

Transformational Graphing

Reporting Category Functions

Topic Exploring transformational graphing

Primary SOL All.6 The student will recognize the general shapes of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

Related SOL All.7b

Materials

- Graph paper
- Graphing calculators
- Function Family Matching Cards (attached)
- Transformational Graphing handout (attached)

Vocabulary

transformation, vertex, absolute value, quadratic term, reflection (earlier grades)

logarithmic function, exponential function (All.6)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Have students create a table of values in order to graph the following parent functions on graph paper without using their calculators.

$$f(x)=x^2, \quad f(x)=x^3, \quad f(x)=\sqrt{x}, \quad f(x)=\sqrt[3]{x}, \quad f(x)=|x|, \quad f(x)=2^x$$

After students have graphed each function, discuss the general shape of the graph and the zeros of the function.

2. Have students graph the following functions, referencing the parent function $f(x)=x^2$ and using a table of values: $f(x)=x^2+3$, $f(x)=(x+3)^2$, $f(x)=x^2-3$, $f(x)=(x-3)^2$. Have them discuss with partners how each function differs from the parent function. Ask them to consider why the horizontal shift would be to the left when a number is being *added* to x , and to the right when a number is being *subtracted*. After they have made some educated guesses, suggest that they think about what makes the value of the *quadratic term* zero. Guide them to relate that to the vertex of each parabola. Stress that the parent function, $y=x^2$, has a double zero at $(0,0)$. Since a horizontal shift has no vertical change, the new function still has a double zero, and the vertex is on the x -axis.

3. Have students hypothesize what the graphs of the following functions would look like:

$$f(x)=(x+5)^2, \quad f(x)=x^2+5, \quad f(x)=5x^2, \quad f(x)=\frac{1}{5}x^2, \quad \text{and} \quad f(x)=-x^2.$$

Direct them to write down the transformations they believe will occur. Then, have them check by graphing the functions on their calculators.

4. Transition into a discussion about the similarities of transformations on a quadratic function to absolute value, square root, and cubic and cube root functions. Lead students to surmise the same for other polynomial functions.
5. Introduce the graph of a logarithmic function, and, if you have already taught inverse functions, define it as the inverse of an exponential function (given that the bases are the same). Otherwise, just name it, and have students graph it on their calculators and discuss the behavior of the graph of the function. Compare it to the graph they drew of $y = 2^x$.
6. Matching Activity: You can do this activity in two different ways:
 - Give each group of three or four students an entire set of attached Function Family Matching Cards, and have them match each function to its corresponding graph. If they have difficulty, encourage them to separate the cards into the different function families.
 - Give each student one card. Have each student who is holding a graph card write the equation that corresponds to the graph. Have each student holding an algebraic function card draw a rough sketch of the graph that corresponds to the function. Then, direct students to find their partners and check their responses.
7. Distribute copies of the attached Transformational Graphing handout, and have students work with partners to complete it. Have one partner do #1 while the other does #2. When each student has completed one problem, have partners exchange papers and check each other's work. If corrections are necessary, the student who did the problem should make the changes. When both students agree on the first two problems, then the student who did #1 does #3, and the student who did #2 does #4. Have them continue in this manner until the handout is complete. Be sure to check students' work along the way to be sure they are on the right track.

Assessment

- **Questions**
 - What transformations will map $y = x^2$ onto $y = -2(x + 4)^2 - 7$?
 - I am a function. My parent function is $y = |x|$. My parent function is mapped onto me by a reflection over the line $y = 0$, then a horizontal shift 3 units to the right, a vertical shift 4 units up, and finally a horizontal stretch with a factor of 2. Who am I?
- **Journal/Writing Prompts**
 - Given that a quadratic function is written in vertex form, explain how the quadratic term can help you determine the vertex of a quadratic function.
 - Explain the fact that the graphs of $y = |-2x|$ and $y = |2x|$ are the same, yet the graphs of $y = 2|x|$ and $y = -2|x|$ are different.

