Math 1 Unit 3
Acquisition Lesson Concept 4
Plan for the Concept, Topic, or Skill – Not for the Day
Key Standards addressed in this Lesson: MM1G3c
Time allotted for this Lesson: 6 Hours

Essential Question:

How can I justify that two triangles are congruent?

Activating Strategies: (Learners Mentally Active)
Math 1 and Math 1 Support

Four Corners:

- Have adhesive chart paper placed around the room with the titles: SSS, SAS, ASA, AAS
- Give students pairs of congruent triangles which have been marked with 3 pairs of tic marks each
- Students will place each pair on the chart paper where they think it belongs

Acceleration/Previewing: (Key Vocabulary)

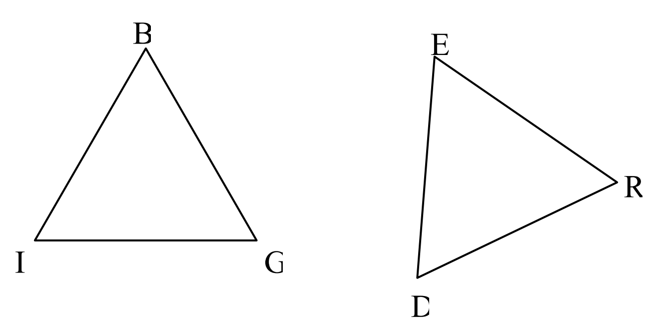
Vocabulary: (SSS) Side-side-side, (SAS) Side-angle-side, (ASA) Angle-side-angle, (AAS) Angle-angle-side, (LL) Leg-leg, (HA) Hypotenuse-angle, (LA) Leg-angle, (HL) Hypotenuse-leg

Everyday examples of congruent items, shapes. Assembly line products: cars, water bottles, vending machine packs of M&Ms, ceiling tiles, floor tiles, concrete blocks.

As you have studied in your previous math courses, triangles can also be congruent. Use the four corners activator to discuss the corresponding sides and angles that are congruent in each set of triangles. Also, discuss why the students chose to place the triangle pairs in each of the four corners.

Based on what you know and the examples from the activator, name each pair of corresponding sides and corresponding angles if $\Delta BIG \cong \Delta RED$.

- Corresponding sides –
- Corresponding angles -



Discussion: What do you think is needed to prove that two triangles are congruent?

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

Vocabulary Teaching Strategies: Word Wall

Task : Triangles Learning Task (High School Pennants) (Questions #1-8 only.) Collaborative Pairs (Think-Pair-Share)

Part 1:

- Have the students read through the excerpt on hometown pennants and answer question #1 in groups.
- Next, have students “think” through question #2 individually.
- Then have them construct triangles as per question #2 using prior knowledge.
- Have students “pair” with partners to discuss discoveries about “SSS” .
- After students have completed # 2, move on to read #3 and draw the triangles detailed in problem.
- Next, have students “pair” with partners to discuss discoveries about “SAS”.
- Emphasize that “SAS” only works when angle is between the given sides (Problem #4).
- Direct initial groups to pair with another group “pairs squared” and share their answers.
- Each group should compare answers and choose the best answer or incorporate their answers for a more precise answer.
- Before moving on to # 5, have one person from each group contribute answers from #2-4.
- Discuss each answer as a class.
- Next, have students work through #5 and #6 discovering “ASA” and “AAS”. Encourage students to “think” about their answers then “pair” with their partners to ensure understanding.
- Lastly, have students read through # 7 and “think” about how to draw triangles that work with three angles given “AAA”.
- Compare triangles with partners. Discuss whether this could be used to prove triangles congruent.
- Instruct groups to be prepared to share discoveries with the class.

Distribute “Graphic Organizer” and “Notes sheet” to ensure that students have gained necessary knowledge to continue with task. (Note: you will only need to work through part of the graphic organizer initially. As you continue with the lesson, you will complete the sheets as you go.)

Proof of AAS Theorem: See attached handout (If more is needed on the proof of this theorem, you will want to add here.)

Part 2: (Discovering special cases of congruencies for right triangles)

- Have students read through #9 - #12 and conjecture possibilities for each. Create a list of ways to prove their findings.
- Next, students will “pair” with partner to discuss their findings.
- Remaining in pair groupings have students move on to problem # 13 and #14 transferring acquired knowledge to application situations.

- Then have “paired groups” pair with another group “paired squared” and share their results.
- Later, have each initial group share with the class.

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Prompt:
Part 1:
What have you discovered about triangle congruency from SSS, SAS, ASA, AAS?

- Using problem #8, have students “summarize” the results from #2 - #7. Emphasize the common abbreviations and how they would help to remember the triangle situations they have just explored.
- See “Triangles Congruencies Worksheet” : Students work through and practice skills learned.

Part 2:
What are the special cases resulting from SSS, SAS, ASA, AAS?

- Worksheet

Summarizing Strategies: Learners Summarize & Answer Essential Question

3-2-1

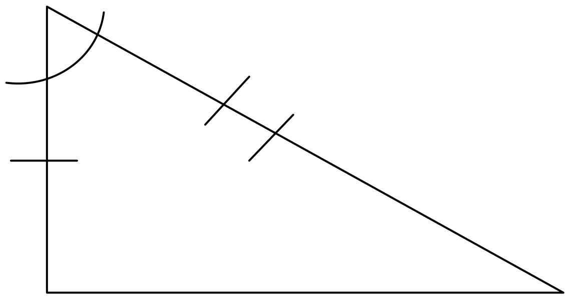
Essential Question: How can I prove that two triangles are congruent?

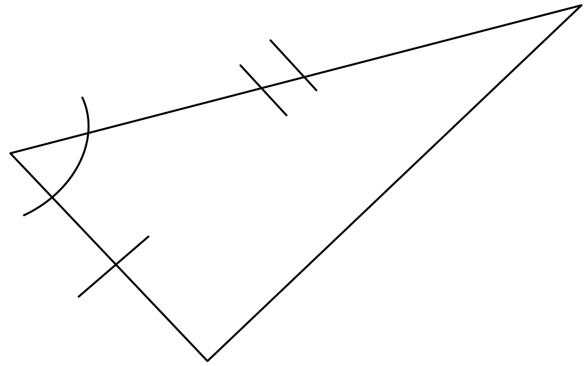
3 Ways to show (prove) triangles are congruent.

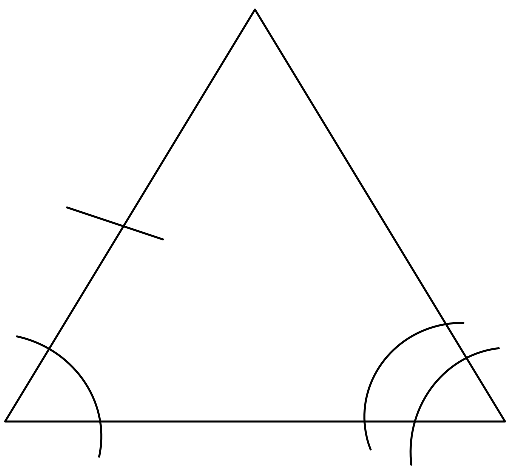
2 Things I know about congruent triangles.

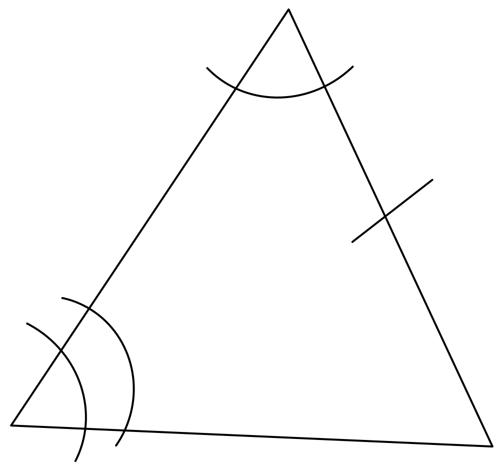
1 Example of congruent triangles ABC and DEF (draw and label congruent parts).

