Unit # __3__ **Name of unit __Circles and Spheres__**

Lesson 1	Properties of circles including lines and line segments Part I
E. Q.	How do I identify lines and line segments that are related to a circle?
Standard	MM2G3. Students will understand the properties of circles. (a)
Opening	Warm-up: Find the diameter of a circle with a radius of 6 mm. (Answer: 12 mm) Warm-up: A right triangle has legs 15 cm and 20 cm. Find the length of the hypotenuse. (Answer: 25 cm)
Work session	Teacher-guided completion of the lines and line segments graphic organizer. Teacher-guided notes and student-guided practice.(Review Theorems)
Closing	Ticket-out-the-Door

Grade __10__

Unit # __3__ **Name of unit __Circles and Spheres__**

Lesson 2	Properties of circles including lines and line segments Part II
E. Q.	How do I identify lines and line segments that are related to a circle?
Standard	MM2G3. Students will understand the properties of circles. (a)
Opening	Warm-up: Address incorrect responses from ticket-out-the-door. Review and answer questions from previous work.
Work session	Sunrise on the First Day of the New Year Learning Task (Questions 1-4) Additional Guided Practice as necessary.
Closing	Class Discussion over Sunrise Learning Task

Opening for Properties of circles including lines and line segments

Warm-up: Find the diameter of a circle with a radius of 6 mm.

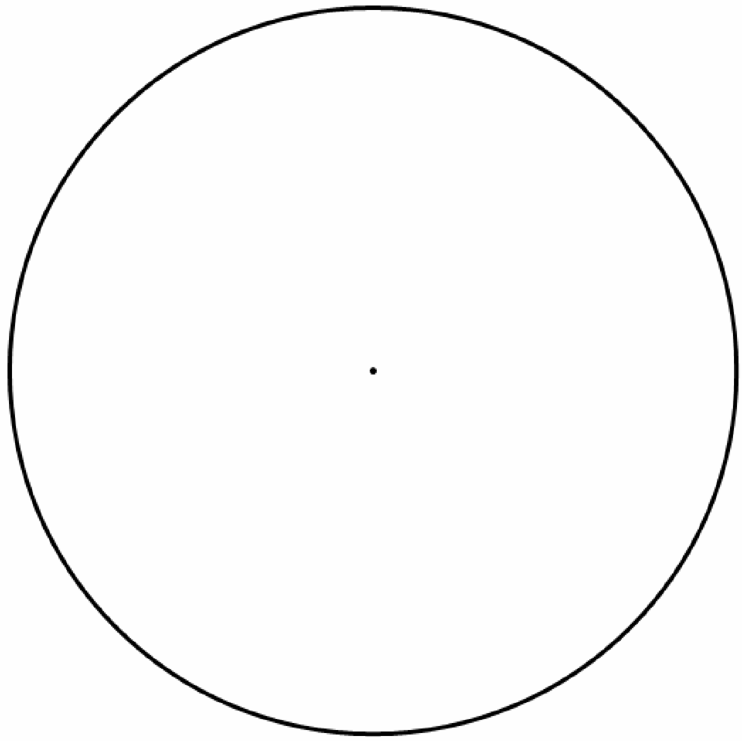
Warm-up: A right triangle has legs 15 cm and 20 cm. Find the length of the hypotenuse.

Opening for Properties of circles including lines and line segments

Warm-up: Find the diameter of a circle with a radius of 6 mm. (Answer: 12 mm)

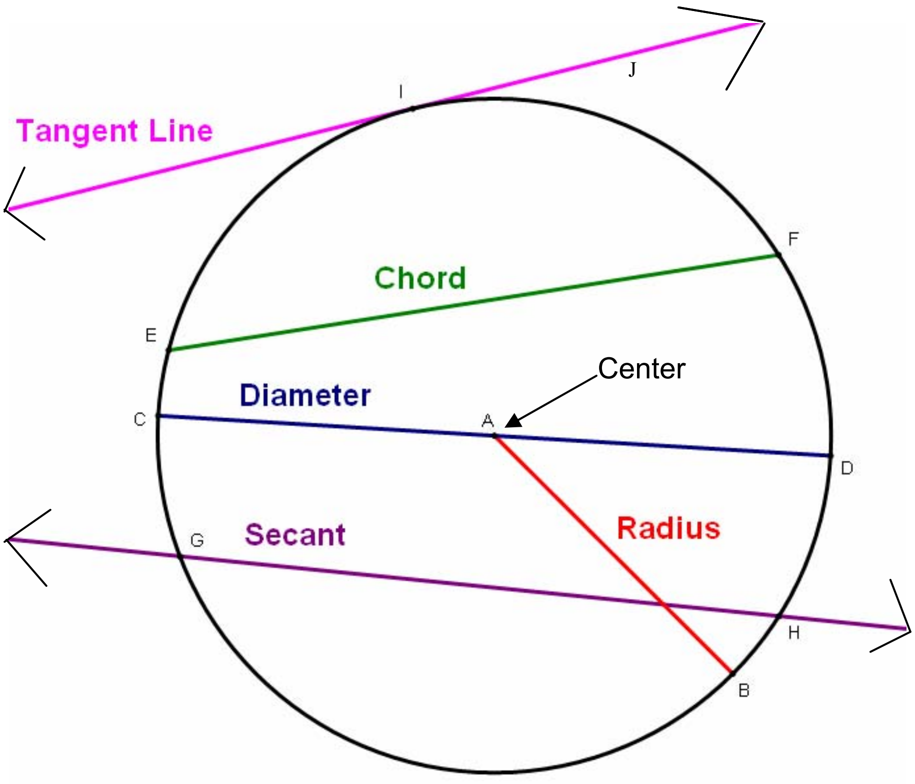
Warm-up: A right triangle has legs 15 cm and 20 cm. Find the length of the hypotenuse.
(Answer: 25 cm)

<u>Lines & Line Segments of Circles Graphic Organizer</u>



Vocabulary Word	Definition	Example
Circle		
Center		
Radius		
Chord		
Diameter		
Secant		
Tangent		

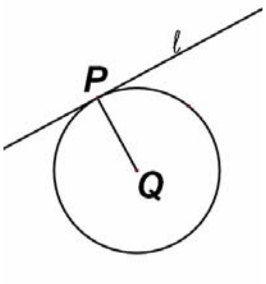
<u>Lines & Line Segments of Circles Graphic Organizer (Key)</u>



Vocabulary Word	Definition	Drawing
Circle	The set of all points in a plane that are equidistant (the length of the radius) from a given point.	Circle C
Center	The point from which all points of the circle are equidistant.	Point C
Radius	The segment between the center of a circle and a point on the circle.	$\overline{AB}$
Chord	A segment on the interior of a circle whose endpoints are on the circle.	$\overline{EF}$
Diameter	A segment between two points on a circle, which passes through the center of the circle. (The diameter is the longest chord of a circle).	$\overline{CD}$
Secant	A line that intersects a circle at two points on the circle.	$\overleftrightarrow{GH}$
Tangent	A line that intersects the circle at exactly one point.	$\overleftrightarrow{IJ}$

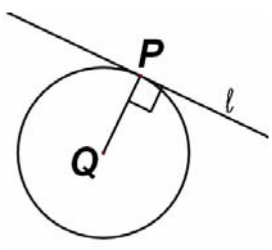
Guided notes for Properties of circles including lines and line segments

Perpendicular Tangent Theorem: If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.



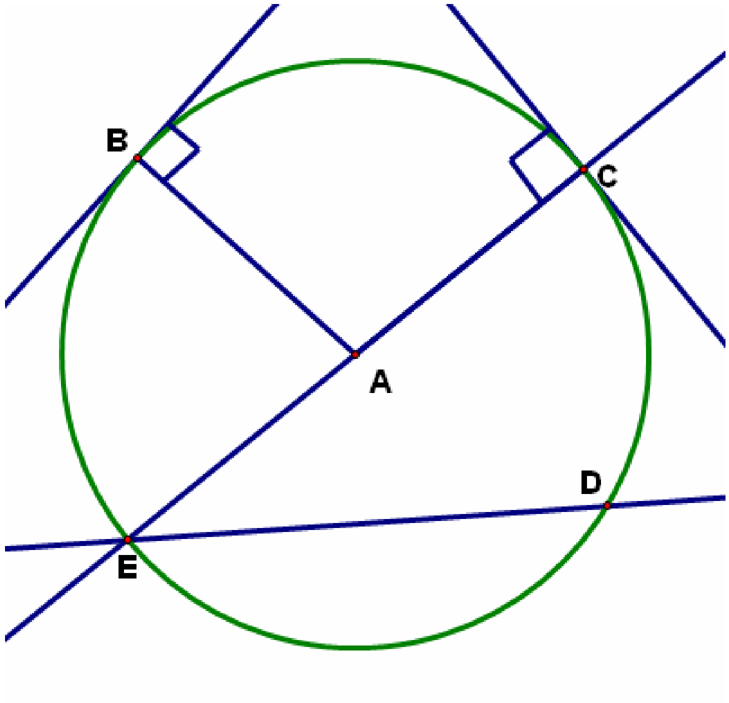
If ℓ is tangent to $\bigcirc$ *Q* at *P*, then $\ell \perp \overline{QP}$.

Converse of the Perpendicular Tangent Theorem: In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.



If $\ell \perp \overline{QP}$ at *P*, then ℓ is tangent to $\bigcirc$ *Q*.

Ticket Out the Door – Central Angles And Arcs



Using the diagram above, give an example of each of the following. Be sure to use proper notation!

1. Center
2. Chord (other than the diameter)
3. Diameter
4. Radius
5. Tangent
6. Point of Tangency
7. Secant

Grade __10__
Unit # __3__

Name of unit __Circles and Spheres__

Lesson 3	Properties of central angles and relationships of arcs
E. Q.	What is the relationship between major arcs, minor arcs, and central angles?
Standard	MM2G3. Students will understand the properties of circles. (b, d)
Opening	Warm-up: Angles A and B are supplementary angles and $m\angle A = 56$. Find $m\angle B$. ($m\angle B = 124$) Warm-up: Angles C and D are complementary angles and $m\angle C = 31$. Find $m\angle D$. ($m\angle D = 59$)
Work session	Teacher-guided completion of the central angle and arcs graphic organizer. Teacher-guided notes and student-guided practice. Sunrise on the First Day of the New Year Learning Task (Question 5) and/or McDougal Littell Mathematics II Assessment Book with Performance Task pg 57 questions (a-e)/pg 58 questions (a-k) Additional resources: http://www.geogebra.org/en/wiki/index.php/Degrees_of_Arcs_and_Angles_in_Circles
Closing	Ticket-out-the-Door

Opening for Properties of central angles and relationships of arcs

Warm-up: Angles A and B are supplementary angles and $m\angle A = 56.$ **Find** $m\angle B.$

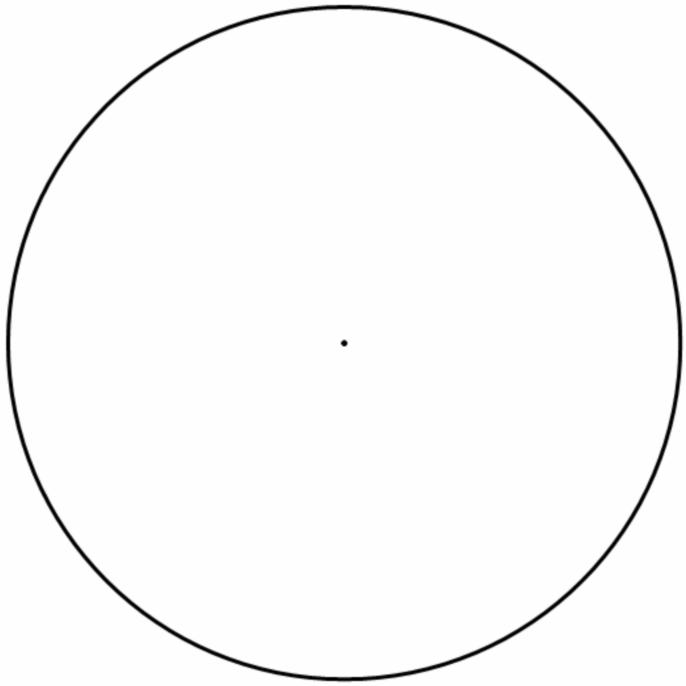
Warm-up: Angles C and D are complementary angles and $m\angle C = 31.$ **Find** $m\angle D$ **.**

Opening for Properties of central angles and relationships of arcs

Warm-up: Angles A and B are supplementary angles and $m\angle A = 56.$ **Find** $m\angle B.$
($m\angle B = 124$ **)**

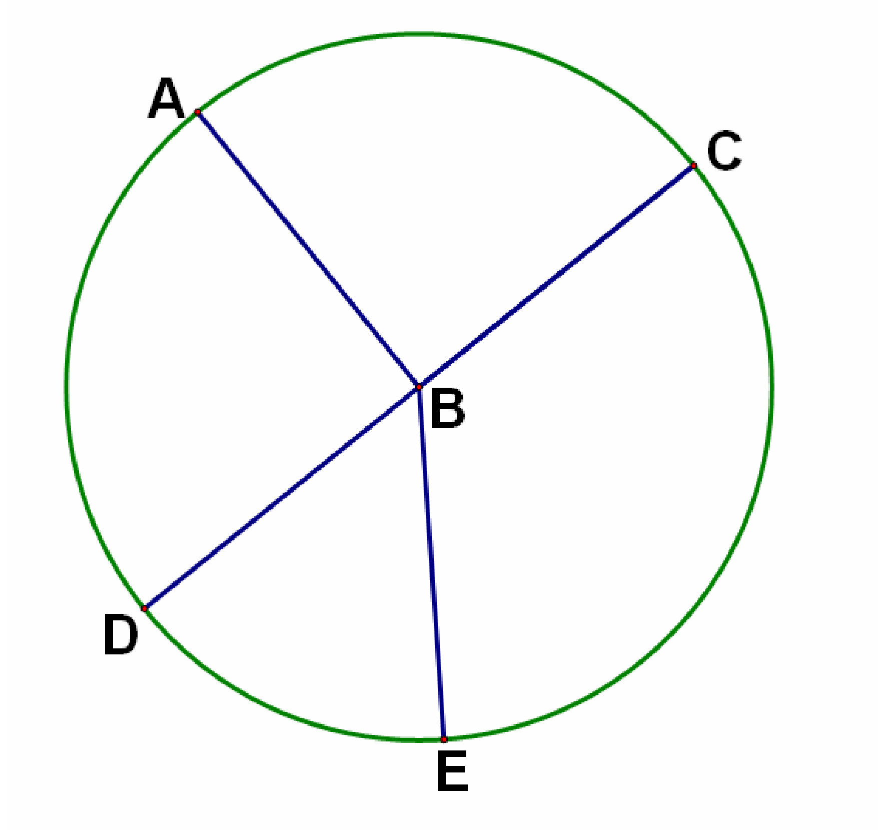
Warm-up: Angles C and D are complementary angles and $m\angle C = 31.$ **Find** $m\angle D$ **.**
($m\angle D = 59$ **)**

Central Angles and Arcs of Circles Graphic Organizer



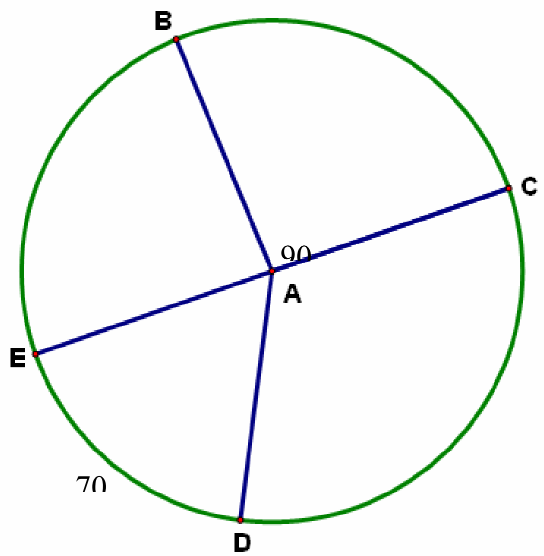
Vocabulary Word	Definition	Example
Central Angle		
Semicircle		
Arc		
Minor Arc		
Major Arc		
Congruent Circles		
Congruent Arcs		

<u>Central Angles and Arcs of Circles Graphic Organizer (Key)</u>



Vocabulary Word	Definition	Drawing
Central Angle	An angle whose vertex is the center of the circle.	<DBE
Semicircle	An arc with endpoints that are the endpoints of a diameter.	$\overset{\frown}{DEC}$
Arc	An unbroken part of a circle.	$\overset{\frown}{DA}$ or $\overset{\frown}{DEA}$
Minor Arc	Part of a circle measuring less than 180º.	$\overset{\frown}{DA}$
Major Arc	Part of a circle measuring between 180º and 360º.	$\overset{\frown}{DEA}$
Congruent Circles	Two circles that have the same radius.	
Congruent Arcs	Two arcs that have the same measure and are arcs of the same circle or of congruent circles.	$\overset{\frown}{DA}$ & $\overset{\frown}{AC}$

<u>Ticket Out the Door – Arcs and Central Angles of Circles</u>



Name each of the following. Be sure to use proper notation! $\overline{EC}$ is a diameter of the circle.

