nonlinear) systems” (Moscow, 2003); on 1st International Conference on Vibro-Impact Systems ICoVIS (Laughborough, 2006); on XV International Symposium “Dynamics of vibro-impact (strongly nonlinear) systems” (Moscow, 2006); on XXXV International School-Conference “Advanced Problems in Mechanics” (St. Petersburg, 2007); on International Scientific Conference “Modern Achievements of Science and Education” (Natania, 2007); on International Scientific Conference “Modern Achievements of Science and Education” (Hurgada, 2007); on International Scientific Conference “Chaotic Modeling, Simulation and Applications” (Chania, 2008); regularly from 2000 till 2007 on seminars of Institute of Mechanics of RTU.

Statement on protection

In the dissertation paper there is presented a statement that using of the complete bifurcation groups method allows to implement global bifurcation analysis of strongly nonlinear oscillators and vibro-impact systems, find new nonlinear effects and bifurcation groups unknown before periodic and chaotic regimes, what is shown on typical piece-wise linear and smooth systems with one or two degrees-of-freedom (which have theoretical and practical value) and shown an opportunity of using new results in tasks of vibroengineering.

Structure and volume of thesis

Dissertation paper consists of introduction, seven chapters, conclusion, bibliography (156 sources) and three appendixes. Volume of the thesis is 146 pages of text, 64 figures.

Content of thesis

The complete bifurcation groups method allows to implement global bifurcation analysis of strongly nonlinear oscillators and vibro-impact systems.

In introduction the urgency of the theme of the dissertation is proved and the overall aim of the thesis is formulated: to show that using of the complete bifurcation groups method allows to implement global bifurcation analysis of strongly nonlinear oscillators and vibro-impact systems and to find new bifurcation groups and unknown before forced regular and chaotic regimes on examples of simple typical piece-wise linear and smooth systems with one or two degrees-of-freedom. There is presented short summary of the thesis and basic scientific-practical results.

In the first chapter there are investigated state of the issue and formulated task of dissertation paper. Basis of theory of nonlinear oscillators is worked out by such great scientists as I.Newton, A.Poincaré, A.M.Lyapunov, J.D.Birkhoff, van der Pol, A.A.Andronov, N.M.Krylov, N.N.Bogolyubov, L.I.Mandelshtamm, C.Hayashi and others. Essential contribution in development of nonlinear dynamics was made by modern scientists, such as J.G.Panovko, I.I.Blehmann, M.Z.Kozlovsky, V.I.Babitsky, J.I.Neimark, A.J.Kobrinsky, F.Moon, Ph.Holms, K.Ragulskis, V.Ragulskiene, R.Bansevičius, L.M.Litvin, F.Peterka, M.I.Feigin, P.S.Landa, A.L.Fradkov, Y.Ueda, D.I.Trubetskov, V.S.Anischenko, V.K.Astashev, K.V.Frolov, A.A.Zevin, M.F.Dimentberg, J.M.T.Thompson, H.B.Stewart, Albert C.J.Luo, D.A.Indeycev, E.Kreuzer, G.A.Leonov, L.Chua, S.J.Hogan, W.Szemplińska-Stupnicka, E.Mosekilde, J.Awrejcewicz, T.Kapitaniak, M.Wiercigroch, G.Stepan, L.Pust F.L.Chernousko, L.I.Manevich, N.A.Magnitsky, G.G.Malenetsky, Jon J.Thomsen, Dick H. van Campen, G.Rega, H.Troger, S.W.Shaw and many others. Serious researches about nonlinear dynamic system are performed also in Riga Technical university by: E.Lavendelis, J.Vība, S.L.Tsifansky, J.Auziņš, A.Januševskis and others. The present thesis is one of the dissertation papers written in the scientific group of Institute of Mechanics on the course "Nonlinear dynamics, chaos, catastrophes and control" under guidance of M.V.Zakrzhevsky and is a continuation and development of results of doctor dissertations of V.Frolov (1997), R.Smirnova (2002), I.Schukin (2005).

Well-known that at variety of modern methods, approximate analytical as well as numerical, by using high-speed computers, for investigation of nonlinear systems, many important regimes remains unnoticed. This conclusion is right even for simple nonlinear models, which are, for example, Duffing equation, piece-wise bilinear and trilinear models, system with one-sided or two-sided impact interactions and others.

Thus in the first chapter there are analyzed reasons of deficiency of nonlinear dynamics traditional methods, are presented basic theses of new alternative approach, worked out by M.V.Zakrzhevsky – complete bifurcation groups method, and formulated set of tasks for dissertation paper: to show that using of the complete bifurcation groups method allows to implement global bifurcation analysis of strongly nonlinear oscillators and vibro-impact systems and to find new bifurcation groups and unknown before forced regular and chaotic regimes.

In accordance with tasks, **in the second chapter** there is described complete bifurcation groups method and methodology of its application for global complex analysis of simple piece-wise linear and smooth systems with polynomial characteristics with one or two degrees-of-freedom.

Complete bifurcation groups method consist in direct numerical modeling of originate existing nonlinear model, that is, without its simplification. Under complete bifurcation groups method we understand complex of approaches to analysis of dynamic systems, which involves the following procedures: at fixed system parameters – search of all periodic stable and unstable regimes and bifurcation subgroups with unstable periodic infinitiums (UPI) (fig.2) on plane of states, constructing of regimes' basins of attraction on plane of states; at varying of system parameters – constructing of bifurcation diagrams and bifurcation maps. Special importance in the method is for continuation of parameter solution (in one-parameter task) along solution branch of definite regime (not along parameter), and that allows to find new unknown before stable regimes in broadly used dynamic models of strongly nonlinear oscillation systems.

Complete bifurcation groups method is worked out recently and tied to it complex of approaches aren't worked out entirely to the present time. That is why in the present chapter there are worked out basic recommendations on methodology of using complete bifurcation groups method for strongly nonlinear oscillators and vibro-impact systems.

Basic new opportunities related to complete bifurcation groups method are shown in the third and fourth chapters on examples of forced oscillators in simple piece-wise linear oscillation system – bilinear, i.e., system, elastic characteristic of which has two linear sub-regions. With a help of complete bifurcation groups method can be made global analysis with complicated protuberances, with rare periodic and chaotic regimes.

In spite of simplicity, bilinear system is strongly nonlinear. Furthermore, that dynamic model has broad usage in investigation of various mechanical systems. Equation of investigated model is (fig.1.):

$$m\ddot{x} + b\dot{x} + \begin{cases} c1x & if\ x \le 0 \\ c2x & if\ x > 0 \end{cases} = h1\cos(\omega t + \varphi 0)\,, \qquad (1)$$

where x – generalized coordinate; m - mass; b – coefficient of linear dissipation; $c1$, $c2$ – coefficients of nonlinear elastic characteristic stiffness on linear regions; $h1$, ω, $\varphi 0$ – amplitude, frequency and phase of excitation.

In the third chapter there is performed bifurcation system analysis on the basis of complete bifurcation groups method, described in equation 1, at varying of initial conditions and few parameters of the system. For solving the task of global analysis as varying parameters was: coefficient of linear dissipation b, coefficient of nonlinear elastic characteristic stiffness on linear sub-region $c2$ and frequency of excitation ω.

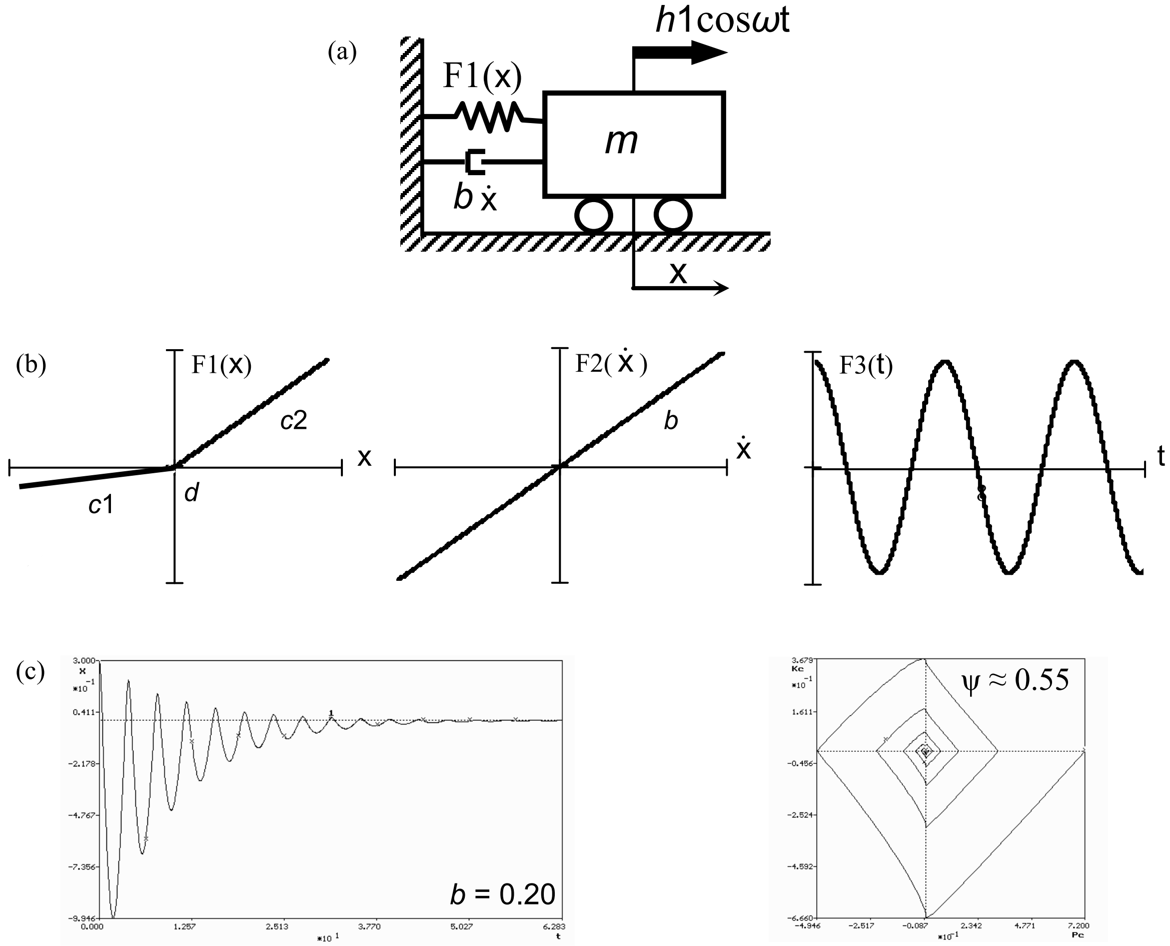

Fig. 1. Dynamical model and characteristics of system with bilinear elastic characteristic and linear dissipation at harmonic excitation (eq.1): (a) dynamic model; (b) characteristics of elastic and dissipation forces and diagram of external harmonic excitation; (c) die-away curve and energy diagram of free oscillators, coefficient of losses $\psi = (\Pi_1 - \Pi_0) / \Pi_0$. Parameters of system: $m = 1$, $c1 = 1$, $c2 = 16$, $d = 0$, $b = 0.20$, $k = 7$

