

# Writing in the Mathematics Class

Mathematics Education Workshop for 8-12 Teachers

Lake Ritter

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# Acknowledgements

The following works contributed to the content of this presentation:

- A Guide to Writing in Mathematics Classes, Dr. Annalisa Crannell, Franklin & Marshall College, [http://edisk.fandm.edu/annalisa.crannell/writing\\_in\\_math/guide.html#sec2](http://edisk.fandm.edu/annalisa.crannell/writing_in_math/guide.html#sec2)
- National Council of Teachers of Mathematics <http://www.nctm.org/>
- The Writing Site <http://www.thewritingsite.org/articles/vol2num2a.asp>



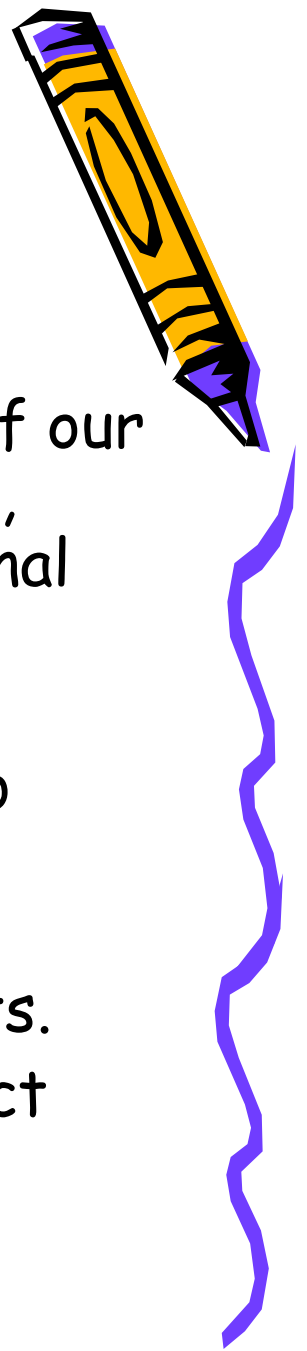
# Acknowledgements

The following works contributed to the content of this presentation:

- Calibrated Peer Review (CPR) Web based writing and peer review <http://cpr.molsci.ucla.edu/>
- Platonic Realms MiniTexts <http://www.mathacademy.com/pr/minitext/writing/index.asp>
- WritingFix <http://writingfix.com/index.htm>
- An excellent first hand account of experience with Journaling <http://www.oncourseworkshop.com/Awareness005.htm>



# Why bother with writing in a math class?



- Written communication is key to all aspects of our lives: job applications, composing web content, business, government, science, and international affairs.
- Writing actually helps us learn mathematics better! It is a form of explaining a concept to another.
- Proper use of mathematical symbols reflects accurate understanding of underlying concepts.
- "Writing is how we think our way into a subject and make it our own." --William Zinsser



# Why bother with writing in a math class?



From the National Council of Teachers of Mathematics:

**Instructional programs from prekindergarten through grade 12 should enable all students to-**

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.



# Why bother with writing in a math class?



[I]mplementation of the Georgia Performance Standards places a greater emphasis on the process standards from the National Council of Teachers of Mathematics: problem solving, reasoning, representation, connections, and communication.

From the Georgia Department of Education  
Secondary Mathematics GPS Course Descriptions



# Why bother with writing in a math class?



GPS Process Standards Math I through Math IV

MMIP3: Students will communicate mathematically

(I = 1, 2, 3, and 4)

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- c. Analyze and evaluate the mathematical thinking and strategies of others.
- d. Use the language of mathematics to express mathematical ideas precisely.



From the **Georgia Department of Education**  
**Secondary Mathematics GPS Course Descriptions**

# Why bother with writing in a math class?



*Every year, we buy ten cases of paper at \$35 each; and every year we sell them for about \$1 million each. Writing well is very important to us.*

- Bill Browning, President of *Applied Mathematics, Inc.*





# The difficulties with writing in mathematics



- Finding the time to fit in appropriate assignments
- Clear communication of what a writing assignment entails and what is expected
- Grading and feedback: How to make the grading process as objective as possible



# The difficulties with writing in mathematics



- Finding the time to fit in appropriate assignments

- Think small, consider 1-5 sentence in class assignments
- Consider journaling (works especially well in classes where a portfolio is already kept) in and out of the classroom
- Incorporate into existing assignments or exam questions
- Remember that not everything has to be graded (or perhaps not graded by you)



# The difficulties with writing in mathematics



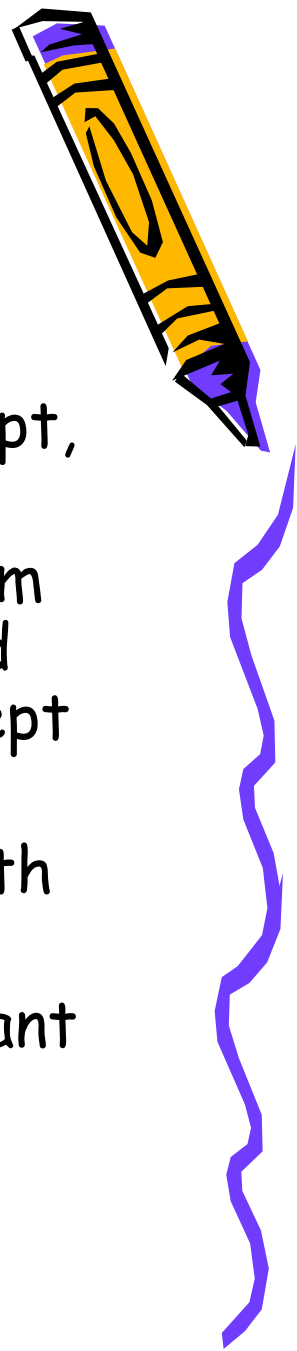
- Clear communication of what a writing assignment entails and what is expected
- Grading and feedback: How to make the grading process objective

- A well thought out and written assignment takes time
- Construct a clear rubric in advance, and give a copy to the students *before* they begin work on the assignment. (For longer term assignments and papers)

[Here is a: Neat Example of Writing Check List!](#)



# Writing assignments BIG and small



- Short in-class assignments: explaining a concept, stating a definition, journaling/reflection
- Short homework assignments: solving a problem and explaining the solution, using real data and explaining charts or graphs, explaining a concept with some depth, journaling
- Long homework assignments: solving an in-depth problem and explaining the solution
- Term papers: conducting some research relevant to course content and reporting ones findings



# In Class Writing Assignments

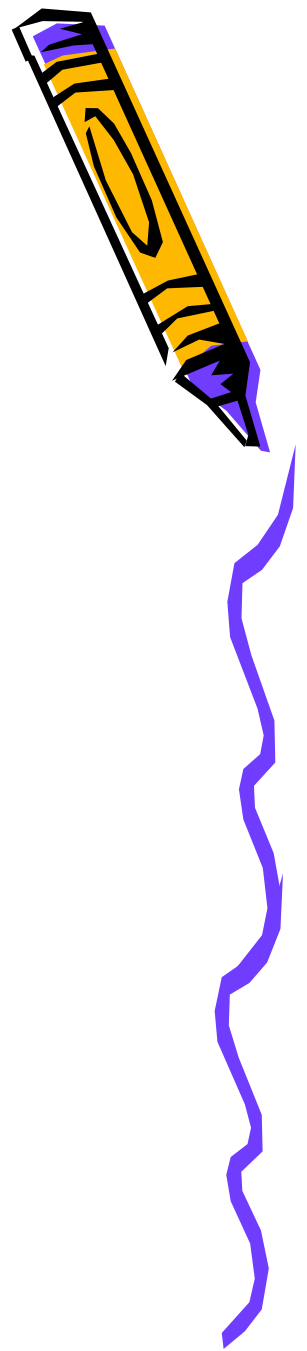


These are short assignments intended to take less than 5 minutes.

- State a specific definition or theorem.
- Explain what a certain symbol means.
- Explain how to do X in three or four sentences.
- Make up a problem that has a predetermined solution (or no solution at all!)
- Identify proper use and misuse of mathematical symbols and properties.