1. Semicircle
2. Minor Arc
3. Major Arc
4. Central Angle

Find each of the following measures on the diagram above.

5. $\angle EAD$
6. $\overset{\frown}{BC}$
7. $\overset{\frown}{BEC}$
8. $\overset{\frown}{ECB}$

Grade __10__
Unit # __3__ **Name of unit __Circles and Spheres__**

Lesson 4	Properties of Chords
E. Q.	How do I apply properties of arcs and chords in a circle?
Standard	MM2G3. Students will understand the properties of circles. (a, d)
Opening	Warm-up: Have the students draw a circle and label the following parts. Center, Radius, Diameter, Chord, Secant, and Tangent
Work session	Teacher-guided completion of the Theorems 1 and 2 About Chords of Circles graphic organizer. Teacher-guided examples of Theorems 1 and 2, and student-guided practice of theorems.
Closing	Classify each arc as a major arc, a minor arc or as a semicircle: $180°, 62°, 240°$. (answers: semicircle, minor arc, major arc)

Grade __10__
Unit # __3__ **Name of unit __Circles and Spheres__**

Lesson 5	Properties of Chords
E. Q.	How do I apply properties of arcs and chords in a circle?
Standard	MM2G3. Students will understand the properties of circles. (a, d)
Opening	Warm-up: Review Theorems 1 and 2
Work session	Teacher-guided completion of the Theorems 3 and 4 About Chords of Circles graphic organizer. Teacher-guided examples of Theorems 3 and 4, and student-guided practice of theorems.
Closing	Find the length of a chord of a circle with radius 8 that is a distance of 5 from the center. (answer: $2\sqrt{39}$)

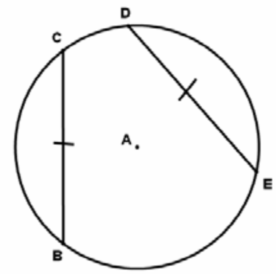
Chords of Circles Theorems Graphic Organizer

Chords of Circles Theorem #1

In the ____________ circle, or in __________________ circles, two _________ __________

are _____________ if and only if their _____________ ______________ are ______________.

≅ if and only if ≅

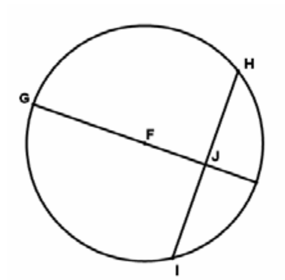


Chords of Circles Theorem #2

If a ____________ of a circle is _____________ to a _____________, then the diameter

______________ the __________ and its _____________.

≅ , ≅

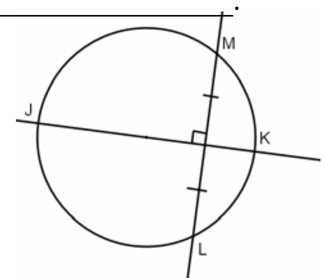


Chords of Circles Theorem #3

If one ___________ is a ________________ ______________ of another _____________,

then the __________ ___________ is a ________________.

is a _____________ of the circle.

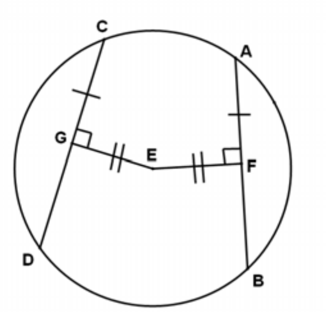


Chords of Circles Theorem #4

In the _________ circle, or in __________________ circles, two __________ are

______________if and only if they are __________________ from the ______________.

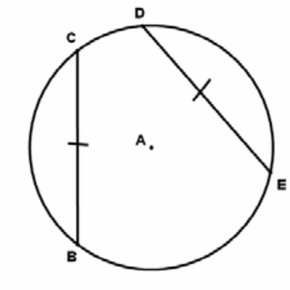
≅ if and only if ≅



Chords of Circles Theorems Graphic Organizer (Key)

Chords of Circles Theorem #1

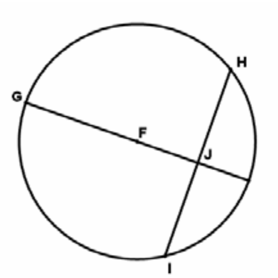
In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.



$\overset{\frown}{AB} \cong \overset{\frown}{BC}$ if and only if $\overline{AB} \cong \overline{BC}$

Chords of Circles Theorem #2

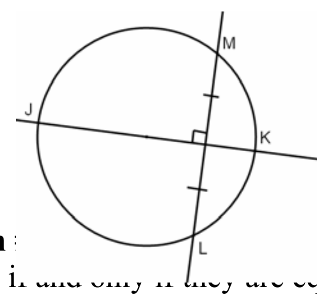
If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.



$\overline{DE} \cong \overline{EF}$, $\overset{\frown}{DG} \cong \overset{\frown}{GF}$

Chords of Circles Theorem

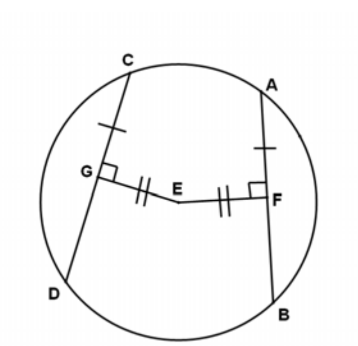
If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.



$\overline{JK}$ is a diameter of the circle.

Chords of Circles Theorem

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



$\overline{AB} \cong \overline{CD}$ if and only if $\overline{EF} \cong \overline{EG}$

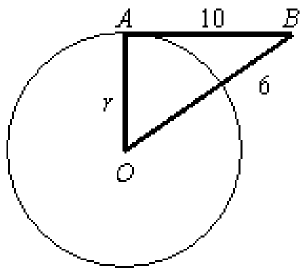
Grade __10__
Unit # __3__ **Name of unit __Circles and Spheres__**

Lesson 6	Properties of Chords
E. Q.	How do I apply properties of arcs and chords in a circle?
Standard	MM2G3. Students will understand the properties of circles. (a, d)
Opening	Warm-up: Review Theorems 3 and 4.
Work session	Complete *Is it Shorter Around or Across Learning Task*. Additional Task: http://www.geogebra.org/en/upload/files/english/Guy/Circles_and_angles/Two_chords_angles.html http://www.geogebra.org/en/upload/files/english/Guy/Circles_and_angles/Two_chords_angles_practice.html
Closing	Describe the properties of congruent chords of a circle. (answer: Congruent chords intercept congruent arcs and they are equidistant from the center of the circle.)

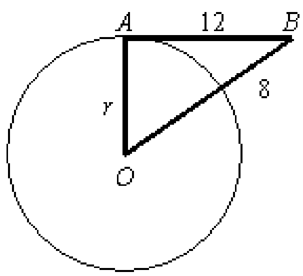
Grade __10__
Unit # __3__ **Name of unit __Circles and Spheres__**

Lesson 7 and 8	Properties of Circles including: line segments, central angles, arcs and chords.
E. Q.	How do I identify and apply all the properties of a circle?
Standard	MM2G3. Students will understand the properties of circles. (a, b, d)
Opening	Warm-up: Review and Assess Students on the Properties of a Circle.
Work session	Give Properties of a Circle – Assessment
Closing	Review and Assess Properties of a Circle

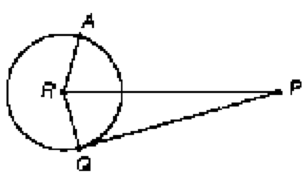
1. $\overline{AB}$ is tangent to $\odot O$ at A (not drawn to scale). Find the length of the radius r, to the nearest tenth.



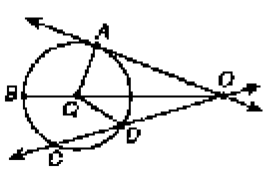
2. $\overline{AB}$ is tangent to $\odot O$ at A (not drawn to scale). Find the length of the radius r, to the nearest tenth.



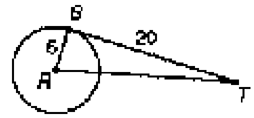
3. Given: $RP = 22$, $RA = 6$, $\overline{PQ}$ is tangent to $\odot R$ at Q
Find PQ.



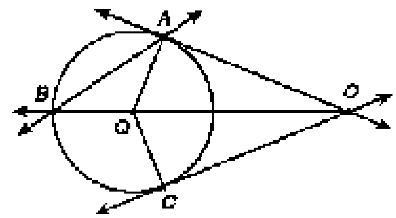
4. Given: $\overleftrightarrow{OA}$ is tangent to $\odot Q$ at A
List any right angles. Explain.



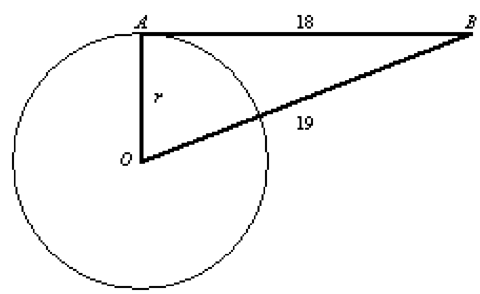
5. Given: $\overline{ST}$ is tangent to $\odot R$ at S
Find RT.



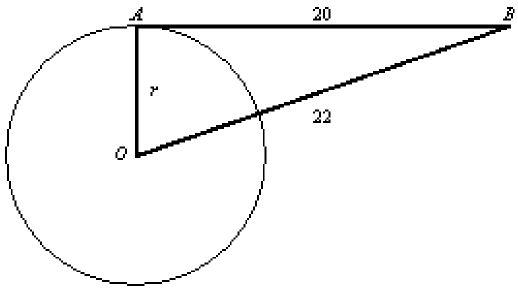
6. Given: $\overleftrightarrow{OA}$ and $\overleftrightarrow{OC}$ are tangent to $\odot Q$ at A and C, respectively. List any right angles.



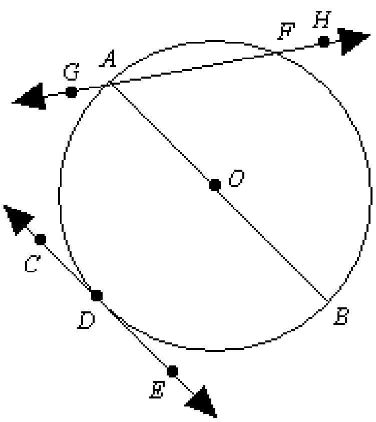
7. You are standing at point B. Point B is 19 feet from the center of the circular water storage tank and 18 feet from point A. $\overline{AB}$ is tangent to $\odot O$ at A. Find the radius of the tank. (Round answer to one decimal place)



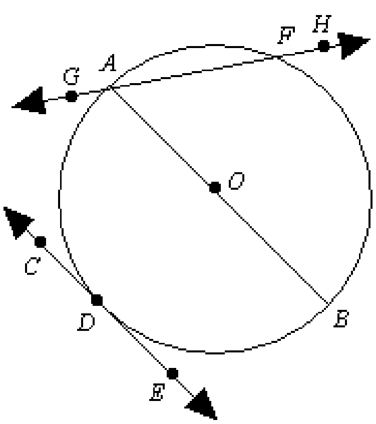
8. You are standing at point B. Point B is 22 feet from the center of the circular water storage tank and 20 feet from point A. $\overline{AB}$ is tangent to $\odot O$ at A. Find the radius of the tank. (Round answer to one decimal place)



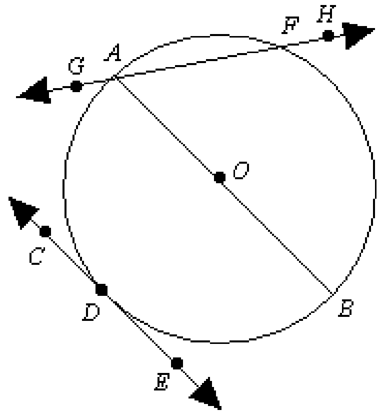
9. Identify all chords for circle O.



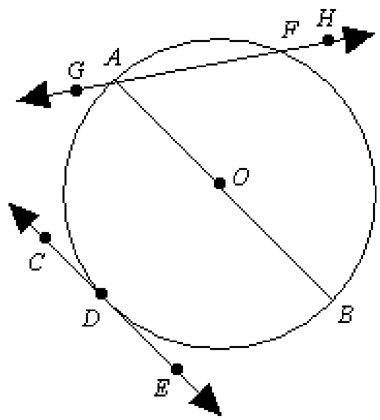
10. Identify all tangents for circle O.



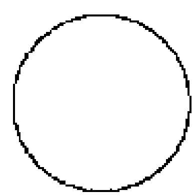
11. Identify all radii for circle O.



12. Identify all secants for circle O.

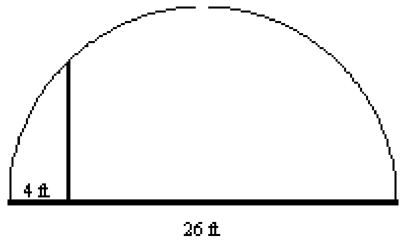


13. Define a secant of a circle and illustrate the definition on the circle below.

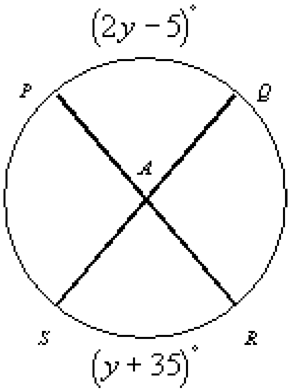


14. Define a tangent line to a circle. Draw a sketch to illustrate the definition.

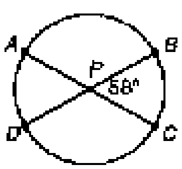
15. Inside a semicircular tunnel of diameter 26 feet, a vertical support beam is placed 4 feet from the side of the tunnel. How tall is the beam? (Round to one decimal place.)



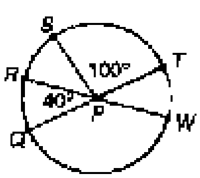
16. Find $m\,\widehat{PQ}$ in $\odot A$. Drawing is not to scale.



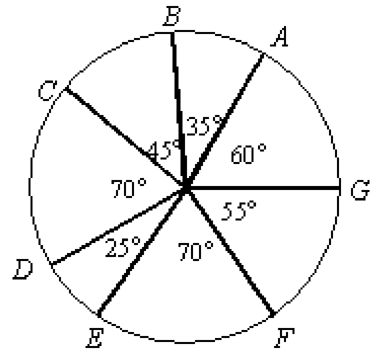
17. Find the measure of $\widehat{DBC}$ in $\odot P$.