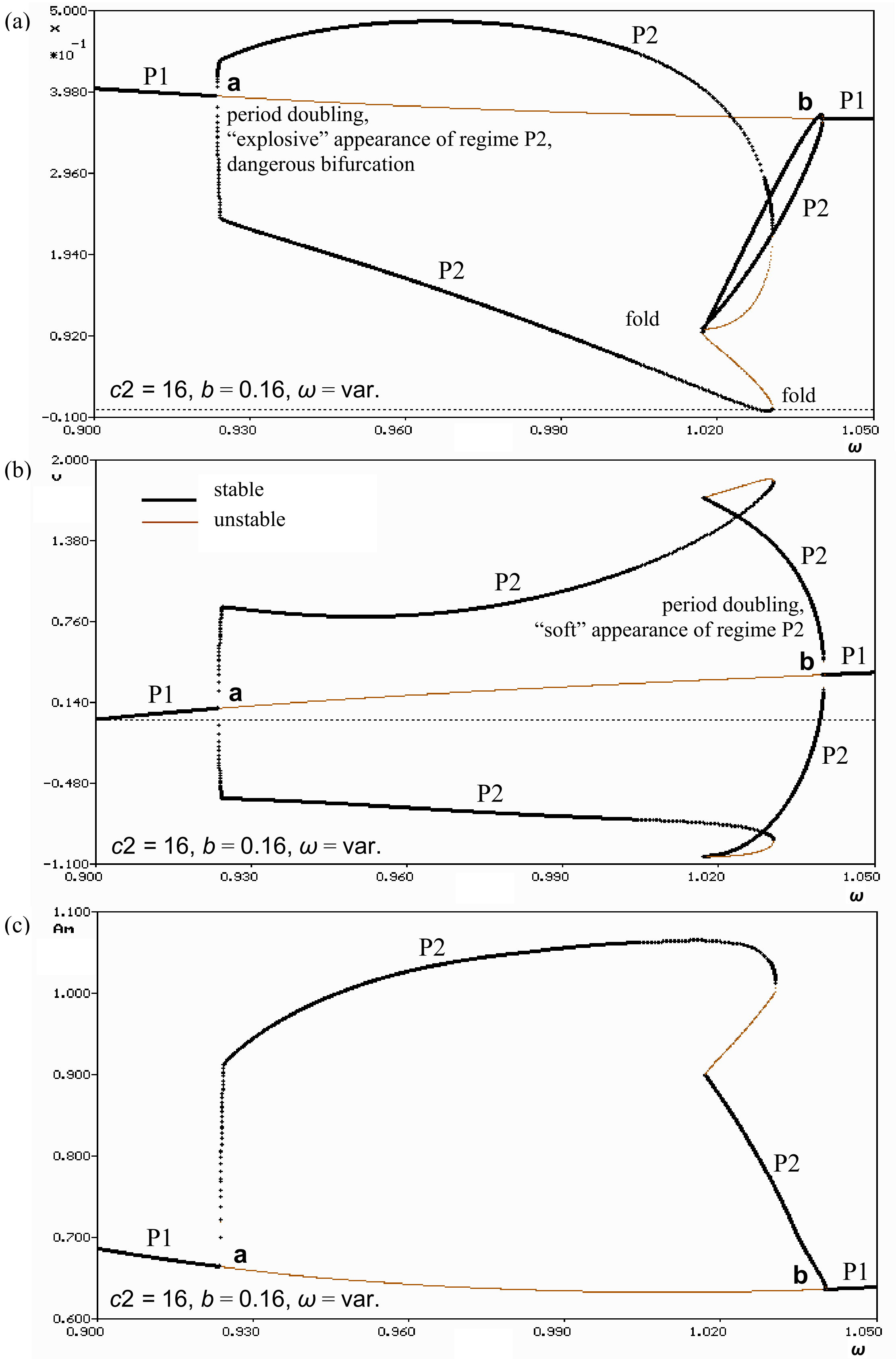

Fig. 2. Example of bifurcation group 1T with simple protuberance (a,b), which has two supercritical period doubling bifurcation and hysteresis with two folds. Complete bifurcation diagrams for system with bilinear elastic characteristic and linear dissipation at harmonic excitation (eq.1): (a), (b) coordinates x, v of fixed point and (c) amplitude of oscillators Am of periodic regime vs excitation frequency ω. Parameters: $m = 1$, $c1 = 1$, $c2 = 16$, $d = 0$, $b = 0.16$, $h1 = 1$, $\varphi 0 = 0$, $k = 7$, $\omega =$ var.

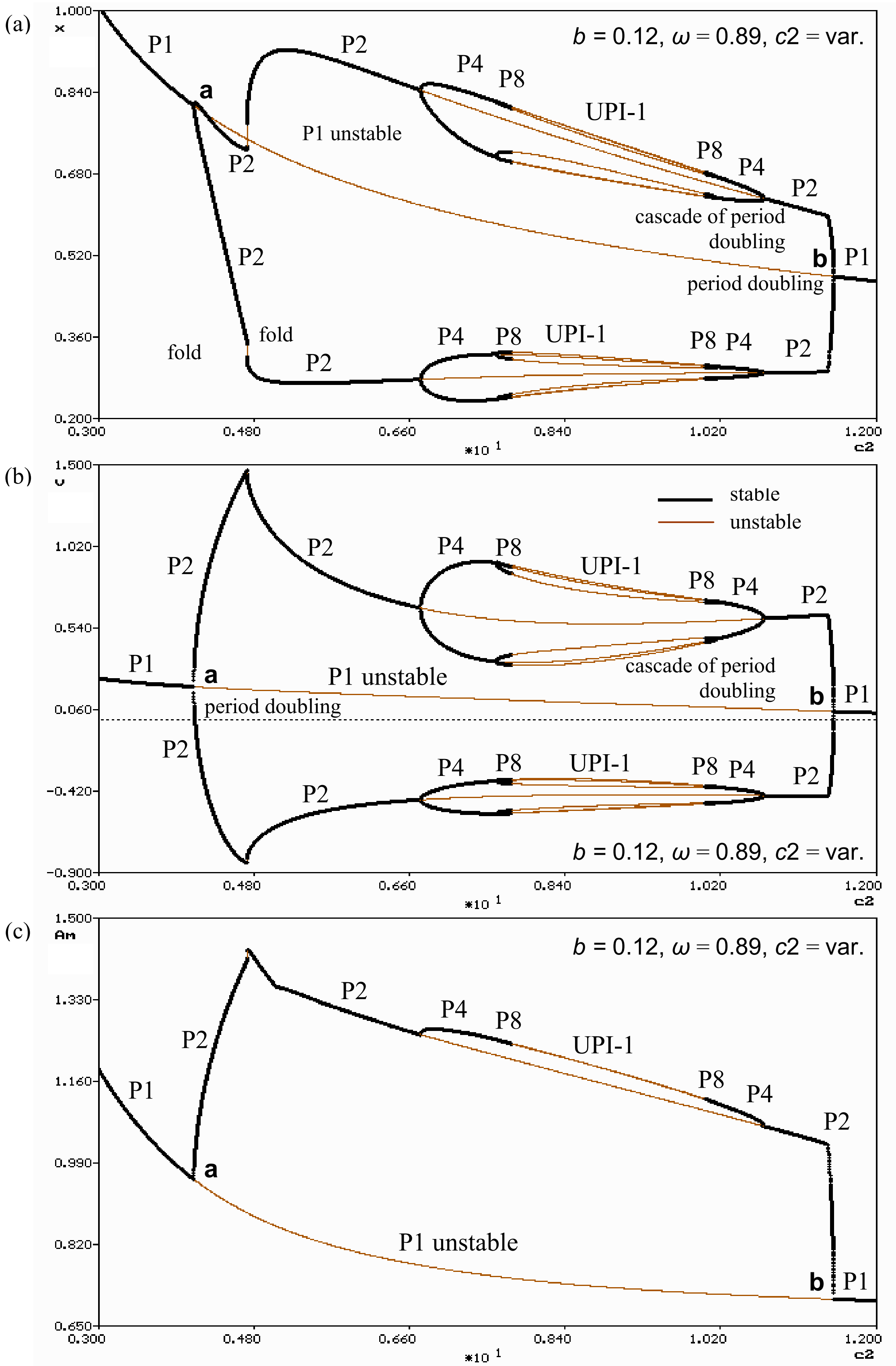

Fig. 3. Example of bifurcation group 1T with protuberance (a,b), which has unstable periodic infinitium UPI-1. Complete bifurcation diagrams for system with bilinear elastic characteristic and linear dissipation at harmonic excitation (eq. 1): (a), (b) coordinates x, v of fixed point and (c) amplitude of oscillators Am of periodic regime vs stiffness coefficient *c*2. Parameters: *m* = 1, *c*1 = 1, *d* = 0, *b* = 0.12, *h*1 = 1, *ω* = 0.89, *φ*0 = 0, *k* = 7, *c*2 = var.

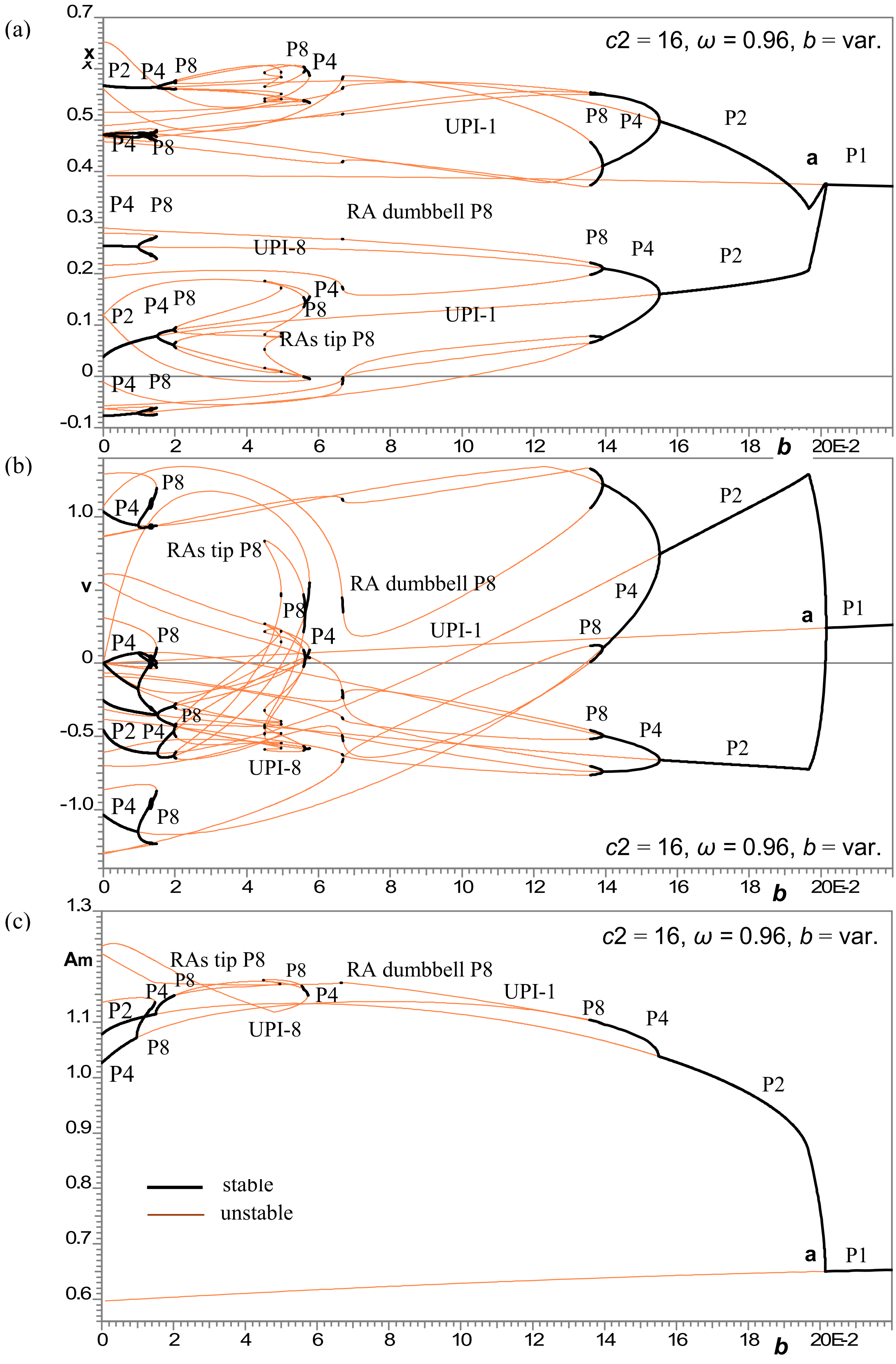

Fig. 4. Example of complicated bifurcation group 1T, which has rare attractors P8 of dumbbell and tip type, complicated protuberances, few various regions with UPI. There is shown on figure 4 *one* bifurcation group. Shown complete bifurcation diagrams for system with bilinear elastic characteristic and linear dissipation at harmonic excitation (eq. 1): (a), (b) coordinates x, v of fixed point and (c) amplitude of oscillators Am of periodic regimes vs linear dissipation coefficient *b*. Investigated regimes till P8 including. Parameters: $m = 1$, $c1 = 1$, $c2 = 16$, $d = 0$, $h1 = 1$, $\omega = 0.96$, $\varphi0 = 0$, $k = 7$, $b =$ var.

The particular results of this chapter are shown on fig. 2-4: example of simple bifurcation group, with simple protuberance; example of bifurcation group with unstable periodic infinitiums (UPI); example of complicated bifurcation group with few regions UPI and rare attractors of dumbbell and tip type.

Thus it is shown that application of method of complete bifurcation groups to investigation of forced oscillators on example of simple oscillation system with bilinear elastic characteristic allowed to find new bifurcation groups with complicated protuberances, with rare periodic and chaotic regimes.

Various complete bifurcation groups interaction issues, existing in the same range of parameters are investigated **in the fourth chapter**. Separate parts of complete bifurcation group 1T can be the only bifurcation group at definite meanings of varying parameter, in general there is coexistence of bifurcation group 1T and bifurcation groups of sub-harmonic regimes.

Coexistence of few bifurcation groups leads to:

1. at coexistence of stable periodic attractors of various groups there can be observed the fact of multiplicity – at equal parameters of system plane of states divides to regions of attraction of coexisting regimes;
2. at coexistence of stable periodic attractor of one bifurcation group with unstable periodic infinitiums (UPI) and other bifurcation group, there are few opportunities possible:
    - multiplicity – coexistence of stable periodic attractor with chaotic attractor;
    - globally stable periodic attractor with chaos transient process to it;
3. analogically the last case, coexistence of unstable periodic infinitiums in both bifurcation groups leads to:
    - multiplicity – coexistence of few chaotic attractors;
    - one globally stable chaotic attractor.

Example of coexistence of stable periodic attractors of one bifurcation group with unstable periodic infinitiums (UPI) and other bifurcation group is shown on the fig. 5. On bifurcation diagram there is shown coexistence of stable sub-harmonic regime P3 and region with UPI of basic regime P1. Here it is a case of multiplicity – coexistence of stable periodic attractor P3 and chaotic attractor Chaos-1, what is demonstrated on fig.6. On this figure there is presented dividing of the plane of states into basins of attraction of two regimes.