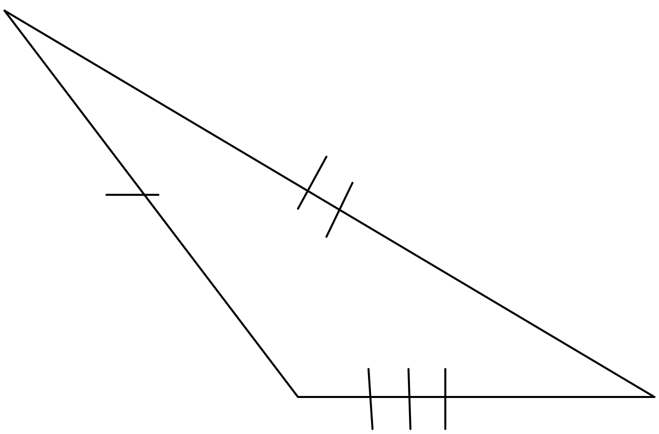
FOUR CORNERS

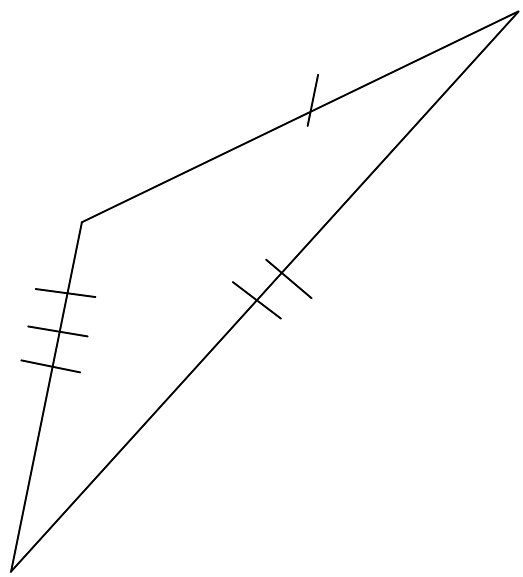




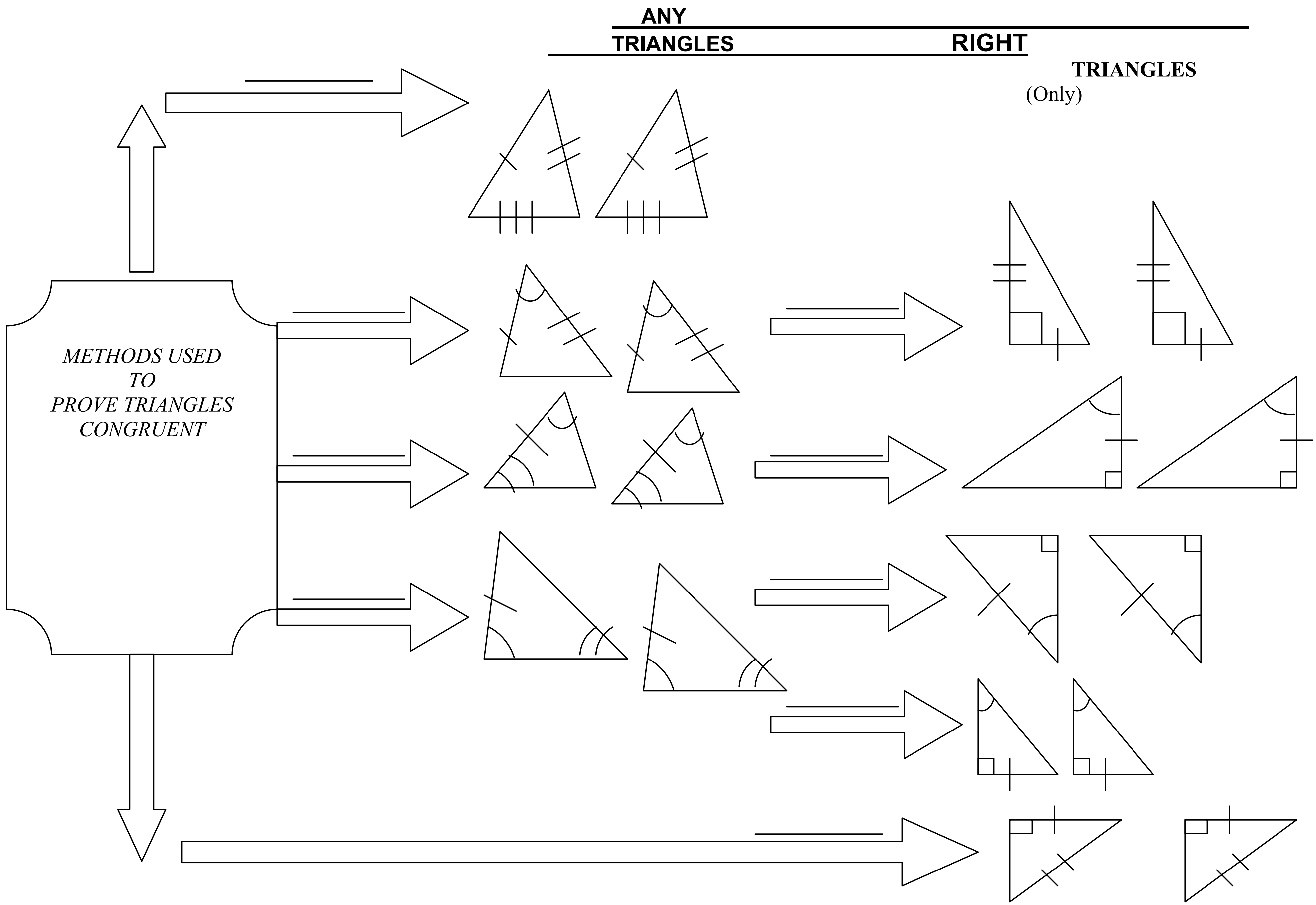




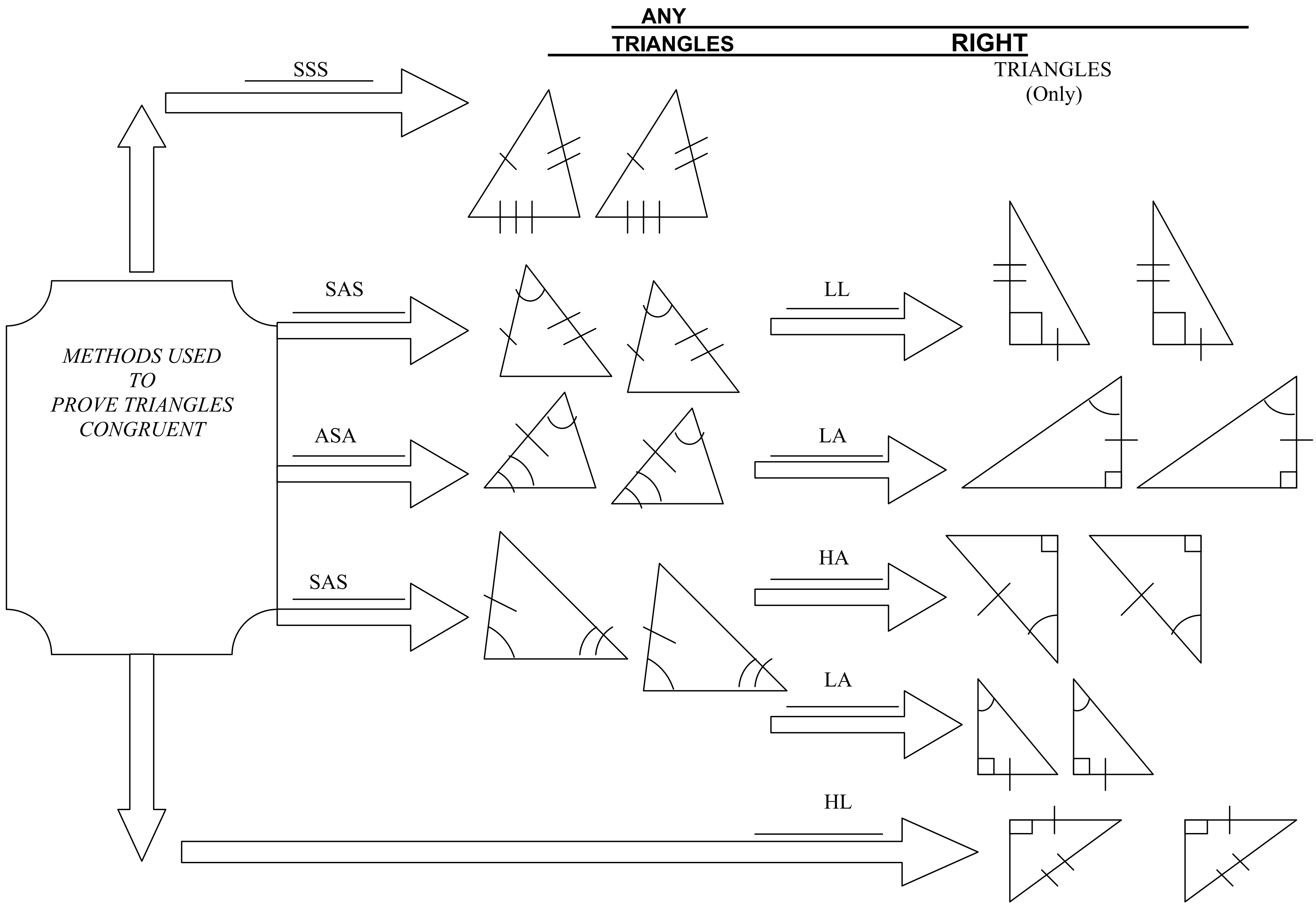




PROVING TRIANGLES CONGRUENT



PROVING TRIANGLES CONGRUENT



VOCABULARY FOR CONGRUENT TRIANGLES

VOCABULARY	DEFINITION	DIAGRAM
	If the sides of one triangle are congruent to the sides of a second triangle, then the two triangles are congruent.	
	If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.	
	If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.	
	If two angles and the non-included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle, then the two triangles are congruent.	
	If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the two triangles are congruent.	
	If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle another right triangle, then the two triangles are congruent.	
	If one leg of and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the two triangles are congruent.	
	If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the two triangles are congruent.	

VOCABULARY FOR CONGRUENT TRIANGLES

VOCABULARY	DEFINITION	DIAGRAM
Side-Side-Side SSS	If the sides of one triangle are congruent to the sides of a second triangle, then the two triangles are congruent.	
Side-Angle-Side SAS	If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.	
Angle-Side-Angle ASA	If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.	
Angle-Angle-Side AAS	If two angles and the non-included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle, then the two triangles are congruent.	
Leg-Leg LL	If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the two triangles are congruent.	
Hypotenuse-Angle HA	If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle another right triangle, then the two triangles are congruent.	
Leg-Angle LA	If one leg of and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the two triangles are congruent.	
Hypotenuse-Leg HL	If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the two triangles are congruent.	

Triangles Learning Task

The students at Hometown High School decided to make large pennants for all 8 high schools in their district. The picture above shows typical team pennants. The Hometown High students wanted their pennants to be shaped differently than the normal isosceles triangles. Each pennant was to be a scalene triangle. They plan to hang the final products in the gym as a welcome to all the schools who visit Hometown High.

Jamie wanted to know how they could make sure that all of the pennants are congruent to each other. The students wondered if they would have to measure all six parts of every triangle to determine if they were congruent. They decided there had to be a shortcut for determining triangle congruence, but they did not know the minimum requirements needed. They decided to find the minimum requirements needed before they started making the pennants.

1. Every triangle has ____ parts, _____ sides and _____ angles.

2. First they picked out 3 sides and each person constructed a triangle using these three sides. Construct a triangle with the 3 sides of 3 in., 4 in., and 6 in. Compare your triangle to others in your class. Are any of the triangles congruent? Are three sides enough to guarantee congruent triangles? Explain.

3. Next the class decided to use only 2 sides and one angle. They choose two sides of 5 in., and 7 in. with an angle of 38°. Using these measures, construct a triangle and compare with your classmates. Are any of the triangles congruent?

4. Joel and Cory ended up with different triangles. Joel argued that Cory put her angle in the wrong place. Joel constructed his triangle with the angle between the two sides. Cory constructed her sides first then constructed her angle at the end of the 7 in. side not touching the 5 in. side. Everybody quickly agreed that these two triangles were different. They all tried Cory's method, what happened? Which method, Joel's or Cory's will always produce the same triangle?

5. Now the class decided to try only 1 side and two angles. They chose a side of 7 in. and angles of 35° and 57°. Construct and compare triangles. What generalization could be made?

6. Jim noticed that Sasha drew her conclusion given two angles and the included side. He wondered if the results would be the same if you were given any two angles and one side. What do you think?

7. The last situation the class decided to try was to use three angles. They chose angles of 20°, 40°, and 120°. How do you think that worked out? Construct a triangle using these three angles and compare with others. Could they prove two triangles congruent by using the three corresponding angles? Explain why or why not.

