Lesson Plans for Use of Algebra Tiles:

Instruction of combining integers and common terms of polynomials

designed for Buffalo City High School Algebra courses
at Grover Cleveland High School
in connection with the
1999-2000 Carnegie Learning Cognitive Tutor Math A Year I Curriculum

compatible for use in Pre-Algebra and Basic Algebra courses: grades 7 – 9

6 (40 minute) lesson plans (spanning 2 weeks in Carnegie Curriculum)

Necessary Tools: Overhead Projector
overhead projection Algebra tiles – see Appendix A
overhead pens and acetate sheets
student notebooks & writing instruments
class sets of algebra tiles – see Appendix A
class set of chalk or markers: yellow, blue, green, & red
(zip-lock bags are useful for tile & marker sets)
Carnegie Assignment and Assessment binder
or other text providing practice examples

submitted by Mary C. Dileas
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Lesson Plans for Use of Algebra Tiles:

Instruction of combining integers and common terms of polynomials

Objectives:

1. To establish a base of prior knowledge on which to build knowledge and concepts of algebra through the use of manipulatives.

2. To build that knowledge through a series of experiences which move through three levels of learning: concrete, visualizing, and theorizing (using symbols).

3. To review/ instruct a knowledge base of combining integers

4. To extend the base of combining integers to combining terms

5. To allow students freedom in exploring math concepts and building their own academic self-confidence.

NCTM Standards:

Numbers and Operations – understanding numbers: **ways of representing numbers, relationships among numbers, and number systems, compute fluently and make reasonable estimates.**

Algebra – represent and analyze: **mathematical situations and structures using algebraic symbols.**

Problem Solving - **build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; monitor and reflect on the process of mathematical problem solving.**

Communication - **organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; use the language of mathematics to express mathematical ideas precisely.**

Connections - **recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics.**

Representation - **create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems.**

New York State Standards:

1. Students will use mathematical analysis, scientific inquiry, and engineering design, as appropriate, to pose questions, seek answers, and develop solutions.
3. Students will understand mathematics and become mathematically confident by communicating and reasoning mathematically, by applying mathematics in real world

situations, and by solving problems through the integrated study of number systems, …, algebra… (etc.)

7. Students will apply the knowledge and thinking skills of mathematics … to address real-life problems and make informed decisions.

Overview of Lesson Plans for Use of Algebra Tiles

This unit was designed for students who are enrolled in a first year high school Algebra course intending to prepare them for the New York State Regents Math A exam, after two years of study. However, this lesson is easily adapted to any pre-algebra or basic algebra course by omitting the computer lab days and using the lesson plans on successive instruction days. As an instructor of classes with inclusion students and limited English proficiency (LEP) students, I have found that measurable differences occur in learning and retention when students participate in a process that affirms and builds on their prior knowledge. In addition, algebra tiles are just one way that students can build their conceptual understanding through the use of models, assisting their learning process from the concrete to the symbolic.

As a fairly short unit of lessons, this plan serves to augment the philosophy and practice of the Cognitive Tutor Curriculum which is designed to capitalize on student exploration and discovery of conceptual learning. (Carnegie, 1999) This primarily occurs through students' repeated experiences using and manipulating math ideas and representations. Guided experiments can and do lead to profound learning experiences. (Dewey, 1938) This is the goal of these lessons. In addition, these algebra tile skills can be extended and reused when students are learning how to solve equations and multiply polynomials. (Howden, 1994)

The unit consists of five stages of learning: first, establishing their prior knowledge of integers; second, learning about algebra tiles as a tool used to represent their ideas of integer concepts; third, utilizing the tiles as a concrete representation of combining integers and demonstrating the zero principle; fourth, utilizing the tiles as a concrete representation of combining algebraic terms; and fifth, extending their concrete understanding into symbols which they can effectively manipulate on paper.

Partner work is utilized during discovery and exploration stages. Individual journaling is intended to solidify learning that has occurred on an individual basis. Student learning will be measured through journal reviews, teacher observation of partner work in class, examination of worksheet practices and student generated examples. Retention will be measured through

ongoing use of the concepts throughout the year's study of algebra, particularly in equation solving.