As a generalization of information about coexistence of various complete bifurcation groups serves bifurcation maps, which divides plane of two system parameters into regions with qualitatively equal dynamical behavior (fig. 7, 8).

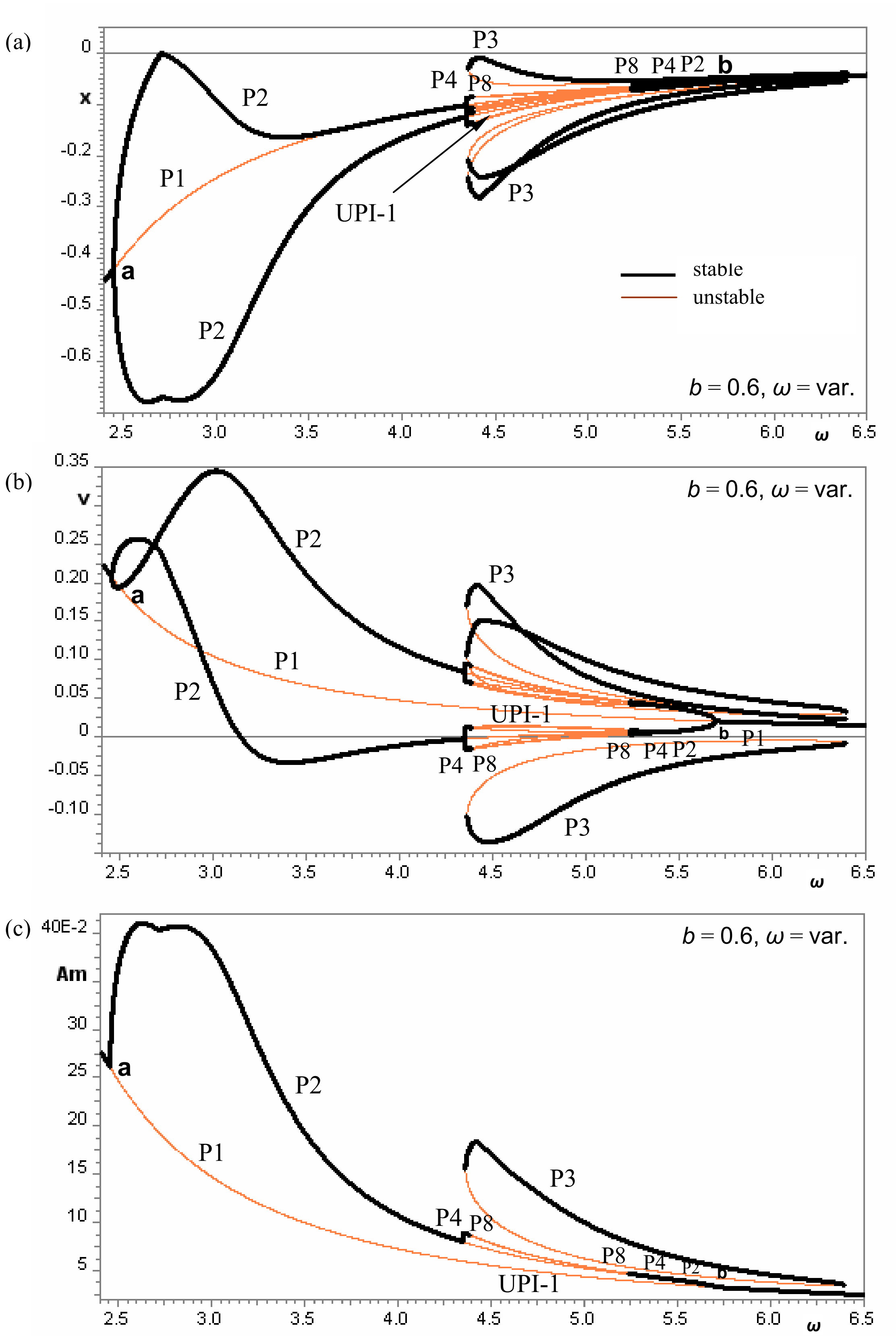

Fig. 5. Example of coexistence of two bifurcation groups: group 1T with protuberance (a,b) and chaotic behavior and sub-harmonic group 3T with regular dynamic behavior. Complete bifurcation diagrams for system with bilinear elastic characteristic and linear dissipation at harmonic excitation (eq. 1): (a), (b) coordinates x, v of fixed point and (c) amplitude of oscillators Am of periodic regime vs excitation frequency ω. System parameters: $m = 1$, $c1 = 1$, $c2 = 16$, $d = 0$, $b = 0.6$, $h1 = 1$, $\varphi 0 = 0$, $k = 7$, ω = var.

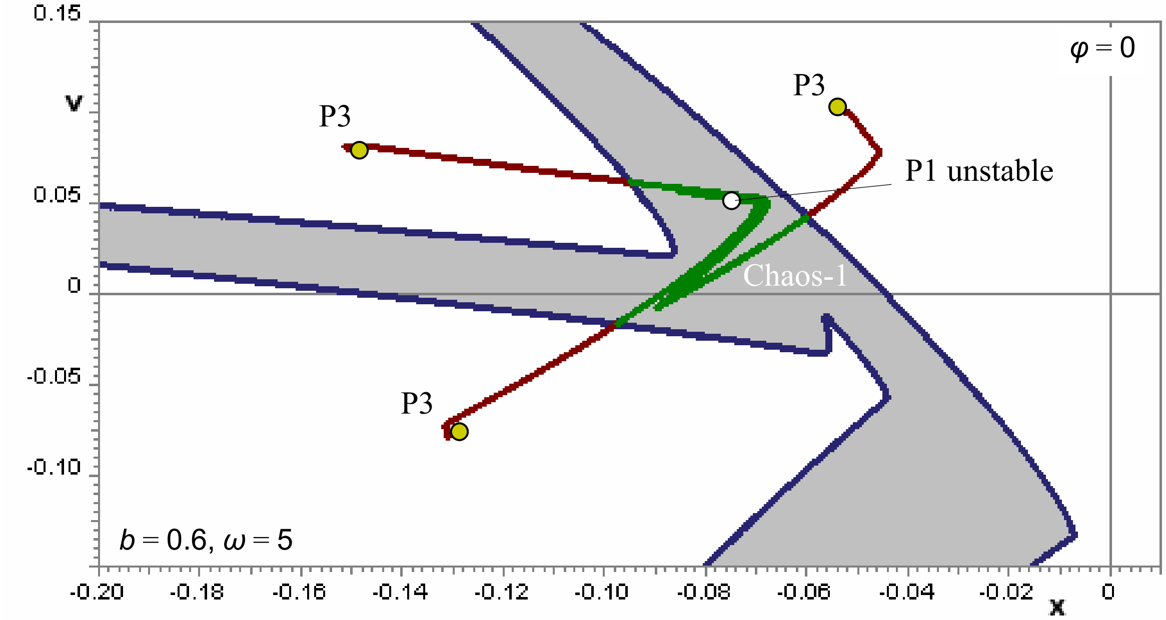

Fig. 6. Basins of attraction of periodic sub-harmonic P3 and chaotic Chaos-1 regimes, related to bifurcation groups 3T and 1T accordingly (fig. 5) for system with bilinear elastic characteristic and linear dissipation at harmonic excitation (eq. 1). Basins of attraction are built on the basis of mapping from line in straight and reverse time in surrounding of saddle point. White color corresponds to sub-harmonic regime P3, grey color – to chaotic regime Chaos-1. Parameters: $m = 1$, $c1 = 1$, $c2 = 16$, $d = 0$, $b = 0.6$, $h1 = 1$, $\omega = 5$, $\varphi 0 = 0$, $k = 7$

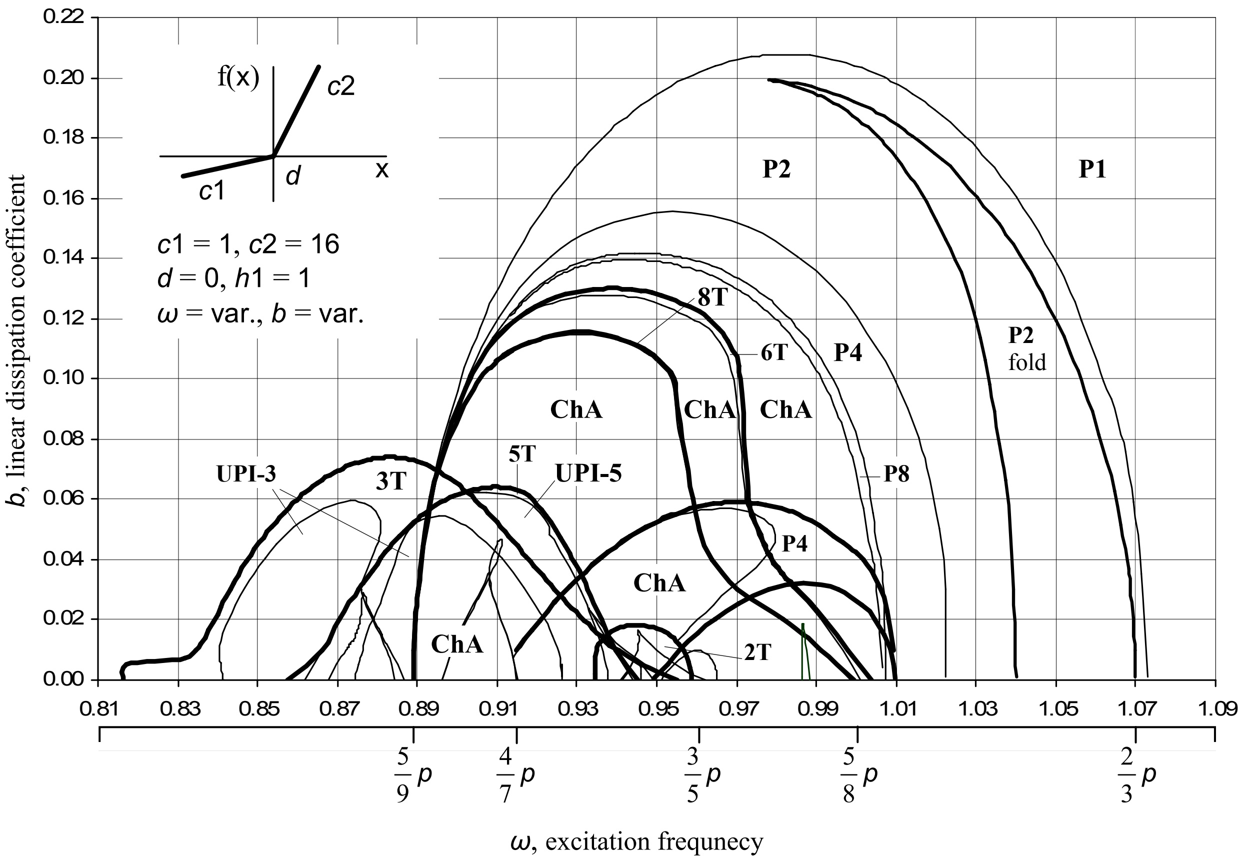

Fig. 7. Bifurcation map with regions of various dynamic behavior of system with bilinear elastic characteristic and linear dissipation at harmonic excitation (eq. 1) on plane of parameters ω-b. Pre-resonance region ($p = 1.6$). Parameters: $m = 1$, $c1 = 1$, $c2 = 16$, $d = 0$, $h1 = 1$, $\varphi 0 = 0$, $k = 7$, ω = var., b = var.

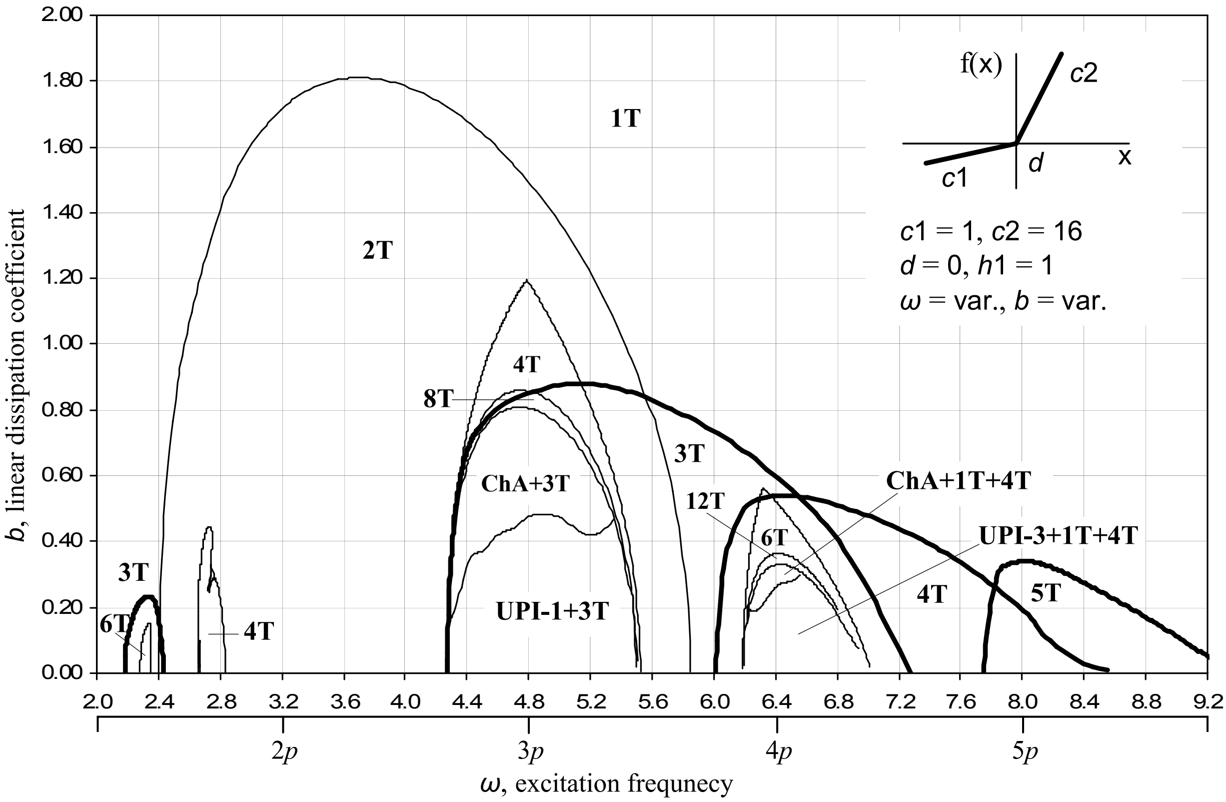

Fig. 8. Bifurcation map with regions of various dynamic behaviour of system with bilinear elastic characteristic and linear dissipation at harmonic excitation (eq. 1) on region of parameters ω-b. Post-resonance region (p = 1.6). Parameters: $m = 1$, $c1 = 1$, $c2 = 16$, $d = 0$, $h1 = 1$, $\varphi 0 = 0$, $k = 7$, ω = var., b = var.