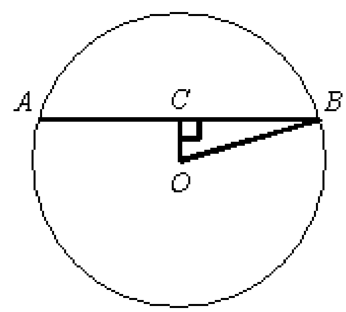
18. If $\overline{QT}$ and $\overline{RW}$ are diameters in $\odot P$, find $m\,\widehat{QW}$.



19. Identify the minor congruent arcs in the figure.

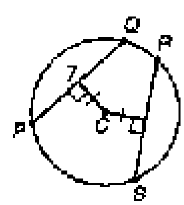


20. Given circle O with radius 25 and $OC = 7$. Find the measure of $\overline{AB}$.

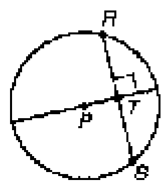


21. A footbridge is in the shape of an arc of a circle. The bridge is 11 ft tall and 27 ft wide. What is the radius of the circle that contains the bridge? Round your answer to the nearest tenth.

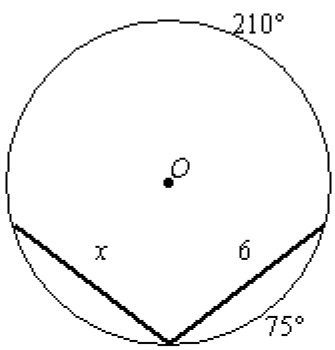
22. Find RS in $\odot C$. Explain your reasoning.



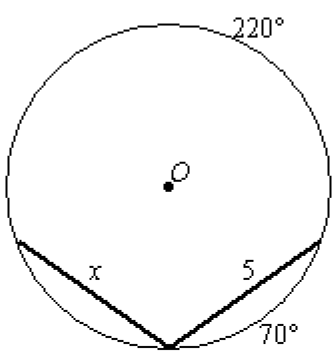
23. Given: $\odot P$ and $\overline{PT} \perp$ to chord $\overline{RS}$ at T. Decide whether or not $RT = TS$. Explain your reasoning.



24. Find the value of x to the nearest tenth.



25. Find the value of x to the nearest tenth.



Answers to Assessment Properties of Circles

1. 5.3
2. 5.0
3. $\sqrt{448} = 8\sqrt{7} \approx 21.2$
4. $\angle QAO$. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency; two lines are perpendicular if they intersect to form a right angle.
5. $\sqrt{425} = 5\sqrt{17} \approx 20.6$
6. $\angle QAO \ \ and \ \ \angle QCO$
7. 6.1 ft.
8. 9.2 ft
9. $\overline{AF}, \ \overline{AB}$
10. $\overleftrightarrow{CE}$
11. $\overline{OA}, \ \overline{OB}$
12. $\overleftrightarrow{HG}$
13. A secant of a circle is a line that intersects a circle twice. Sketches vary.
14. A tangent of a circle is a line that intersects a circle at exactly one point. Sketches vary.
15. 9.4 ft.
16. $75°$
17. $238°$
18. $140°$
19. $\overset{\frown}{CD} \cong \overset{\frown}{EF}; \overset{\frown}{BG} \cong \overset{\frown}{FD} \cong \overset{\frown}{EC}; \overset{\frown}{BF} \cong \overset{\frown}{GD} \cong \overset{\frown}{DA};$
$\overset{\frown}{AF} \cong \overset{\frown}{DB}; \overset{\frown}{CG} \cong \overset{\frown}{EB}$
20. 48
21. 13.8 ft.
22. $RS = 7$. In a circle, two chords that are equidistant from the center are congruent (Theorem 4).
23. Yes, *RT* = *TS*. A diameter that is perpendicular to a chord bisects the chord and its arc (Theorem 2).
24. 6
25. 5

Grade 10

Unit # **3** Name of unit **Circles and Spheres**-

Lesson 9	Using Inscribed Angles
E. Q. –	How do you use inscribed angles to solve problems?
Standard –	**MM2G3b:** Understand and use properties of chords, tangents, and secants as an application of triangle similarity. **MM2G3d:** Justify measurements and relationships in circles using geometric and algebraic properties.
Opening –	**Class Opener:** Comparing central angle and inscribed angle measures (see attached handout). Hopefully, the students will see that the relationship between the inscribed angle (the angle formed by the two chords is half the measure of the intercepted arc). **Vocabulary:** Inscribed angle Teacher will use http://www.geogebra.org/en/wiki/index.php/Circles_%28Angles%29 to introduce inscribed angles. **Collaborative pairs:** Have students draw a circle and an inscribed angle inside their circle. The students will then measure their angle and its intercepted arc. Using the endpoints of their intercepted arc, have them draw another inscribed angle. The student will then measure their newly formed angle and compare its measure to their previously drawn inscribed angle. Ask the students how the two angles compare and what relationship they see. (If two angles intercept the same arc, then the two angles are congruent.) **Powerpoint**—Inscribed angles and intercepted arcs. *Optional—Use if you like.* **Extension**—Paul is attending the musical production of "Cat on a Hot Tin Roof". You have a job as an usher at this particular theater. Paul has asked you to find him a seat such that he will have a 90° viewing angle of the stage. If the stage is 40 feet long, find the location(s) in which Paul will have the viewing angle that he has asked for. Sketch a picture and identify the locations for Paul's seat on your sketch.
Work session –	Student Practice A worksheet (attached)
Closing –	**Ticket Out the Door** – Have the students complete the top part of the attached graphic organizer.

Lesson 10	**Properties of Inscribed Polygons**
E. Q. –	How do you use the properties of Inscribed Polygons?
Standard –	**MM2G3d.** Justify measurements and relationships in circles using geometric and algebraic properties
Opening –	**Warm-up:** Give the students some review problems to reinforce inscribed angles and their measures. The student should check their answers with their partner and discuss any problems that they did not agree on. As a class go over any problems the students had trouble with. **Vocabulary:** inscribed angle (reinforce) **Discovery Learning or Guided Discovered Learning:** **(A)** Each student should draw a circle and its diameter. Using the endpoints of the diameter, have the students draw an inscribed angle inside the circle. The students then need to measure their inscribed angle and its intercepted arc. They should compare their findings with a partner. Together the student and his/her partner should come up with a conjecture about their findings. Hopefully, they should see that an angle inscribed inside a semicircle should always be a right angle. (B) Have students draw a circle and inscribe a quadrilateral inside of the circle. Partners will trade papers and find the angle measure for each angle in the circle. The partners will discuss each quadrilateral's angle measurements, and then form with a small group (4) to determine if they can find a relationship about the angles of a quadrilateral inscribed in a circle. If a pair or small group of students conjectured that the opposite angles are supplementary, let them present their conjecture. If no students discovered the relationship that opposite angles in a quadrilateral are supplementary, guide them into the relationship. **Practice the Skill:** Draw different quadrilaterals and assign numerical values to three of the angles then have students determine if the quadrilateral could be inscribed in a circle. Draw circles with different quadrilaterals inscribed in them. Give some quadrilaterals numerical values (less than 180°) for two consecutive angles. Have students determine the missing angle measures. Next, have some or all of the angle measures as algebraic expressions where the students have enough information to solve for at least one of the angles.
Work session –	Student Practice B worksheets (attached). Homework: assign Practice Problems similar to the ones for Practice the Skill or your preference.
Closing –	Writing Assignment: Have students write "What I Thought you Taught"

Warm-Up

Inscribed Angles

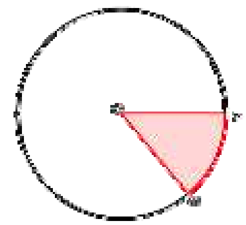
Directions:

1) With a compass, draw a circle. Draw a central angle.
2) Estimate the measure of your central angle.
3) What is the measure of your central angle?
4) What is the measure of the intercepted arc?
5) What is the relationship between the central angle and its intercepted arc?
6) Using the endpoints of the intercepted arc of your central angle, draw two chords that intersect at a point on the circle but not on the intercepted arc.
7) Make a prediction about the measure of this angle.
8) Make a prediction of the relationship between the measure of the central angle and the angle formed from the intersection of the two chords.
9) What is the measure of the angle formed by the two chords?
10) Write a comparison about your prediction and actual measurements of the two angles. Compare your data with your partner. (Optional)
11) Write a conclusion about the relationship between the angle formed by the 2 chords and its intercepted arc. Share your conclusion with your partner

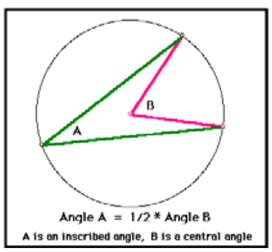
Warm-Up
Inscribed Angles Solutions

Directions:

1) With a compass, draw a circle. Draw a central angle.



2) Estimate the measure of your central angle.
3) What is the measure of your central angle?
4) What is the measure of the intercepted arc?
5) What is the relationship between the central angle and its intercepted arc? ***The two measurements will be equal***
6) Using the endpoints of the intercepted arc of your central angle, draw two chords that intersect at a point on the circle but not on the intercepted arc.



Angle A = 1/2 * Angle B
A is an inscribed angle, B is a central angle

7) Make a prediction about the measure of this angle. ***It should be approximately equal to ½ of the measure of the central angle.***
8) Make a prediction of the relationship between the measure of the central angle and the angle formed from the intersection of the two chords. ***The measure of the central angle is twice the measure of this angle.***
9) What is the measure of the angle formed by the two chords? ***It should be ½ the measure of the central angle.***

10) Write a comparison about your prediction and actual measurements of the two angles. Compare your data with your partner. (Optional)

11) Write a conclusion about the relationship between the angle formed by chords and its intercepted arc. Share your conclusion with your partner**.** ***The measure of this angle is ½ the measure of its intercepted arc, or the measure of the intercepted arc is twice the measure of the (inscribed) angle.***

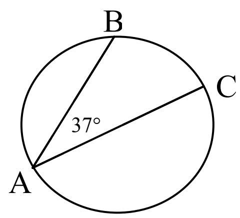
Name ___________________________________

Date ___________________________________

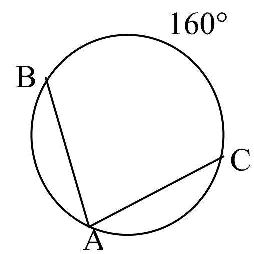
Practice A   Inscribed Angles

Find the measure of the indicated angle or arc.

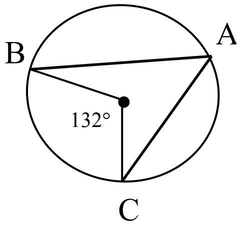
1. $m\overset{\frown}{BC}$ = ______



2. $m\angle BAC$ = _______

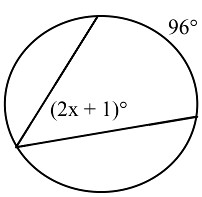


3. $m\angle BAC$ = _______

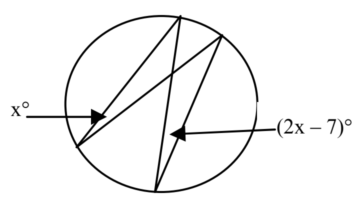


Find the value of x.

4. x = ________



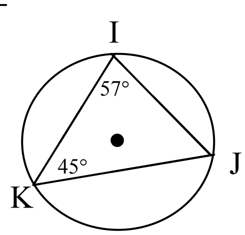
5. x = _________



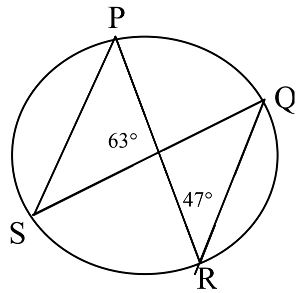
6. Find: $m\overset{\frown}{IJ}$ = ________

$m\overset{\frown}{JK}$ = ________

$m\overset{\frown}{IK}$ = ________



7. Find: m∠Q = ________
$m\overset{\frown}{SR}$ = ________

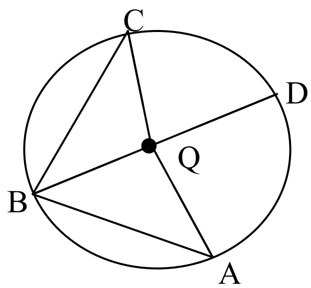


In Circle Q, m∠ABC = 72° and $m\overset{\frown}{CD}$ = 46°. Find each measure.

8. $m\,\overset{\frown}{CA}$ = ________

9. $m\,\overset{\frown}{AD}$ = ________

10. m∠C = ________



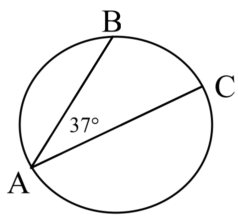
Name ___________________________________

Practice A   Inscribed Angles Answer Key

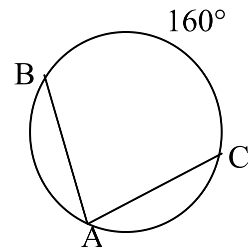
Date ____________________________________

Find the measure of the indicated angle or arc.

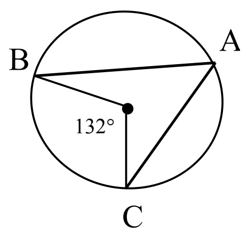
1. $m\overset{\frown}{BC}$ = 74°



2. m∠BAC = 80°

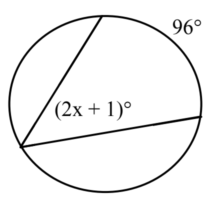


3. m∠BAC = 66°

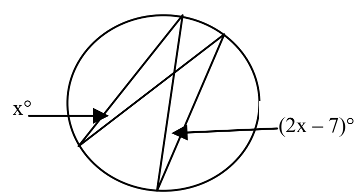


Find the value of x.

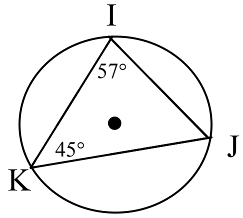
4. x = 23.5°



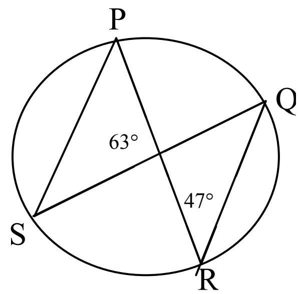
5. x = 7°



6. Find: $m\overset{\frown}{IJ}$ = 90°
$m\overset{\frown}{JK}$ = 114°
$m\overset{\frown}{IK}$ = 156°



7. Find: $m\angle Q$ = 70°
$m\overset{\frown}{SR}$ = 140°

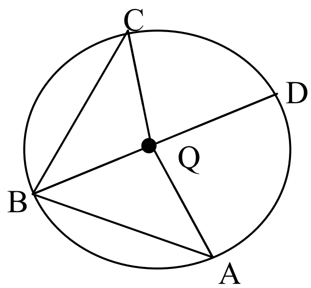


In Circle Q, $m\angle ABC = 72°$ and $m\overset{\frown}{CD} = 46°$. Find each measure.