Extensions and Connections (for all students)

- Put students into groups of four, and assign each group one of the following function families: absolute value, quadratic, square root, cube root and cubic, exponential, or logarithmic. Have each group designate two function writers and two function sketchers. The function writers will create algebraic functions that include at least two transformations from a parent function each. The function sketchers will create separate

graphs that have at least two transformational differences from a parent function. The sketchers and the writers will then exchange work and write or sketch the function that corresponds to what they are now holding. Creators will check the work of their group members.

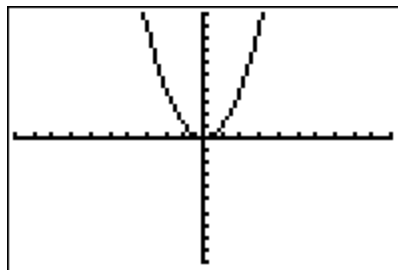
- Demonstrate the transformations from $y = f(x)$ onto $y = f(x+h)+k$.
- Have students write $y = 2x^2 + 6x + 11$ in vertex form and then graph the function by using transformational mapping from the parent function $y = x^2$.
- Present students with the following problem for solving. Projectile motion refers to the motion of an object acted on only by the force of gravity. The height, $h(t)$, of a projectile at any time, t , can be found by using the quadratic function $h(t) = -\frac{1}{2}gt^2 + v_0t + h_0$, where g is the acceleration due to gravity v_0 , is the initial upward velocity of the object, and h_0 is the initial height of the object. If two projectiles are thrown with the same initial velocity, but one is thrown from a height of 10 m while the other is thrown from a height of 42 m, how would the graphs of the motions of the two objects differ?

Strategies for Differentiation

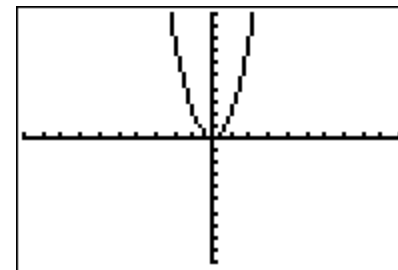
- Have students model horizontal and vertical shifts, using body movement.
- Use an overlay, either with an interactive whiteboard or transparencies, to demonstrate transformations on a parent (anchor) function.
- Have students create and use vocabulary flash cards with illustrations to reinforce transformational terminology.

Function Family Matching Cards

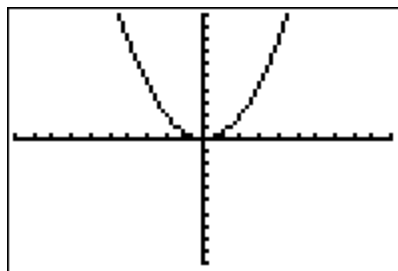
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 $Y_3 =$
 $Y_4 =$
 $Y_5 =$
 $Y_6 =$
 $Y_7 =$