Thus in the present dissertation paper there is investigated interaction between various coexisting bifurcation groups (1T and sub-harmonic nT) on example of oscillation system with bilinear elastic characteristic. Obviously, for the first time, there are built bifurcation maps of existence of various regimes on plane of two parameters (excitation frequency – dissipation coefficient). There is shown that existence of subgroups with unstable periodic infinitiums (UPI) in main bifurcation group always leads to chaotic behavior of system: chaotic attractor and transient chaos.

In the fifth chapter is made a transition to global analysis of forced oscillations in simple vibro-impact systems with one-sided impact on the basis of application of complete bifurcation groups method. As it is shown in the first chapter, global analysis with using of bifurcation diagrams and direct methods of investigation of similar systems wasn't made.

Also there was done a comparison of results of implemented in the thesis of bifurcation analysis of stiff and soft vibro-impact system dynamics, and was made a conclusion that using of hypothesis of momentary impact leads to qualitative mistakes even at high values of stop stiffness.

There are considering two models, describing vibro-impact systems. First – stiff impact model (introduction of coefficient of restitution), second – soft impact model, by using F.Peterka terminology.

In case of stiff impact (fig. 9) equation is the following:

$$\begin{cases} m\ddot{x} + b\dot{x} + cx = h1\cos(\omega t + \varphi 0) \\ \quad if\ \ x = d \quad \dot{x}^{+} = R\dot{x}^{-} \end{cases}, \tag{2}$$

where x – generalized coordinate; m – mass of oscillating body; b – linear dissipation coefficient; c – stiffness coefficient of linear elastic characteristic; $h1$, ω, $\varphi 0$ – amplitude, frequency and phase of excitation ; d – impact coordinate; R – coefficient of restitution.

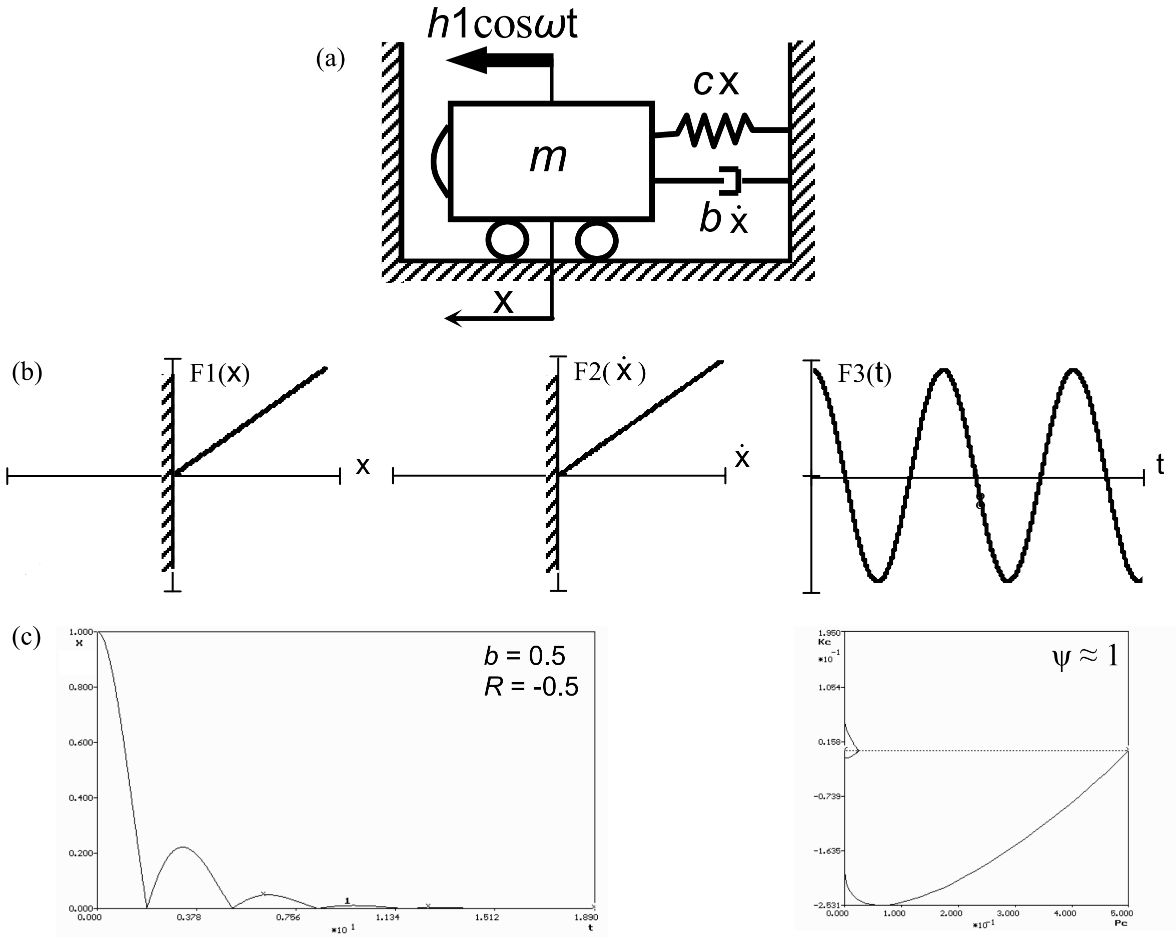

Fig. 9. Dynamical model and characteristics of vibro-impact system with stiff one-sided impact with linear elastic and dissipative characteristics at harmonic excitation (eq. 2): (a) dynamic model; (b) characteristics of elastic and dissipative forces and diagram of external harmonic excitation; (c) die-away curve and energy diagram of free oscillations. Parameters: $m = 1$, $c = 1$, $b = 0.5$, $d = 0$, $R = -0.5$, $k = 7$

In case of soft impact (fig. 10) mathematic model of investigated system is analogical to bilinear system model (eq. 1) except of dissipation forces. Linear dissipation coefficient is system state function – each linear part of elastic characteristic has its own sticky dissipation coefficient. Equation of movement:

$$m\ddot{x}+\begin{cases} b1\dot{x} & if\ x\le 0 \\ b2\dot{x} & if\ x>0 \end{cases}+\begin{cases} c1x & if\ x\le 0 \\ c2x & if\ x>0 \end{cases}=h1\cos(\omega t+\varphi 0), \tag{3}$$

where x – generalized coordinate; m – mass of oscillating body; $b1$, $b2$ – linear dissipation coefficient, accordingly to linear subregions of elastic characteristic; $c1$, $c2$ – stiffness coefficients of nonlinear elastic characteristic on linear subregions; d – break point of elastic characteristic; $h1$, ω, $\varphi 0$ – amplitude, frequency and phase of excitation.

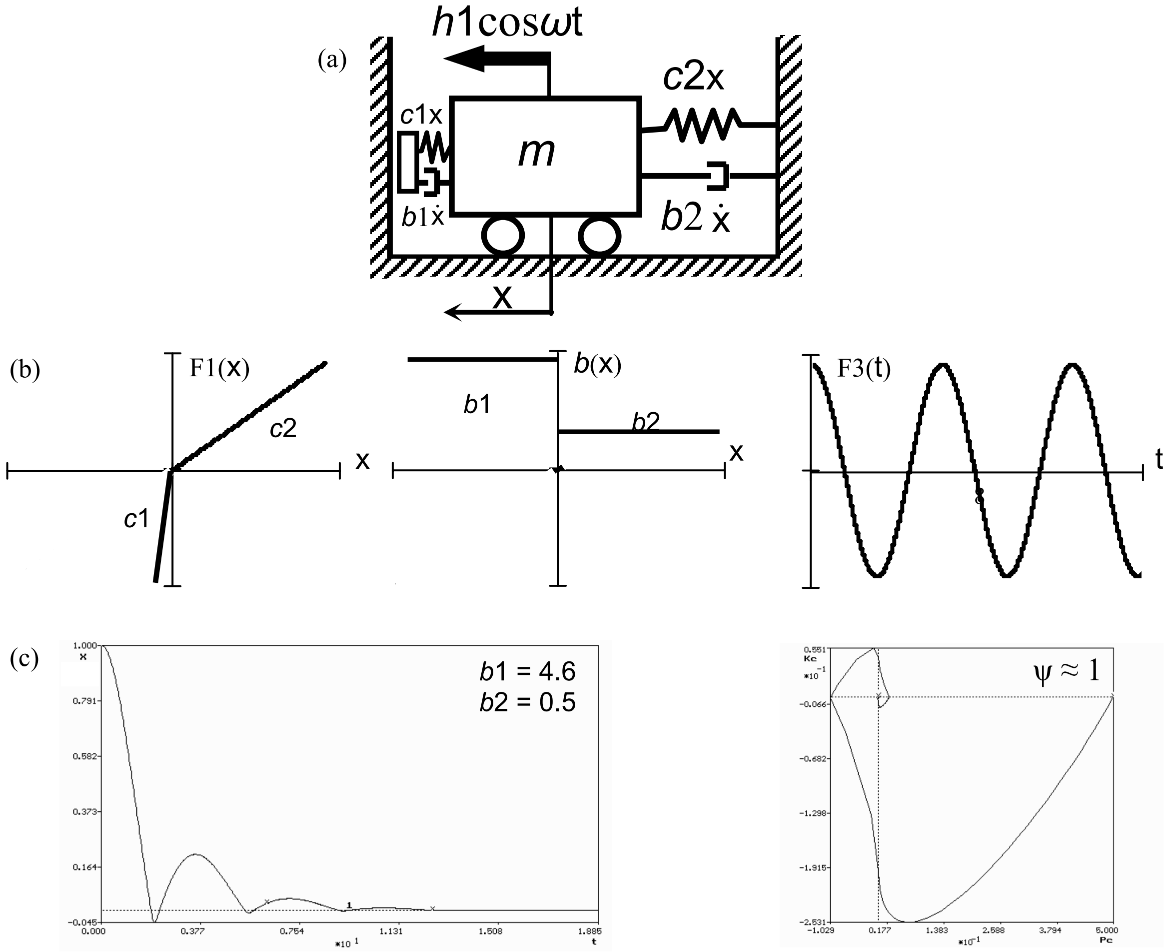

Fig. 10. Dynamic model and characteristics of vibro-impact system with soft one-sided impact with bilinear elastic characteristic and proportional dissipation at harmonic excitation (eq. 3): (a) dynamic model; (b) elastic forces characteristics, dependence of dissipation coefficient on state of system and a diagram of external harmonic excitation; (c) die-away curve and energy diagram of free oscillations. Parameters: m = 1, $c1$ = 100, $c2$ = 1, d = 0, $b1$ = 4.6, $b2$ = 0.5, k = 7

For models, described in eq. 2 and eq. 3, there is made a global bifurcation analysis on the basis of complete bifurcation groups method at varying of initial conditions and parameter – excitation frequency ω. Also is made a comparative analysis of stiff and soft impact models. Bifurcation diagrams of sub-harmonic groups 3T in stiff and soft systems are presented on fig. 11, 12 accordingly. Not all the solutions in stiff impact model it was possible to elongate on parameter, what is related to occurrence of grazing phenomenon.