8. Summarize the results using the chart below. Discuss what is meant by the common abbreviations and how they would help to remember the triangle situations you have just explored.

Common Abbreviation	Explanation of meaning and significance
SSS	
SAS	
SSA	
ASA	
AAS	
AAA	

9. The methods listed in the table, that can be used for proving two triangles congruent, require three parts of one triangle to be congruent to three corresponding parts of another triangle. Nakita thought she could summarize the results but she wanted to try one more experiment. She wondered if the methods might be a bit shorter for right triangles since it always has one angle of 90_o. She said: "I remember the Pythagorean Theorem for finding the length of a side of a right triangle. Could this help? My father is a carpenter and he always tells me that he can determine if a corner is square if it makes a 3 – 4 – 5 triangle." Nakita chose a hypotenuse of 6 in., and a leg of 4in. Does her conjecture work? Why or why not?

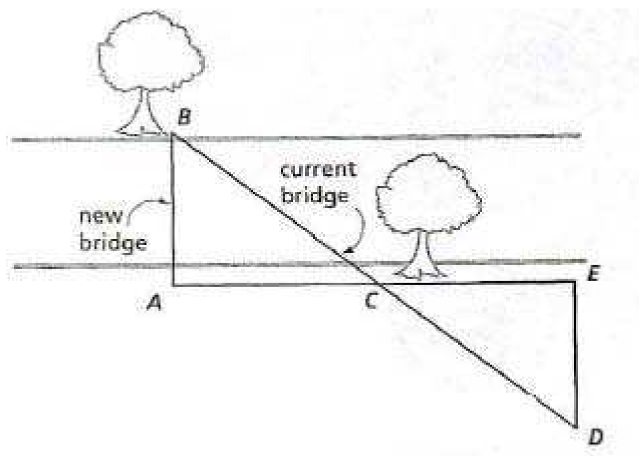
10. What if Nakita had chosen 6 inches and 4 inches to be the length of the legs. Does her conjecture work? Why or why not?

11. What are the minimum parts needed to justify that two right triangles are congruent? Using the list that you already made, consider whether these could be shortened if you knew one angle was a right angle. Create a list of ways to prove congruence for right triangles only.

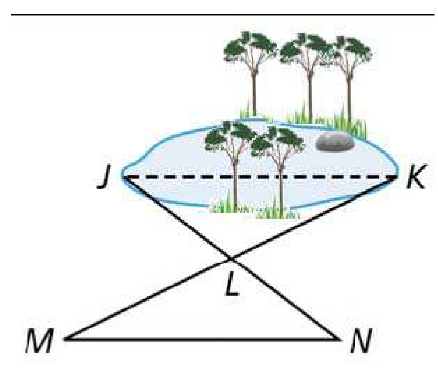
12. Once it is known that two triangles are congruent, what can be said about the parts of the triangles? Write a statement relating the parts of congruent triangles.

Congruent triangles can be used to solve problems encountered in everyday life. The next two situations are examples of these types of problems.

13. An engineer needs to determine the distance across a river without swimming to the other side. The engineer notices a tree on the other side of the river and suddenly has an idea. She quickly sketched her idea and was able to determine the distance. Her sketch is below. How was she able to do this?



14. A landscape architect needed to determine the distance across a pond. Why can't he measure this directly? He drew the following sketch as an indirect method of measuring the distance. He stretched a string from point J to point N and found the midpoint of this string, point L. He then stretched a string from M to K making sure it had the same center. He found the length of MN was 43 feet and the length of segment LK was 19 feet. Find the distance across the pond. Justify your answer.

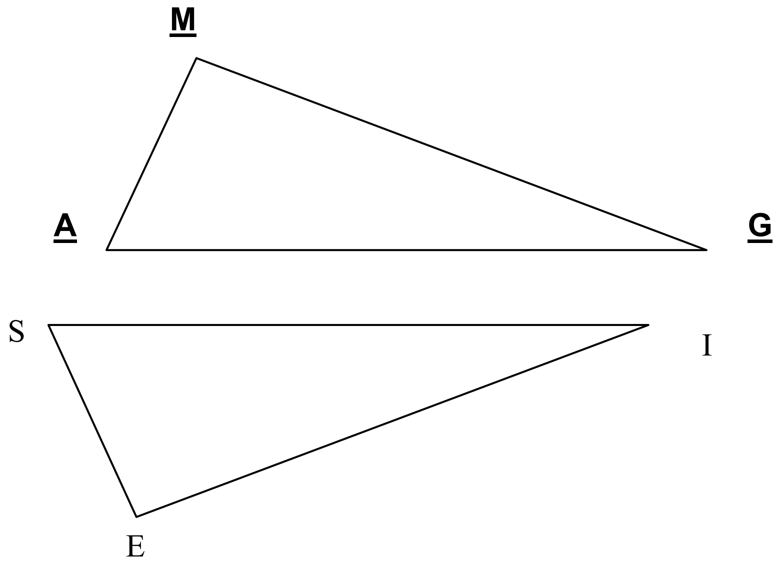


Given: $\angle M \cong \angle E$

$\angle G \cong \angle I$

$\overline{AM} \cong \overline{SE}$

Prove: $\Delta AMG \cong \Delta SEI$



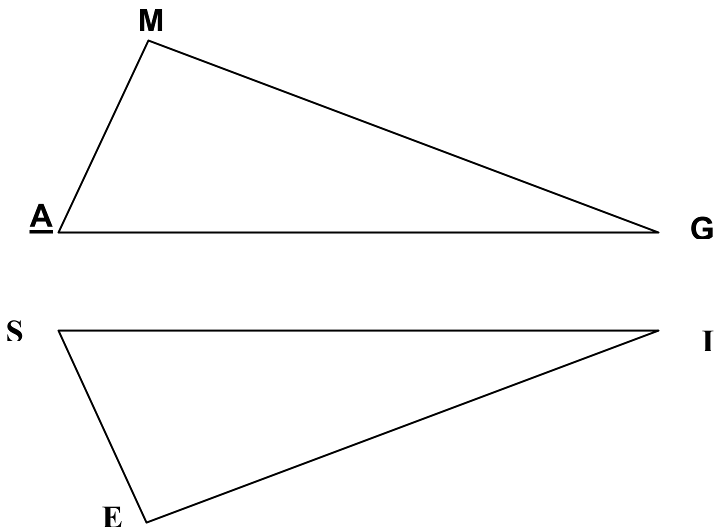
Proof:

Statements	Reasons

Given: $\angle M \cong \angle E$

$\angle G \cong \angle I$

$\overline{AM} \cong \overline{SE}$



Prove: $\Delta AMG \cong \Delta SEI$

Proof:

Statements	Reasons
1. $\angle M \equiv \angle E$; $\angle G \equiv \angle I$; $\overline{AM} \cong \overline{SE}$	1. Given
2. $\angle A \equiv \angle S$	2. Third Angle Theorem (If two angles of one triangle are congruent to two angles of another, then the third angles are congruent.
3. $\Delta AMG \cong \Delta SEI$	3. ASA Postulate

Math I Unit 3 Geometry

Name______________________ Date____________

Congruent Triangle Guided Practice

Part I: **State the third congruence that must be given to prove the given triangles congruent using the indicated postulate or theorem.**

(1) $\overline{AB} \cong \overline{XY}$; $\overline{AC} \cong \overline{XZ}$; _______________; $\Delta ABC \cong \Delta XYZ$ by SSS.

(2) $\overline{PQ} \cong \overline{AB}$; $\overline{QR} \cong \overline{BC}$; _______________; $\Delta PQR \cong \Delta ABC$ by SAS.

(3) $\angle D \cong \angle G$; $\angle F \cong \angle I$; _______________; $\Delta DEF \cong \Delta GHI$ by ASA.

(4) $\angle P \cong \angle A$; $\angle R \cong \angle C$; _______________; $\Delta ABC \cong \Delta PQR$ by AAS.

(5) $\overline{BD} \cong \overline{SU}$; $\angle D \cong \angle U$; _______________; $\Delta BCD \cong \Delta STU$ by AAS.

(6) $\overline{MN} \cong \overline{TU}$; $\angle N \cong \angle U$; _______________; $\Delta MNO \cong \Delta TUV$ by SAS.

(7) $\overline{JL} \cong \overline{EG}$; $\overline{KL} \cong \overline{FG}$; _______________; $\Delta JKL \cong \Delta EFG$ by SAS.

(8) $\overline{FG} \cong \overline{IJ}$; $\overline{EG} \cong \overline{HJ}$; _______________; $\Delta EFG \cong \Delta HIJ$ by SSS.

(9) $\overline{JK} \cong \overline{WX}$; $\angle K \cong \angle X$; _______________; $\Delta JKL \cong \Delta WXY$ by ASA.

(10) $\angle P \cong \angle S$; $\overline{PR} \cong \overline{SU}$; _______________; $\Delta PQR \cong \Delta STU$ by ASA.

Part II: **For each pair of triangles, determine if they can be proven congruent and state the congruence postulate or theorem that justifies your answer. If the triangles cannot be proven congruent write "not possible."**

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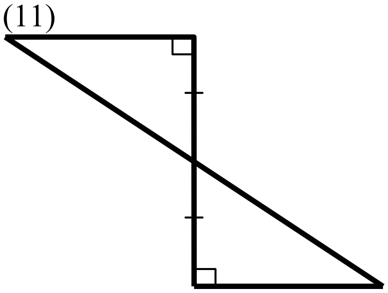

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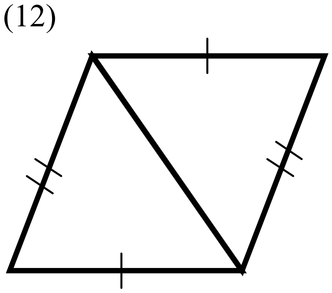

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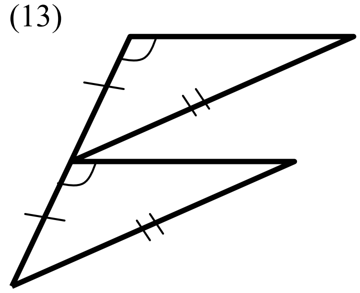

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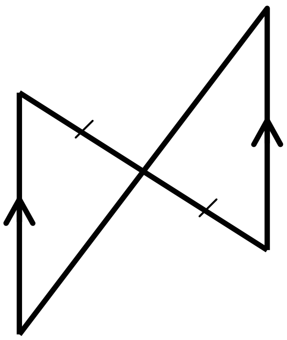