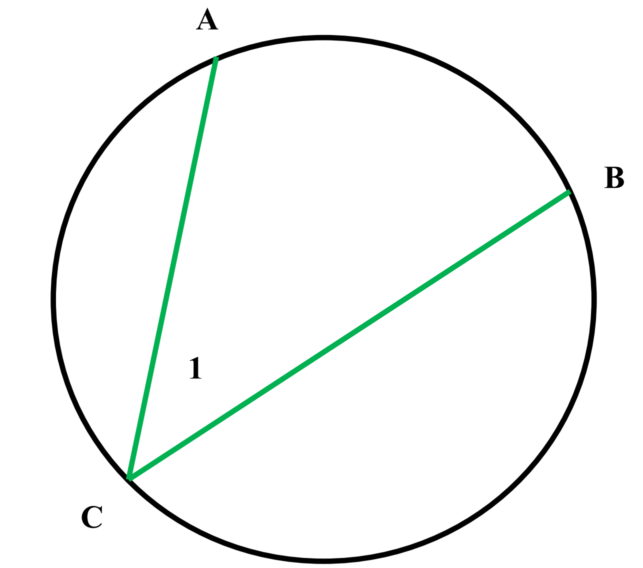
8. $m\overset{\frown}{CA}$ = 144°

9. $m\overset{\frown}{AD}$ = 98°

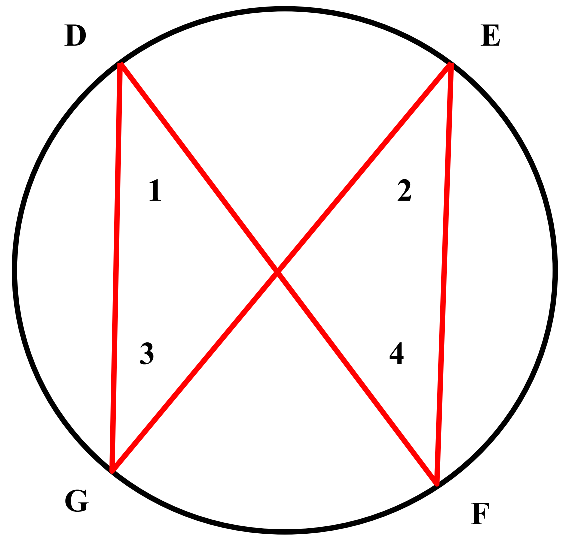
10. $m\angle C$ = 23°



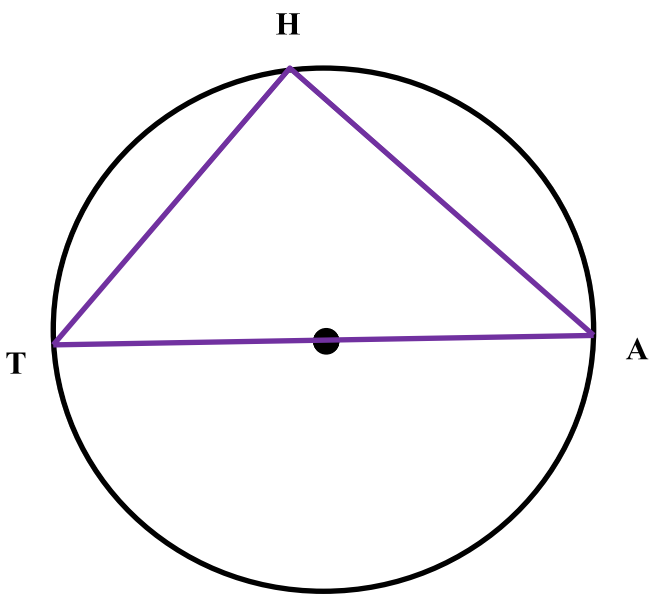
Inscribed Angles Graphic Organizer



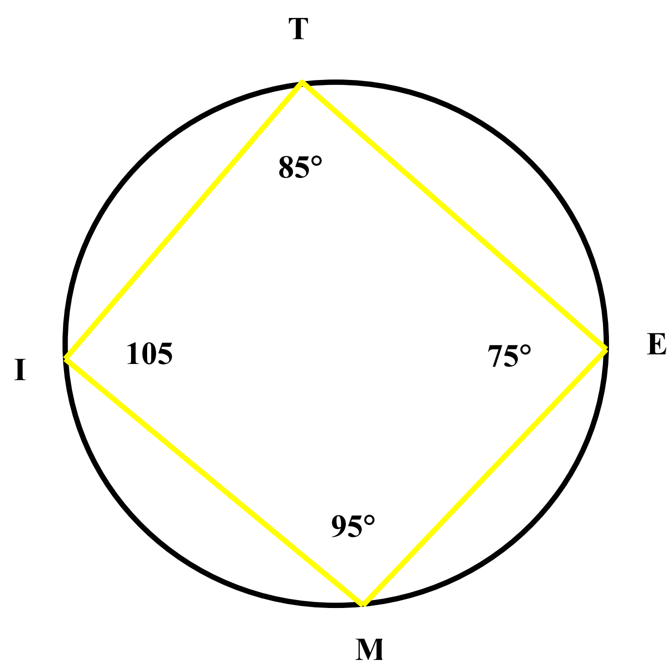
Inscribed angle = _______________
Intercepted arc = _______________
Give an example: _______________



Angles intercepting the same arc are __________
Give an example: ________________________



An angle inscribed in a semicircle = ______



Opposite angles in an inscribed quadrilateral are always __________________________

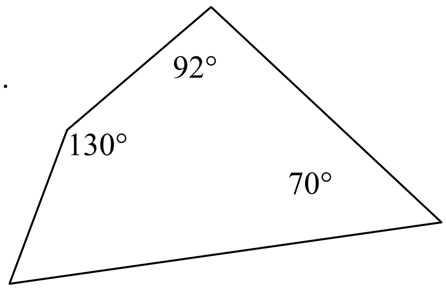
Name ____

Date ____

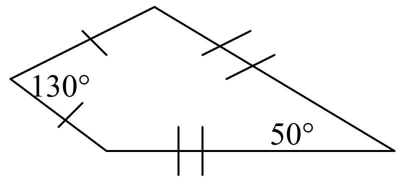
Practice B Inscribed Angles

Decide whether a circle can be circumscribed about the quadrilateral. Explain why or why not.

1.

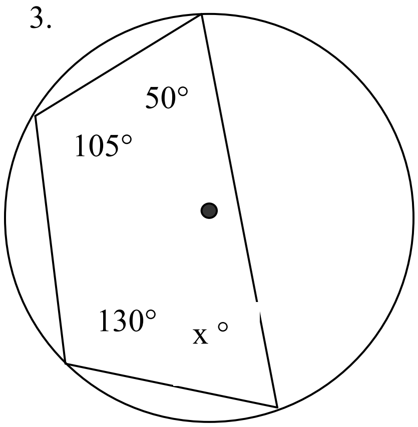


2.

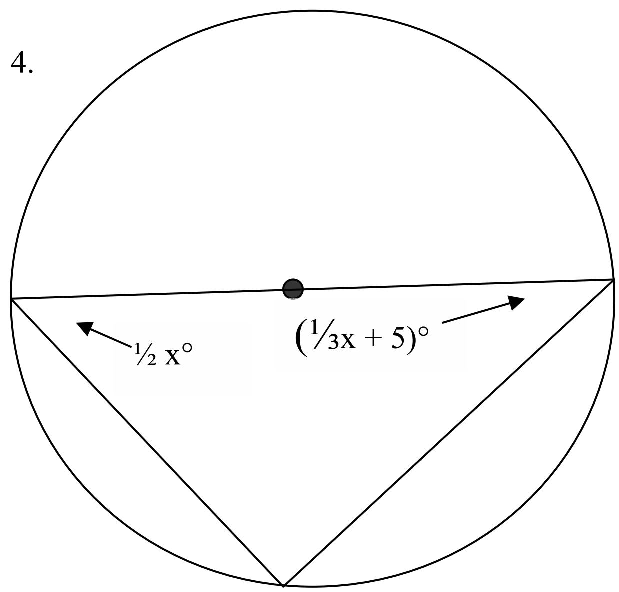


Find the value of each variable.

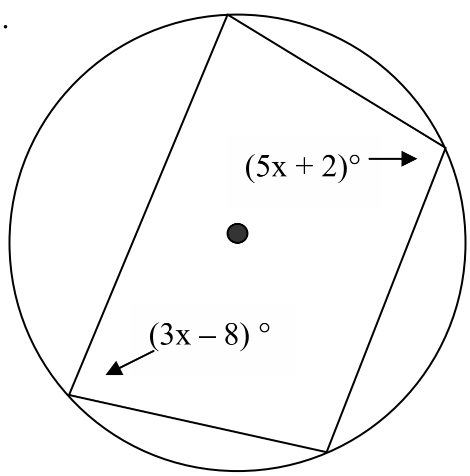
3.



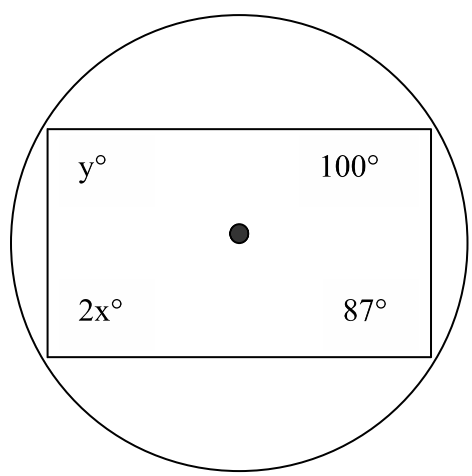
4.



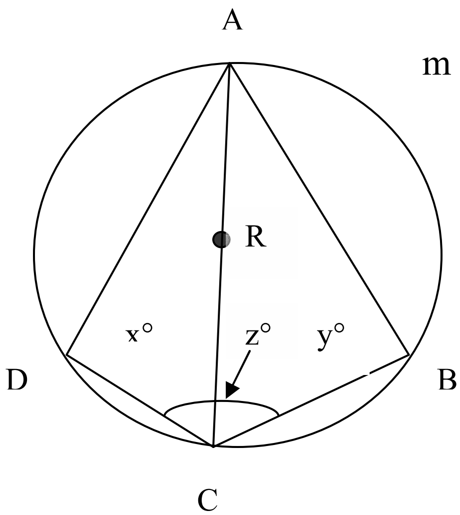
5.



6.

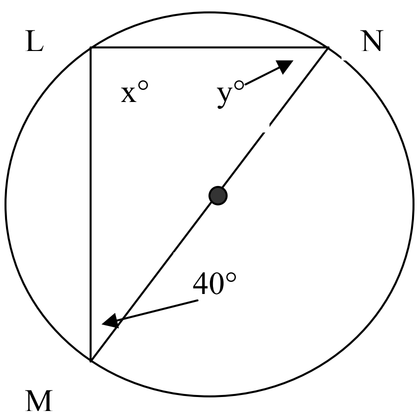


7. Find the value of x, y and z.

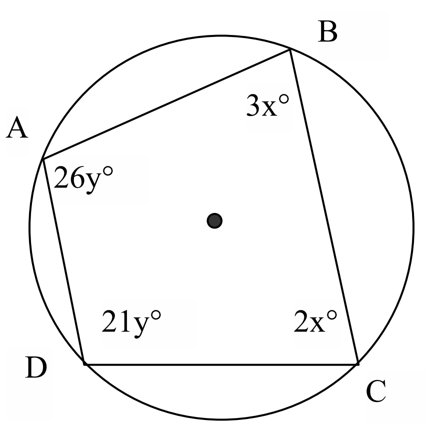


$m\overset{\frown}{BCD} = 136°$

8. Find the value of each variable.



9. Find the values of x and y. Then find the measures of the interior angles of the polygon.



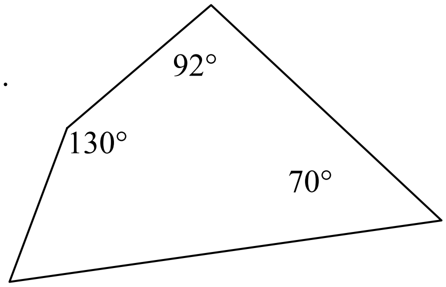
Name ____

Date ____

Practice B Inscribed Angles Answer Key

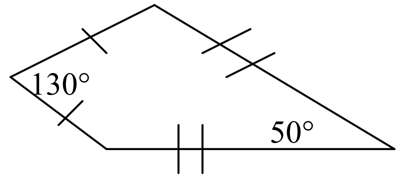
Decide whether a circle can be circumscribed about the quadrilateral. Explain why or why not.

1.



No, because opposite angles are not supplementary.

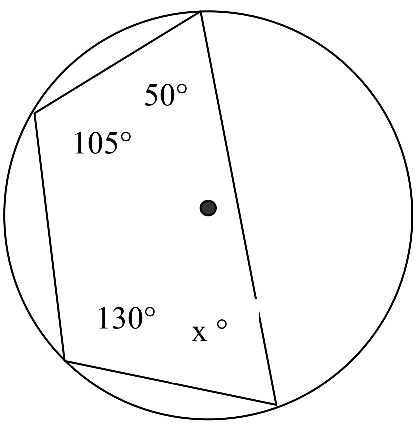
2.



Yes, because opposite angles in the quadrilateral are supplementary.

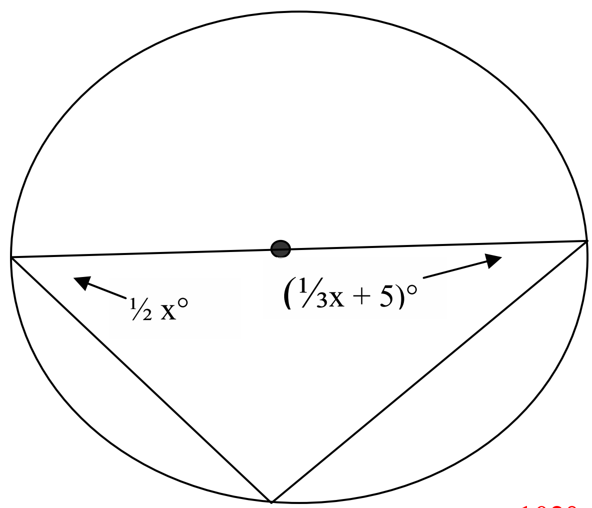
Find the value of each variable.

3.



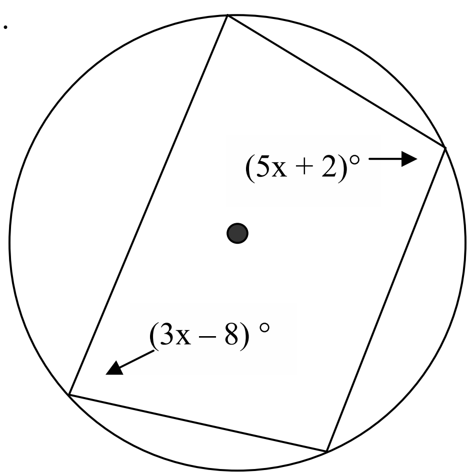
$x = 75°$

4.



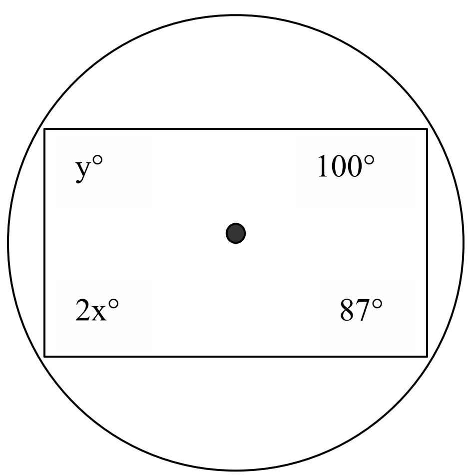
$x = 102°$

5.



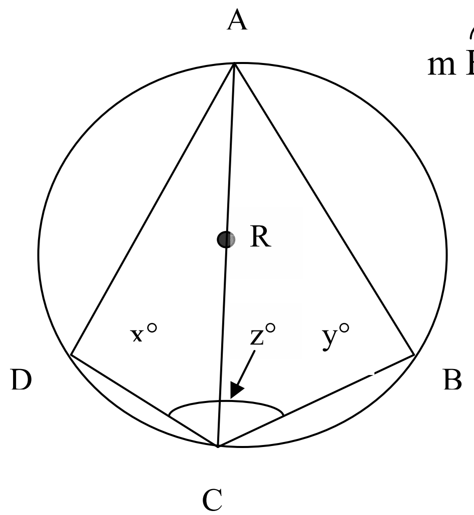
$x = 23.25°$

6.



