

Walk This Way**1. The Problem**

A person walks away from a motion detector at various speeds for a period of 5 seconds. We will use the calculus concepts of Riemann sums and the Fundamental Theorem of Calculus to enhance our understanding of integral calculus and the relationship between velocity and distance functions.

2. Calculating the Distance Walked

The data we have collected and graphed is a distance vs. time scatterplot of the person as he/she walks away from the motion detector. Trace on the scatterplot to determine the starting distance and the ending distance from the motion detector. Record these values below, rounded to the nearest 0.01.

Starting distance: _____ ft. Ending distance: _____ ft.

Subtract these distances to find the total distance walked. Record this value below.

Total distance walked: _____ ft. (We will come back to this!)

3. The Velocity vs. Time Data and the Velocity Function

We also have the “velocity” data in the stat/data editor of our calculator. Set up and graph a scatterplot of the velocity vs. time data. This scatterplot shows the velocity of the person at any time. This is the data we want to work with! The scatterplot appears to produce a pattern that can be modeled by a quadratic function, so we will attempt to find the “best fitting” function $v(t)$ in the form:

$$v(t) = at^2 + bt + c.$$

To do this we will use the power of the technology and perform a quadratic regression on our calculator. (See notes, if necessary, to obtain the quadratic regression equation.) Record the equation below, with the values of **a**, **b**, and **c**, rounded to 0.01.

$v(t) =$ _____

Enter this function into **Y1** of your calculator and graph it. Hopefully it fits the data pretty well!

4. A Few Questions

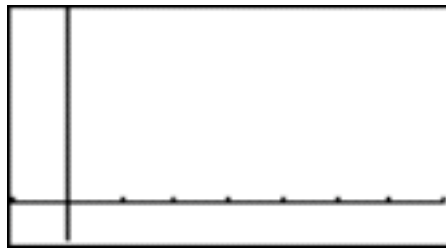
(Note: Turn off the scatterplot, and answer the following questions with the function $v(t)$ that you found in part 3.)

- What was the initial velocity of the walker? _____
- On what time interval was the walker slowing down? _____
- What was the walker's slowest speed during the 5 second period? _____
- What was the acceleration of the walker when $t=3$? _____

5. Using the Velocity Function to Calculate the Total Distance Walked

The total distance traveled by the walker can be calculated by finding the area of the region under the velocity function and above the t (time) axis. We can approximate this area by dividing the region into rectangular sections and calculating the sum of the areas of the rectangles.

Sketch the graph of the velocity function below.



Our first approximation will be to divide the horizontal time axis into four equal parts. Since the total time for the data collection was 5 seconds, what is the width of each of the four rectangles?

Rectangle width = _____ secs.

Mark these values on your graph.

We will use the right endpoint of each subinterval to determine the height of each rectangle. Draw the four rectangles on your graph, and on the “top” of each rectangle write it’s height (rounded to 0.01).

Finally, fill in the blanks below which show the sum of the areas of the four rectangles, and evaluate it.

Rectangle Sum = _____ + _____ + _____ + _____ = _____

Of course, if we used more rectangles to approximate the area under the velocity function, we should get a better approximation for the total distance traveled by the walker. But, the more rectangles we use, the more work it is for us to calculate the sum of the areas of the rectangles. Unless, we can write the sum using sigma notation!

If we use 10 rectangles (of equal width) to approximate the area of the region, the width of each rectangle, which we call Δx , would be:

Rectangle width = _____ secs.

Again, if we use the right endpoint of each subinterval to determine the height of each rectangle, then the height of the first rectangle would be $v(\text{_____}) = \text{_____}$, the height of the second rectangle would be $v(\text{_____}) = \text{_____}$, the third $v(\text{_____}) = \text{_____}$, and the tenth rectangle $v(\text{_____}) = \text{_____}$. In general, the height of “rectangle #i” would be calculated by finding $v(\text{_____})$.

Now, write and evaluate an expression involving sigma, that could be used to calculate the area of the 10 rectangles.

Rectangle Sum = $\sum$ _____ = _____

Let’s try one more approximation. If we use 40 rectangles, $\Delta x = \text{_____}$ and the height of “rectangle #i” would be calculated by finding $v(\text{_____})$. Again, write and evaluate, an expression involving sigma, that could be used to calculate the area of the 40 rectangles.

Rectangle Sum = $\sum$ _____ = _____

Rectangle Sum = $\sum$ _____ = _____

Area = _____ = _____

Time =