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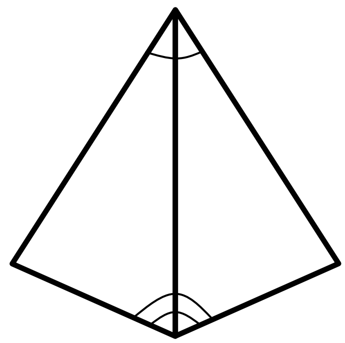

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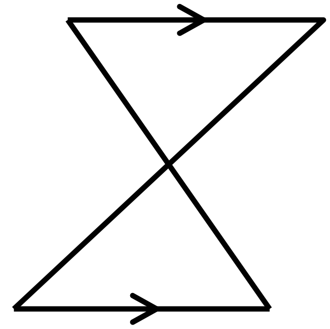

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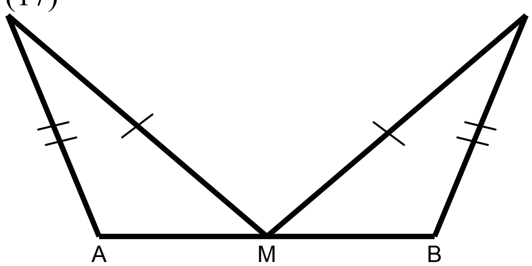

Given: Point M is a midpoint of $\overline{AB}$

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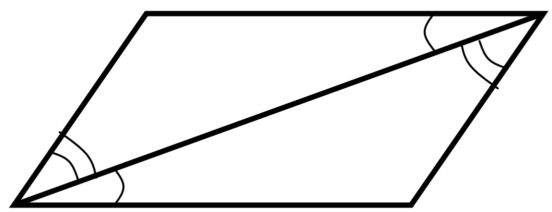

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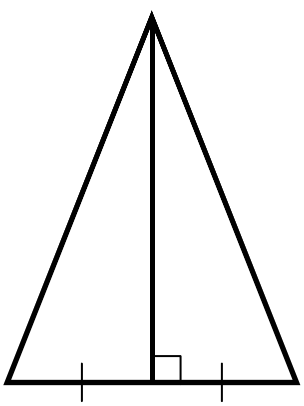

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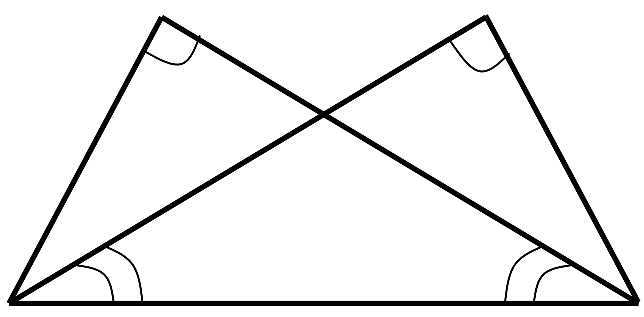

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Name_______________________ Date____________

Congruent Triangle Guided Practice

Part I: State the third congruence that must be given to prove the given triangles congruent using the indicated postulate or theorem.

(1) $\overline{AB} \cong \overline{XY}$; $\overline{AC} \cong \overline{XZ}$; $\overline{BC} \cong \overline{YZ}$; $\Delta ABC \cong \Delta XYZ$ by SSS.

(2) $\overline{PQ} \cong \overline{AB}$; $\overline{QR} \cong \overline{BC}$; $\angle Q \cong \angle B$; $\Delta PQR \cong \Delta ABC$ by SAS.

(3) $\angle D \cong \angle G$; $\angle F \cong \angle I$; $\overline{DF} \cong \overline{GI}$; $\Delta DEF \cong \Delta GHI$ by ASA.

(4) $\angle P \cong \angle A$; $\angle R \cong \angle C$; $\overline{AB} \cong \overline{PQ}$ or $\overline{BC} \cong \overline{QR}$; $\Delta ABC \cong \Delta PQR$ by AAS.

(5) $\overline{BD} \cong \overline{SU}$; $\angle D \cong \angle U$; $\angle C \cong \angle T$; $\Delta BCD \cong \Delta STU$ by AAS.

(6) $\overline{MN} \cong \overline{TU}$; $\angle N \cong \angle U$; $\overline{NO} \cong \overline{UV}$; $\Delta MNO \cong \Delta TUV$ by SAS.

(7) $\overline{JL} \cong \overline{EG}$; $\overline{KL} \cong \overline{FG}$; $\angle L \cong \angle G$; $\Delta JKL \cong \Delta EFG$ by SAS.

(8) $\overline{FG} \cong \overline{IJ}$; $\overline{EG} \cong \overline{HJ}$; $\overline{EF} \cong \overline{HI}$; $\Delta EFG \cong \Delta HIJ$ by SSS.

(9) $\overline{JK} \cong \overline{WX}$; $\angle K \cong \angle X$; $\angle J \cong \angle W$; $\Delta JKL \cong \Delta WXY$ by ASA.

(10) $\angle P \cong \angle S$; $\overline{PR} \cong \overline{SU}$; $\angle R \cong \angle Q$; $\Delta PQR \cong \Delta STU$ by ASA.

Part II: For each pair of triangles, determine if they can be proven congruent and state the congruence postulate or theorem that justifies your answer. If the triangles cannot be proven congruent write "not possible."

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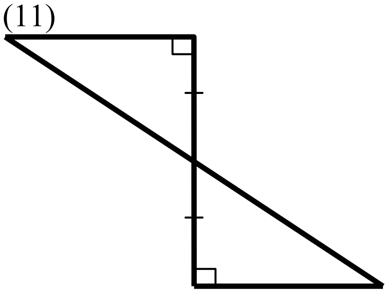

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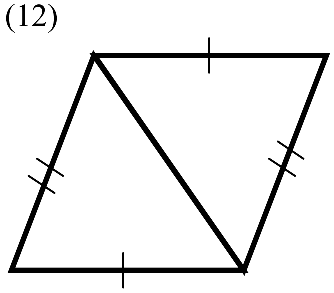

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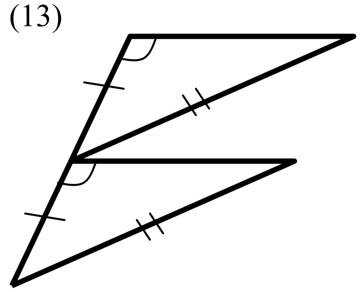

_____Not Possible________

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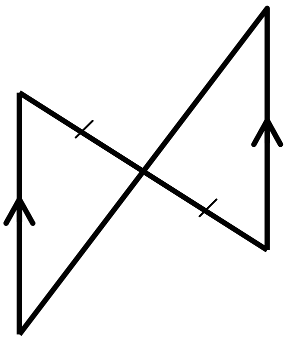

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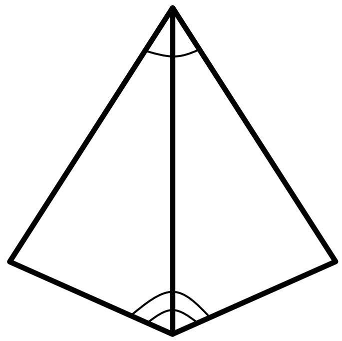

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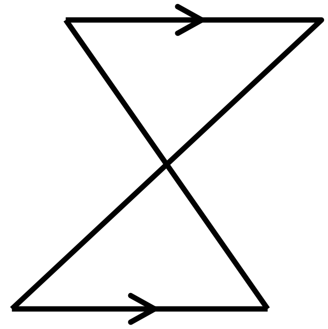

_____Not Possible_______

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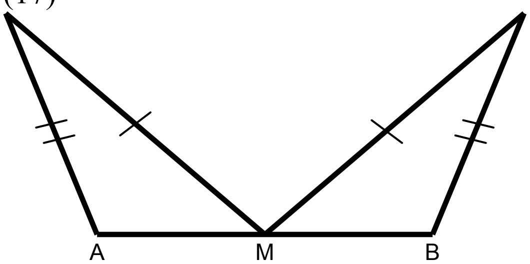

Given: Point M is a midpoint of $\overline{AB}$

_____________SSS____________________

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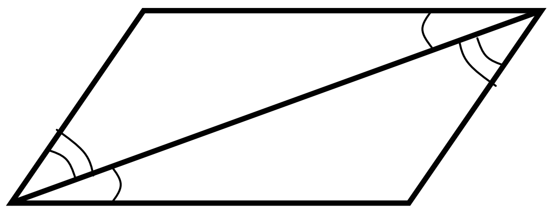

_____________ASA___________________

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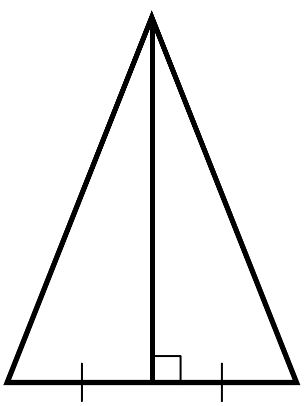

_______SAS_____________

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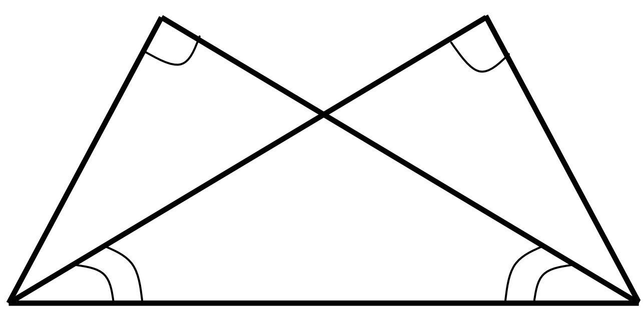


______________AAS_________________________

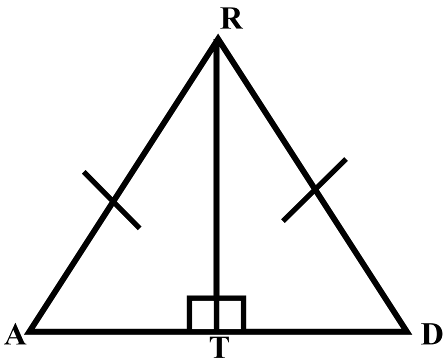
Math I Unit 3 Geometry

Name ____ **Date** ___________________________________

Right Triangle Congruence

Is it possible to prove that the two triangles are congruent? If so, state the right triangle congruence theorem you would use to prove the two triangles are congruent and the congruence statement.

1.