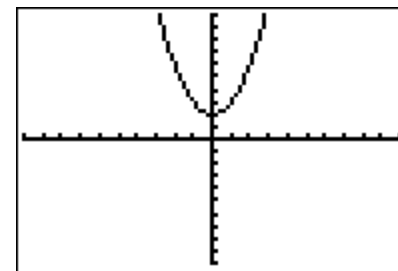
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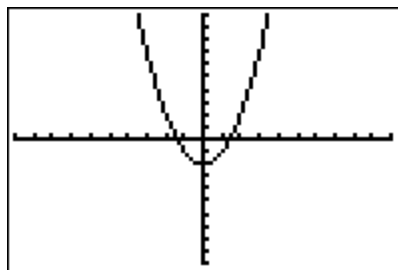
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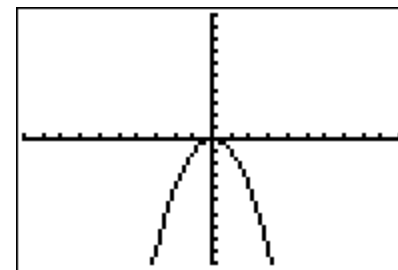
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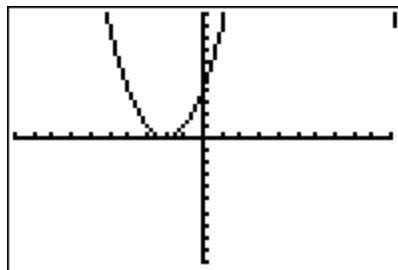
Plot1 Plot2 Plot3
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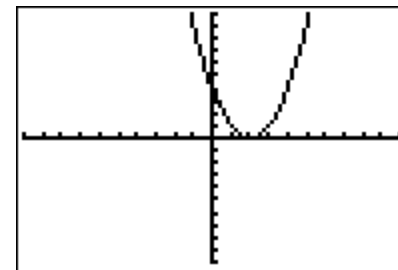
Plot1 Plot2 Plot3
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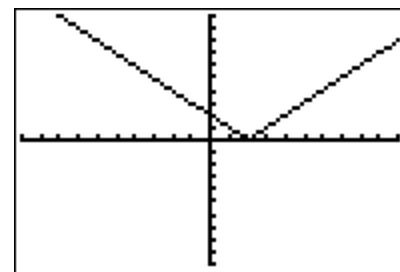
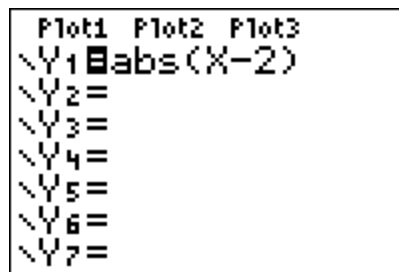