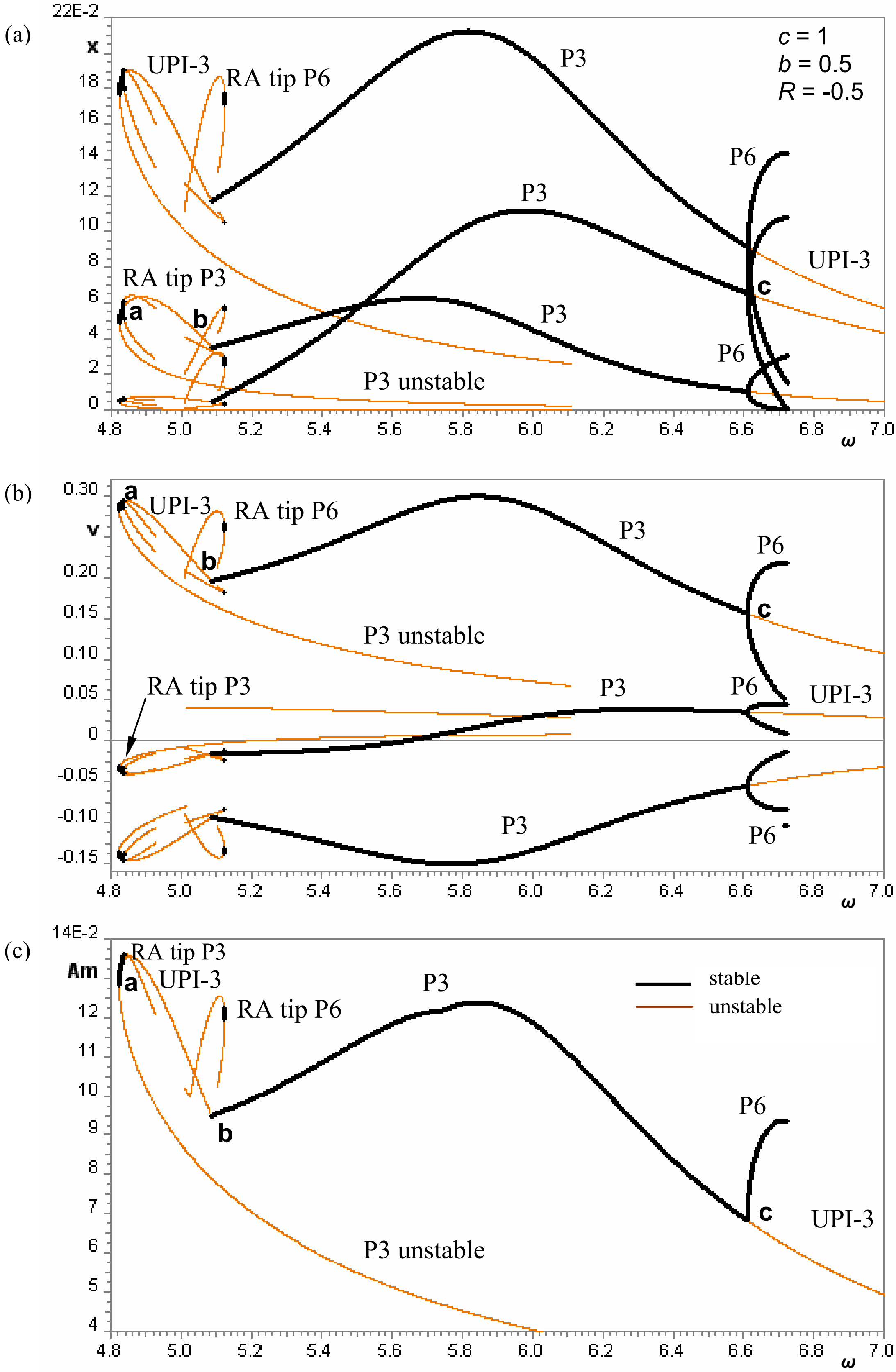

Fig. 11. Bifurcation group of sub-harmonic regime 3T with two protuberances, two regions with UPI and rare attractors of tip type P3 and P6. Not all the solutions it was possible to elongate on parameter. Bifurcation diagrams for vibro-impact system with stiff one-sided impact with linear elastic and dissipative characteristics at harmonic excitation (eq. 2): (a), (b) coordinates x, v of fixed point and (c) oscillation amplitude Am of periodic regime vs excitation frequency ω. Parameters: $m = 1$, $c = 1$, $b = 0.5$, $d = 0$, $R = -0.5$, $h1 = 1$, $\varphi 0 = 0$, $k = 7$, ω = var.

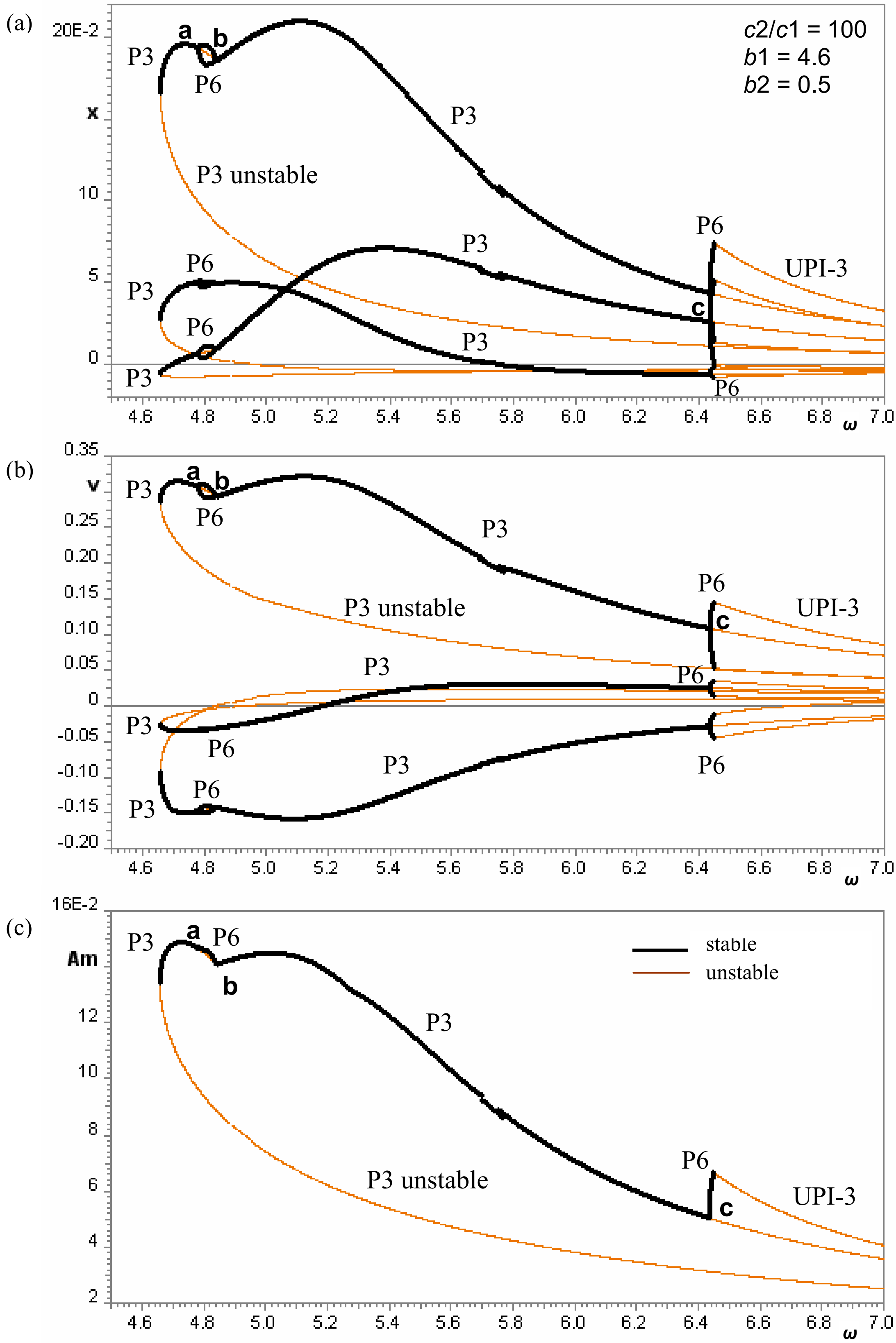

Fig. 12. Bifurcation group of sub-harmonic regime 3T with two protuberances and a region with UPI. Complete bifurcation diagrams for vibro-impact system with soft one-sided impact with bilinear elastic characteristic and proportional dissipation at harmonic excitation (eq. 3): (a), (b) coordinates x, v of fixed point and (c) oscillation amplitude Am of periodic regime vs excitation frequency ω. Parameters: $m = 1$, $c1 = 100$, $c2 = 1$, $d = 0$, $b1 = 4.6$, $b2 = 0.5$, $h1 = 1$, $\varphi 0 = 0$, $k = 7$, $\omega = \text{var}$.

Thus on the basis of complete bifurcation groups method there are built typical bifurcation diagrams for vibro-impact systems with one-sided impact. Comparison of results, performed in thesis bifurcation analysis of stiff and soft vibro-impact systems dynamics has shown that using of momentary impact hypothesis leads to qualitative mistakes even at high values of stop stiffness.

In the previous chapters the was investigated the application of complete bifurcation groups method for strongly nonlinear systems with one degree-of-freedom. However, perhaps the application of the method for global analysis of forced oscillations is also possible in systems with few degrees-of-freedom.

In the sixth chapter there are investigated issues related to application of complete bifurcation groups method for global analysis of forced oscillation in nonlinear systems with few degrees-of-freedom on an example of system with two degrees-of-freedom with two potential wells. The objective of the present chapter – to show that complete bifurcation groups method allows to find new unnoticed before regimes in a system with few degrees-of-freedom, in particular on an example of sub-harmonic isle, consisting only of unstable solutions.

Equation of movement model (fig. 13)

$$\begin{cases} m1\ddot{x}1 + b1\dot{x}1 + c1x1 - b2\dot{x} - c21x - c22x = h1\cos(\omega t + \varphi 0) \\ m2\ddot{x}2 + b2\dot{x} + c21x + c22x = 0 \end{cases}, \qquad (4)$$

where $x1$, $x2$ – generalized coordinates ($x = x2 - x1$); $m1$, $m2$ – mass of oscillating bodies; $b1$, $b2$ – linear dissipation coefficients; $c1$ – stiffness coefficient of the first linear elastic spring; $c21$, $c22$ - stiffness coefficient of the second nonlinear elastic spring; $h1$, ω, $\varphi 0$ – amplitude, frequency and phase of excitation.

On the base of complete bifurcation groups method there is made bifurcation analysis at varying of initial conditions and parameter – excitation amplitude $h1$. Results of bifurcation analysis are demonstrated on fig. 14-16.

On the fig. 14 there are presented two bifurcation groups: basic regime groups P1 with rare attractor, region UPI and almost periodic oscillations and new bifurcation group of sub-harmonic isle P2. Separately new bifurcation group is presented on the fig. 15. New is that all branches of solutions related to particular group are unstable, that is, at least one of indicators of stability is always out of unit circle borders.

Varying of other system parameters, mass of oscillating bodies, linear dissipation coefficients, particular stiffness coefficients or excitation frequency, leads to appearance of rare attractors on unstable branches of stable periodic regimes. As example, on fig. 16, there are shown bifurcation diagrams of sub-harmonic isle 2T with rare attractors, regions with UPI and almost periodic oscillations at varying of parameter $m1$. Rare attractor of tip type at $m1 \approx 0.9$ leads to appearance of unstable periodic infinitiums (UPI-2), but at $m1 \approx 1.2$ beyond rare attractor the region of almost periodic oscillations.