# In Class Writing Assignments Exercise



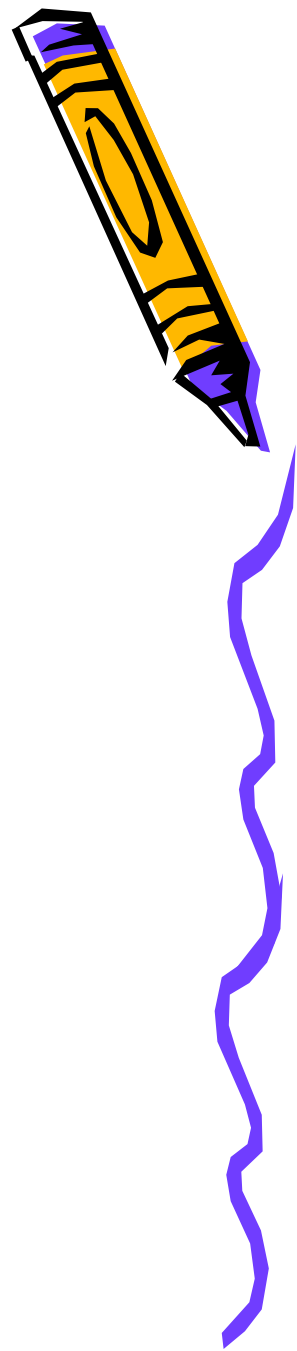
Context: Consider a Math II class unit on quadratic equations (finding real and complex roots MM2A3 & MM2A4.)

Task: State the quadratic formula.



# In Class Writing Assignments

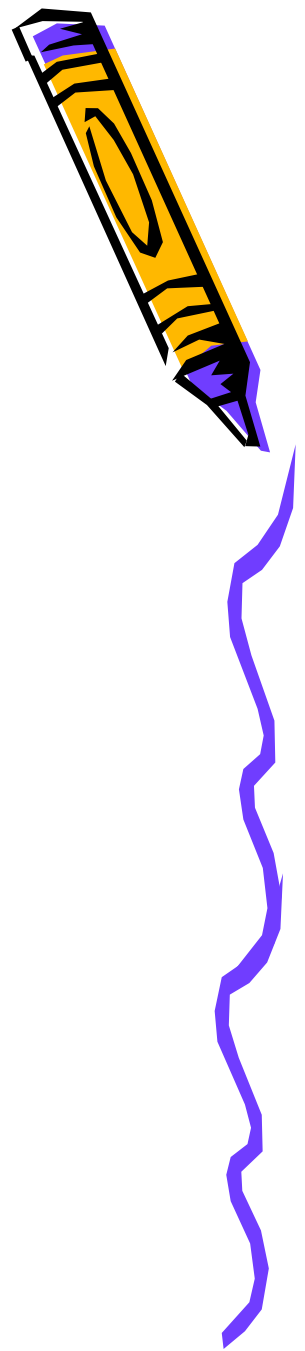
State the Pythagorean Theorem:



# In Class Writing Assignments

State the Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

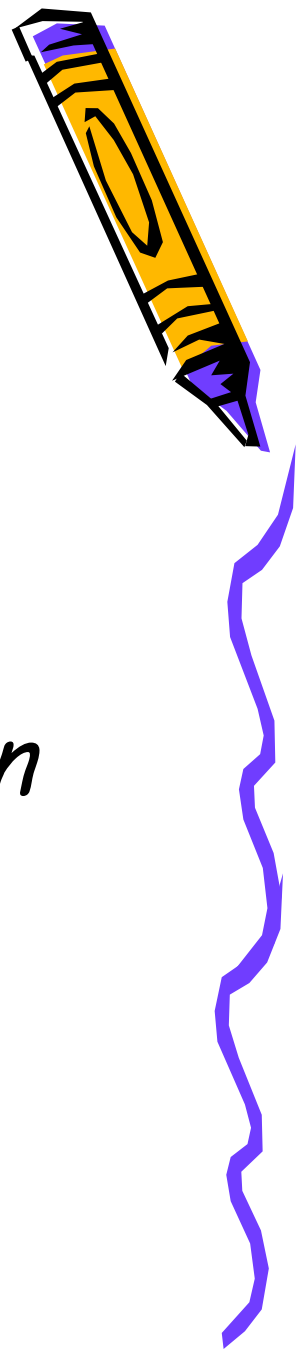




# In Class Writing Assignments

State the Pythagorean Theorem:

*If  $\triangle ABC$  is a right triangle, then  $a^2 + b^2 = c^2$ .*



# In Class Writing Assignments

State the Pythagorean Theorem:

*Suppose  $\triangle ABC$  is a right triangle with legs of lengths  $a$  and  $b$  and hypotenuse of length  $c$ . Then  $a^2 + b^2 = c^2$ .*



# In Class Writing Assignments

Writing theorems and definitions requires us to

- analyze hypothesis/conclusion relationships*
- clearly communicate what a symbol is used to denote*
- recognize the difference between a convention/definition that needs no proof, and a result that can be proved*



# In Class Writing Assignments



**Definition:** If  $a$  is a nonnegative real number, the nonnegative number  $b$  such that  $b^2 = a$  is the **principal square root** of  $a$ . It is denoted

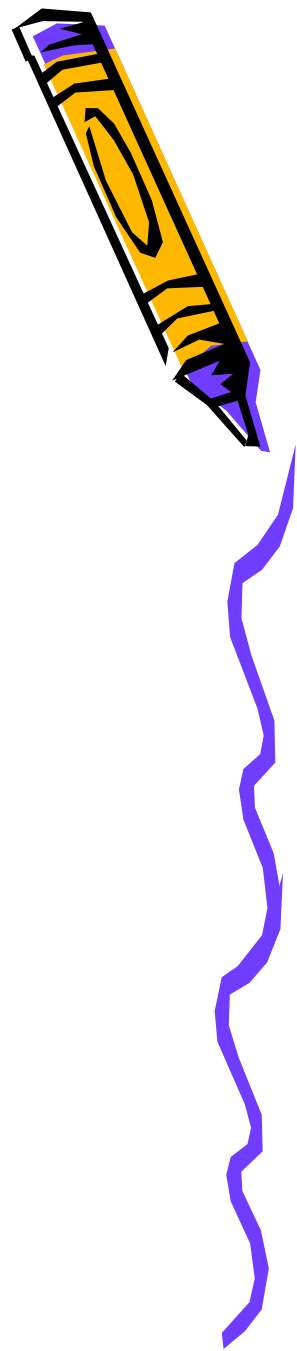
$$b = \sqrt{a}$$

**Prompt:** While it is true that  $(-2)^2 = 4$ , it is also true that  $\sqrt{4} \neq -2$ .

**Writing Assignment:** Explain in three to five sentences why both statements are true. Why is there no contradiction?



# In Class Writing Assignments



Short writing assignments can be used to get students to give critical focus to their own thinking process. There can be the element of "self-communication" and turning away from the hunt for the **almighty algorithm.**



# In Class Writing Assignments



## Example:

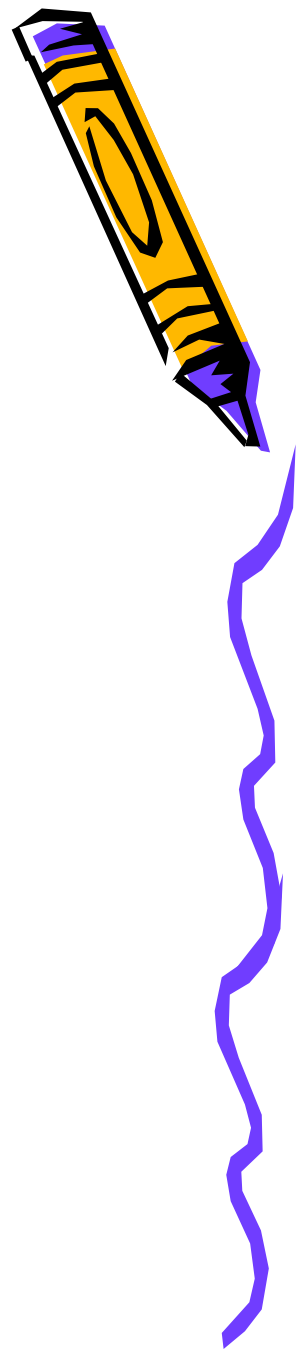
Describe how you would proceed to find the domain and range of a function if you were given its graph. How would your strategy change if you were given the equation defining the function instead of its graph?