Overview of Lesson Plans for Use of Algebra Tiles

Calendar Outline

Monday	Tuesday	Wednesday	Thursday	Friday
Computer Lab	Computer Lab	Introduce the meaning and use of algebraic tiles Partner work: representing integers with worksheet practice All: Journal Entry	Review meaning of algebraic tiles Partner work: reverse representing and combining integers Journal Entry and worksheet practice	Introduce algebraic tiles as representing algebraic terms Partner work: representing and combining terms All: Journal Entry
Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Review meaning of algebraic tiles & representation exercises Visualizing & using symbols Journal Entry and worksheet practice	Computer Lab	Computer Lab	Review Visualizing & use of symbols; Have students share general rules and extend to combining terms; also, reverse process. Journal Entry and worksheet practice	Review general rules and combining terms; extend to using distributive and division work Journal Entry and worksheet practice

Lesson Plans for Use of Algebra Tiles – week #1

Monday – Computer Lab

Use of the Cognitive Tutor Math A Year I program – ongoing

Students work at their own pace, establishing understanding through guided practice of math concepts in real life situations. Initial lessons are designed to represent simple linear functions and equation solving via designing a table of values, identifying the independent variable, writing a formula and graphing the resulting data to draw a line.

This unit is particularly helpful if presented co-currently with the initial elements of the computer program or when most students are about to begin unit 3 which assumes knowledge and understanding of combining integers and terms in order to begin solving linear equations.

Tuesday – Computer Lab

same as Monday.

Lesson Plans for Use of Algebra Tiles – week #1
(continued)

Wednesday – Introduction of Algebra Tiles

Topics: Establish prior knowledge of work with integers (signed numbers)

            Definition and use of Algebra tiles as concrete representations of integers

            Journal on the learning process.

Materials: Overhead projector with pens, acetate sheets and tile set, algebra tile sets for students,

            worksheets (copied), student notebooks and writing instruments.

Process: Outline the three part lesson and materials on the board or overhead. [prepared ahead]
            Partner students & handout Assignment 2-5 Task 1, together students answer # 1-12. [5 min.]
               (p.52 Carnegie Learning – New York Regents v1.0)
            Review student answers as a class. [3 min.]
            Introduce algebra tiles as a tool, a concrete representation of answers given –
               modeling # 1-5 on the overhead. [6 min.]
            Then pass out sets to partners and have them model # 6 – 12. [6 min.]
            Observe student process and progress by walking throughout the room.
            After regaining full class attention, read #2 aloud and have students experimentally
               model the situation with their algebra tiles. (*Notus Bene*: If you pass this worksheet
               out ahead of time, students will lose the opportunity to experience their learning
               transfer from concrete to symbolic.) Demonstrate the correct use of algebra tiles
               for #2 on the overhead or gain this information from student volunteers while
               modeling their suggestions on the overhead. A discussion concerning 'finding the
               answer' will very likely surface. This can be tabled for next lesson or directed by
               question posing (Brown, 1993) into a discussion of the zero principle. [5-15 min.]
            If discussion is pursued, have students journal for last 5 minutes of class. This can be
               directed through problem posing or simply be thoughts shared about the days
               lesson.
            If discussion is tabled, use the same process again with #3. [5 min.]
            Hand out Assignment 2-5 Task 2 (p.53) and have students work in pairs to explore
               and record their tile representations of # 4 & 6. [5 min.]
            Students return materials and journal for last 5 minutes of the period. They can
               complete p.53 and their journal for homework.

Suggested Journal Questions to Pose – choose one or two:

        Can zero be demonstrated using algebra tiles? Explain using examples.
        What are algebra tiles? Can they be used to represent an unknown?
        Given a specific problem situation, will all student representations look the same?
        What did you learn from your partner today?

Lesson Plans for Use of Algebra Tiles – week #1
(continued)

Thursday – Use of Algebra Tiles to represent combining integers and the zero principle

Topics: Review use of algebra tiles to represent integers and situations.

Experiment with application of understanding by reversing the process.