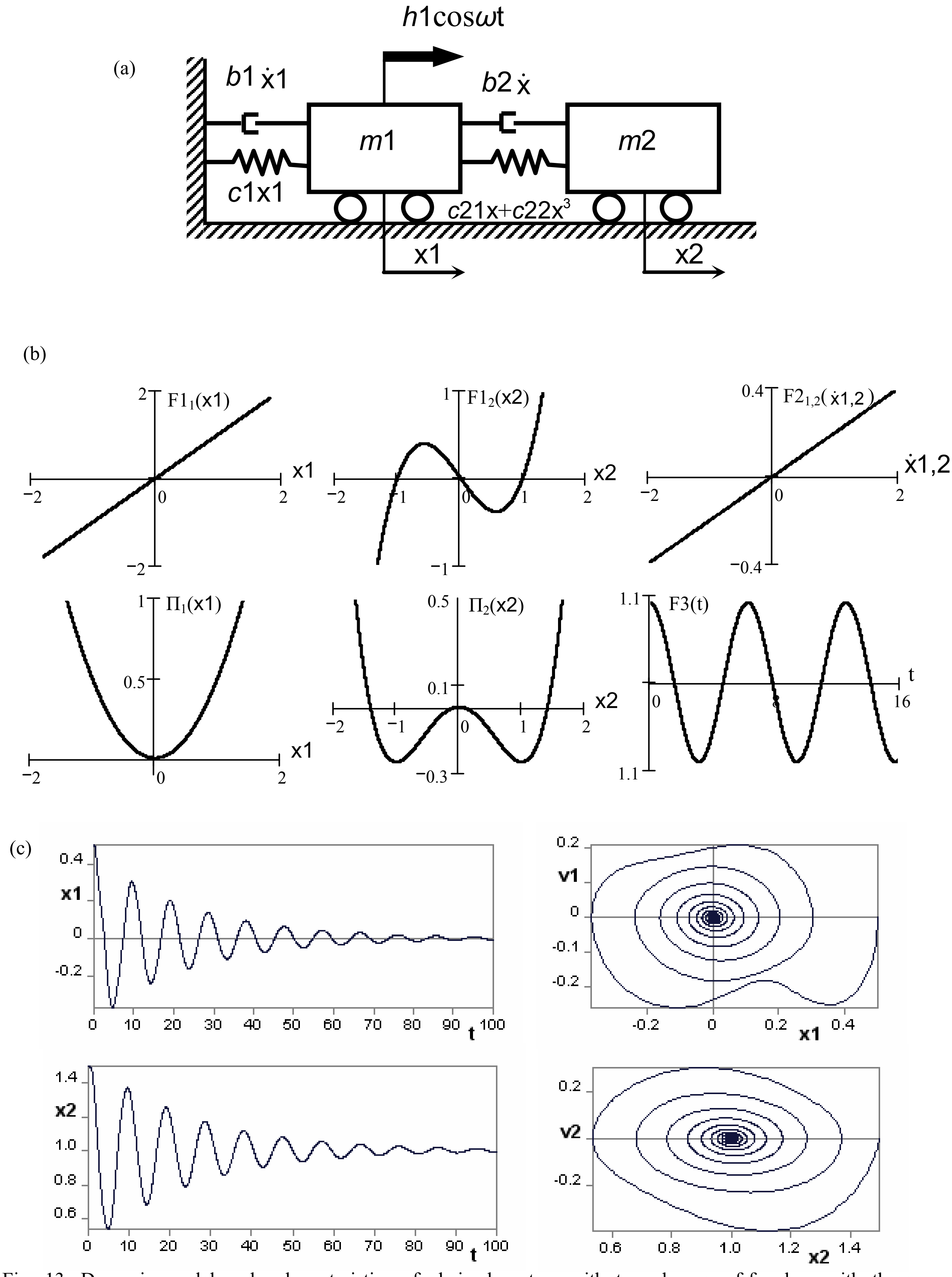

Fig. 13. Dynamic model and characteristics of chained system with two degrees-of-freedom with three equilibrium positions of second mass *m*2 and linear dissipation at harmonic excitation (eq. 4). (a) dynamic model; (b) characteristics of elastic and dissipative forces, acting in a model, characteristics of potential energies, corresponding to elastic forces and diagram of external harmonic excitation, $x = x2 - x1$; (c) combined free oscillations of the first and second mass. Parameters: $m1 = m2 = 1$, $b1 = b2 = 0.2$, $c1 = 1$, $c21 = -1$, $c22 = 1$, $k = 7$

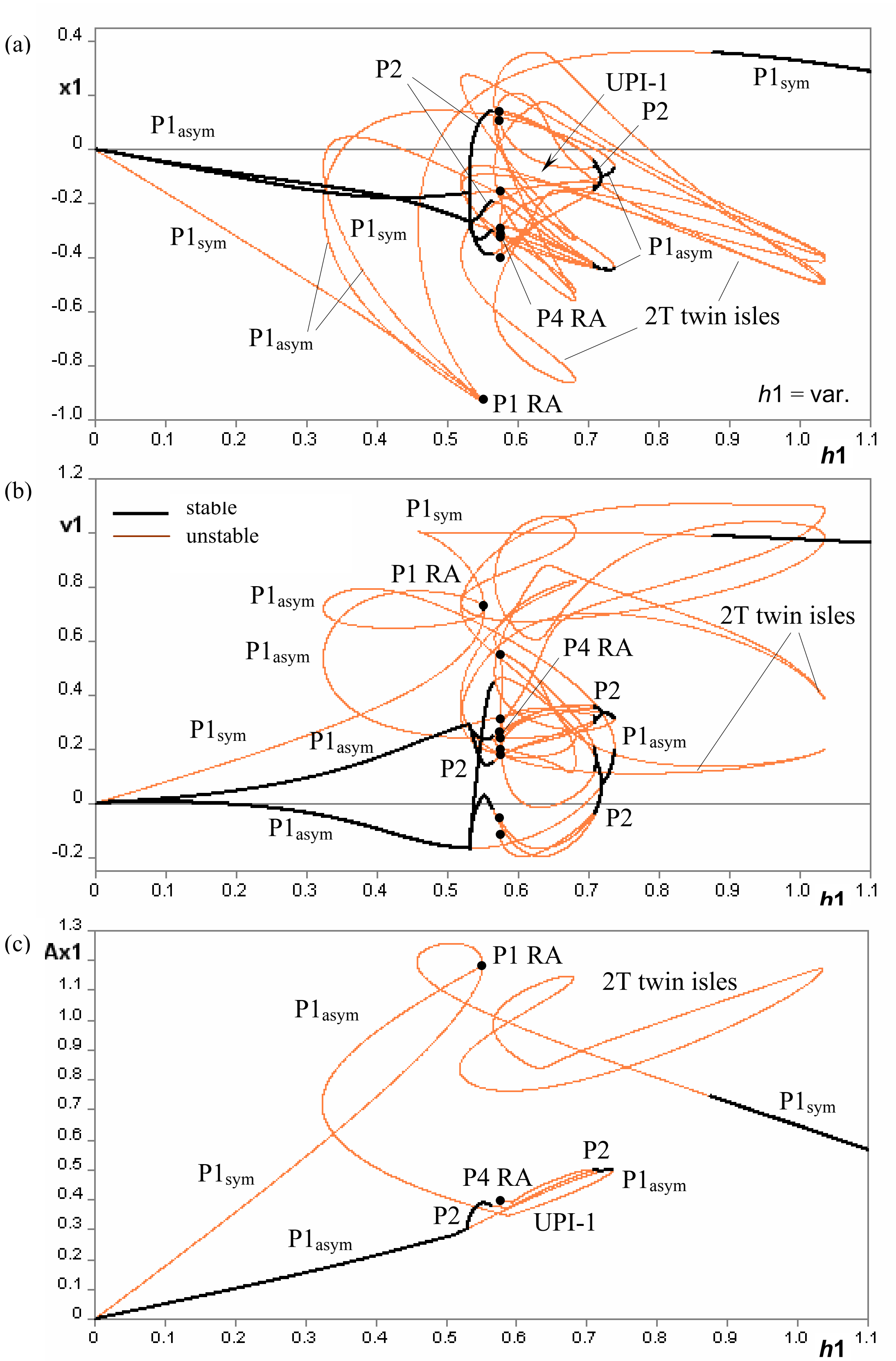

Fig. 14. Example of complicated bifurcation groups. There are shown two bifurcation groups: group 1T with rare attractor, region UPI and almost periodic oscillations and group of completely unstable sub-harmonic isles 2T. Bifurcation diagrams for chained system with two degrees-of-freedom with three equilibrium positions of mass *m*2 and linear dissipation at harmonic excitation (eq. 4). (a), (b) coordinates x1, v1 of fixed point of the first mass of excitation amplitude *h*1; (c) oscillation amplitude Ax1 of the first mass vs *h*1. Parameters: *m*1 = *m*2 = 1, *b*1 = *b*2 = 0.2, *c*1 = 1, *c*21 = -1, *c*22 = 1, *ω* = 1, *φ*0 = 0, *k* = 7, *h*1 = var.

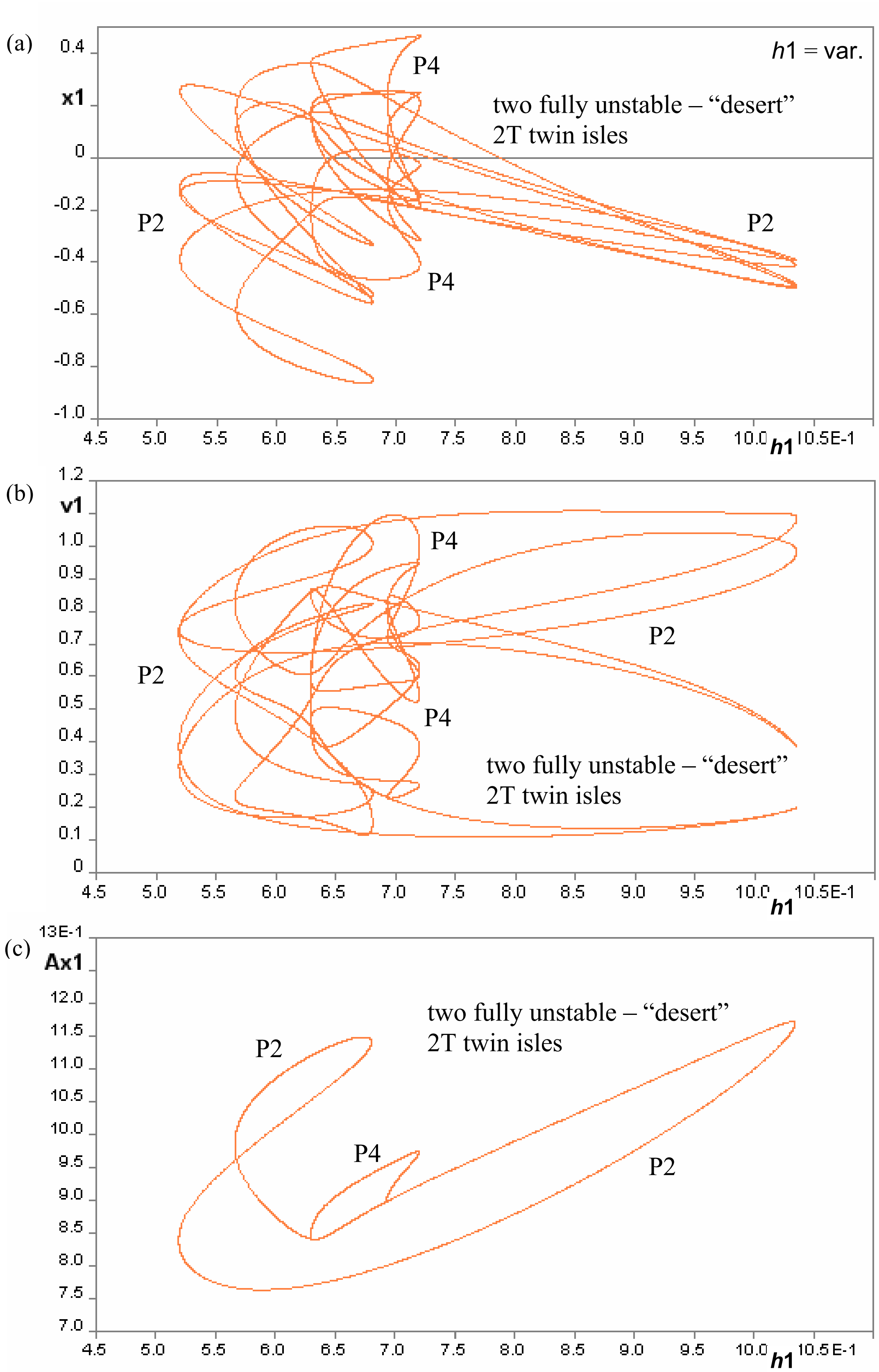

Fig. 15. New bifurcation groups of two completely unstable twins sub-harmonic isles 2T. Bifurcation diagrams for chained system with two degrees-of-freedom with three equilibrium positions of the second mass $m2$ and linear dissipation at harmonic excitation (eq. 4). (a), (b) coordinates x1, v1 of fixed point of the first mass of excitation amplitude $h1$; (c) oscillation amplitude Ax1 of the first mass vs $h1$. Parameters: $m1 = m2 = 1$, $b1 = b2 = 0.2$, $c1 = 1$, $c21 = -1$, $c22 = 1$, $\omega = 1$, $\varphi 0 = 0$, $k = 7$, $h1 = \text{var}$