From *Algebra & Trigonometry* 8<sup>th</sup> edition, Sullivan



# In Class Writing Assignments



Example:

Verify the identity. Keep a log of what motivates you at each step.

$$\frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$$



# In Class Writing Assignments



Example:

Verify the identity. Keep a log of what motivates you at each step.

$$\frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$$

$$\frac{\sin \theta}{1 + \cos \theta} \left( \frac{1 - \cos \theta}{1 - \cos \theta} \right) = \frac{\sin \theta (1 - \cos \theta)}{1 - \cos^2 \theta}$$

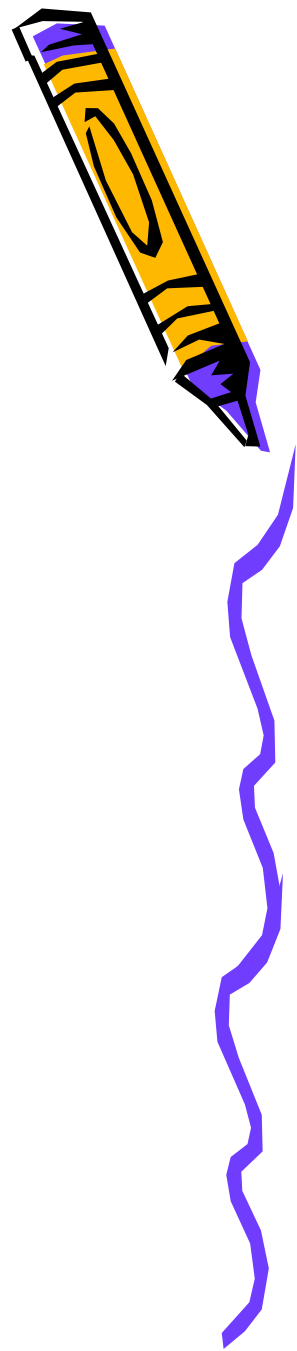
I need  $1 - \cos \theta$   
in the numerator.



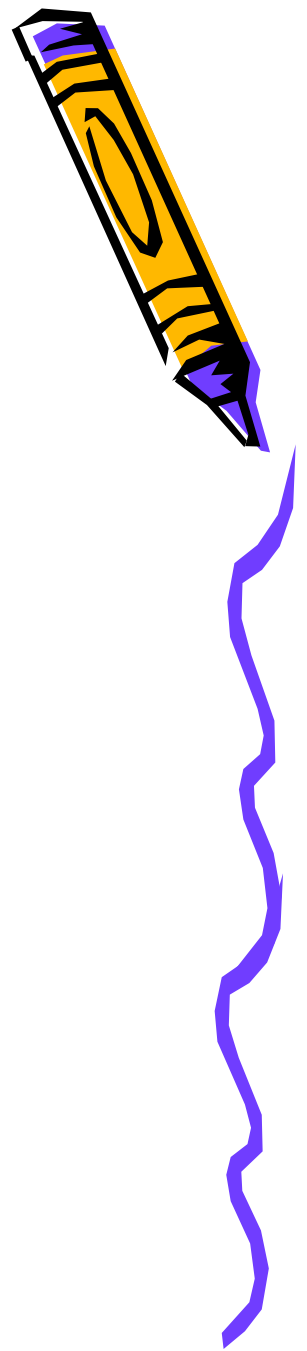


# In Class Writing Assignments

- ❑ Writing in mathematics is often about writing in *mathematese*--correctly!
- ❑ We demonstrate proper use in our texts and on our blackboards.
- ❑ Asking students to assess mathematical expressions and statements can highlight the importance of proper use in communication.



# In Class Writing Assignments



(3) Determine if the following contain properly used mathematical notation. Try to translate the expression into English; note any errors that you find.

(a)  $\frac{\tan x}{x} = \tan$

(b)  $\sin^2 \theta + \cos^2 \theta = 1$

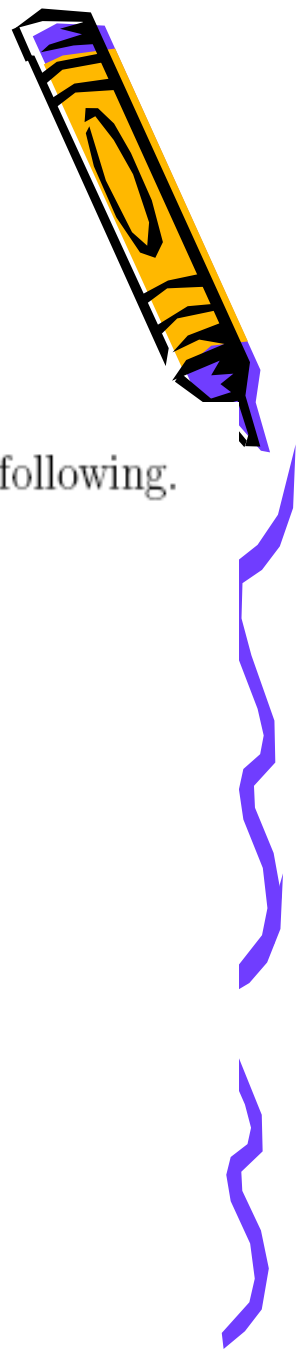
(c)  $x^2 - 2x + 1 \implies 0 \implies x \implies 1$

(d)  $|a| > 1 \implies -1 > a > 1$

(e)  $|a| < 1 \implies -1 < a < 1$



# In Class Writing Assignments



(10) (6 points 2 each) Explain in one or two sentences what is wrong with the following.

(a)  $\cos(2x+1) = 2 \cos x + \cos 1$

(b)  $\frac{16}{64} = \frac{16}{64} = \frac{1}{4}$

(c)  $\tan^{-1} x = \cot x$



# In Class Writing Assignments



$$(b) \frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4} - \text{nothing}$$

" > "DID TO COS ✓

$$(b) \frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}$$

16 + 64 ARE NUMBERS THAT YOU CANNOT SPLIT UP  
THERE IS NO SIGN BETWEEN THEM, THEY ARE  
A SINGLE ENTITY

$$(b) \frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}$$

it is 16 divided by 64 not  $1.6/6.4$



# In Class Writing Assignments



Journal Keeping:

Journal entries are short ( $\leq 5$  minute) writing exercises that offer the student an opportunity to think about the problem solving process and self-assess.



# In Class Writing Assignments



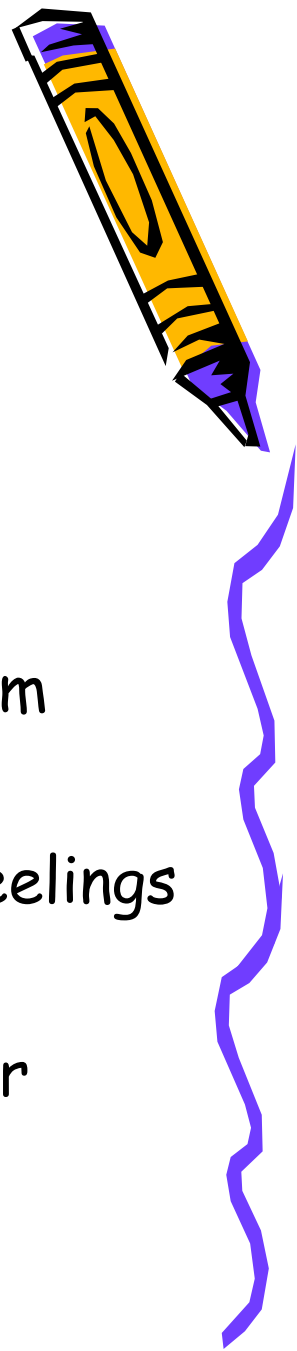
## Journal Keeping:

### Some Advantages:

- Provides a low-risk writing venue (when not graded for content)
- Can be used as a focus tool to begin or end class time
- Can provide the teacher with feedback about whether the material is understood or needs revisiting



# In Class Writing Assignments



## Journal Keeping:

### Some Complaints:

- Students may look to math class as a refuge from writing.
- There is concern over replacing content with "feelings about math"
- Pressure to cover content takes precedence over pressure to include writing.