$x = 40°$
$x = 93°$

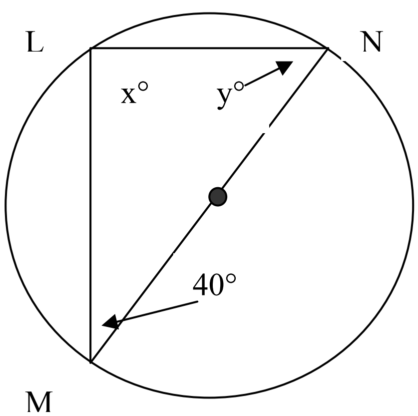
7. Find the value of x, y and z



m $\overset{\frown}{BCD}$ = 136°

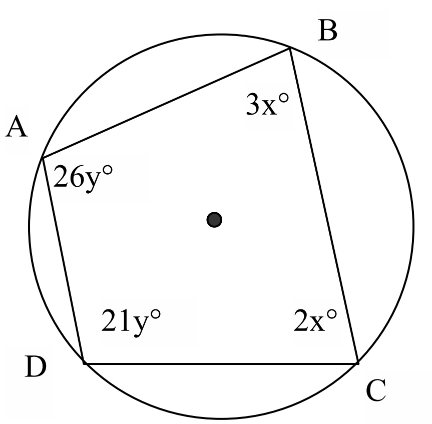
x = 90°
y = 90°
z = 112°

8. Find the value of each variable.



x = 90°
y = 50°

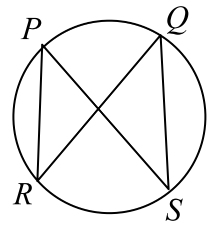
9. Find the values of x and y. Then find the measures of the interior angles of the polygon.



x = 25°
y = 5°
m∠A = 130°
m∠B = 75°
m∠C = 50°
m∠D = 105°

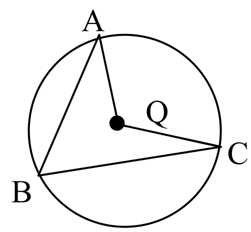
Possible Test Questions

1. Find $m\angle PSQ$ if $m\angle PSQ = 3y - 15$ and $m\angle PRQ = 2y + 10$.



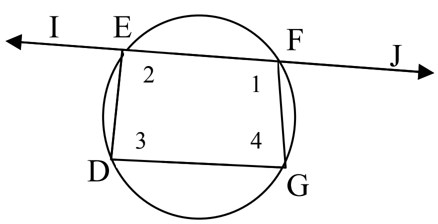
a. 30° b. 25° c. 0° d. 60°

2. Given: Circle Q and $m\angle B = 62°$, find $m\,\widehat{AC}$.



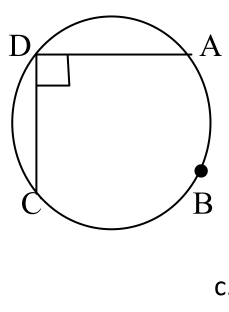
a. 248° b. 124° c. 236° d. 62°

3. Given: $m\angle IED = 91°$ and $m\angle JFG = 97°$
   Find the measure of each unknown angle. (not drawn to scale)



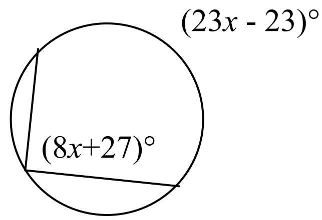
a. $m\angle 1 = 83°$, $m\angle 2 = 89°$, $m\angle 3 = 97°$, $m\angle 4 = 91°$
b. $m\angle 1 = 89°$, $m\angle 2 = 83°$, $m\angle 3 = 97°$, $m\angle 4 = 91°$
c. $m\angle 1 = 83°$, $m\angle 2 = 89°$, $m\angle 3 = 91°$, $m\angle 4 = 97°$
d. $m\angle 1 = 89°$, $m\angle 2 = 83°$, $m\angle 3 = 91°$, $m\angle 4 = 97°$

4. Use the diagram to find $m\,\widehat{ABC}$.



a. 270° b. 90° c. 180° d. 230°

5. Use the diagram to find the value of *x*.



a. 20/3

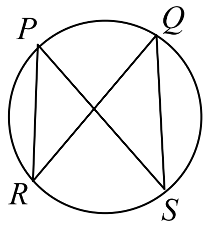
b. 15

c. 3/20

d. 11

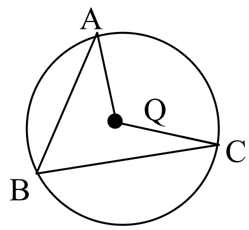
Possible Test Questions (Solutions)

1. Find $m\angle PSQ$ if $m\angle PSQ = 3y - 15$ and $m\angle PRQ = 2y + 10$.



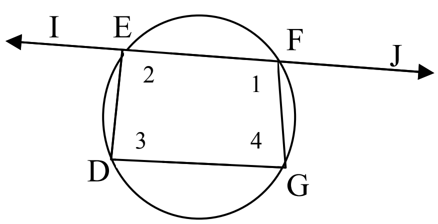
a. 30°   b. 25°   c. 0°   d. 60°

2. Given: Circle Q and $m\angle B = 62°$, find $m\overset{\frown}{AC}$.



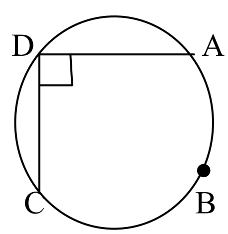
a. 248°   b. 124°   c. 236°   d. 62°

3. Given: $m\angle IED = 91°$ and $m\angle JFG = 97°$
Find the measure of each unknown angle. (not drawn to scale)



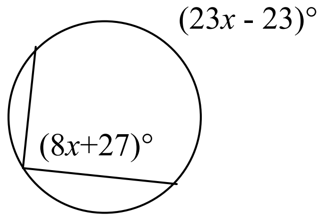
a. $m\angle 1 = 83°$, $m\angle 2 = 89°$, $m\angle 3 = 97°$, $m\angle 4 = 91°$
b. $m\angle 1 = 89°$, $m\angle 2 = 83°$, $m\angle 3 = 97°$, $m\angle 4 = 91°$
c. $m\angle 1 = 83°$, $m\angle 2 = 89°$, $m\angle 3 = 91°$, $m\angle 4 = 97°$
d. $m\angle 1 = 89°$, $m\angle 2 = 83°$, $m\angle 3 = 91°$, $m\angle 4 = 97°$

4. Use the diagram to find $m\overset{\frown}{ABC}$.



a. 270°   b. 90°   c. 180°   d. 230°

5. Use the diagram to find the value of *x*.



a. 20/3

b. 15

c. 3/20

d. 11

Unit # ___3___ Name of unit __Circles and Spheres___

Lesson 11	Properties of Special Angles
E. Q. –	Based on your investigations, what can you conclude about the relationships between a circle and the vertex of an angle?
Standard –	MM2G3. Students will understand the properties of circles b. Understand and use properties of central, inscribed, and related angles.
Opening –	Have students complete "Properties of Special Angles" Anticipation Guide. As a class, discuss results. **Angles of a Circle Learning Task** – Use the Special Angles Graphic Organizer and Geogebra's "Inscribed Angle" to guide students through discovery of the relationship between an inscribed angle and its intercepted arc ("On" section of graphic organizer)
Work session –	Students will complete the "Properties of Special Angles – Inscribed Angles" practice sheet.
Closing –	As a ticket-out-the-door, students will sketch the 5 combinations of chords, secants, and/or tangents that will create an inscribed angle.

Lesson 12	Properties of Special Angles
E. Q. –	Based on your investigations, what can you conclude about the relationships of a circle and the vertex of an angle?
Standard –	MM2G3. Students will understand the properties of circles b. Understand and use properties of central, inscribed, and related angles.
Opening –	**Angles of a Circle Learning Task** – Using the Special Angles Graphic Organizer and Geogebra's "Two Chords" and "Two Secants" to guide students through discovery of the relationship between interior angles and their intercepted arcs and the relationship between an angle formed in the exterior of a circle and their intercepted arcs ("Inside" and "Outside" sections of graphic organizer)
Work session –	Students will complete the "Properties of Special Angles" practice sheet to review the properties of all special angles.
Closing –	Have students summarize this lesson by writing a letter to the absent student detailing their conclusions about the relationships between a circle and the vertex of an angle. Summaries should include examples of each of the three possible vertex locations and should be written using the language of the standards.

Properties of Special Angles

Anticipation Guide

Name: ___________________________ Date: ___________ Period: ___________

Answer TRUE or FALSE for each scenario below. If TRUE, sketch an example.

_________1. Two chords can intersect in the exterior of a circle.

_________2. A secant and a chord can intersect in the interior of a circle.

_________3. Two tangents can intersect in the exterior of a circle.

_________4. The intersection of two secants can lie on a circle.

_________5. A tangent and a chord can intersect in the exterior of a circle.

Properties of Special Angles

Anticipation Guide

Name: ___________________________ Date: ___________ Period: ___________

Answer TRUE or FALSE for each scenario below. If TRUE, sketch an example.

_________1. Two chords can intersect in the exterior of a circle.

_________2. A secant and a chord can intersect in the interior of a circle.

_________3. Two tangents can intersect in the exterior of a circle.

_________4. The intersection of two secants can lie on a circle.

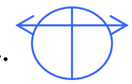
_________5. A tangent and a chord can intersect in the exterior of a circle.

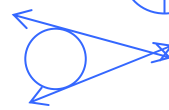
Properties of Special Angles
Anticipation Guide

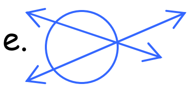
Name: ____________________________ Date: ___________ Period: ___________

Answer TRUE or FALSE for each scenario below. If TRUE, sketch an example.

FALSE 1. Two chords can intersect in the exterior of a circle.

TRUE 2. A secant and a chord can intersect in the interior of a circle. 

TRUE 3. Two tangents can intersect in the exterior of a circle. 

TRUE 4. The intersection of two secants can lie on a circle. 

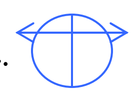
FALSE 5. A tangent and a chord can intersect in the exterior of a circle.

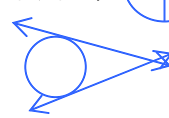
Properties of Special Angles
Anticipation Guide

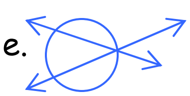
Name: ____________________________ Date: ___________ Period: ___________

Answer TRUE or FALSE for each scenario below. If TRUE, sketch an example.

FALSE 1. Two chords can intersect in the exterior of a circle.

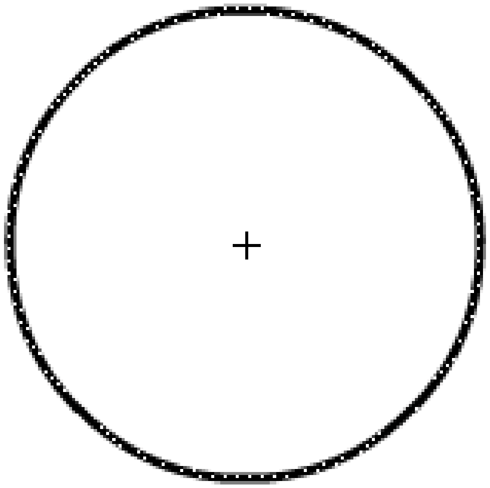
TRUE 2. A secant and a chord can intersect in the interior of a circle. 

TRUE 3. Two tangents can intersect in the exterior of a circle. 

TRUE 4. The intersection of two secants can lie on a circle. 

FALSE 5. A tangent and a chord can intersect in the exterior of a circle

Properties of Special Angles

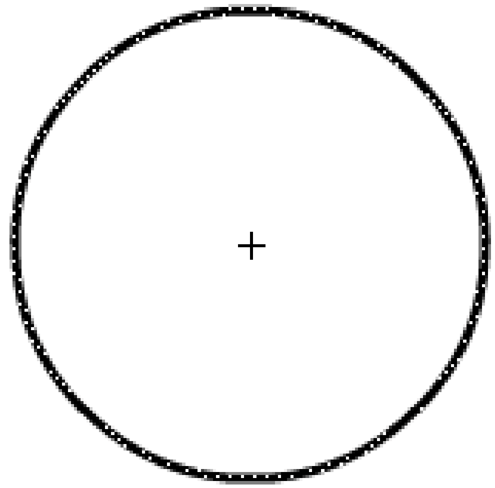


On

What is the relationship between an inscribed angle and the intercepted arc?

What combinations of chords, secants, and/or tangents would create an inscribed angle?

Choose one combination from your list above and draw an illustration on the diagram above.

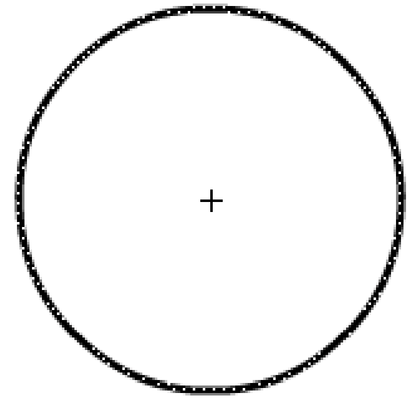


Interior

What is the relationship between the interior angles and their intercepted arcs?

What combinations of chords, secants, and/or tangents would create interior angles?

Choose one combination from your list above and draw an illustration on the diagram above.



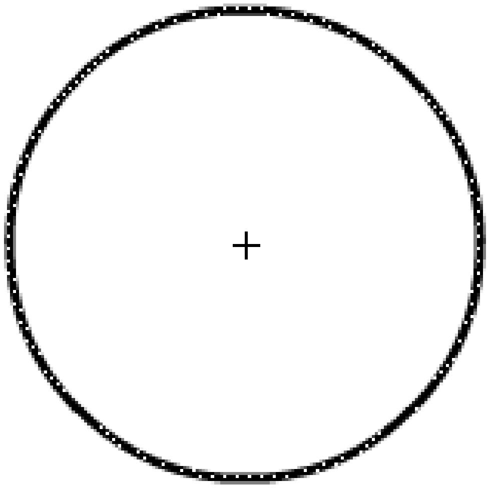
Exterior

What is the relationship between an angle formed in the exterior of a circle and its intercepted arcs?

What combinations of chords, secants, and/or tangents would create interior angles?

Choose one combination from your list above and draw an illustration on the diagram above.

Properties of Special Angles



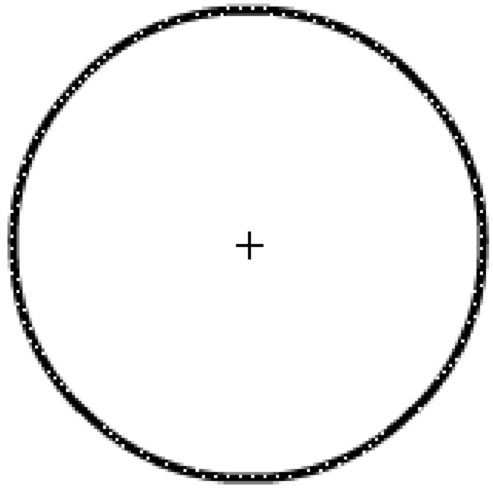
On

Use Geogebra's "Inscribed Angle".

Link from students' prior knowledge of central angles to help them explore the relationship of an inscribed angle and the intercepted arc. Have students write this relationship in their own words. ($m\angle = \frac{1}{2}$ intercepted arc)

Brainstorm combinations of chords, secants, and/or tangents that create an inscribed angle. (*chord/chord, tangent/secant, chord/tangent, chord/secant, secant/secant*)

Then have students choose one of these combinations and create an example on the diagram above.