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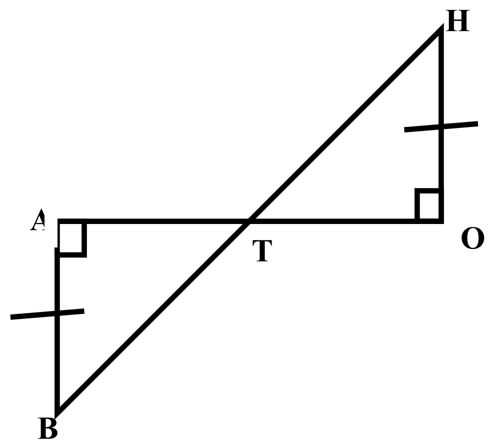
2.



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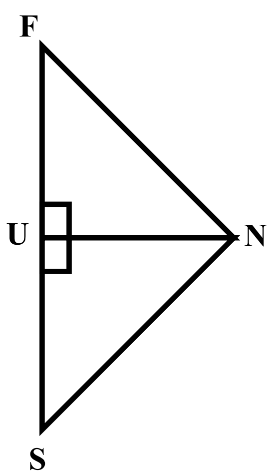
3.



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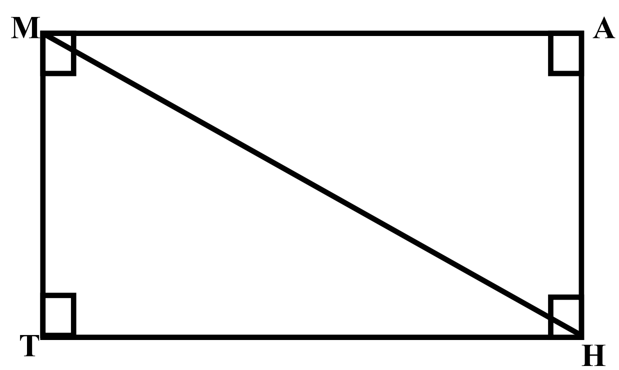
4.



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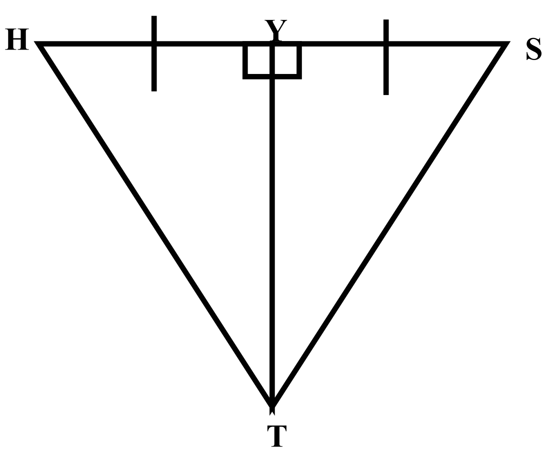
5.



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State the second congruence that must be given to prove that ΔDEF ≅ ΔPQR using the indicated right triangle congruence theorem if ∠D and ∠P are right angles.

6. $\overline{ED} \cong \overline{QP}$, by LL Theorem

7. $\overline{PR} \cong \overline{DF}$, by HL Theorem

8. $\overline{EF} \cong \overline{QR}$, by HA Theorem

9. $\angle F \cong \angle R$, by LA Theorem

10. $\angle E \cong \angle Q$, by HA Theorem

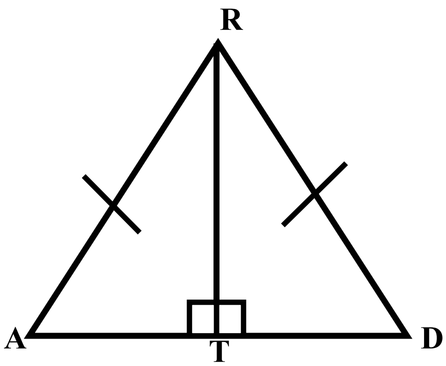
Math I Unit 3 Geometry

Name ____ **Date** ___________________________________

Right Triangle Congruence

Is it possible to prove that the two triangles are congruent? If so, state the right triangle congruence theorem you would use to prove the two triangles are congruent and the congruence statement.

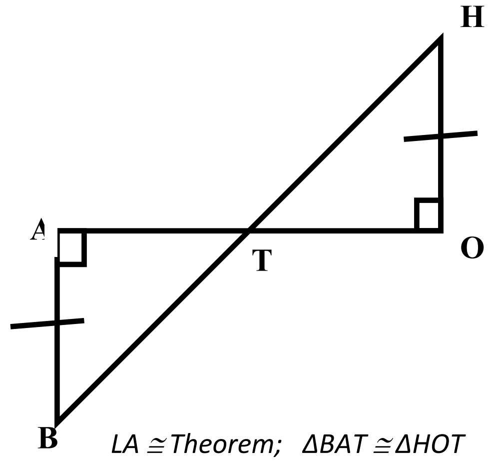
1.



HL ≅ Theorem; ΔRAT ≅ ΔRDT

____

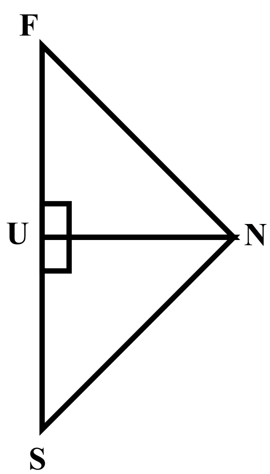
2.



LA ≅ Theorem; ΔBAT ≅ ΔHOT

____

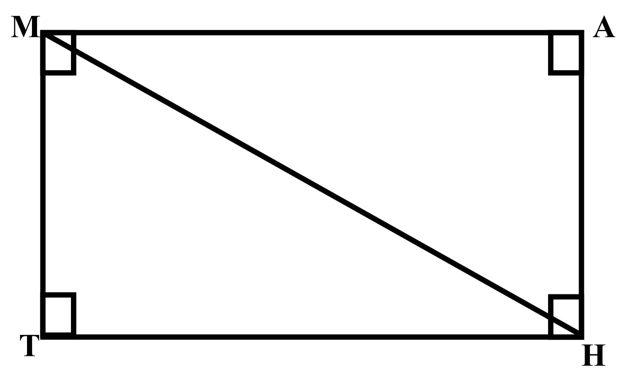
3.



Not possible

____

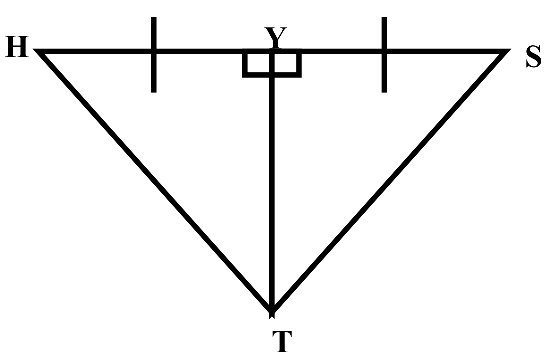
4.



HA ≅ Theorem; ΔMHT ≅ ΔHMA

____

5.



LL ≅ Theorem; ΔHYT ≅ ΔSYT

____

State the second congruence that must be given to prove that ΔDEF ≅ ΔPQR using the indicated right triangle congruence theorem if ∠D and ∠P are right angles.

6. $\overline{ED} \cong \overline{QP}$**, by LL Theorem**

7. $\overline{PR} \cong \overline{DF}$**, by HL Theorem**

$\overline{BF} \cong \overline{PR}$

$\overline{EF} \cong \overline{QR}$

8. $\overline{EF} \cong \overline{QR}$, by HA Theorem

$\angle F \cong \angle R$ *or* $\angle E \cong \angle Q$

9. $\angle F \cong \angle R$, by LA Theorem

$\overline{DF} \cong \overline{PR}$ *or* $\overline{DE} \cong \overline{QR}$

10. $\angle E \cong \angle Q$, by HA Theorem

$\overline{EF} \cong \overline{QR}$

Concept 5: Quadrilaterals

Unit Essential Question: How are the parallelogram, rectangle, rhombus, square, trapezoid, and kite alike and different?

Lesson Essential Question:
What characteristics differentiate the following quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite?
If 3 parallel lines cut off equal segments of one transversal, how would lengths of segments of any transversal be related?

Activating Strategies: (Learners Mentally Active)

Students complete anticipation guide: "Can you name these special quadrilaterals?" independently, then in a group go through and compare ideas and come to a consensus.

Acceleration/Previewing: (Key Vocabulary)
Quadrilateral, parallelogram, rectangle, rhombus, square, trapezoid, diagonal, kite.

Use the graphic organizer to teach properties of quadrilaterals, then answer the following questions:

1. **What are the 3 categories of Quadrilaterals and what are they classified by?**
2. **Which category has many more sub-categories.**

Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

- **Lead students through characteristics of all quadrilaterals.**
- **Think, Pair, Share: have students think individually and complete attached chart (Figure 2), then direct students to pair with their partner and share their answers.**
- **Pair squared: after each pair have shared, direct pairs to discuss with other groups and compare answers and come to a consensus for best answer.**
- **Review answers with class, let them decide which answers are correct.**

Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)

Prompt:
Lead students through "Constructing with Diagonals Learning Tasks" Questions 1-11