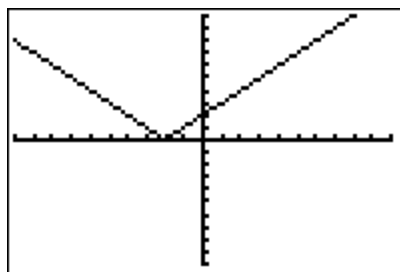
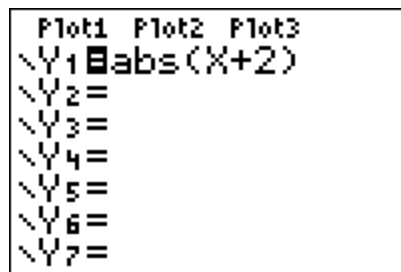
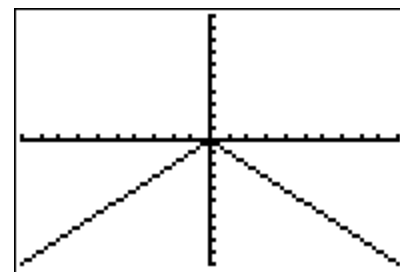
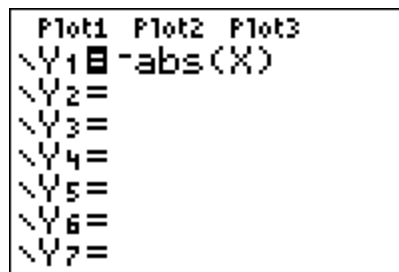
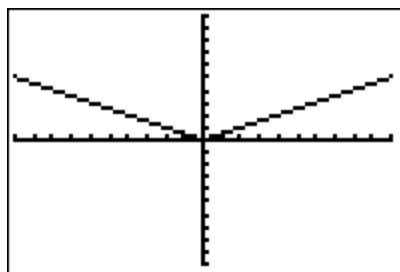
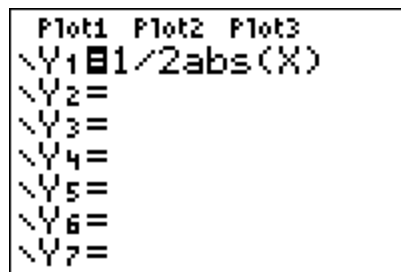
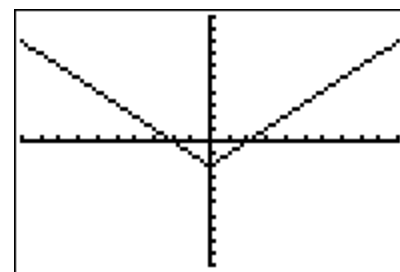
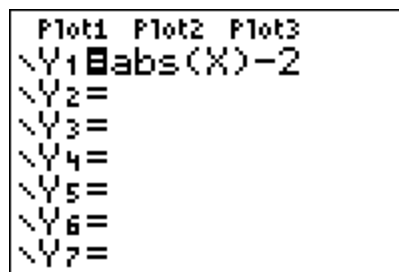
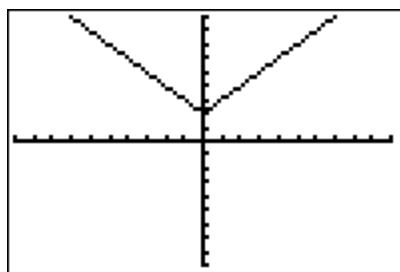
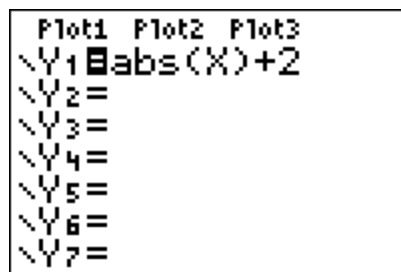
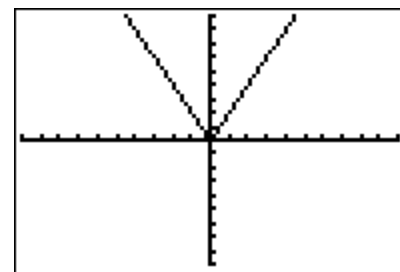
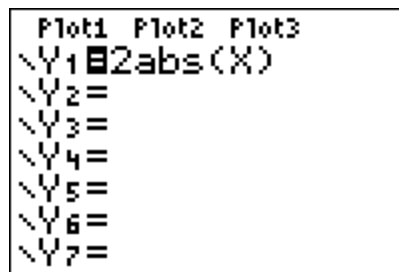
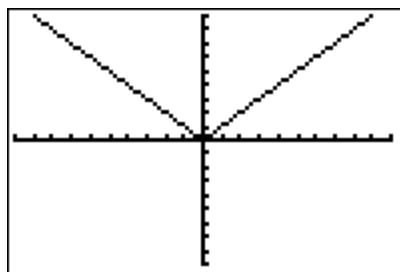
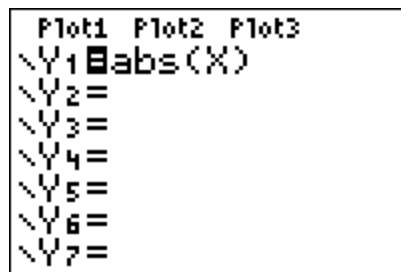