# In Class Writing Assignments



## Journal Keeping:

There are many available lists of prompts for journaling. Some examples include:

- Identify something in today's lesson that is similar to something you already knew. Write about these similarities.
- The main idea of today's lesson was \_\_\_\_\_
- I need more help understanding \_\_\_\_\_
- Write an explanation of how to (factor a quadratic) for a student who was absent today.





# Journal Keeping

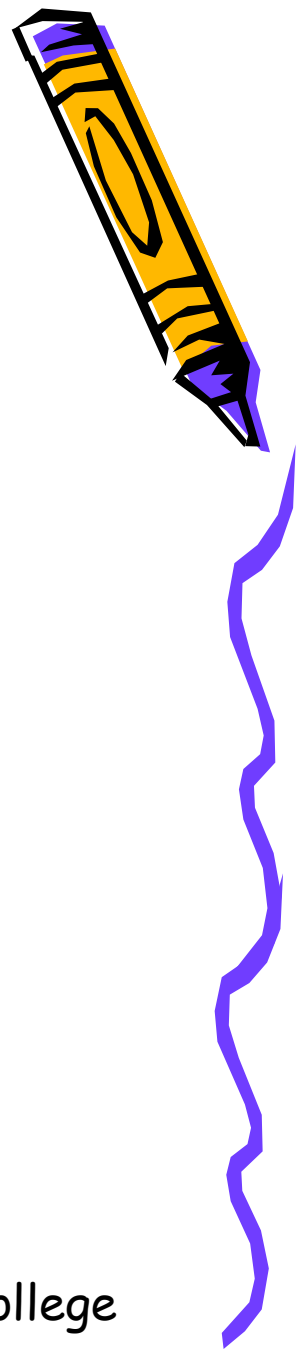
Periodic prompts can be given for journal entries done outside of class and due at a specified time. Journaling can be submitted through email or by submission to an online class discussion board.

Example Prompts:

- How much time did you spend this week studying for this course? Was it enough?
- How do you plan to prepare for next weeks exam?
- Give an example in which you have used, seen, or can find (**matrices**) outside of this or other math classes.



With permission from Debra Poese, Montgomery College



# Journal Keeping

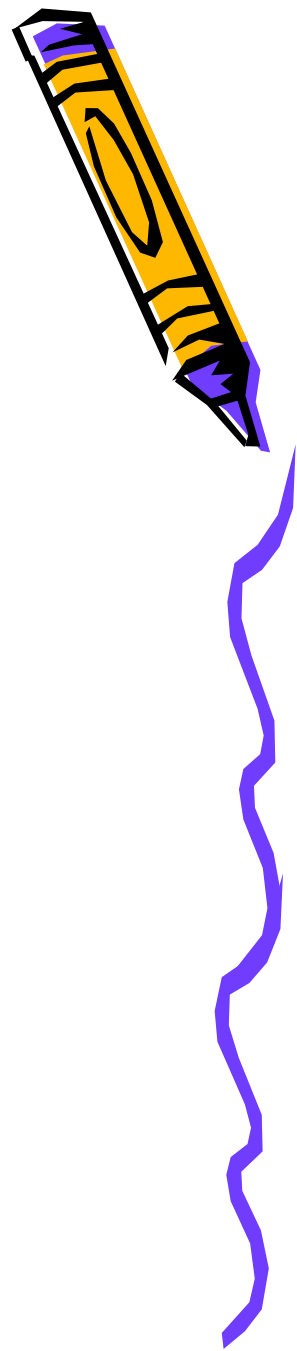
Debra Poese of Montgomery College gives a detailed first hand account, complete with prompts and student responses, of her use of journaling in a Survey of Mathematics course.

<http://www.oncourseworkshop.com/Awareness005.htm>



# Homework Assignments

- Can incorporate some degree of research (of terminology, convention, or historically important individuals);
- May be used to encourage students to think beyond the text book (real life rarely presents us with "end of the chapter" exercises);
- May be assigned with a clear rubric for grading constructed in advance and available to students



# Homework Assignments

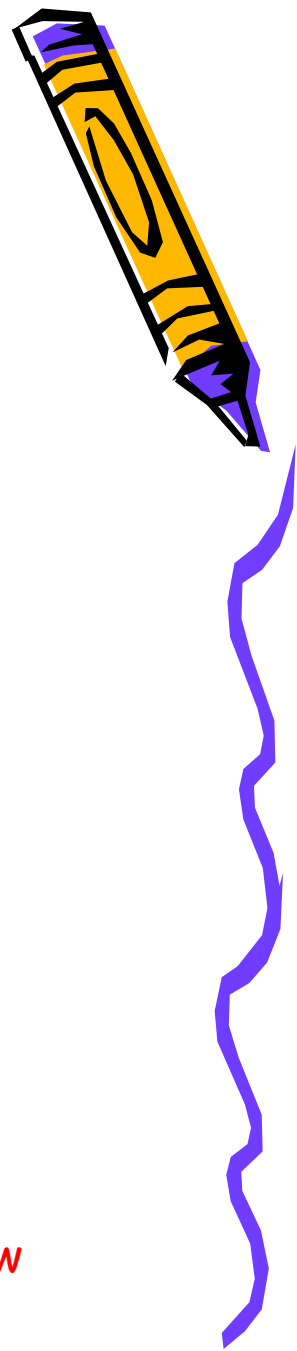
## Constructing Clear Assignments

Some or all of the following guidelines may apply:

- List goals of the assignment
- Clarify appropriate or required source material
- Consider including a list of questions intended to guide the student
- List any specifications (required length, format for figures, requirements for cited works if applicable)



Adapted from *Calibrated Peer Review*

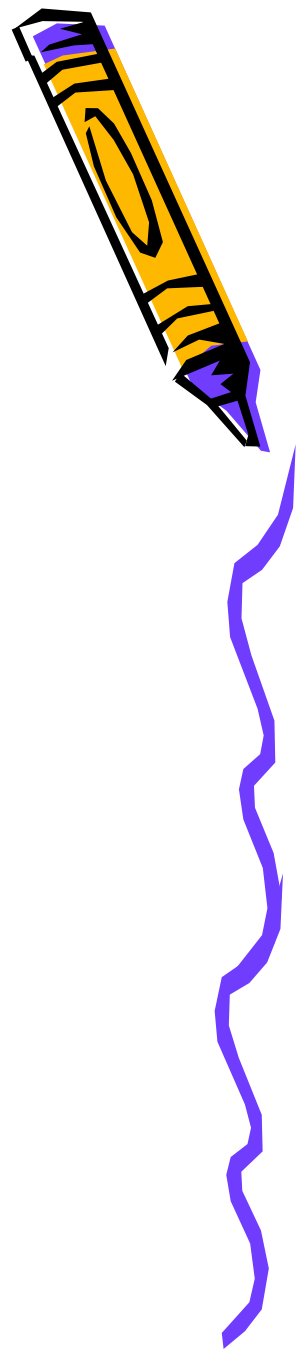


# Homework Assignments

Example:

Investigating Exponential  
Functions

In the Math Class and in Real Life



# Homework Assignments

Assignment Goals:

In this assignment, you will

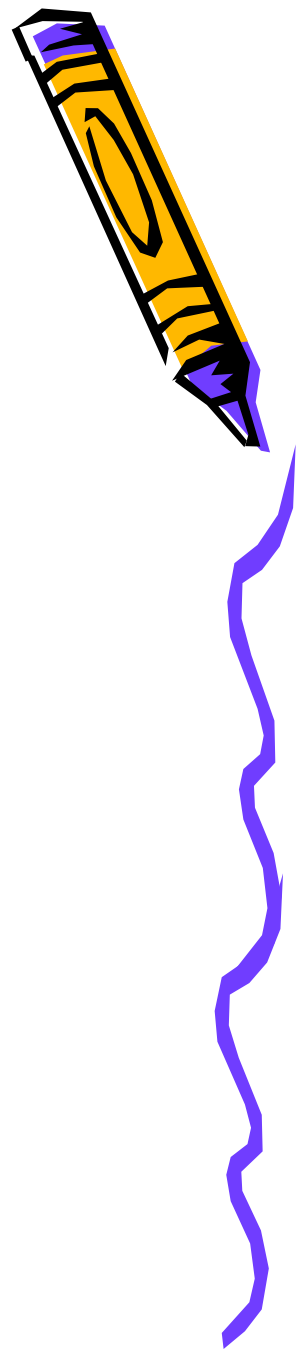
- 1) study exponential functions
- 2) compare exponential function with other types of functions (linear, quadratic, etc.)
- 3) look for natural processes that might be described by exponential functions



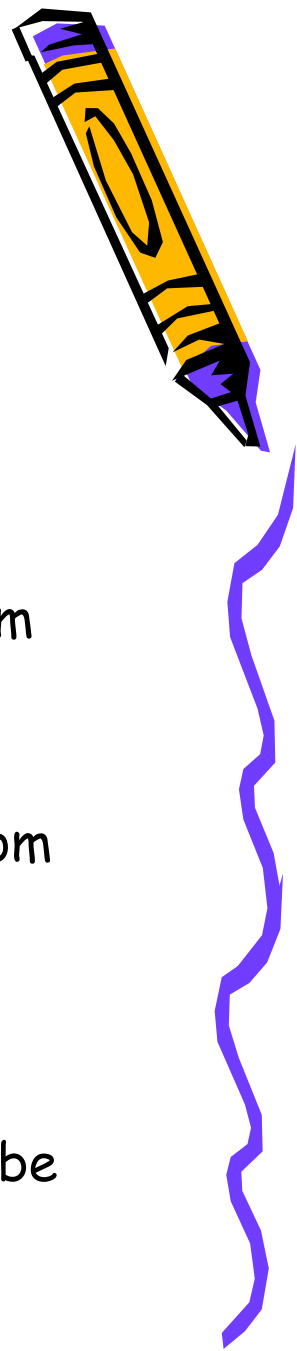
# Homework Assignments

Sources:

- 1) Your text book chapter (or sections) XXX
- 2) The Purple Math Forum  
<http://www.purplemath.com/modules/expofcns.htm>
- 3) The Math Page (on Logarithms and Exponentials)  
<http://www.themathpage.com/aPreCalc/logarithmic-exponential-functions.htm>
- 4) Algebra Lab (on Applications of Exponential Functions)  
[http://www.algebra-lab.org/lessons/lesson.aspx?file=Algebra\\_ExponentsApps.xml](http://www.algebra-lab.org/lessons/lesson.aspx?file=Algebra_ExponentsApps.xml)



# Homework Assignments



Guiding Questions: Think about the following questions as you organize your writing.

- 1) What makes exponential functions different from linear, quadratic and other functions you are familiar with?
- 2) How can you identify an exponential function from its graph?
- 3) Are there any uses for exponential functions in your life?
- 4) Can you identify any natural process that might be described as "exponential"?





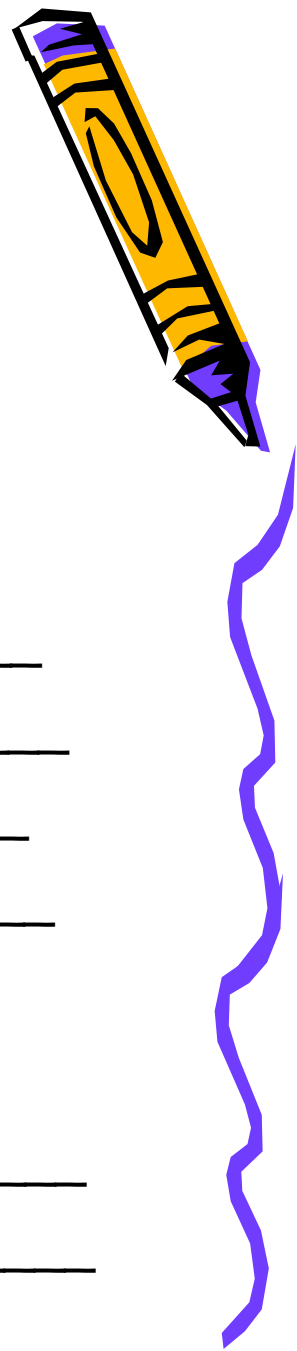
# Homework Assignments



Write about exponential functions in three to five paragraphs. Be certain to define what an exponential function is in your own words or a variation on the definitions you've read in the sources. Address all of the issues raised in the Guiding Questions. Include a well labeled graph that illustrates the difference between exponential and other types of functions or illustrates real life data modeled by an exponential function. Remember, you are writing paragraphs, not giving answers to a list of questions. Check your writing for accurate spelling and grammar.



# Homework Assignments



Rubric:

Does this paper:

- 1) Include clear definition(s) \_\_\_\_\_
- 2) Include a well labeled appropriate graph \_\_\_\_\_
- 3) Include proper mathematical notation \_\_\_\_\_
- 4) Address each of the guiding questions \_\_\_\_\_

Does this paper:

- 5) Include proper spelling punctuation and grammar \_\_\_\_\_
- TOTAL \_\_\_\_\_



# Homework Assignments

## R.A.F.T.

An approach that combines problem solving (relevant to a given learning unit) and realistic, communicative writing.

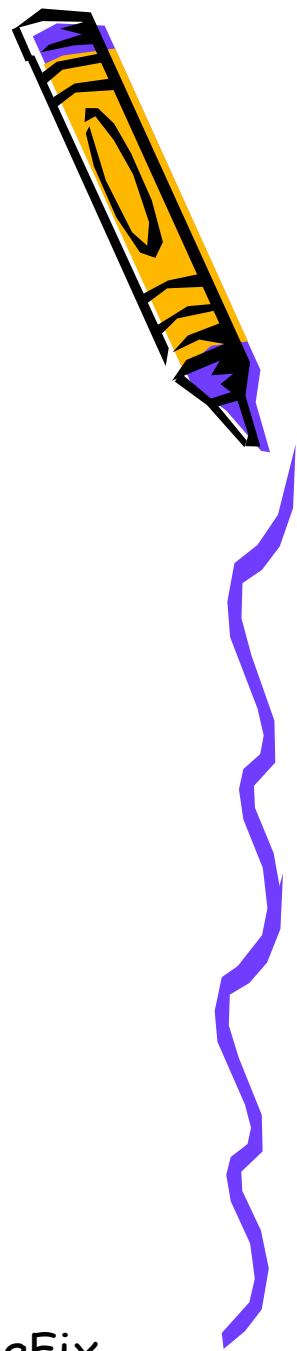
Give the student a **R**ole,

Define an **A**udience,

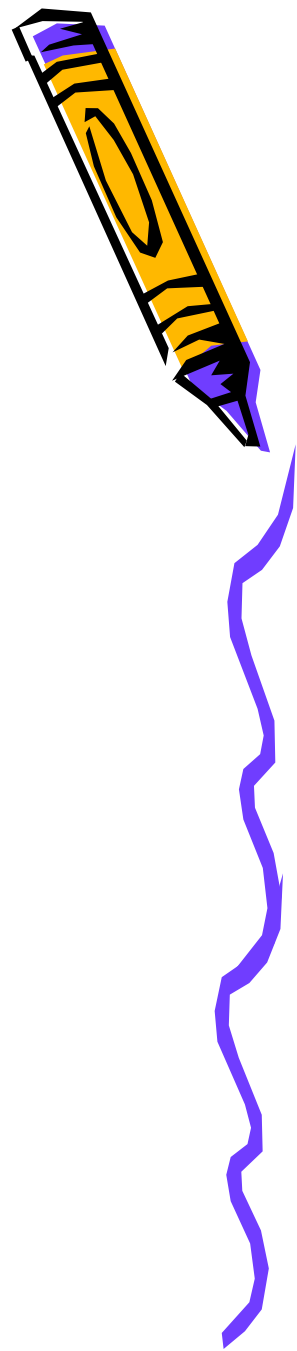
Specify a **F**ormat,

And a **T**opic

With permission from WritingFix



# Homework Assignments



Prompts:

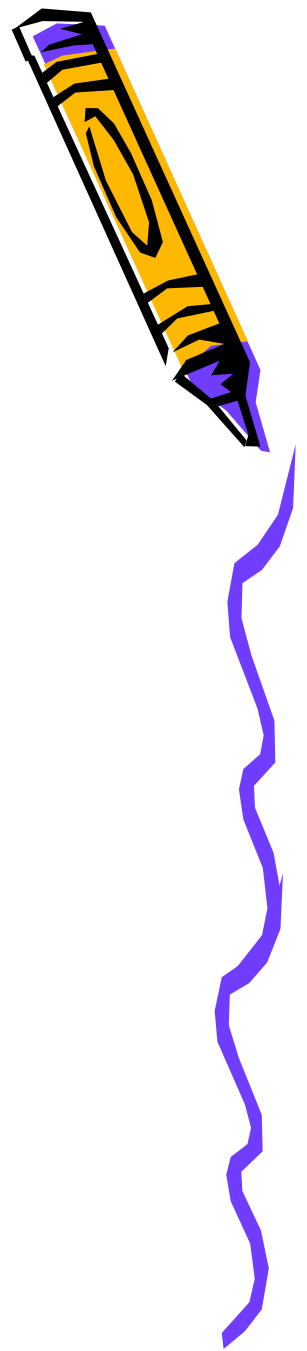
- You will write as though you are...
- writing something to be read by...
- The writing will take the form of...
- and will be about...



# Homework Assignments

## Example

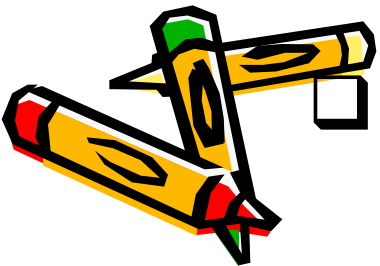
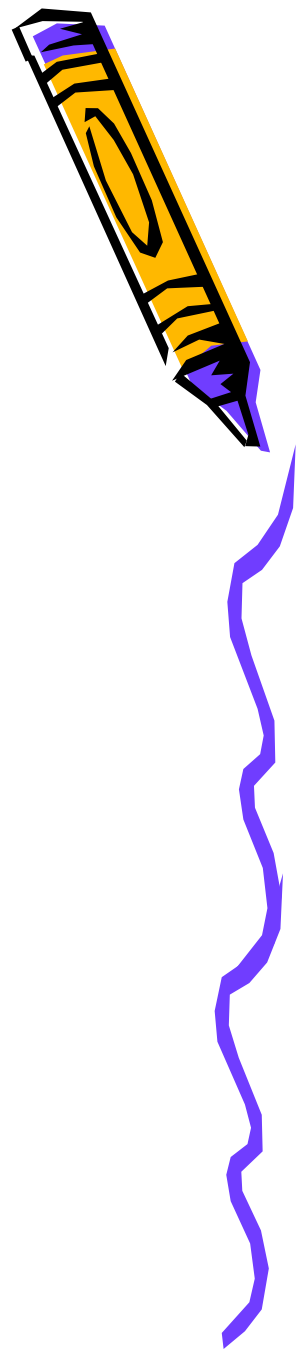
- You will write as though you are a **civil engineer**
- writing something to be read by **space aliens**
- The writing will take the form of a **top ten reasons why list**,
- and will be about **trigonometry**.



# Homework Assignments

## Example

- You will write as though you are a **computer programmer**
- writing something to be read by a **textbook publisher**
- The writing will take the form of **a request for funding,**
- and will be about **prime numbers.**



# Homework Assignments

The WritingFix offers web based interactive writing prompts that generates the components of RAFT writing:

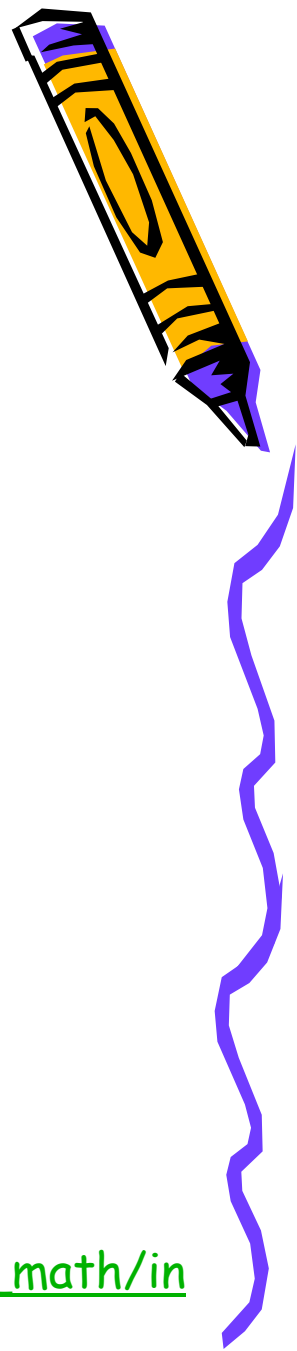
[http://www.writingfix.com/WAC/Writing  
Across Curriculum RAFTS Math.htm](http://www.writingfix.com/WAC/Writing_Across_Curriculum_RAFTS_Math.htm)



# Homework Assignments

An excellent example of *realistic, communicative writing* in mathematics is given by Annalisa Crannel of Franklin & Marshall College. These are sets of writing assignments given over a period of time (a semester) that share a theme.

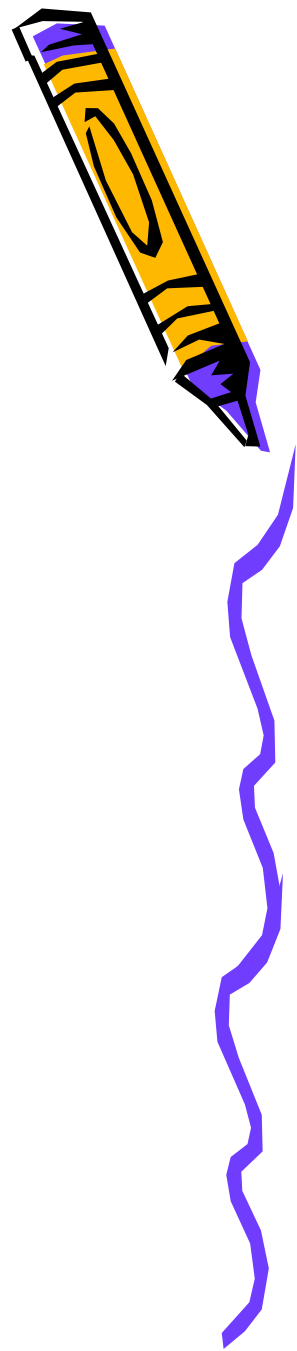
[http://edisk.fandm.edu/annalisa.crannell/writing\\_in\\_math/index.html](http://edisk.fandm.edu/annalisa.crannell/writing_in_math/index.html)





# Homework Assignments

These assignments take *typical word problems* and engage the student in not only the problem solving process but also the exercise of explaining that process and the mathematical concepts used.