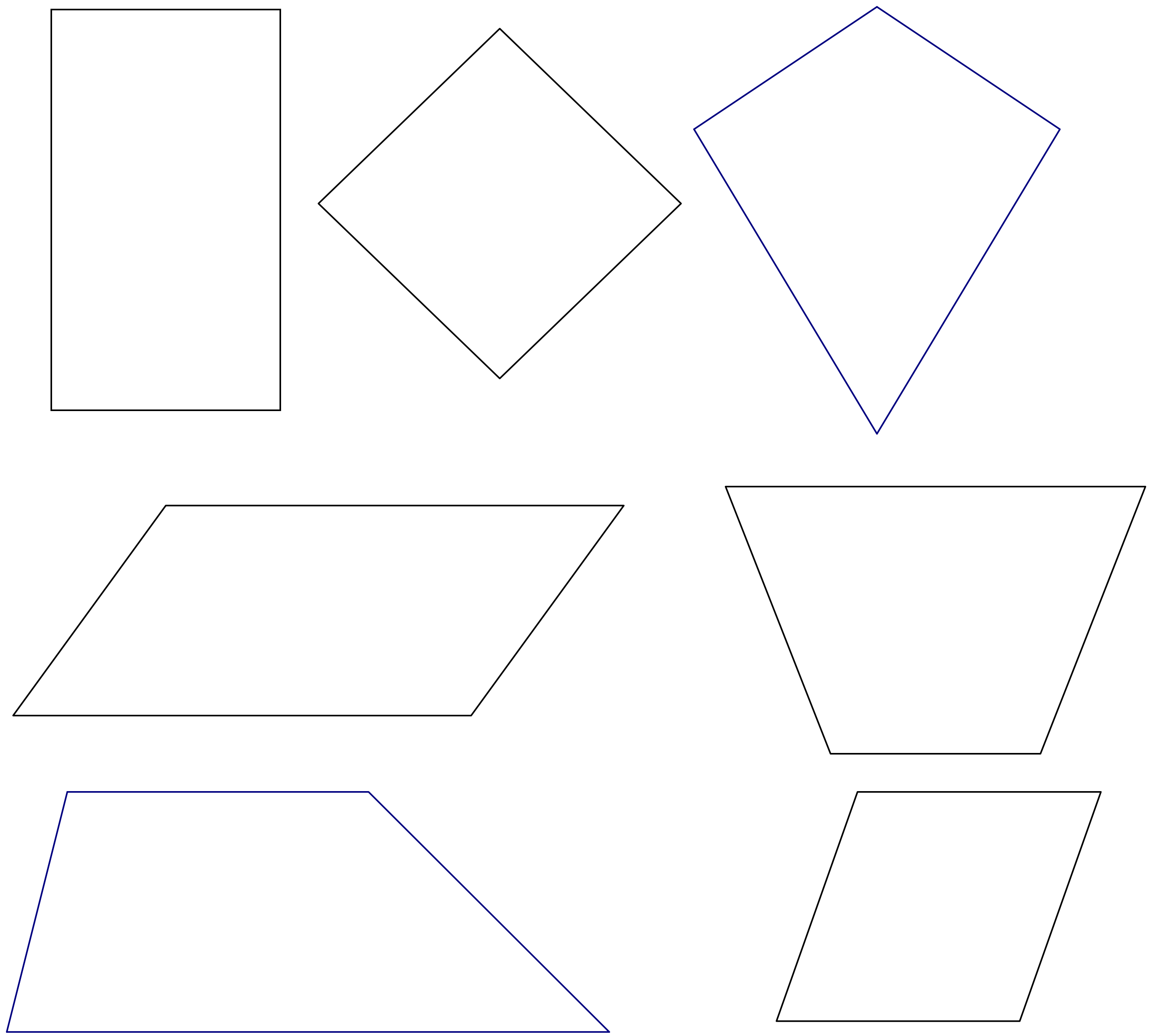
Prompt:
Lead students through "Floor/Tape activity" to discover

Summarizing Strategies: Learners Summarize & Answer Essential Question

Assessment

Discovery:

Can you name these special quadrilaterals?



Conclusions:

Based on what you have discovered about the properties of special quadrilaterals, write a definition of each figure. Compare your definitions with your classmates.

1. Parallelogram
2. Rhombus
3. Rectangle
4. Square
5. Trapezoid
6. Isosceles Trapezoid
7. Kite

Math I Unit 3 Geometry

GRAPHIC ORGANIZER:

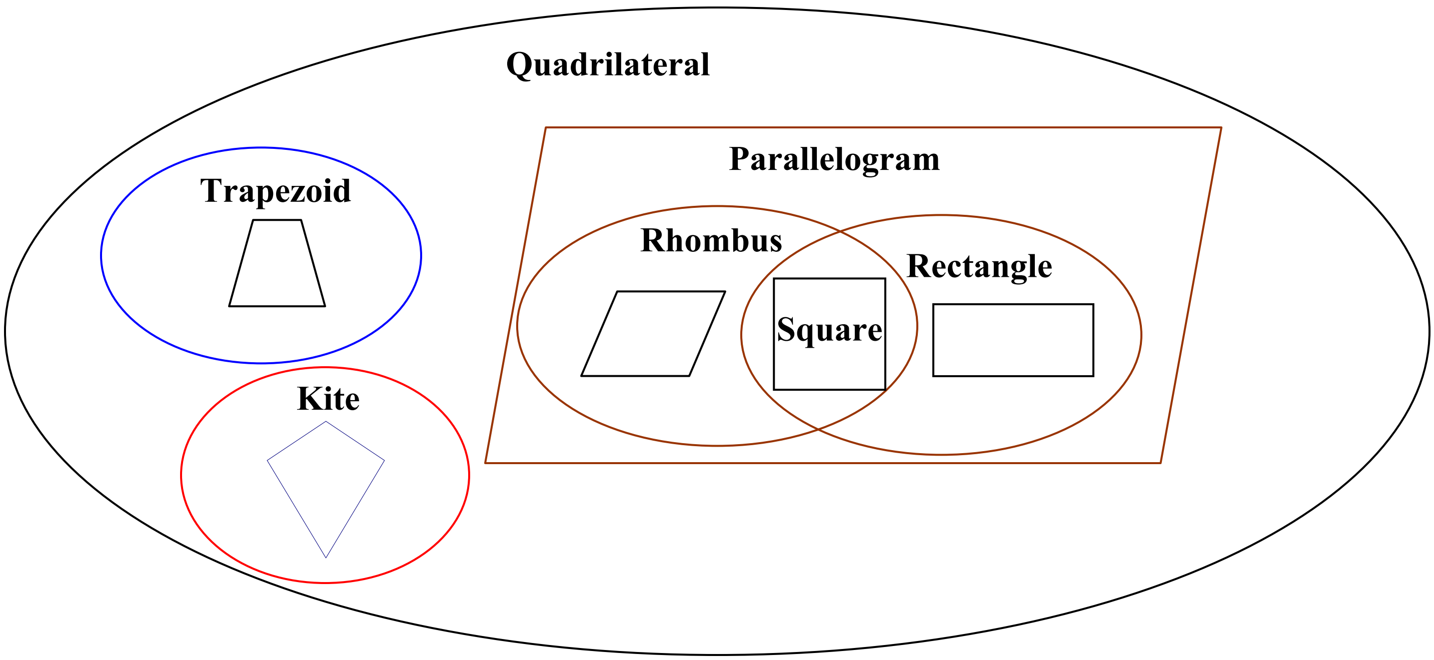
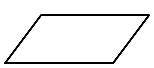
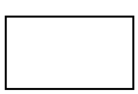
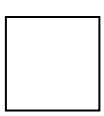
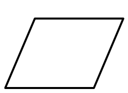
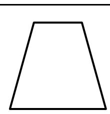
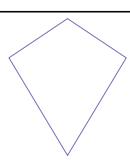


FIGURE #2

Quadrilateral	Figure	No Parallel Sides	Only 1 Pair of Parallel Sides	2 Pairs of Parallel Sides	All Sides Congruent	Opposite Sides are Congruent	All Angles are Right Angles
Parallelogram							
Rectangle							
Square							
Rhombus							
Trapezoid							
Kite							

PARALLELOGRAM

A parallelogram is a quadrilateral with opposite sides parallel and congruent. It is the "parent" of some other quadrilaterals, which are obtained by adding restrictions of various kinds:

- A rectangle is a parallelogram but with all angles fixed at 90°
- A rhombus is a parallelogram but with all sides equal in length
- A square is a parallelogram but with all sides equal in length and all angles fixed at 90°

RHOMBUS

A rhombus is actually just a special type of parallelogram. Recall that in a parallelogram each pair of opposite sides are equal in length. With a rhombus, *all four* sides are the same length. It therefore has all the properties of a parallelogram.

Its a bit like a square that can 'lean over' and the interior angles need *not* be 90°. Sometimes called a 'diamond' or 'lozenge' shape.

RECTANGLE

The rectangle, like the square, is one of the most commonly known quadrilaterals. It is defined as having opposite sides parallel and its corner angles all right angles (90°) from this it follows that the opposite sides will always be the same length. Adjust the rectangle above and satisfy yourself that this is so.

Since it is also a specific case of a parallelogram, like a parallelogram, its diagonals bisect each other. In a rectangle the diagonals are the same length.

A rectangle can be classified in more than one way, for example a square is a special case of a rectangle. It is also a special case of a parallelogram (but where the angles are fixed at 90°)

SQUARE

The square is probably the best known of the quadrilaterals. It is defined as having all sides equal, and its interior angles all right angles (90°). From this it follows that the opposite sides are also parallel.

A square can be thought of as a special case of other quadrilaterals, for example

- a rectangle but with opposite sides equal
- a parallelogram but with opposite sides equal and the angles all 90°
- a rhombus but with angles all 90°

TRAPEZOID

A quadrilateral which has one set of opposite sides parallel.

ISOSCELES TRAPEZOID

If both legs of a trapezoid are the same length, this is called an **isosceles trapezoid**, and both base angles are the same.

KITE

No sides parallel, diagonals are perpendicular, one diagonal is bisected, two distinct pairs of equal, adjacent sides.

Constructing with Diagonals Learning Task

It is possible to construct special quadrilaterals with only given information about the diagonals. Using descriptions below determine which special quadrilateral is formed and defend your answer using congruent triangles.

1. Construct two segments of different length that are perpendicular bisectors of each other. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints?

2. Repeat #1 with two congruent segments. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints?

3. Construct two segments that bisect each other but are not perpendicular. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints?

4. What if the two segments in #3 above are congruent in length? What type of quadrilateral is formed? What names can be used to describe the quadrilaterals formed using these constraints?

5. Construct a segment and mark the midpoint. Now construct a segment that is perpendicular to the first segment at the midpoint but is not bisected by the original segment. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints?