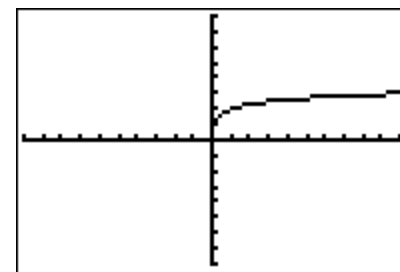
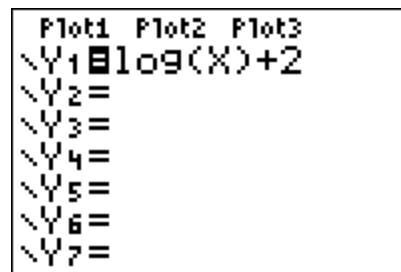
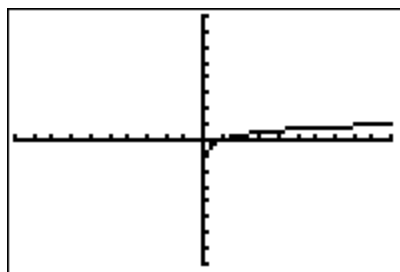
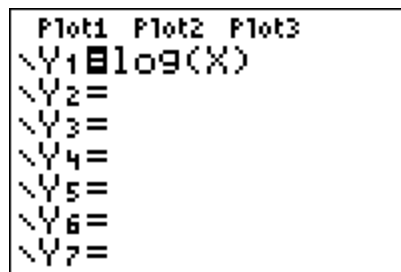
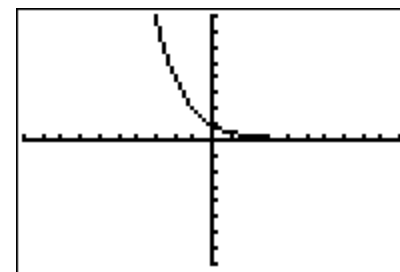
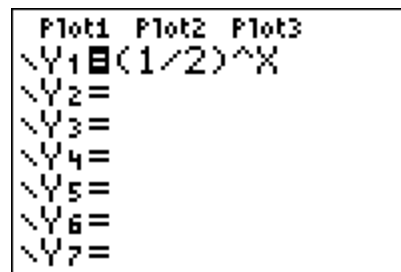
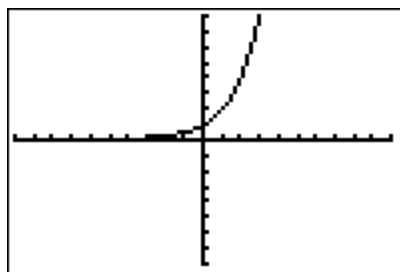
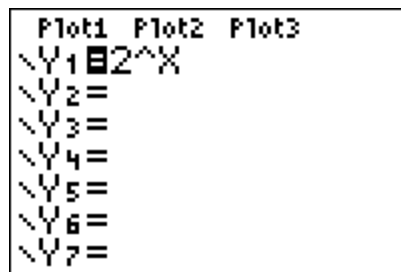
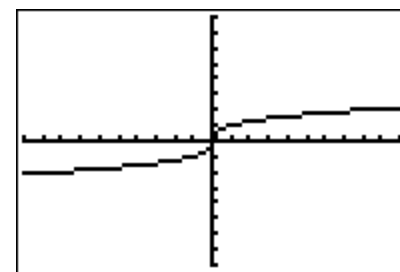