Interior

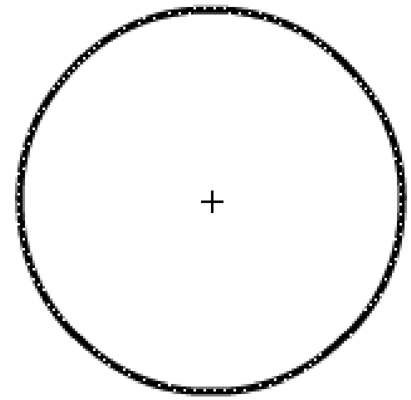
Use Geogebra's "Two Chords"

Students will discover the relationship between interior angles and their intercepted arcs. Be sure to stress which arcs correspond to which interior angles. Have students write this relationship in their own words.

($m\angle = \frac{1}{2}$(sum of their intercepted arcs))

Brainstorm combinations of chords, secants, and/or tangents that create an interior angle. (*chord/chord, secant/secant, chord/secant*)

Then have students choose one of these combinations and create an example on the diagram above.



Exterior

Use Geogebra's "Two Secants"

Students will discover the relationship between an angle formed in the exterior of a circle and their intercepted arcs. Have students write this relationship in their own words.

($m\angle = \frac{1}{2}$(major arc – minor arc))

Brainstorm combinations of chords, secants, and/or tangents that create an angle in the exterior of a circle (*tangent/tangent, tangent/secant, secant/secant*)

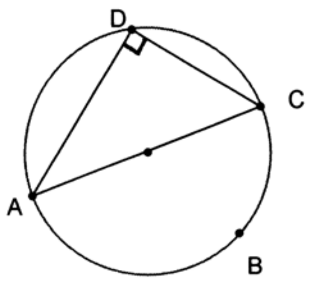
Then have students choose one of these combinations and create an example on the diagram above.

Properties of Special Angles
Inscribed Angles

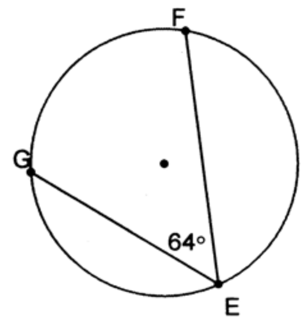
Name:______________________________ Date:______________________

Determine the following measures.

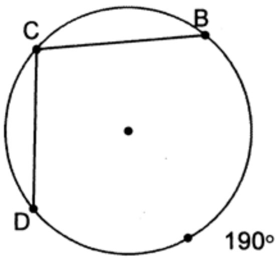
1. $m\widehat{ABC}$ =____________



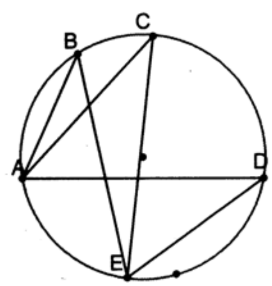
2. $m\widehat{GF}$=_____________



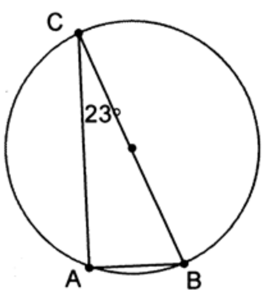
3. $m\angle DCB$=_____________



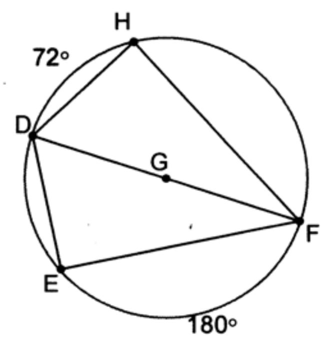
4. If the $m\widehat{AE}$ = 72°, find $m\angle$ ACE, $m\angle$ ABE, and $m\angle$ ADE



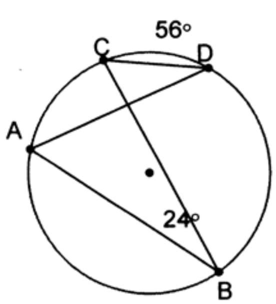
5. $m\angle$ B = _____________



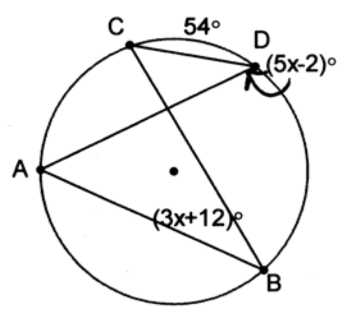
6. $m\angle$ H =___ $m\angle$ HFD = _____ $m\angle$ HDF = _____
$m\angle$ FDE = ___ $m\angle$ EFD = ____



7. $m\angle$ A = _____ $m\angle$ D =_____

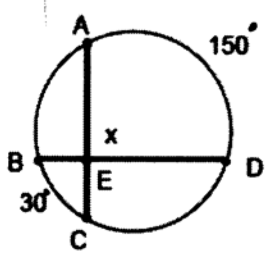


8. $m\angle$ A = _____ $m\angle$ D =_____



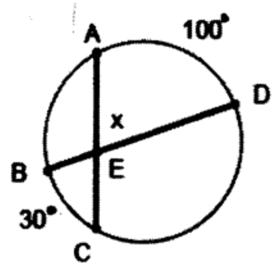
Properties of Special Angles

1.



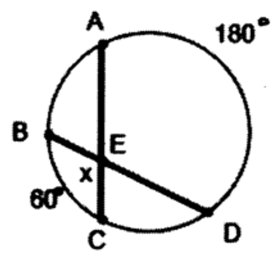
$\overset{\frown}{AD} = 150°$
$\overset{\frown}{BC} = 30°$
$\angle BEC = x$

2.



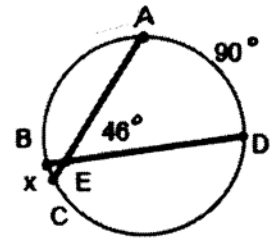
$\overset{\frown}{AD} = 100°$
$\overset{\frown}{BC} = 30°$
$\angle BEC = x$

3.



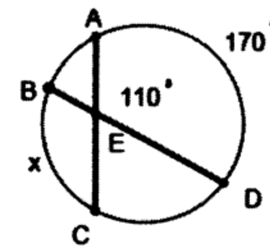
$\overset{\frown}{AD} = 180°$
$\overset{\frown}{BC} = 60°$
$\angle BEC = x$

4.



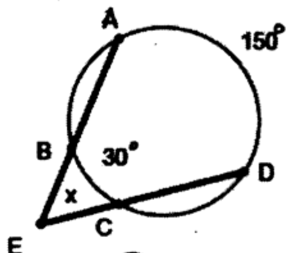
$\overset{\frown}{AD} = 90°$
$\overset{\frown}{BC} = x$
$\angle BEC = 46°$

5.



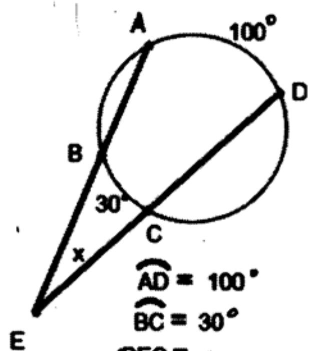
$\overset{\frown}{AD} = 170°$
$\overset{\frown}{BC} = x$
$\angle BEC = 110°$

6.



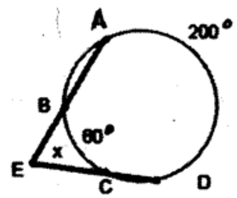
$\overset{\frown}{AD} = 150°$
$\overset{\frown}{BC} = 30°$
$\angle BEC = x$

7.



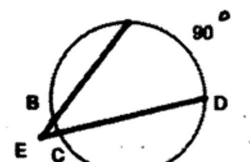
$\overset{\frown}{AD} = 100°$
$\overset{\frown}{BC} = 30°$
$\angle BEC = x$

8.



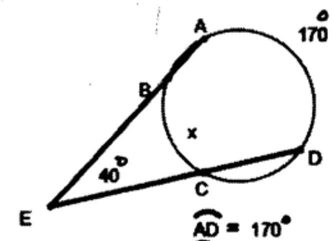
$\overset{\frown}{AD} = 200°$
$\overset{\frown}{BC} = 60°$
$\angle BEC = x$

9.



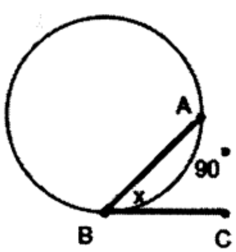
$\overset{\frown}{AD} = 90°$
$\overset{\frown}{BC} = x$
$\angle BEC = 43°$

10.



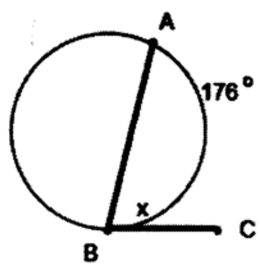
$\overset{\frown}{AD} = 170°$
$\overset{\frown}{BC} = x$
$\angle BEC = 40°$

11.



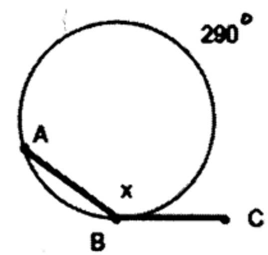
$\angle ABC = x$
$\overset{\frown}{AB} = 90°$

12.



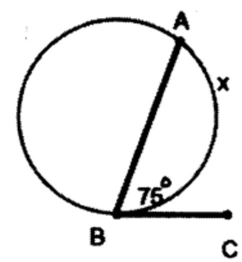
$\angle ABC = x$
$\overset{\frown}{AB} = 176°$

13.



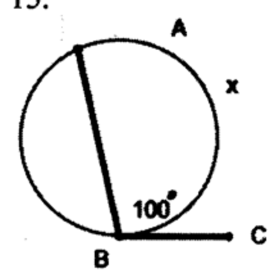
$\angle ABC = x$
$\overset{\frown}{AB} = 290°$

14.



$\angle ABC = 75°$
$\overset{\frown}{AB} = x$

15.



$\angle ABC = 100°$
$\overset{\frown}{AB} = x$

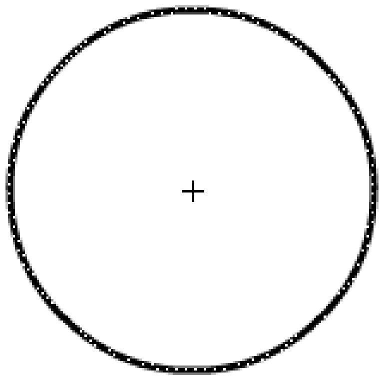
Name____________________________ Date________________

Grade ___10______
Unit # ___3___ Name of unit ___Circles and Spheres___

Lesson 13	Segment Lengths
E. Q. –	How can we apply the properties of chords, tangents, and secants to determine the length of various segments?
Standard –	MM2G3. Students will understand the properties of circles a. Understand and use properties of chords, tangents, and secants as an application of triangle similarity. d. Justify measurements and relationships in circles using geometric and algebraic properties.
Opening –	Have students go ahead and draw and label the intersecting chords, intersecting secants, and intersecting secant/tangent on the graphic organizer. This will help reinforce the concept of a chord, secant, and tangent. Then, on a separate sheet of paper (or on the back of the graphic organizer), have students quickly hypothesize a relationship between the segments formed in each circle. **Lines and Line Segments of a Circle Task** – Use the Segment Lengths Graphic Organizer and Geogebra's "Chord Lengths in a Circle" and "Exterior Segments in Circles" to guide students through examples of the relationship between chords, tangents, and secant segments.
Work session –	Students will complete the "Segment Lengths" practice sheet. The "Circles" practice sheet could be used to review both properties of special angles and segment lengths.
Closing –	As a summarizer, have students complete the "Create-Your-Own" section of the "Segment Lengths" graphic organizer. Once they have constructed and labeled their diagrams, students will exchange papers with a partner to find the length of the missing segment. Students will exchange papers back and check that their partner calculated the missing length correctly.

Segment Lengths in Circles

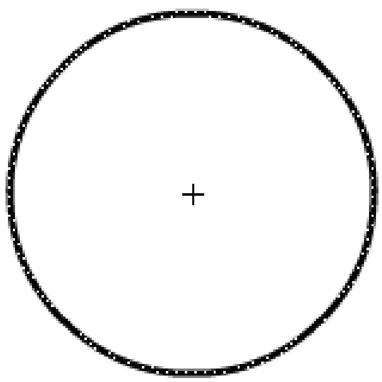
Chords



Mathematical Statement:

Calculations:

Secants



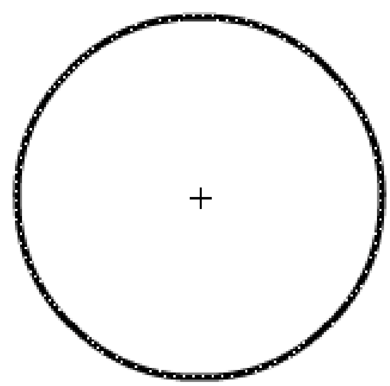
Mathematical Statement:

Calculations:

Secant/Tangent

Mathematical Statement:

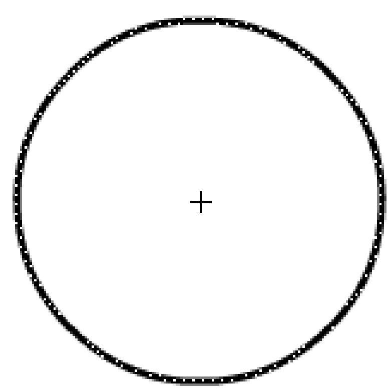
Calculations:



Create-Your-Own

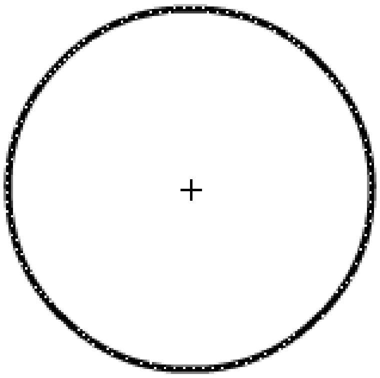
Mathematical Statement:

Calculations:



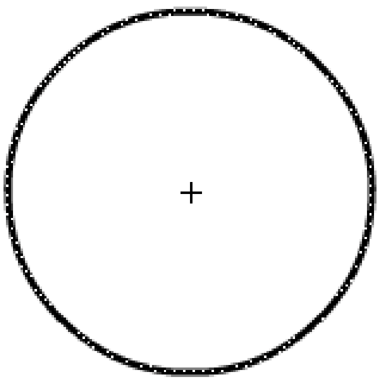
Segment Lengths in Circles

Chords



Use Geogebra's "Chord Lengths in a Circle".
Using explicit instruction, have students practice calculating the missing segment length. Repeat the exercise in Geogebra with several different measurements.
Once students understand the concept, have them state the theorem using a mathematical statement that relates to their diagram (this diagram will have been drawn and labeled during the opener). The student will measure three of the four segments created by the intersecting chords, then exchange papers with a partner and find the missing segment from their partners diagram.

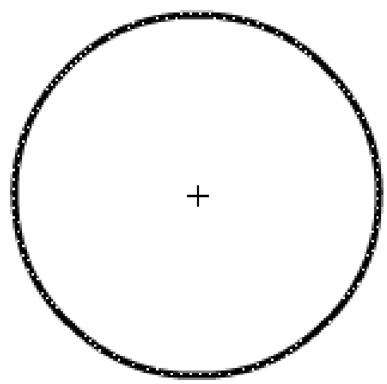
Secants



Use Geogebra's "Exterior Segments in Circles". (example on the ***right***)
Using explicit instruction, have students practice calculating the missing segment length. Repeat the exercise in Geogebra with several different measurements.
Once students understand the concept, have them state the theorem using a mathematical statement that relates to their diagram (this diagram will have been drawn and labeled during the opener). The student will measure three of the four segments created by the intersecting secants, then exchange papers with a partner and find the missing segment from their partners diagram.