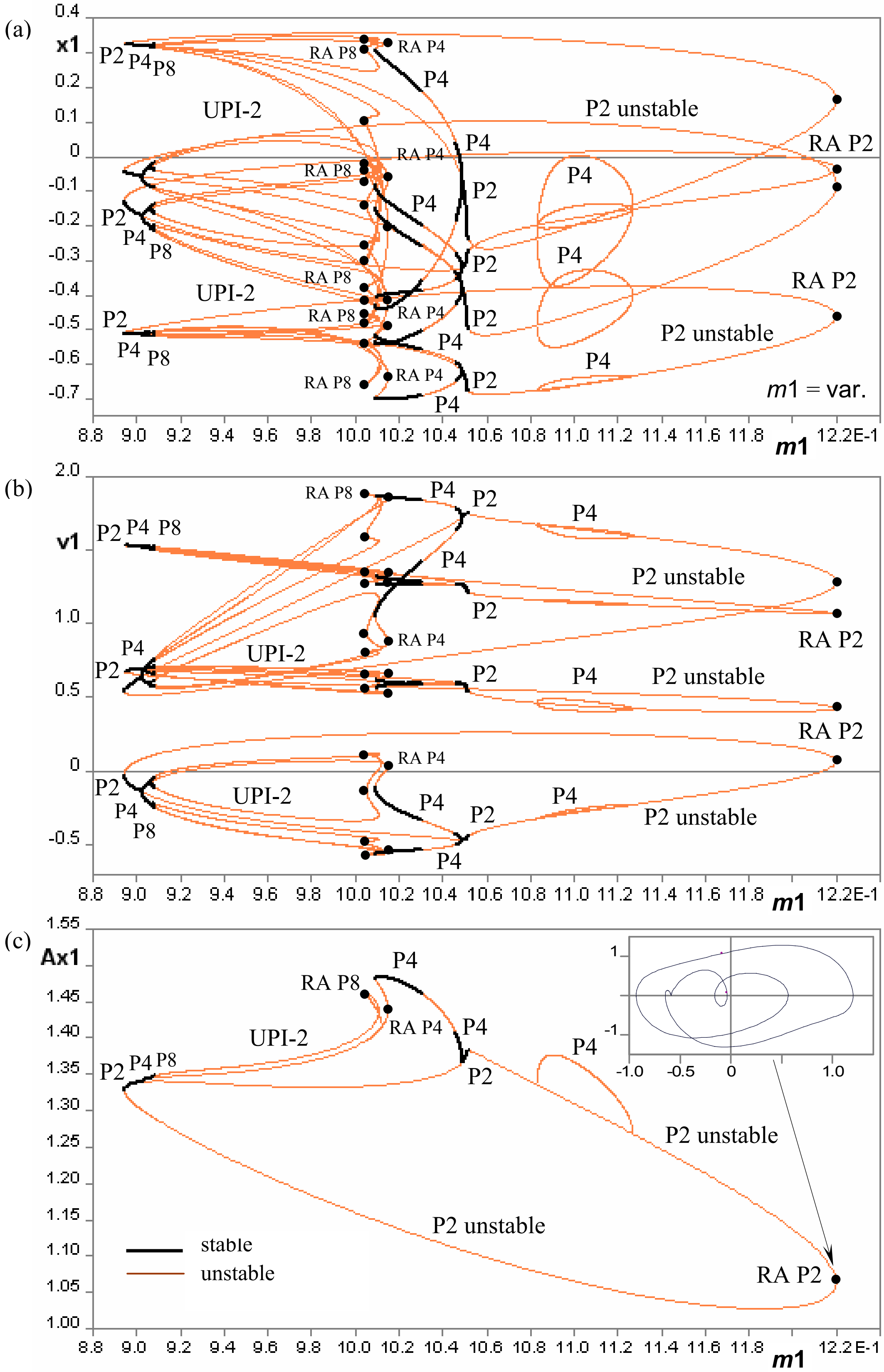

Fig. 16. On the completely unstable sub-harmonic isles 2T (fig. 15) appeared stable regimes – rare attractors at varying of the first mass *m*1. Bifurcation diagrams for chained system with two degrees-of-freedom with three equilibrium positions of the second mass *m*2 and linear dissipation at harmonic excitation (eq. 4). (a), (b) coordinates x1, v1 of fixed point of the first mass vs *m*1; (c) oscillation amplitude Ax1 of the first mass vs *m*1. Parameters: *m*2 = 1, *b*1 = *b*2 = 0.2, *c*1 = 1, *c*21 = -1, *c*22 = 1, *h*1 = 1, *ω* = 1, *φ*0 = 0, *k* = 7, *m*1 = var.

So, by the using of the complete bifurcation groups method for a system with two degrees-of-freedom there was found new nonlinear effect. In particular, for a system with three equilibrium positions there was found unknown before bifurcation group of sub-harmonic regimes – “desolate” isle, which consists only of unstable periodic regimes. From this unstable isle, at changing of system parameters, there appears rare stable periodic and chaotic regimes.

Complete bifurcation groups method allowed to find a series of complicated regular and chaotic regimes by investigating of simple strongly nonlinear oscillating and vibro-impact systems, which are able to find their application in vibroengineering. However, the results, presented in previous chapters, are obtained without taking into account of drive power, to precise it, it was taken as unlimited high. So, oscillations of a system itself didn’t have any influence on a drive.

The objective of **the seventh chapter** – a substantiation of using opportunity of multiplicity and controlling of practical system in terms of multiplicity.

For those objectives there where created an experimental rig, which implemented a model with one degree-of-freedom with bilinear elastic characteristic and linear dissipation at inertial excitation (fig. 17, 18, 19).

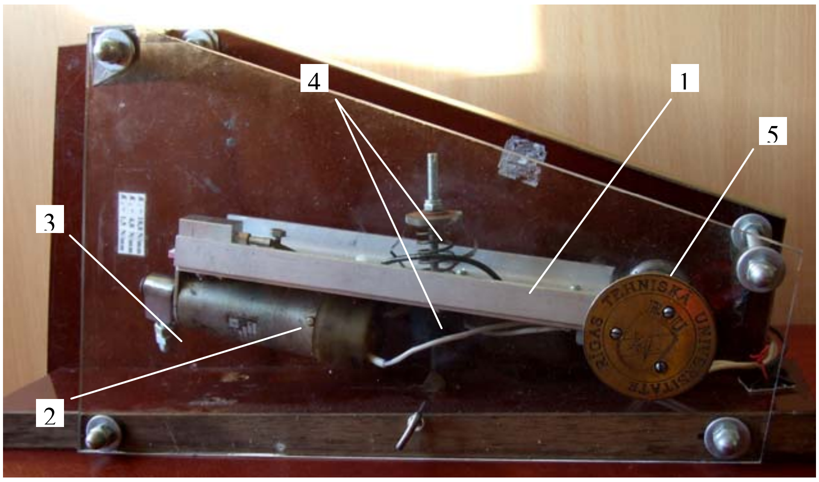

Fig. 17. Experimental rig, which implemented a model with one degree-of-freedom with bilinear elastic characteristic and linear dissipation at inertial excitation: 1– stiff beam; 2 – motor; 3 – eccentric; 4 – springs; 5 – joint support

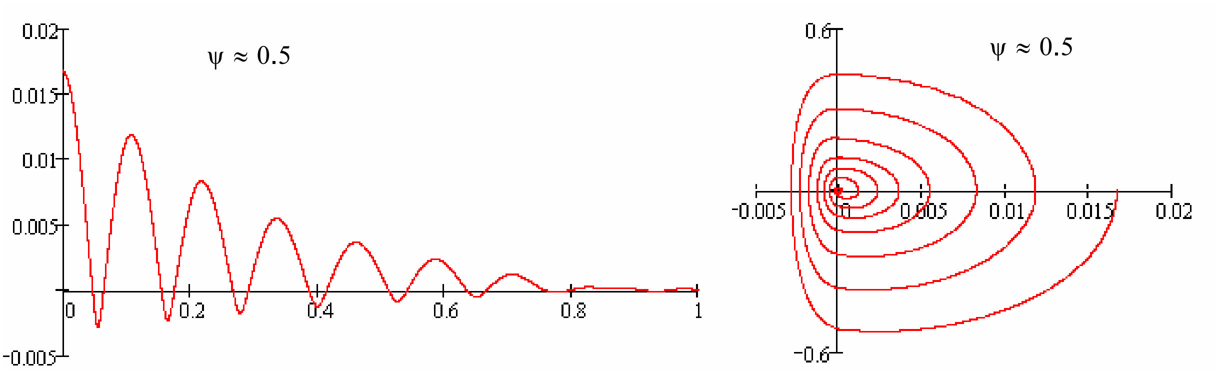

Fig. 18. Die-away curve and phase portrait of experimental free oscillations in experimental rig (fig.17). Resonant frequency $p \approx 10$ Hz

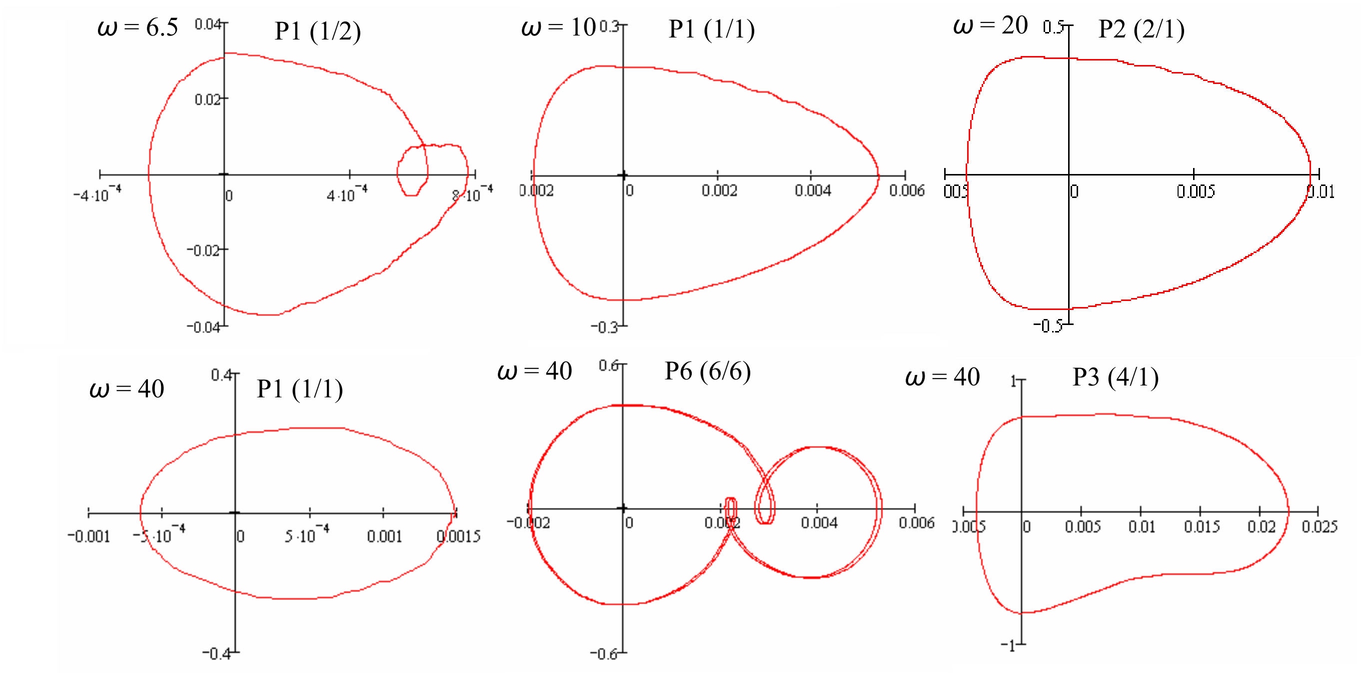

Fig. 19. Phase projections of periodic regimes, registered experimentally on experimental rig (fig.17). At excitation frequency ω = 40 (4p) there was fixed the fact of multiplicity of regimes P1, P6 and P4, that is in good agreement with the results of theoretical investigations (fig. 8)

There was also observed an important issue of influence of drive power on forced oscillations of strongly nonlinear oscillation systems. Task of the current research included a clarification of the question – if it is possible the implementation of particular nonlinear effects, taking into consideration of static characteristics of energy source, in particular of complicated regimes and chaotic oscillation processes.

If to take into consideration of interaction of nonlinear oscillation system and drive, system of equations for movement seems as the following (fig. 20):

$$\begin{cases} (m1+m2)\ddot{x}+b\dot{x}+\mathrm{f}(x)=m2r\dot{\varphi}^2\cos\varphi+m2r\ddot{\varphi}\sin\varphi \\ m2r^2\ddot{\varphi}=\mathrm{L}(\dot{\varphi})-\mathrm{H}(\dot{\varphi})+m2r\ddot{x}\sin\varphi \end{cases}, \qquad (5)$$

where $m1$, $m2$ – inertial coefficients; x – positional coordinate; φ – cyclical coordinate; r – radius of inertia; b – linear dissipation coefficient; f(x) – bilinear elastic force; $\mathrm{L}(\dot{\varphi})$ – static characteristic of torque on power supply, $\mathrm{H}(\dot{\varphi})$ – moment of resistance.

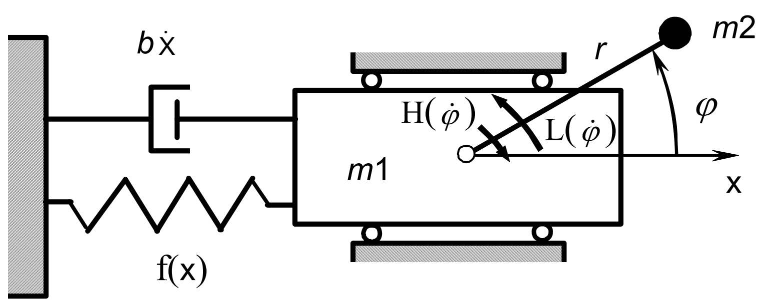

Fig. 20. Dynamic model of system with bilinear elastic characteristic and linear dissipation at inertial excitation considering the limited power supply (eq. 5).