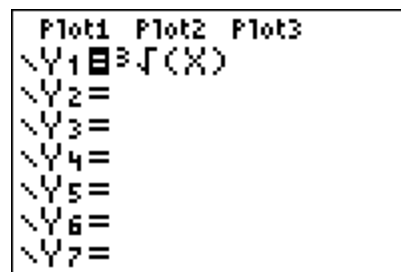
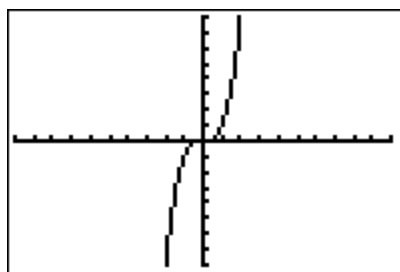
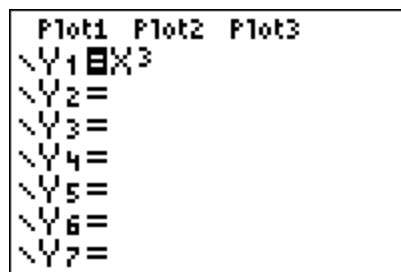
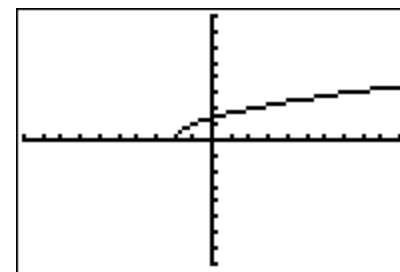
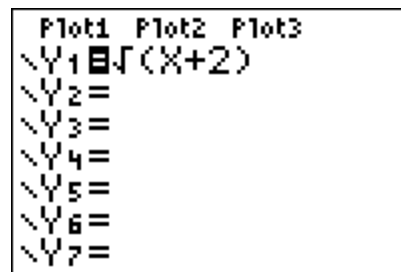
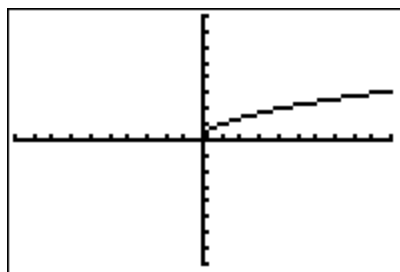
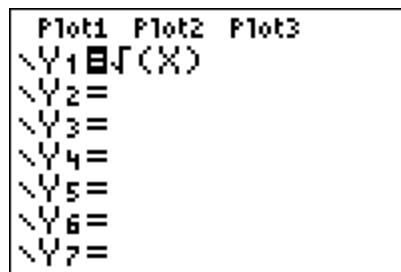
Plot1 Plot2 Plot3
 $Y_1 = (X+2)^2$
 $Y_2 =$
 $Y_3 =$
 $Y_4 =$
 $Y_5 =$
 $Y_6 =$
 $Y_7 =$