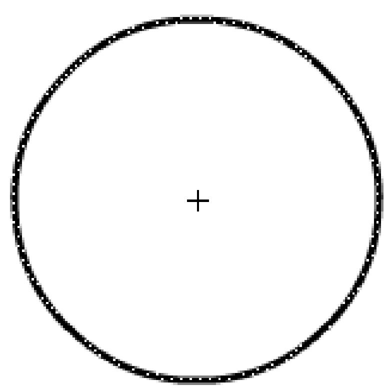
Secant/Tangent

Use Geogebra's "Exterior Segments in Circles". (example on the ***left***)
Using explicit instruction, have students practice calculating the missing segment length. Repeat the exercise in Geogebra with several different measurements.
Once students understand the concept, have them state the theorem using a mathematical statement that relates to their diagram (this diagram will have been drawn and labeled during the opener). The student will measure three of the four segments created by the intersecting secant and tangent, then exchange papers with a partner and find the missing segment from their partners diagram.



Create-Your-Own

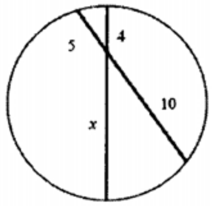
As a summarizer, have students complete this "Create-Your-Own" section. Once they have constructed and labeled their diagrams, students will exchange papers with a partner to find the length of the missing segment. Students will exchange papers back and check that their partner calculated the missing length correctly.



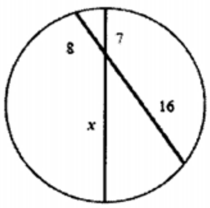
Segment Lengths

Name ____________________________ Date____________

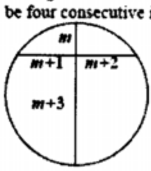
1. Find the value of x.



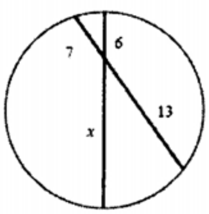
2. Find the value of x.



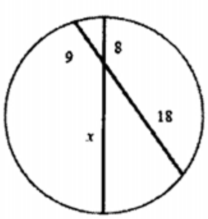
3. Show that it is not possible for the lengths of the segments of the two intersecting chords to be four consecutive integers.



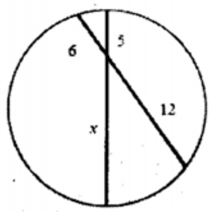
4. Find the value of x.



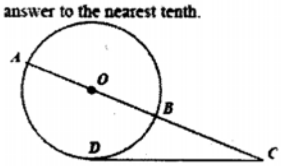
5. Find the value of x.



6. Find the value of x.

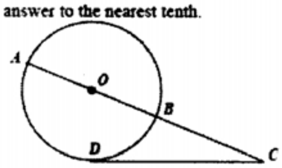


7. Find the diameter of the circle (not drawn to scale). $BC = 18$, and $DC = 21$. Round your answer to the nearest tenth.



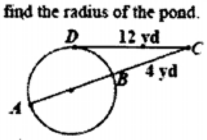
[A] 6.5 [B] 9.2 [C] 5.6 [D] 42.5

8. Find the diameter of the circle (not drawn to scale). $BC = 12$, and $DC = 20$. Round your answer to the nearest tenth.

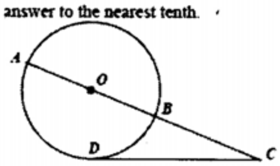


[A] 12.8 [B] 18.6 [C] 21.3 [D] 45.3

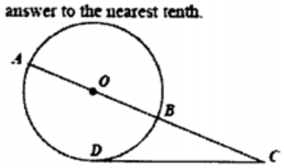
9. To find the radius of a circular pond, Maria measured $\overline{CD}$ and $\overline{BC}$. If $\overline{CD}$ is a tangent, find the radius of the pond.



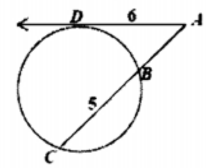
10. Find the diameter of the circle (not drawn to scale). $BC = 10$, and $DC = 16$. Round your answer to the nearest tenth.



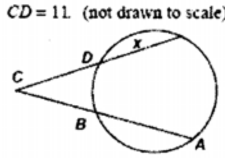
11. Find the diameter of the circle (not drawn to scale). $BC = 17$, and $DC = 23$. Round your answer to the nearest tenth.



12. Determine AB.

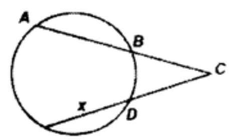


13. Find the value of x if $AB = 17$, $BC = 10$, and $CD = 11$. (not drawn to scale)

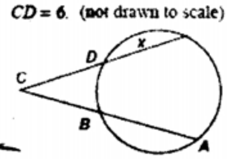


[A] 10.4 [B] 15.1 [C] 15.8 [D] 13.5

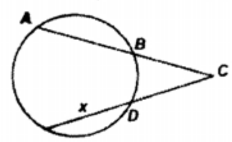
14. Find the value of x if $AB = 23$, $BC = 14$, and $CD = 13$. (not drawn to scale)



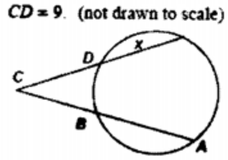
15. Find the value of x if $AB = 20$, $BC = 8$, and $CD = 6$. (not drawn to scale)



16. Find the value of x if $AB = 17$, $BC = 9$, and $CD = 10$. (not drawn to scale)



17. Find the value of x if $AB = 16$, $BC = 11$, and $CD = 9$. (not drawn to scale)

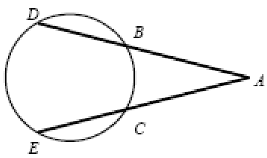


Circles

Name___________________________ Date__________

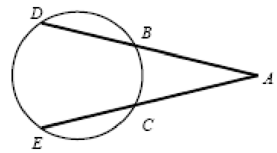
1.

If $m\widehat{DE} = 121$ and $m\widehat{BC} = 83$, find $m\angle A$.



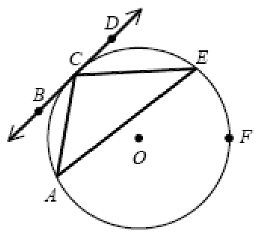
2.

If $m\widehat{DE} = 113$ and $m\widehat{BC} = 67$, find $m\angle A$.



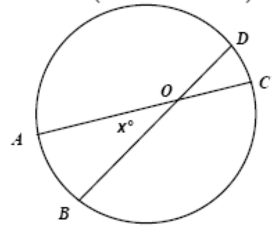
3.

Given: $\overleftrightarrow{BD}$ is tangent to $\odot O$ at C. The measure of $\widehat{EFA} = 206$ and $m\angle ECD = 42$. Find $m\widehat{AEC}$.



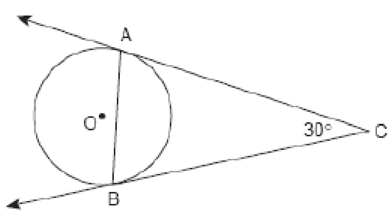
4.

Find the value of x if $m\widehat{AB} = 62$ and $m\widehat{CD} = 56$ (not drawn to scale).



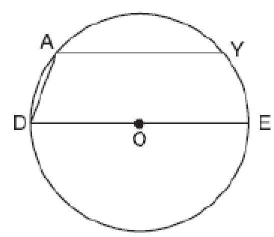
5.

The accompanying diagram represents circular pond O with docks located at points A and B. From a cabin located at C, two sightings are taken that determine an angle of 30° for tangents $\overline{CA}$ and $\overline{CB}$.



6.

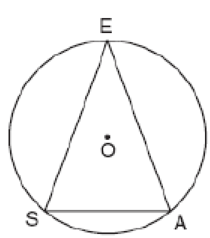
In the accompanying diagram of circle O, chord $\overline{AY}$ is parallel to diameter $\overline{DOE}$, $\overline{AD}$ is drawn, and $m\widehat{AD} = 40$.



What is $m\angle DAY$?

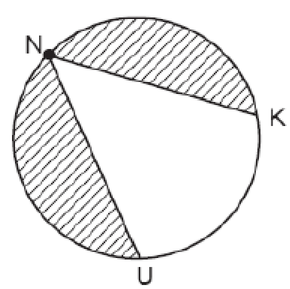
7.

A machine part consists of a circular wheel with an inscribed triangular plate, as shown in the accompanying diagram. If $\overline{SE} \cong \overline{EA}$, $SE = 10$, and $m\widehat{SE} = 140$, find the length of $\overline{SA}$ to the *nearest tenth*.



8.

The NUK Energy Company is designing a new logo, as shown in the accompanying diagram, with $m\widehat{NK} = 130$ and $m\widehat{NK} = m\widehat{NU}$.



What is the measure of $\angle KNU$?

Test Items

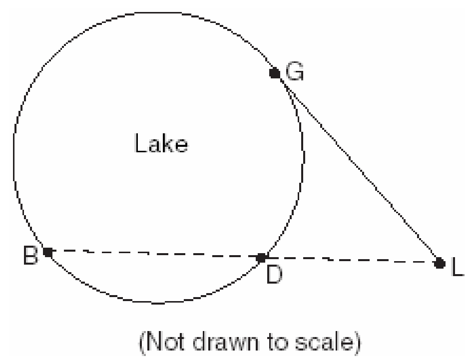
Special Angles and Segment Lengths

- (**Angles**) A small fragment of something brittle, such as pottery, is called a shard. The accompanying diagram represents the outline of a shard from a small round plate that was found at an archaeological dig.

If ray BC is a tangent to arc AB at B and m$\angle$ABC = 45, what is the measure of arc AB (the outside edge of the shard)?

a. 45°
b. 90°
c. 135°
d. 225°

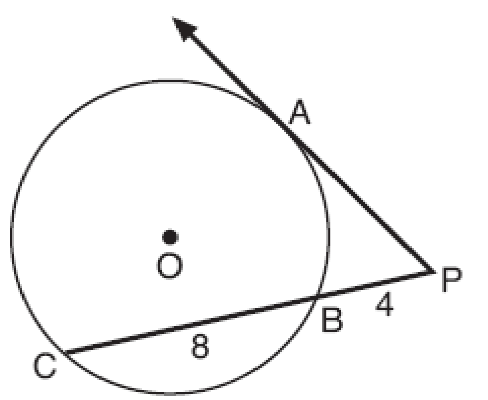
- (**Lengths**) In the accompanying diagram, cabins B and G are located on the shore of a circular lake and cabin L is located near the lake. Point D is a dock on the lake shore and is collinear with cabins B and L. The road between cabins G and L is 8 miles long and is tangent to the lake. The path between cabins L and dock D is 4 miles long.



What is the length, in miles, of $\overline{BD}$?

a. 24
b. 12
c. 8
d. 4

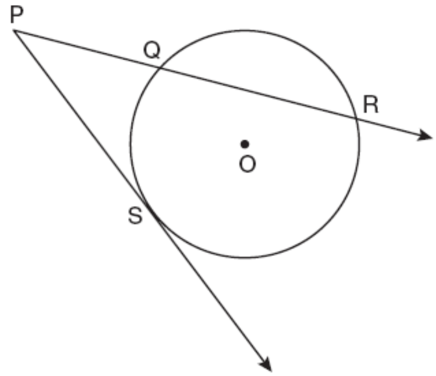
- (**Lengths**) In the accompanying diagram, $\overrightarrow{PA}$ is tangent to circle O at A, $\overline{PBC}$ is a secant, PB = 4, and BC = 8



What is the length of $\overline{PA}$?

a. $4\sqrt{6}$
b. $4\sqrt{2}$
c. $4\sqrt{3}$
d. 4

- (**Lengths**) In the diagram below, $\overline{PS}$ is a tangent to circle O at point S. $\overline{PQR}$ is a secant, PS = x, PQ = 3, and PR = x + 18

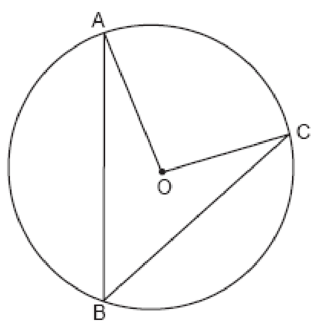


(Not drawn to scale)

What is the length of $\overline{PS}$?

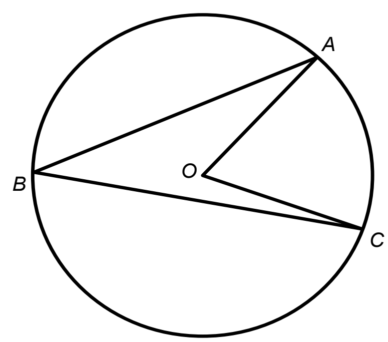
a. 6
b. 9
c. 3
d. 27

- (**Angles**) In the accompanying diagram of circle O, $\overline{AB}$ and $\overline{BC}$ are chords and m$\angle$ AOC = 96. What is the m$\angle$ ABC?



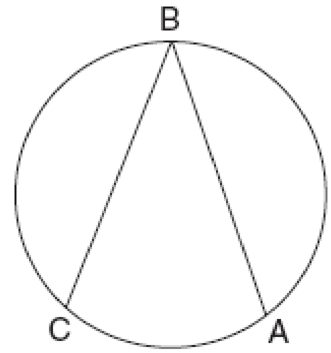
a. 32
b. 48
c. 96
d. 192

- (**Angles**) In the diagram below, circle O has m$\angle$ ABC = z. What is the m$\angle$ AOC?



a. z
b. 2z
c. $\frac{1}{2}$z
d. z^2

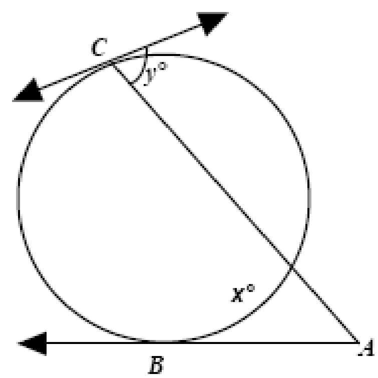
• (**Angles**) The new corporate logo created by the design engineers at Magic Motors is shown in the accompanying diagram.