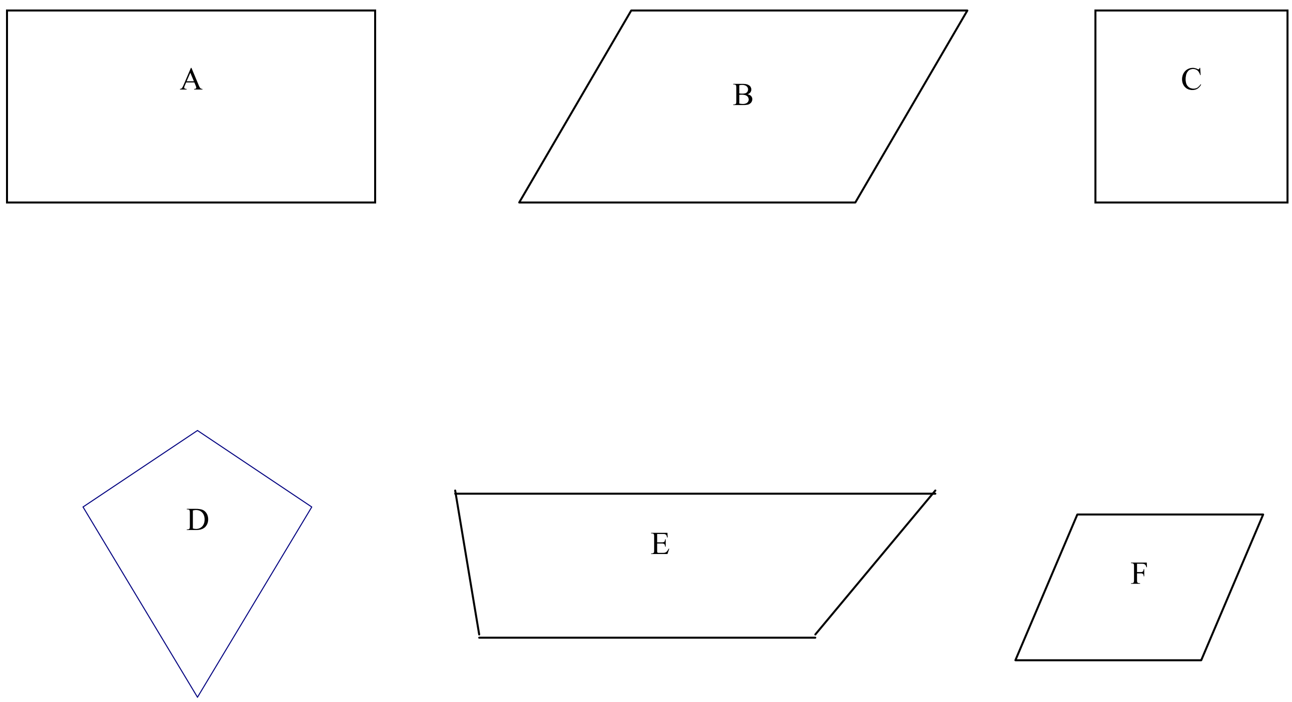
6. In the above constructions you have been discovering the properties of the diagonals of each member of the quadrilateral family. Stop and look at each construction. Summarize any observations you can make about the special quadrilaterals you constructed. If there are any quadrilaterals that have not been constructed yet, investigate any special properties of their diagonals.

7. Complete the chart below.

Conditions	Description of the type(s) of quadrilateral(s) possible	Explain your reasoning
Diagonals are perpendicular.		
Diagonals are perpendicular and one is bisected.		
Diagonals are congruent and intersect but are not perpendicular.		
Diagonals bisect each other.		
Diagonals are perpendicular and bisect each other.		
Diagonals are congruent and bisect each other.		
Diagonals are congruent, perpendicular and bisect each other		

8. As you add more conditions to describe the diagonals, how does it change the types of quadrilaterals possible? Why does this make sense?

9. Name each of the figures below using as many names as possible and state as many properties as you can about each figure.



10. Complete the table below by measuring the quadrilaterals on the previous page to determine which properties are true for each kind of quadrilateral.

Properties of Quadrilaterals

	Parallelogram	Rhombus	Rectangle	Square	Isosceles Trapezoid	Trapezoid	Kite
Exactly 1 pair of parallel sides							
2 pairs of parallel sides							
Exactly 1 pair of congruent sides							
2 pairs of congruent sides							
All sides congruent							
Opposite angles congruent							
Diagonals form two congruent triangles							
Diagonals bisect each other							
Diagonals are perpendicular							
Diagonals are congruent							
Diagonals bisect vertex angles							
All angles are right angles							

11. Knowing the properties in the table above, list the **minimum** conditions necessary to prove that a quadrilateral is:

a trapezoid

a parallelogram

a kite

a rhombus

a rectangle

a square

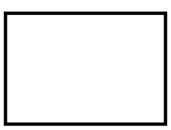
Assessment:

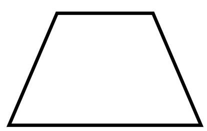
Determine if each statement is Always, Sometimes, or Never True and explain how you know.

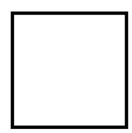
1. A rhombus is a parallelogram.
2. A square is a trapezoid.
3. A kite is a quadrilateral.
4. A parallelogram is a rectangle.
5. A square is a rectangle.
6. An isosceles trapezoid is a trapezoid.
7. A rectangle is a parallelogram.
8. A square is a rhombus.

Quadrilaterals

Judging by appearance, state all correct names for each quadrilateral. Then circle the best.

1. 

2. 

3. 

4. Write a sentence that uses the word *all* and some of the following words: trapezoids, parallelograms, rectangles, rhombuses, squares. Repeat for the word *some* and then for the word *no*.

5. Can a quadrilateral be both a rhombus and a rectangle? Explain.

Quiz ~ Quadrilaterals

Match the following concepts with their definition to the right.

_____ 1. Parallelogram:

_____ 2. Rectangle:

_____ 3. Square:

_____ 4. Rhombus:

_____ 5. Trapezoid:

_____ 6. Kite

A. parallelogram with 4 right angles and four congruent sides.

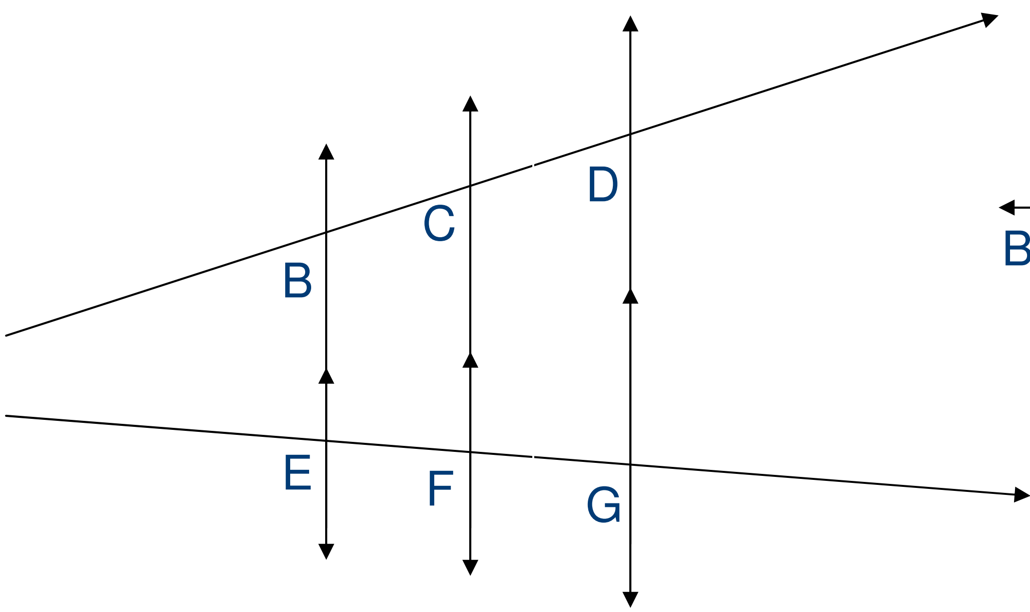
B. quadrilateral that has exactly one pair of parallel sides.

C. quadrilateral with two pairs of opposite sides that are parallel.

D. parallelogram with 4 right angles.

E. parallelogram with 4 congruent sides.

F. quadrilateral with no parallel sides



$\overleftrightarrow{BE} \parallel \overleftrightarrow{CF} \parallel \overleftrightarrow{GD}$

If three parallel lines cut a transversal into two congruent segments, then they will cut EVERY transversal crossing them into two congruent segments no matter how slanted the transversal is.

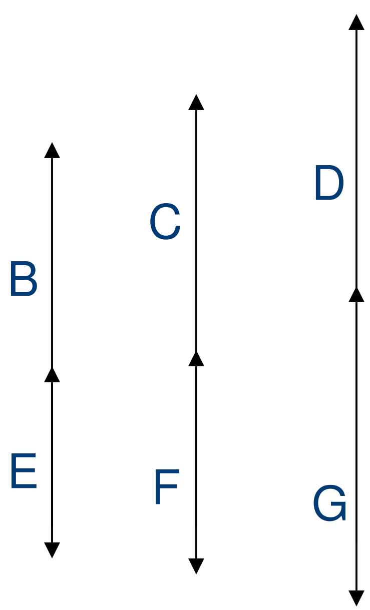
Activity:

Tools: Teacher, tape, floor that tape will stick to and stay
Teacher will place tape on floor to form three parallel lines. Each outside line will be the same distance from the middle line as the other outside line.

Tools: permanent marker, string, students
Students will be given a length of string long enough to transverse the three parallel lines. Two students use the string to transverse the parallel lines making sure to pull the string taut. Another student will mark the string at the points where the string intercepts the three parallel lines.

Students will fold the line at the point where the line intercepts the middle parallel line. They should then be left to discover that the two segments created are congruent.

Once they have discovered that the two segments created are congruent, they should then complete this task using other strings of various lengths. They should then be asked to work individually to write the reason that the theorem above is true. Once they have their answers, the teacher can call on random students, or ask for volunteers to share their answers. As a group, students construct a unified explanation/proof while the teacher writes their decisions on the board.



$\overleftrightarrow{BE} \parallel \overleftrightarrow{CF} \parallel \overleftrightarrow{GD}$

Mathematics I
Unit 3 Assessment

1. What is the measure of an exterior angle of a regular hexagon? (MM1G3a.)

A. 30°
B. 60°
C. 120°
D. 180°

2. A polygon was drawn on a piece of paper. Each of its interior angles has the same measure. The sum of the measures of its interior angles is 360°. Which of the following could be the polygon? (MM1G3a.)

A. a rectangle
B. a regular hexagon
C. a regular pentagon
D. an equilateral triangle

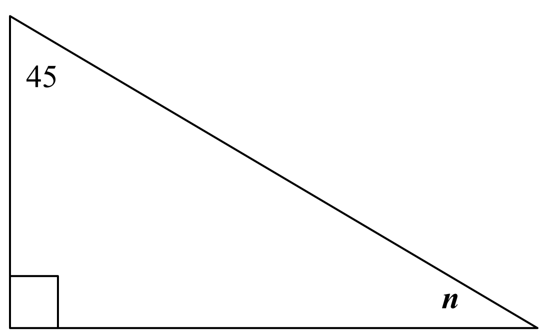
3. A triangle has angles of 32 degrees and 48 degrees. What is the measure of the remaining angle? (MM1G3a.)