Plot1 Plot2 Plot3
 $Y_1 = (X-2)^2$
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 $Y_4 =$
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 $Y_6 =$
 $Y_7 =$



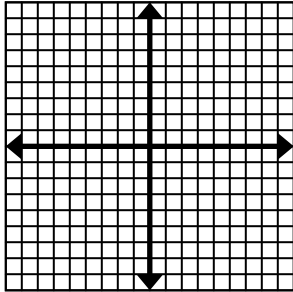




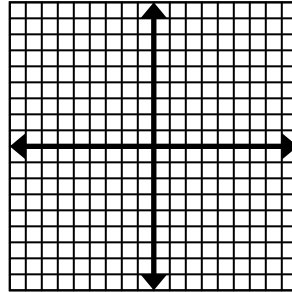
Transformational Graphing

Determine and graph the parent function in pen, then the given function in pencil.

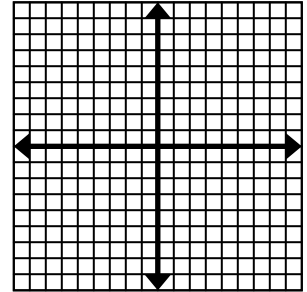
1. $y = (x-1)^2 + 2$



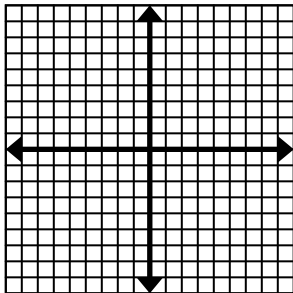
2. $y = -2x^2$



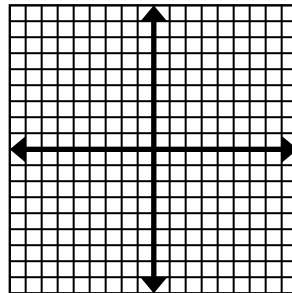
3. $y = \sqrt{x} - 4$



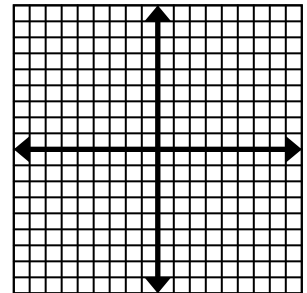
4. $y = \frac{1}{2}|x+3|$



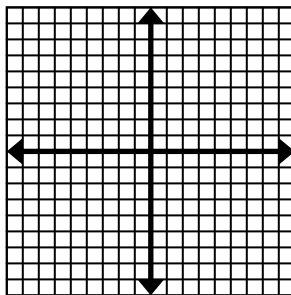
5. $y = \log x + 2$



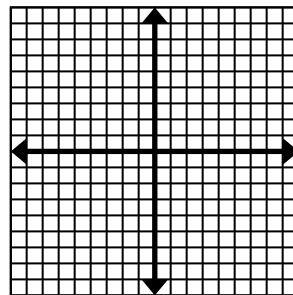
6. $y = \left(\frac{1}{2}\right)^x - 1$



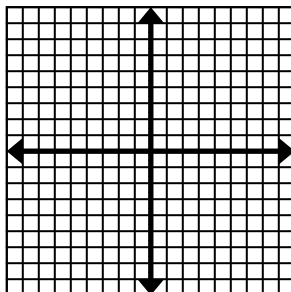
7. $y = \sqrt[3]{x+2}$



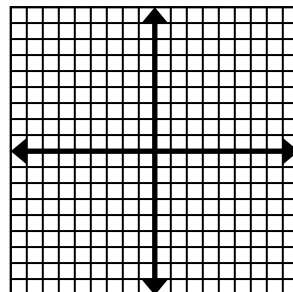
8. $y = x^3 - 3$



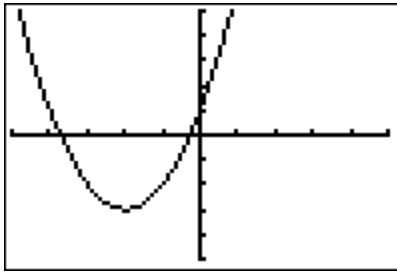
9. $y = -|x| + 1$



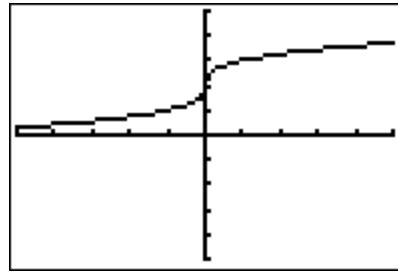
10. $y = |-x| + 1$



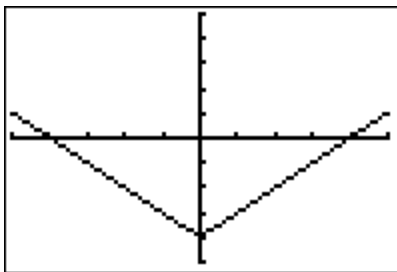
Write the algebraic function represented by the graph.



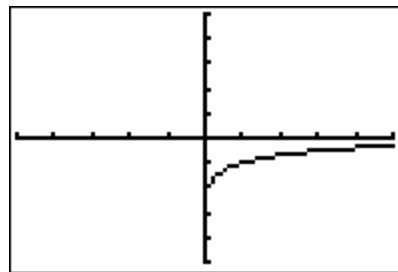
11. _____



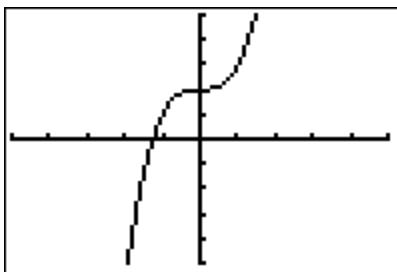
12. _____



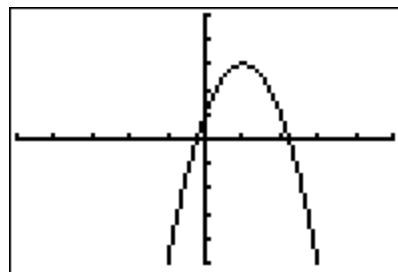
13. _____



14. _____



15. _____



16. _____