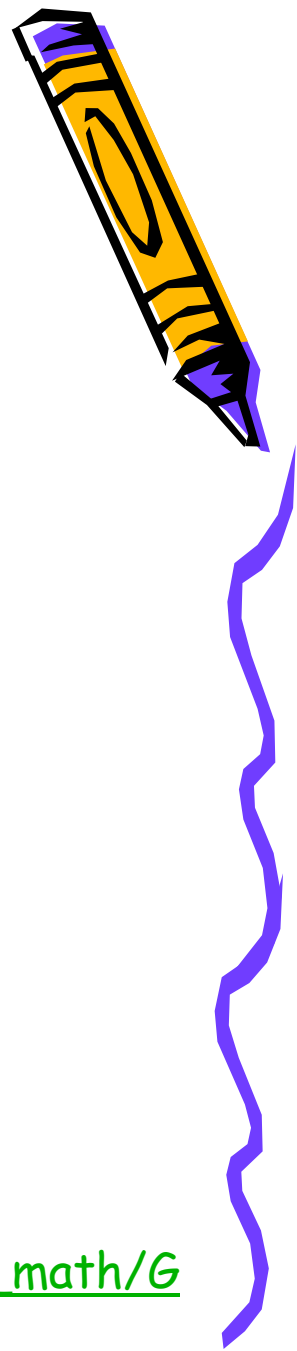


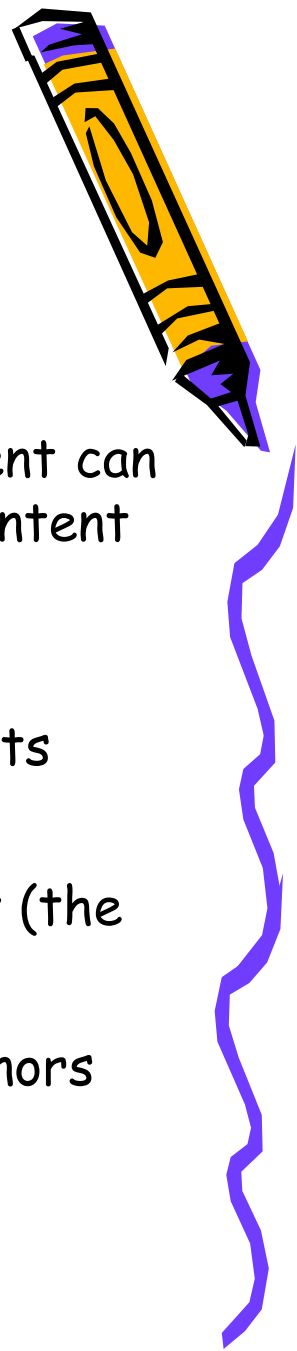
# Homework Assignments

**Word Problem:** A train travels 60 miles in one hour. It travels at a constant 50 mph for the first 20 miles and a constant 60 mph for the next 20 miles. Assuming its speed is constant for the remaining 20 miles, find the speed the train traveled at for the last leg of the journey.



[http://edisk.fandm.edu/annalisa.crannell/writing\\_in\\_math/G\\_OOF/goof1.htm](http://edisk.fandm.edu/annalisa.crannell/writing_in_math/G_OOF/goof1.htm)

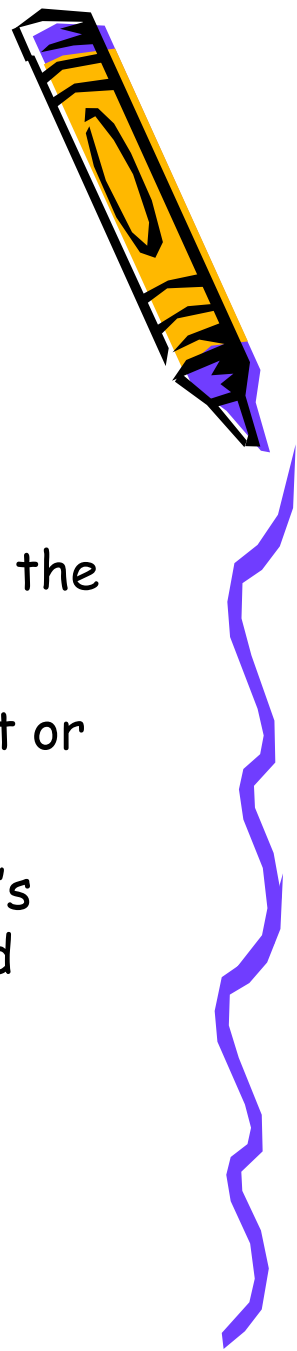




# Research or Term Papers

- The objective for this intensive writing assignment can be to learn something new or learn about course content in greater depth.
- Such projects challenge students to investigate mathematics sources on-line and in the library (texts beyond the class text).
- Can be used to enlighten students on the history (the human endeavor side) of mathematics.
- Can serve as an extra challenge for gifted or honors level students





# Research or Term Papers

Topics: Generally fall into one of three categories

1. Historical and Biographical Papers (discovering  $\pi$ , or the number  $e$ , the contributions of Isaac Newton...)
2. Applications (of complex numbers, applications in art or music, scientific applications ...)
3. Well known problems, objects, and curiosities (Zeno's Paradox, Fibonacci numbers and  $\phi$ , magic squares and sudoku...)





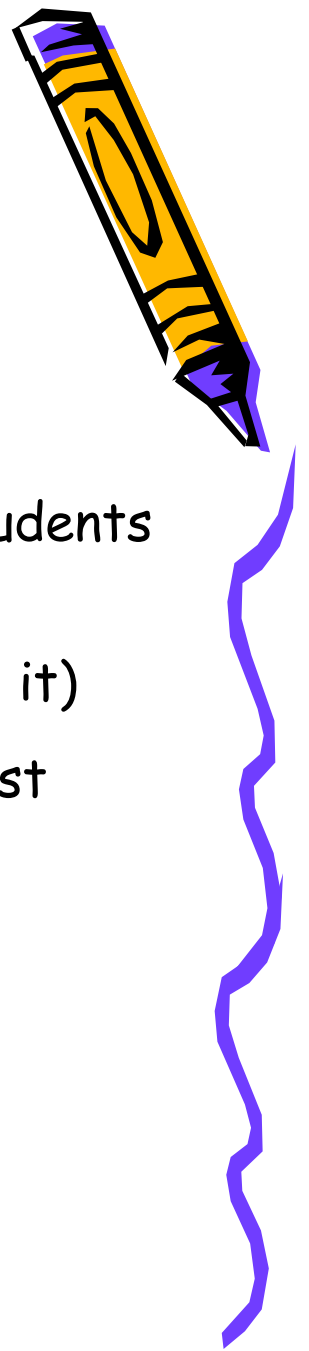
# Research or Term Papers

Writing the Assignment:

A written description of the assignment provided to the students may contain:

- A format guide---for paper structure, for citations, for including equations, figures, and tables...
- A list of potential/allowed topics
- A list of allowed or required references
- Notes on how the paper may differ from a paper for an English class





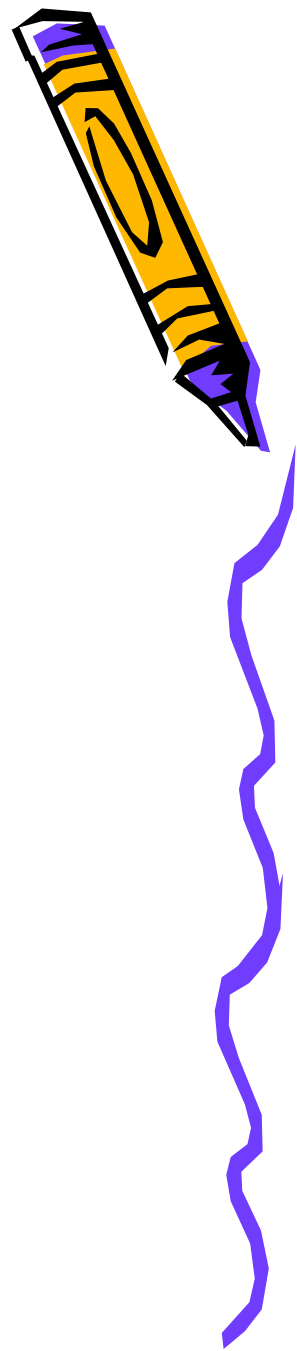
# Research or Term Papers

Writing the Assignment:

A written description of the assignment provided to the students may contain:

- Information about plagiarism (what it is and how to avoid it)
- Example (or information about accessing examples) of past student work