A. 80
B. 180
C. 100
D. 90

4. Two of the angle measures of an obtuse triangle are congruent. The third angle measure 120°. What does one of the other angles measure?

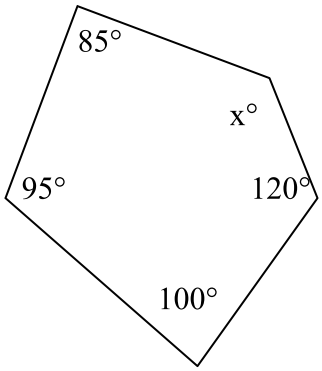
A. 60°
B. 30°
C. 180°
D. 90°

5. What is the measure of angle ***n***. (MM1G3a.)



A. 45°
B. 135°
C. 90°
D. 60°

6. What is the value of x? (MM1G3a.)



A. 100°
B. 140°
C. 120°
D. 150°

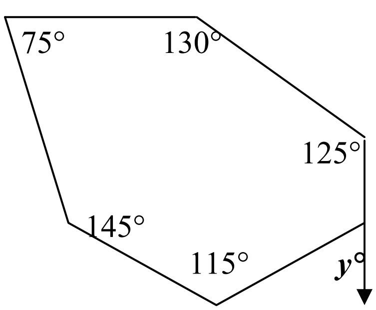
7. Sarah's flower garden is in the shape of a hexagon. What is the sum of the degree measures of the interior angles of her garden? (MM1G3a.)

A. 120°
B. 180°
C. 360°
D. 720°

8. Two of the angle measure of an isosceles triangle are congruent. The third angle measures 70°. What is the measurement of one of the other two angles? (MM1G3a.)

A. 60°
B. 40°
C. 35°
D. 55°

9. What is the value of ***y***? (MM1G3a.)

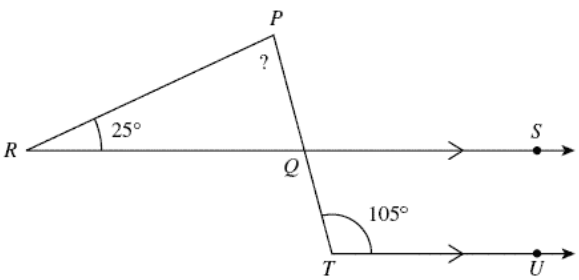


A. 50
B. 100
C. 130
D. 75

10. The measure of each interior angle of a regular polygon is eight times that of an exterior angle of the polygon. How many sides does the polygon have? (MM1G3a.)

A. 20
B. 17
C. 18
D. 19

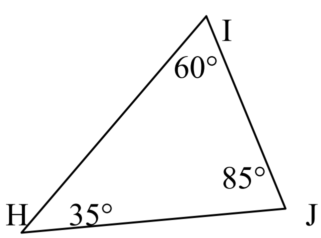
11. In the figure shown below, $\overrightarrow{RS}$ is parallel to $\overrightarrow{TU}$, and $\overline{PT}$ intersects $\overrightarrow{RS}$ at Q



What is the measure of $\angle RPQ$? (MM1G3a.)

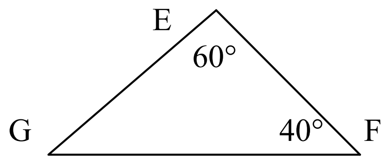
A. 80 degrees
B. 90 degrees
C. 65 degrees
D. 57 degrees

12. Which side is the shortest? (MM1G3b.)



A. HJ
B. IJ
C. HI
D. cannot be determined

13. List the sides in order from the longest to the shortest. (MM1G3b.)



A. EF, GF, EG
B. GF, EG, EF
C. EG, EF, GF
D. GF, EF, EG

14. In triangle ABC, AC = 6, AB = 7, and BC = 5. Which is true? (MM1G3b.)

A. The measure of <C is the least of the three angles.
A. The measure of <C is the greatest of the three angles.
A. The measure of <B is the greatest of the three angles.
A. The measure of <B is the least of the three angles.

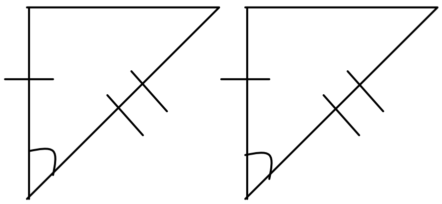
15. In any Δ ABC, which statement is always true? (MM1Gb.)

A. m< A + m< B = 90°
B. m< A + m< B < 90°
C. AB + BC > AC
D. AB + BC < AC

16. Which of the following sets of numbers could represent the lengths of the sides of a triangle? (MM1Gb.)

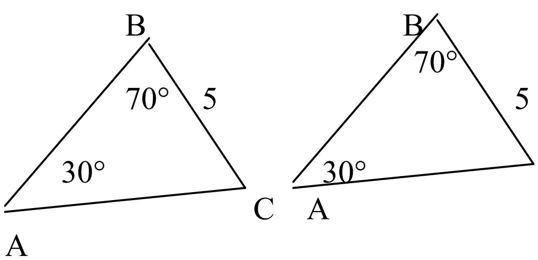
A. 2, 2, 5
B. 3, 3, 5
C. 4, 4, 8
D. 5, 5, 15

17. Determine if the triangles are congruent. If they are congruent, by which theorem can you prove the two triangles congruent? (MM1G3c.)



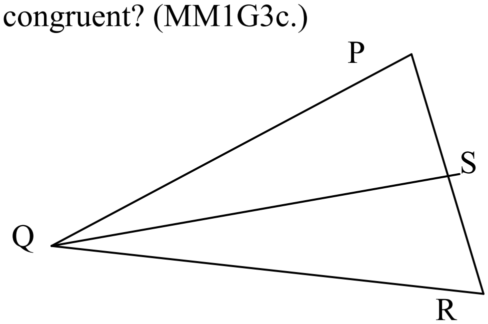
A. SAS
B. ASA
C. SSA
D. AAS

18. Choose the theorem that proves the congruence. (MM1G3c.)



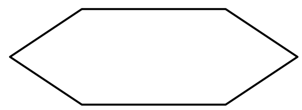
A. SAS
B. SSS
C. AAS
D. The triangles are not congruent.

19. Given $\overline{PQ} \cong \overline{RQ}$; S is the midpoint of segment PR. What reason can be given to prove that the triangles are congruent? (MM1G3c.)



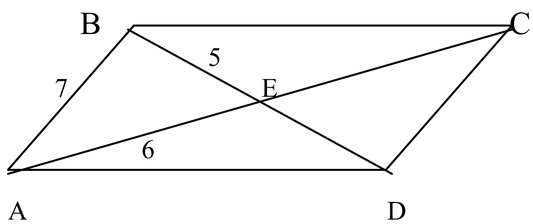
A. AAS
B. ASA
C. SAS
D. SSS

20. The figure below is a ___________. (MM1G3d.)



A. concave hexagon
B. convex hexagon
C. concave heptagon
D. concave heptagon

21. If ABCD is a parallelogram, what is the length of segment BD? (MM1G3d.)



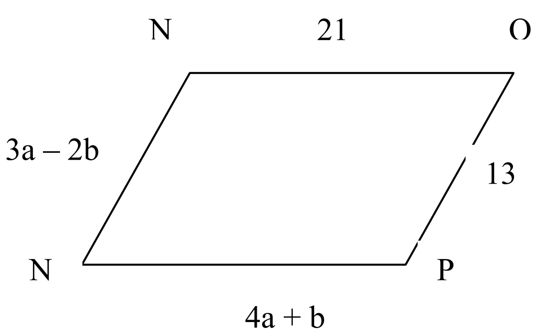
A. 10

B. 11

C. 12

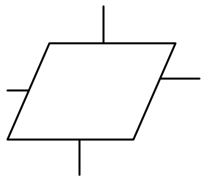
D. 14

22. What values of *a* and *b* make quadrilateral *MNOP* a parallelogram? (MM1G3d.)



A. a = 1, b = 5
B. a = 5, b = 1
C. a = 11/7, b = 34/7
D. a = 34/7, b = 11/7

23. The quadrilateral below is most specifically a ______________. (MM1G3d.)

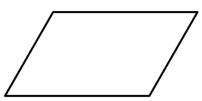


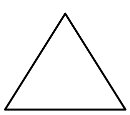
A. parallelogram
B. square
C. rectangle
D. rhombus

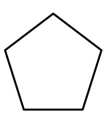
24. A rectangle is __________ a square. (MM1G3d.)

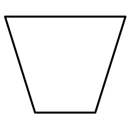
A. sometimes
B. never
C. always
D. not enough info

25. Which of the following shapes is a parallelogram? (MM1G3d.)

A. 

B. 

C. 

D. 

26. What is a parallelogram with four congruent sides? (MM1G3d.)

A. a rectangle
B. a square
C. a rhombus
D. both B and C

27. Altitudes of a triangle have a point of concurrency called ________. (MM1G3e.)

A. centroid
B. incenter
C. orthocenter
D. circumcenter

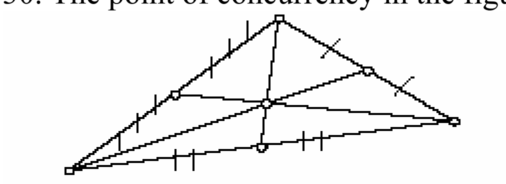
28. In a triangle, perpendicular bisectors of the sides meet at the __________. (MM1G3e.)

A. centroid
B. incenter
C. orthocenter
D. circumcenter

29. The incenter of a circle inscribed in a triangle is the point of intersection of the triangle's _________. (MM1G3e.)

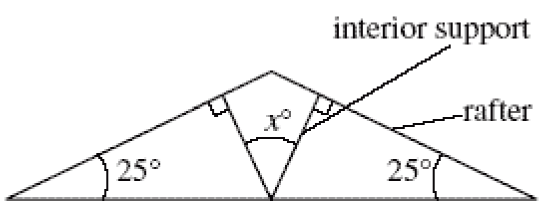
A. medians
B. altitudes
C. angle bisectors
D. perpendicular bisectors

30. The point of concurrency in the figure is:



A. centroid
B. circumcenter
C. incenter
D. orthocenter

31. Each of the two interior supports for part of a roof is perpendicular to a rafter, as shown below.



What is x, the measure, in degrees, of the angle formed by the two interior supports? (MM1G3a.)

A.50
B.65
C.90
D.130