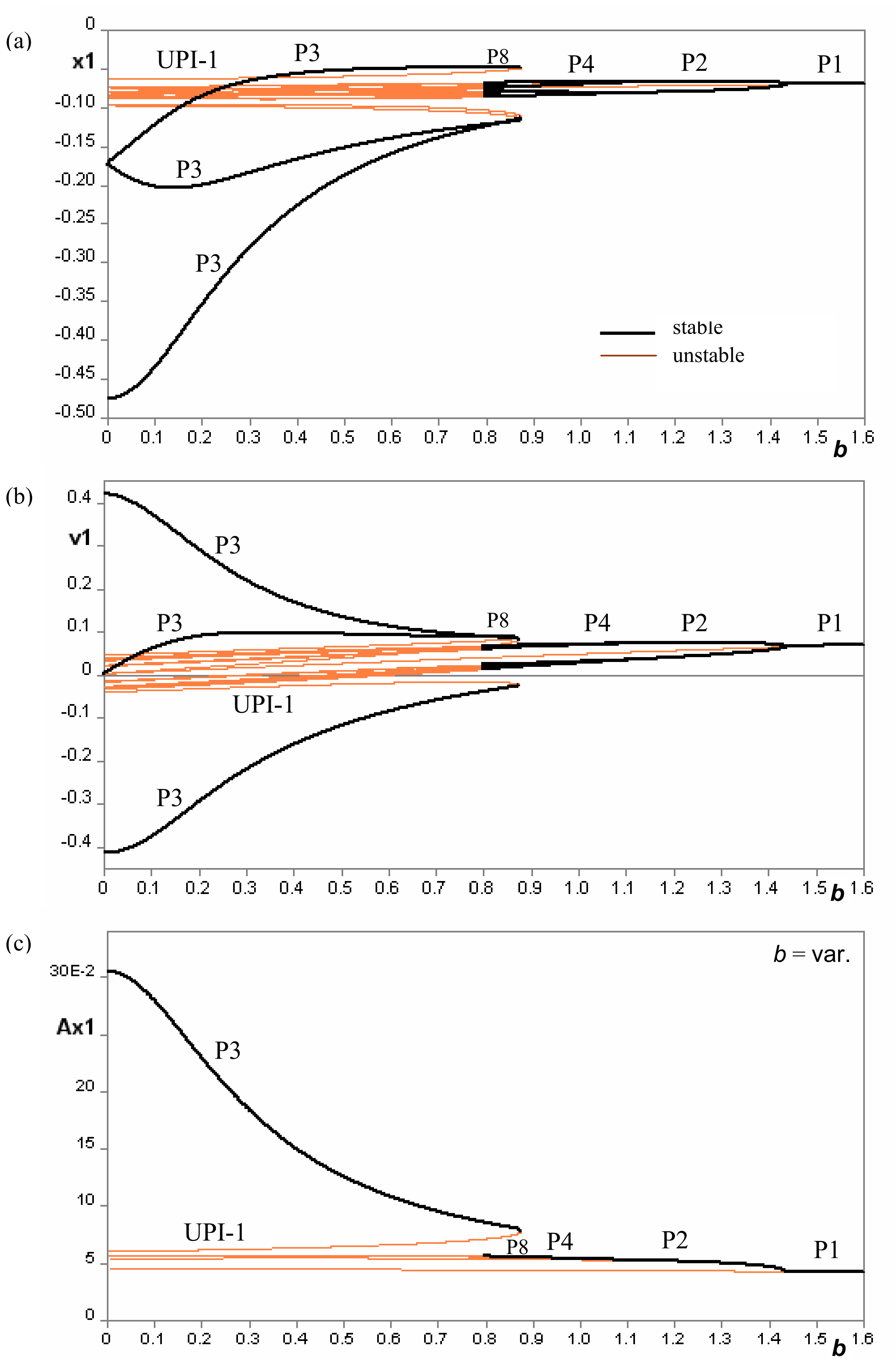

Fig. 21. Complete bifurcation groups 1T with a UPI region and 3T. Bifurcation diagrams for system with bilinear elastic characteristic and linear dissipation at inertial excitation considering the limited power supply (eq. 5). (a), (b) coordinates x1, v1 of fixed point of working mass at vs linear dissipation coefficient b; (c) oscillation amplitude Ax1 of working mass vs b. Parameters: $m1 = m2 = 0.5$, $c1 = 1$, $c2 = 16$, $d = 0$, $r = 0.08$, $k = 7$, b = var.

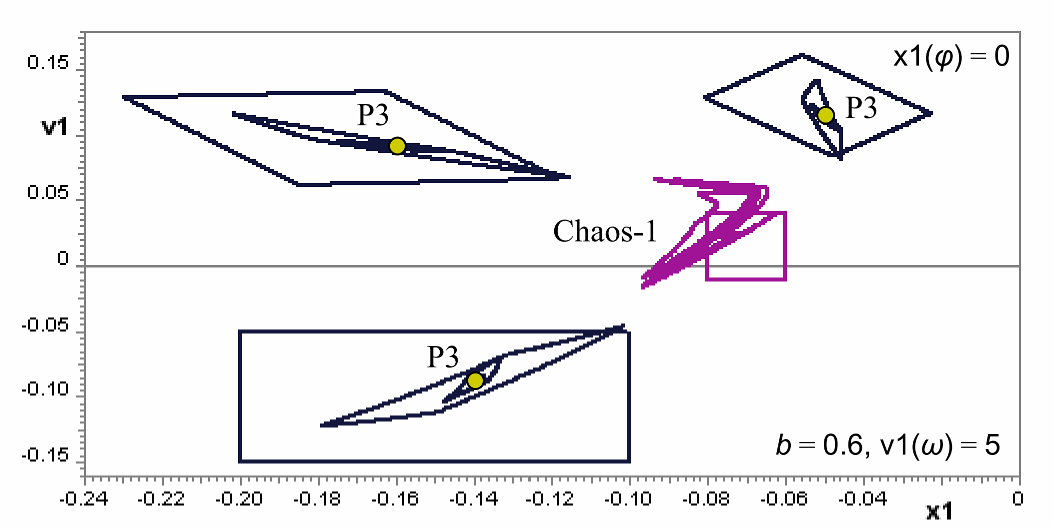

Fig. 22. Multiplicity: cores of basins of attraction of periodic P3 and chaotic Chaos-1 regimes, related to bifurcation groups 3T and 1T accordingly, for a system with bilinear elastic characteristic and linear dissipation at inertial excitation, considering the limited power supply (eq. 5). Constructing of cores of basin of attraction was made by line mapping. Parameters: $m1 = m2 = 0.5$, $c1 = 1$, $c2 = 16$, $d = 0$, $b = 0.6$, $r = 0.08$, $k = 7$

Experimentally and theoretically, considering motor power there is shown an opportunity of using results, obtained in investigations of simple strongly nonlinear oscillations and vibro-impact systems, in tasks of vibroengineering. There was analyzed influence of electric drive.

Thus, in the present dissertation paper is shown that application of complete bifurcation groups method allows to implement a global bifurcation analysis of strongly nonlinear oscillations and vibro-impact systems, to find new nonlinear effects, bifurcation groups, unknown before periodic and chaotic regimes and is shown an opportunity to use new results in tasks of vibroengineering.

Conclusion

The dissertation paper is dedicated to application of complete bifurcation groups method, which allows to find new nonlinear effects, bifurcation groups, unknown before periodic and chaotic regimes, for a global analysis of strongly nonlinear oscillations and vibro-impact systems, and for an opportunity to use new results in tasks of vibroengineering.

In the dissertation paper there are obtained the following basic new results:

1. Shown that using of the complete bifurcation groups method and on its base worked out complex approach for qualitative global analysis of forced oscillations in strongly nonlinear oscillators and vibro-impact systems allows to implement full qualitative topological analysis of various bifurcation groups and to find new periodic and chaotic regimes.
2. Using of the complete bifurcation groups method for investigation of forced oscillations on the example of simple oscillation system with bilinear elastic characteristics allowed to find new bifurcation groups with complex protuberances, with rare periodical and chaotic regimes.

3. There is studied an interaction between various existing bifurcation groups (1T and sub-harmonic nT) on the example of oscillation system with bilinear elastic characteristic. Obviously for the first time, there are built bifurcation maps of existing various regimes on a plane of two parameters (frequency of excitation – coefficient of dissipation). There is shown that existing subgroups with unstable periodic infinitum (UPI) in a main bifurcation group always leads to chaotic behavior of a system: chaotic attractor and transient chaos.
4. On the base of complete bifurcation groups method there are built typical bifurcation diagrams for vibro-impact systems with one-sided impact. Comparison of results of implemented bifurcation analysis of dynamics of stiff and soft vibro-impact systems has shown that using of hypothesis of momentary impact leads to qualitative mistakes even at high values of stiffness of stop.
5. At the using of complete bifurcation groups method for system with two degrees-of-freedom there is found new nonlinear effect. Particularly for system with three equilibrium positions there is found unknown before bifurcation group of sub-harmonic regimes – "desert" isle, which consists just of unstable periodical regimes. On this unstable isle at parameters changing there born rare stable periodical and chaotic regimes.
6. From analysis of various oscillators and vibro-impact systems with bifurcation groups with rare attractors of tip type, obviously for the first time, is obtained that there are always its own chaotic attractors in those groups.
7. An opportunity of practical using of new research results in tasks of nonlinear vibroengineering: vibromoving, vibromixing, vibropolishing, vibrowelding etc. is shown. There is analyzed the influence of electro-drive at the using of phenomenon of multiplicity.

Thus, the present thesis has shown that using of the method of complete bifurcation groups allows to implement a global bifurcation analysis of strongly nonlinear oscillating and vibro-impact systems, to find new nonlinear effects, unknown before periodic and chaotic regimes, what is demonstrated on typical piece-wise linear and smooth nonlinear systems with one or two degrees-of-freedom and shown an opportunity to use new results in tasks of vibroengineering.

Publications on the dissertation theme

Main content of the dissertation paper is published in the following scientific researches:

1. Yevstignejev V. *Control of Nonlinear Vibro-reducer of Motion*, in M.Zakrzhevsky (ed.) Proceedings of International Students' Conference on Nonlinear Dynamics, Chaos, Catastrophes and Control, RTU, Riga, 2001, pp. 98-101.
2. Zakrzhevsky M., Smirnova R., Schukin I., Ivanov Yu., Yevstignejev V. *Global Dynamics and Bifurcation Analysis of the Forced Oscillations in the Piecewise Linear Systems with Nonlinear Dissipation*, - In V.Astashev and V.Krupenin (eds.) "Dynamics of Vibroimpact (Strongly Nonlinear) Systems", Russian Academy of Sciences, Moscow-Zvenigorod, 2001, pp. 163-167.
3. Zakrzhevsky M., Yevstignejev V., Smirnova R., Schukin I., Ivanov Yu., *Control of Nonlinear Oscillatory Mechanisms: Nonlinear Smart Vibro-reducer NIV1R,* - In V.Astashev and V.Krupenin

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4. Jevstigņejevs V., *Svārstību enerģētika – enerģētiskās plūsmas un to vadība*, Tēžu krājumā II Pasaules latviešu zinātnieku kongresā, Latvijas Zinātņu akadēmija, Rīga, 2001, p. 550.
5. Zakrzhevsky M., Smirnova R., Schukin I., Yevstignejev V. *Bifurcation Analysis of Forced Oscillations in the Trilinear System with Nonlinear Damping*, Scientific Proceedings of Riga Technical University – Transport and Engineering, sērija 6, sējums 7, Riga, 2002, pp. 102-113.
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8. Yevstignejev V., Zakrzhevsky M., Nikishin V., Smirnova R., Schukin I., Dobkevich M., Shilvan E., Zaborovsky V. *Теоретические и экспериментальные исследования по использованию хаотических колебаний в вибротехнике*, в кн. В.К.Асташева, В.Л.Крупенина и Е.Б.Семёновой «Динамика виброударных (сильно-нелинейных) систем», Российская академия наук, Москва-Звенигород, 2006, pp. 102-106.
9. Zakrzhevsky M., Yevstignejev V., Schukin I., Smirnova R., Shilvan E. *Хаотические, субгармонические и виброударные режимы в билинейных системах*, в кн. В.К.Асташева, В.Л.Крупенина и Е.Б.Семёновой «Динамика виброударных (сильно-нелинейных) систем», Российская академия наук, Москва-Звенигород, 2006, pp. 112-116.
10. Schukin I., Zakrzhevsky M., Yevstignejev V. *Редкие аттракторы в системе с двумя степенями свободы с тремя положениями равновесия*, в кн. В.К.Асташева, В.Л.Крупенина и Е.Б.Семёновой «Динамика виброударных (сильно-нелинейных) систем», Российская академия наук, Москва-Звенигород, 2006, pp. 315-323.
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12. Zakrzhevsky M., Schukin I., Smirnova R., Mironov V., Yevstignejev V., Klokov A. *Проблемы виброустойчивости, безопасности и надёжности медицинских центрифуг*, Proceedings of International conference on the Modern Achievements of Science and Education, Natanya, Israel, 2007, pp. 191-196.
13. Smirnova R., Zakrzhevsky M., Schukin I., Yevstignejev V. *Хаотические колебания в нелинейных динамических системах: типы хаотических аттракторов, сценарии рождения и многорежимность*, Proceedings of VI International Conference on the Improvement of the Quality, Reliability and Long Usage of Technical Systems and Technological Processes, Hurghada, Egypt, 2007, pp. 67-70.
14. Zakrzhevsky M.V., Schukin I.T., Smirnova R.S., Yevstignejev V.Yu. *Method of Complete Bifurcation Groups and its Applications for Prediction of Unknown Chaotic and Regular Attractors in Nonlinear Dynamical Systems*, International Conference on Chaotic Modeling, Simulation and Applications, Chania, Greece, 2 p. (in print).
15. Zakrzhevsky M.V., Yevstignejev V.Yu., Shilvan E.P. *Rare Chaotic Attractors and Unknown Multiplicity in the Bilinear and Vibro-Impact Driven Systems. Bifurcation Analysis and Applications*, International Conference on Chaotic Modeling, Simulation and Applications, Chania, Greece, 2 p. (in print).