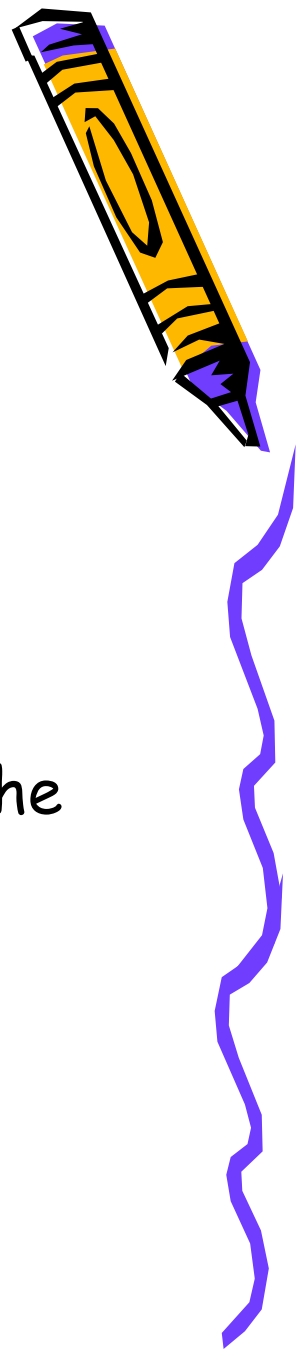


# Research or Term Papers

Possible GPS related topics: Math II

- Complex numbers their use and some history
- The Pythagorean Theorem and how the Sine, Cosine and Tangent are well defined
- $\pi$  and Circles
- Linear and Quadratic Regression

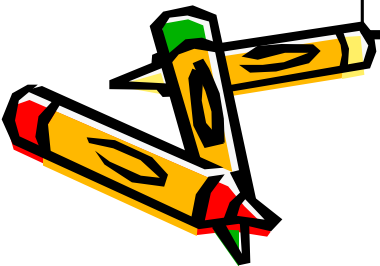




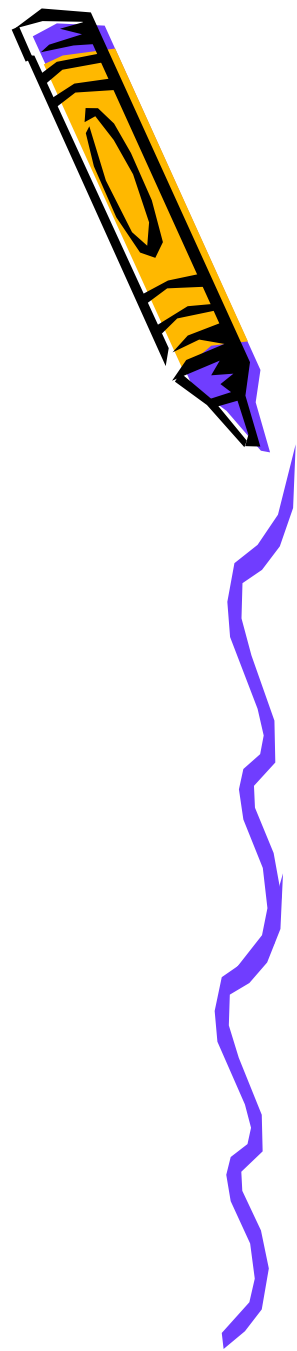
# Research or Term Papers

Possible *GPS* related topics: Math III

- Symmetry
- Use of Logarithms in Arithmetic -or- The Natural Log and the Number  $e$
- Matrices and Applications Science or Economics
- Conic Section







# Research or Term Papers

Possible *GPS* related topics: Math IV

- A History of Function Notation
- Applications of Trigonometry in Music
- The Fibonacci Sequence and the Golden Ratio
- Descartes and Graphing





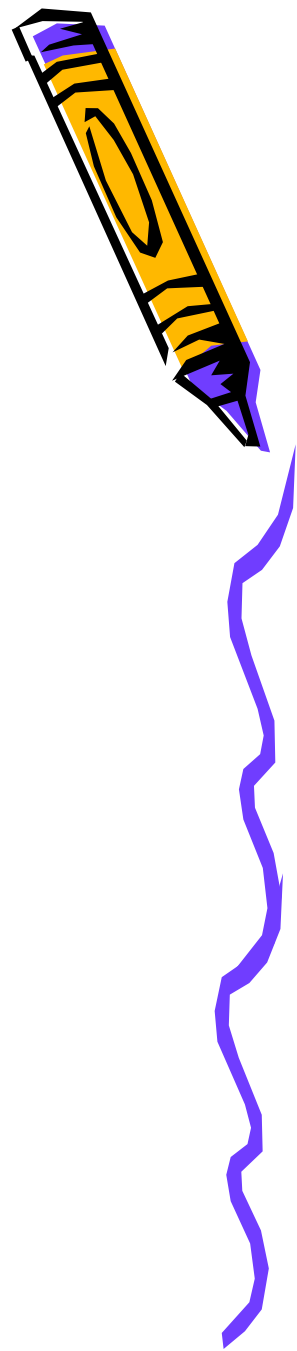
# Research or Term Papers

Possible *GPS* related topics: Calculus

- The Contributions of Isaac Newton -or- Gottfried Leibniz
- Applications of Differential Equations
- Archimedes, Eudoxus and Integration
- Infinite Series and Infinities



# Research Paper Assignments from my own Calculus Classes



Honors Calculus I

Writing Assignment I

Rubric for writing assignment I

Writing Assignment II

Rubric for writing assignment II

Honors Calculus II

Writing Assignment

(The rubric used is the same as for assignment II above.)



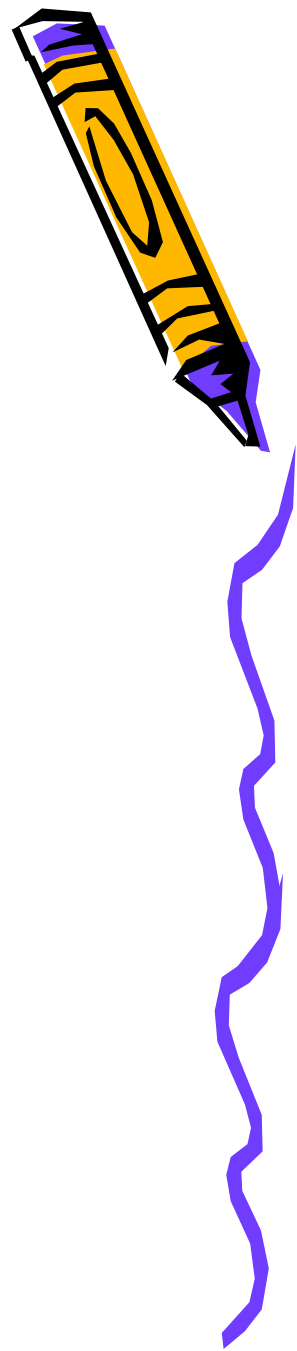
# Research Paper Assignments from my own Calculus Classes

Examples of my responses and grading:

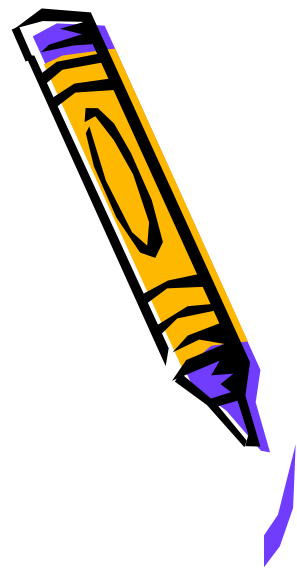
[Feedback for an Excellent Paper](#)

[Feedback for a Good Paper](#)

[Feedback for a Poor Paper](#)



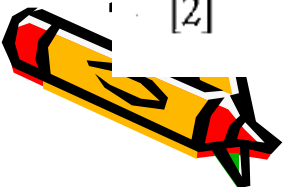
# Research Paper Assignments from my own Calculus Classes



## Newton's Method

Newton's Method was made in 1700's with a collaborative effort by Sir Isaac Newton and Joseph Raphson as a way to finding the zeros of real number functions. It is one of many root finding algorithms in existence. It uses the principles of tangent lines near the zero points of the graph to approximate the solution to an equation  $f(x)$ . The overall goal of the process is to be able to produce a sequence of approximate answers that approaches the solution to an equation.

[2]



is meant  
for "7"

# THE END

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Annalisa Crannell, Franklin and Marshall College

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This presentation can be accessed on the workshop web page:

[Workshop Webpage March 2011](#)

