Math 52 - Winter 2010 - Final Exam

Please circle your TA's name: **Jack Hall** **Xiannan Li** **David Sher**

Circle the time your TTh **section** meets: **10:00** **11:00** **1:15** **2:15**

Your name (print): Student ID:

Please sign the following:

"On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature:____________________________________

Instructions:

- Print your name and student ID number, circle your TA's name, the time that you attend the TTh section and sign to indicate that you accept the Honor Code.
- Read each question carefully. In order to receive full credit, please show all of your work and justify your answers.
- **You have 3 hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of teaching staff.

Problem	1	2	3	4	5	6	7	8	9	10	11	Total
Max	16	16	18	22	20	18	18	16	20	18	18	200
Score												

Formulas you may use: Relating rectangular coordinates x, y, z, cylindrical coordinates r, θ, z and spherical coordinates ρ, θ, φ:

$$\begin{cases} z = \rho \cos\varphi, \\ r = \rho \sin\varphi, \\ x = r\cos\theta = \rho\sin\varphi\cos\theta \\ y = r\sin\theta = \rho\sin\varphi\sin\theta \end{cases} \quad \text{and} \quad dxdydz = rdrd\theta dz = \rho^2 \sin\varphi d\rho d\theta d\varphi.$$

Trig identities: $2\cos^2 x = 1 + \cos 2x$, $\quad 2\sin^2 x = 1 - \cos 2x$.

Problem 1. (16 pts) Evaluate $\int_0^1 \int_x^1 \int_0^y y \sin(xy)\, dz dy dx$.

Problem 2. (16 pts) Consider the force field $\vec{F} = (x^2 + y^2)\vec{i} + xy\vec{j}$ in the plane. Find the work (circulation) done by the force $\vec{F}$ on a particle that moves along the graph of $x = y^2$ from $(0, 0)$ to $(4, 2)$.

Problem 3. (18 pts total) Let $F(x, y, z) = (2xy + z^2,\ 2yz + x^2,\ 2zx + y^2)$.
(a) (12 pts) Find a potential for F.

(b) (6 pts) Let C be the curve parametrized by $r(t) = (\cos t, \sin t, t)$ for $0 \leq t \leq 1$. Compute $\int_C \vec{F} \cdot \vec{T} ds$

Problem 4. (22 pts total) Let S be surface parametrized by $X(s,t) = (s\sin t,\ s\cos t,\ s^2)$ for $1 \le s \le 5$ and $0 \le t \le \pi$.
(a) (16 pts) Compute the surface area of S.

(b) (6 pts) SET UP, but **do not** calculate a DOUBLE integral computing the moment of inertia about the y axis of an object in the shape of the surface S if the density function is $\delta(x,y,z) = \sqrt{x^2+y^2}$.

Problem 5. a) (10 pts) Evaluate

$$\int_{-R}^{R} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{dydx}{1+x^2+y^2}$$

b) (10 pts) Determine all the values of the constant c such that the following integral is finite:

$$\iint_{\mathbb{R}^2} \frac{dA}{(1+x^2+y^2)^c}$$

Problem 6. (18 pts) Let C be the triangle formed by intersecting the plane $x + y + z = 1$ with the three coordinate planes, oriented counterclockwise as we look at it standing at the origin. Use Stokes' Theorem to calculate

$$\int_C z dx + x dy + y dz.$$

Problem 7. (18 pts) If $\vec{F} = 2xy\,\vec{i} + z\cos x\,\vec{j} + (z^2 + xy)\,\vec{k}$, calculate the outwards flux of $\vec{F}$ through the part of the surface $z = x^2 + y^2$ situated below the plane $z = 4$.

Problem 8. (16 pts) Use Green's Theorem to evaluate $\int_C (x^2 + y)dx + xy^2 dy$ where C is the closed curve determined by $y^2 = x$ form $(0, 0)$ to $(1, -1)$ and then $y = -x$ from $(1, -1)$ back to $(0, 0)$.

Problem 9. (20 pts) Find the volume of the region inside both the sphere of radius 2 centered at the origin and the vertical cylinder of radius 1 centered at (1,0,0).

Problem 10. (18 pts total) Consider the vector field $\vec{F}(x,y,z) = \left(x,\ \frac{-z}{y^2+z^2},\ \frac{y}{y^2+z^2}\right)$.

a) (6 pts) Show that the curl of $\vec{F}$ is zero wherever $\vec{F}$ is defined.

b) (6 pts) Show that $\vec{F}$ has nonzero circulation along the curve C given by $x(t) = (\cos(t)\sin(t), \cos(t), \sin(t))$, with for $0 \leq t \leq 2\pi$.

c) (6 pts) **Carefully explain** why the results of part a) and b) do not contradict Stokes' theorem.

Problem 11. (18 pts) Let $\vec{F}$ be a vector field defined everywhere except at the origin. Suppose the divergence of $\vec{F}$ is 3 everywhere $\vec{F}$ is defined. Let S_1 and S_2 be the surface of the cubes centered at the origin and of sides 2 and 4 respectively. Assuming the outwards flux through S_1 is 5, find the outwards flux through S_2. Please **carefully explain** your reasoning.