If chords $\overline{BA}$ and $\overline{BC}$ are congruent and $m\widehat{BC} = 140$, what is m $\angle$ B?

a. 40
b. 80
c. 140
d. 280

• (**Angles**) Find the measure of x and y if m $\angle$ A = 19 and $m\widehat{BC}$ =118 (not drawn to scale)



a. x = 80; y = 162
b. x = 99; y = 81
c. x = 80; y = 81
d. x = 99; y = 162

• (**Angles**)

An angle inscribed in a semicircle is

a. 180°
b. a right angle
c. equal to the measure of its arc
d. an acute angle

The opposite angles of an inscribed quadrilateral are

a. equal in measure
b. right angles
c. each obtuse angles
d. supplementary

The measure of an angle formed by two chords intersecting within a circle is

a. the sum of the intercepted arcs
b. the difference of the intercepted arcs
c. half the difference of the intercepted arcs
d. half the sum of the intercepted arcs

The measure of an angle formed by two secants drawn to a circle from the same external point is

a. the different of the intercepted arcs
b. half the sum of the intercepted arcs
c. half the difference of the intercepted arcs
d. the sum of the intercepted arcs

An angle formed by a tangent to a circle and a chord contains 64 degrees. How many degrees are in its intercepted arc?

a. 32
b. 64
c. 128
d. 180

If an angle inscribed in a circle has a measure of 64°, then its intercepted arc has a measure of

a. 32
b. 64
c. 128
d. 164

Lesson 14	Arc lengths of circles
E. Q. –	How do we find the length of an arc of a circle?
Standard –	MM2G3: How do we use properties of circles to solve problems involving length of an arc of a circle?
Opening –	Key Vocabulary: Circumference, Arc Length Option 1 (Hands-On): Teacher will open class with a review of Circumference and Arcs of Circles by giving each pair of students a round object such as a soup can, a pipe cleaner and a ruler. The students will be asked to recall what circumference of a circle means in relation to the object. Hopefully, they will know to wrap the pipe cleaner around the object, cut off the excess and then lay out the pipe cleaner in a linear fashion to measure its length (cm units work best). If at least one student does not suggest this, then the teacher should pose questions to lead them to this conclusion. Students should be directed to check their circumference measure using the circumference formula ($C = 2\pi r$) learned previously. Next, the student should either color with a marker a section of the pipe cleaner or actually cut a piece of the pipe cleaner and wrap it around the object to show an arc of the circle. The class will discuss at this time that the colored part (or the cut piece) of the pipe cleaner represents a part of (fraction of) the circumference of the original circle. The cut piece should be measured and then expressed as a fraction of the original circumference. Allow several students to share their findings. Option 2 (Technology): Teacher will open class with a review of Circumference and Arcs of Circles by giving each student a KWL chart (attached) and providing circle and arc examples via Math Open Reference (http://www.mathopenref.com/circumference.html), Smartboard, Geometer's Sketchpad or some other investigative tool using technology. Students will complete the K and W, leaving the L to be completed at the end of this lesson.
Work session –	• Go to http://www.mathopenref.com/arclength.html and preview how arc length changes as the central angle changes and vice-versa. Do this as teacher-directed using LCD projector. • Explain that our lesson will involve using proportions related to arc lengths, central angles and circumference. We will: (1) Find the arc length given the central angle and radius/diameter; (2) Find the central angle (measure of the arc in degrees) given the arc length and the radius/diameter; (3) Find the radius of a circle given the arc length and central angle measure; (4) Find the circumference of a circle given the central angle and the arc length • Use Powerpoint: Arc Length to present the lesson with distributed guided practice. The PPT explains how the formula for arc length of a circle is derived using proportions. Then it goes through examples of the arc length formula being transposed to find either radius, circumference or central angle measure. Give students a hard copy of the Powerpoint (15 slides - 6 Slides per page with Notes)
Closing –	Explain on the L portion of the KWL sheet what you have learned about the arc length of a circle and how it is related to the circumference of the circle. The students should respond that the arc length is a part of the circumference, represented as a fraction & they should give an example.

Lesson 15	Arc Lengths of Circles
E. Q. –	How do we find the length of an arc of a circle?
Standard –	MM2G3: How do we use properties of circles to solve problems involving length of an arc of a circle?
Opening –	Big Four (Geometry Mix)
Work session –	• Open the website http://www.mathopenref.com/arclength.html (using an LCD) Direct students to find the following, given radius = 10 cm on each circle. Use the arc length formula that was learned the day before. After students have had time to find the arc lengths described below, have students to use the Smartboard, Interwrite pad, etc. to show the class how to check results at the website. Find the arc length for the following central angles: (1) Central angle = 90 degrees (3) Central angle = 140 degrees (2) Central angle = 75 degrees (4) Central angle = 240 degrees • Math 2 textbook/NTG practice problems (allow students to work in small groups) NoteTaking Guide p. 234-244, Textbook p. 224-236 • If time permits, allow pairs of students to do Kagan Geometry p. 428 (ISBN # 978-1-879097-68-1) Resource Book
Closing –	Ticket Out the Door: (1) Given central angle 50 degrees and radius 8 ft, find the arc length (2) Given arc length 18 cm & central angle 25 degrees, find the radius

Note: Be sure to use the Big Four (Geometry Mix) Power Point with Lesson 15

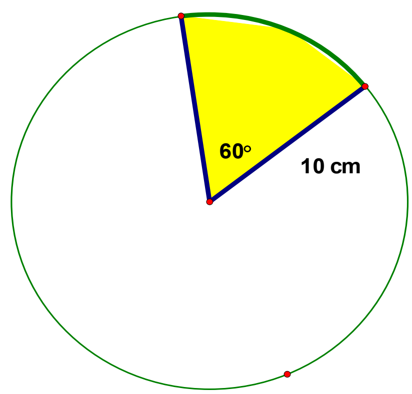
Lesson 16	**Area of Sectors of Circles**
E. Q. –	**How do you find the area of a sector of a circle?**
Standard –	**MM2G3c – Use the properties of circles to solve problems involving the length of an arc and the area of a sector.** **MM2G3d – Justify measurements and relationships in circles using geometric and algebraic properties.**
Opening –	**The teacher will begin class by giving each student graph paper and a compass and have each student draw a circle as close to an exact unit on the grid as possible. Each student will then cut his/her circle out and count each square. If the student remembers the area formula, he/she should check the area algebraically. Then the student will be told to draw and cut out a slice of "pizza" using the center of the circle as the guide. The students will write the ratio of the piece to the whole and this will lead into the Area of Sectors.**
Work session –	**PowerPoint: Area of Sectors of Circles (Notes and Examples are included in PowerPoint)**
Closing – **Bk. Alignment**	**Mathematics 2 – McDougal Littell- Text pages: 230 – 235 (Teacher's choice of problems)**

Lesson 17	**Area of Sectors of Circles**
E. Q. –	**How do you apply the use of the area of a sector of a circle?**
Standard –	**MM2G3c – Use the properties of circles to solve problems involving the length of an arc and the area of a sector.** **MM2G3d – Justify measurements and relationships in circles using geometric and algebraic and algebraic properties.**
Opening –	**Area of Sectors Activator Worksheet Activity**
Work session –	**The students will work on computers on Geometers Sketchpad or www.geoebra.org and complete the Technology Activity on page 236 in the textbook (Mathematics 2- McDougal Littell). The students should also complete practice problems 16 – 29 on page 235. Next the students should access www.mathopenref.com and complete the Area of Sector Activity.**
Closing –	**Rally Table Activity (Kagan Book p. 429) (ISBN # 978-1-879097-68-1)**

Areas of Sectors Activator
Now that arcs have been discovered, a review of the area and circumference of a circle is in order.

The length of an arc equals the circumference times the measure of the central angle divided by 360^{o}. The area of a sector equals the area of the circle times the measure of the central angle divided by 360^{o}.

See circle below and use proportions to find the area of the sector and the length of the arc.



$$C = 20\pi$$
$$A = 10^2\pi = 100\pi$$
$$\frac{60°}{360°} = \frac{x}{20\pi}$$
$$\frac{60°}{360°} = \frac{x}{100\pi}$$

Students made a pie chart using percentages in a previous task. In the game show *Wheel of Fortune,* three contestants compete to earn money and prizes for spinning a wheel and solving a word puzzle. The game requires some understanding of probability and the use of the English language. Make a spinner to use in the *Wheel of Fortune* game.

A spinner can be constructed using a pencil and a paper clip on a circle with the correct sectors. Have students create a playing wheel that has eight spaces (sectors) marked $3000, $750, $900, $400, Bankrupt, $600, $450, and Lose a Turn. It is not necessary to have all the sectors the same size.

Based upon your spinner, calculate the area of each sector and arc length.

Source for this is the link below from *The World's Largest Math Event 8*.

http://my.nctm.org/eresources/view_article.asp?article_id=6221&page=12

Lesson 18	Surface Area of Sphere
E. Q. –	How do we find the surface area of a sphere?
Standard –	MM2G4a: Use and apply surface area and volume of a sphere
Opening –	Key Vocabulary: Sphere, Great Circle, Hemisphere Use the first few slides of the Powerpoint: Surface Area of Sphere to have students brainstorm examples of spheres in the real world and what it means to find the surface area of them
Work session –	• Explain that our lesson will involve a discovery lesson to explore and generate the formula for the Surface Area of a Sphere • Use the reminder of the Powerpoint: Surface Area of Sphere to present the lesson with the discovery activity embedded and also distributed guided practice. • The discovery lesson Orange You Glad….? should be distributed to the students at the appropriate time in the Powerpoint
Closing –	Ticket Out the Door is included on the Powerpoint

Orange You Glad…..?

Objective: Each group of students will use an orange to investigate and make a conjecture about the formula for finding the surface area of a sphere.

Materials for each group of 4 students:

- **1 orange (navel oranges work the best)**
- **2 sheets of cm graph paper**
- **Scissors**
- **Small knife**
- **Plain paper**
- **Wet paper towels**

Notes to the Teacher

Research shows that students commonly have difficulty remembering formulas unless they discover those formulas themselves. This activity offers students the opportunity to discover the formula for the surface area of a sphere through a guided investigation. When you use this activity with your students for the first time, you may find the following notes to be useful.

- Student definition for surface area should include the following: Surface area is the number of square units needed to cover the entire outside of a solid figure.
- The correct unit of measure for surface area is square units.
- Several partial surface areas can be added to find the total surface area for a solid. You may wish to use a rectangular solid as an example to illustrate this point.
- Help students realize that the surface area of the orange equals the surface area of its peel.
- Help students realize that the radius of a great circle of a sphere is the same as the radius of the sphere itself. With an orange, the radius of a great circle is the radius of the orange.
- To reveal a great circle of an orange, students need to cut the orange exactly in half. To help them do this, suggest that they cut through the "equator" of the orange to create two "hemispheres".
- If a group comes up with the wrong formula, such as surface area = $3\pi r$^2, help them find their error. Most often students leave gaps between pieces of peel or overlap them.

Orange You Glad…..?

Objective: Each group of students will use an orange to investigate and make a conjecture about the formula for finding the surface area of a sphere.

Materials for each group of 4 students:

- **1 orange (navel oranges work the best)**
- **2 sheets of cm graph paper**
- **Scissors**
- **Tape**
- **Ruler**
- **Small knife**
- **Plain paper**
- **Wet paper towels**

Group Members:

____________________________   ______________________________

____________________________   ______________________________

1. Using the cm graph paper, scissors, and tape, estimate the surface area of the orange by covering it as best as possible. Count the squares to estimate the total surface area.   Estimate: _______________

2. Cut your orange in half to expose a great circle of the orange.
   Measure: Radius ______________   Diameter ________________
   Circumference of great circle = Circumference of Sphere = ___________
3. Trace as many great circles as possible on your plain paper.

4. Estimate how many of these great circles you think you can cover with pieces of your orange's peel.   Estimate: ____________

5. Use the knife to gently score each hemisphere halfway to make the peeling come off easily. Tear off pieces of the orange peel carefully and place them in the great circles, covering as many great circles as possible with the whole peeling of the orange. Flatten each piece out as much as possible. You may have to cut the pieces into smaller pieces. Each great circle must be covered entirely with no overlaps or gaps.

6. What is the formula for the area of any circle? ______________

   What is the area of your great circle? ____________

7. How many great circles did you cover in all, using the entire peel of the orange? ______ How close was this to your original prediction in #4?

So, what is the surface area of the whole sphere? _____________

8. Based on your findings, write the formula for:

Surface area of a Sphere: _____________

9. Do you think your equation (formula) will work to find the surface area of any sphere? Explain your reasoning.

____

10. Compare your total surface area to your original prediction using the cm graph paper in #1. Were you close?

EAT YOUR ORANGES…..CLEAN UP YOUR MESS….WIPE OFF YOUR DESK WITH WET PAPER TOWELS.

Complete the problems assigned on the Powerpoint.

Lesson 19	Surface Area of Sphere
E. Q. –	What is the effect of changing the radius or diameter on the surface area of a sphere?
Standard –	MM2G4b: Determine the effect of changing the radius or diameter on the surface area of a sphere
Opening –	Show a visual display of all the planets in our solar system at the following website: http://www.kidskonnect.com/content/view/95/27/. Scroll down until you see the planets all in a horizontal alignment. (Reminder: My Very Educated Mother Just Served Us Nine Pizzas for the names of the planets in order from the Sun – Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto). Compare and contrast the size of the planets and discuss why the surface areas of these planets are all different. Which of the planets look as though the surface areas would be about the same? Which has the largest surface area? Smallest?
Work session –	Display and give the students a copy of the Microsoft Word document: Planets in Our Solar System (taken from the same web site mentioned above, but condensed into one chart attached) that shows the diameters of each planet in miles. Have students to complete the questions on the worksheets. Allow them to work in pairs so that they may compare results. Balloon Blow-Up: Ask for a volunteer to come up front, blow up a spherical balloon (not all the way) and then explain how to find its circumference, radius and surface area. They should ask for a flexible measuring tape in order to measure the circumference around the sphere. Everyone record results in a chart form that students make on their own paper (described below). Now blow up the balloon a little more. Recalculate the circumference, radius, diameter and surface area. Continue doing this 1-2 more times until the balloon is about to pop. Complete the chart. Circumference Diameter Radius Surface Area ____ Explore: If the original diameter is doubled, then explain the relationship between the surface areas in a complete sentence.____ Explore: If the original diameter is tripled, then explain the relationship between the surface areas. ____ Explore: If the original radius is halved, then explain the relationship between the surface areas. ____ Explore: If the original radius is multiplied by 10, then explain the relationship between the surface areas. ____
Closing –	Ticket Out the Door: If a golfball has a diameter of 4 cm and a bowling ball has a diameter of 20 cm, then explain how the surface area changes.

The Planets in Our Solar System

Planet					Diameter (miles)	Surface Area (sq. miles) = $4\pi(r)^2$ or $\pi(d)^2$		
Mercury					3,031 miles			
Venus					7,521 miles			
Earth					7,926 miles			
Mars					4,222 miles			
Jupiter					88,729 miles			
Saturn					74,600 miles			
Uranus					32,600 miles			
Neptune					30,200 miles			
Pluto (a dwarf planet)					1,413 miles			

Complete the surface area column for each of the planets in the chart.

1. Find the surface area of Earth. ________________

2. Which planet has diameter closest to Earth's diameter? ________
   Find its surface area. ______________

3. About what percentage of Earth's surface area is this planet? ________
   (Compute the ratio of Earth and this planet, round to 2 decimal places)

4. Find the surface area of Jupiter, our largest planet. __________

5. About how many Earth's surfaces would it take to equal
   Jupiter's surface? _______

6. Write a complete sentence to explain the relationship between the Earth's
   surface area and Jupiter's surface area.
   ____

7. Explain the relationship between the surface areas of
   Uranus and Mars. ____________________________________

8. Which planet has close to 9 times the surface area of Neptune? __________

Lesson 20	Volume of Spheres
E. Q. –	How do you find the volume of a sphere?
Standard –	MM2G4a - Use and apply surface area and volume of a sphere MM2G4b – Determine the effect on surface area and volume of changing the radius or diameter of a sphere.
Opening –	The teacher will begin class by having a plastic cylinder and a Styrofoam ball (each being of equal diameter (the base of the cylinder and the ball)). The teacher will call on a volunteer. The volunteer will place the ball into the cylinder and then the teacher will pour water around the ball into the cylinder. The student will then remove the ball without spilling any of the water. The teacher will pour the water out and the student will measure the water left with a ruler. The teacher will ask the class what ratio of the water is left to the whole? This will lead into the volume of Spheres formula.
Work session –	PowerPoint: Volume of Spheres (Notes and Examples are included in PowerPoint)
Closing – Bk. Alignment	Geometry – McDougal Littell - Text pages: 762 – 763 (10 – 17, 20 - 29)

Lesson 21	Volume of Spheres
E. Q. –	How do you find the volume of a sphere?
Standard –	MM2G4a - Use and apply surface area and volume of a sphere MM2G4b – Determine the effect on surface area and volume of changing the radius or diameter of a sphere.
Opening –	Big Four Review Problems (Included on Power Point Presentation)
Work session –	The students will work on the Review Problems and the teacher will provide immediate feedback. The teacher will then check the homework answers (provided on the PowerPoint) and go over any homework questions. At this time the teacher will direct students to work on a Practice worksheet to access student understanding. Students will also access www.geogebra.org under teacher direction and explore the relationships of the radius and diameter of spheres in relation to the volume and complete the practice problems. Students will be able to see visually what happens to the volume as the radius/diameter is changed.
Closing –	Complete the worksheet (Provided on the Powerpoint)