Journal on the learning process.

Materials: Overhead projector with pens, acetate sheets and tile set, algebra tile sets for students,

worksheet #1 (copied), student notebooks and writing instruments.

Process: Outline the three part lesson and materials on the board or overhead. [prepared ahead]
    Review students' responses on homework sheet: have students draw their models on the board
        or overhead sheets to share with the class. [8 min.]
    Utilize one example (#7 is quite effective) to model and discuss the zero principle and
        finding the answer (use the overhead tile set and focus on asking questions of the
        students rather than providing answers). [8 min.]
    Partner students for reviewing some of their homework and finding answers via
        modeling with the tiles. [5 min.]
    Observe student process and progress by walking throughout the room.
    After regaining full class attention, pose the question: How many ways can you
        represent the result "- 5" using two colors of algebra tiles? Have students
        experimentally model the situation with their algebra tiles. Demonstrate at least
        three volunteered examples on the overhead. [10 min.]
    Hand out Worksheet #1 and have students work in pairs to explore and record their
        tile representations of # 2 & #4. [5 min.]
    Students return materials and journal for last 4 minutes of the period. They can
        complete the worksheet and journal for homework.

Suggested Journal Questions to Pose – choose one or two:

Can zero be demonstrated using algebra tiles? Explain using examples.
What are algebra tiles? Can they be used to represent an unknown?
Given a specific problem situation, will all student representations look the same?
What did you discover today?

Carnegie Learning Math A Year I   **Name** _____

Worksheet #1 – period _____   **date** _____

For each integer below, show at two representations that would result in the integer using algebra

    tiles. You can draw the tiles using colors or shading; be sure to indicate a key for your tiles.

       ☐ positive value   ☐ negative value

\+ 7

\- 3

\+ 11

\- 15

0

Lesson Plans for Use of Algebra Tiles – week #1

(continued)

Friday – Use of Algebra Tiles to represent combining algebraic terms

Topics: Review use of algebra tiles to represent combining integers and the zero principle.

Explore the use of algebra tiles to represent and combine algebraic terms.

Journal on the learning process.

Materials: Overhead projector with pens, acetate sheets and tile set, algebra tile sets for students, worksheet #2 (copied), student notebooks and writing instruments.

Process: Outline the three part lesson and materials on the board or overhead. [prepared ahead]
Review students' responses on homework sheet: have students draw their models on the board or overhead sheets to share with the class. Collect homework for review. [10 min.]
Introduce the use of algebra tiles to represent algebraic terms. Handout worksheet #2; model the examples and have students record the tile formations. [5 min.]
Introduce the combination of algebraic terms by asking what would happen if students were to combine examples 1 and 2. Models remaining on overhead can provide useful tools for discussion; encourage students to model their idea in partners and record their results in tiles. Model and discuss combining examples 3 & 4. [10 min.]
Have students copy 5 homework examples from board or overhead and demonstrate the combination of terms in tile recordings. (*Notus Bene*: Be cautious when choosing these examples as they should only allow for the use of the zero principle when combining. Students will later learn how to deal with subtracting a negative or positive value that is not already represented.) [10 min.]
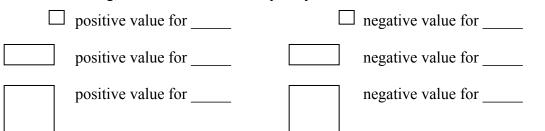Observe student process and progress by walking throughout the room.
Students return materials and journal for last 5 minutes of the period. They can complete journal and copied examples for homework.

Suggested Journal Questions to Pose – choose one or two:

What are algebra tiles?
Given a specific problem situation, will all student representations look the same?
Can algebra tiles be used to solve the equation: $3x - 5 = -7 + 4x$?
What did you learn today?
Reread your last two journal entries; what have you learned this week?

Carnegie Learning Math A Year I   Name _____

Worksheet #2 – period _____   date _____

For each expression below, record an algebra tile representation. You can draw the tiles using

  colors or shading; be sure to indicate a key for your tiles.

☐ positive value for _____   ☐ negative value for _____

[ ] positive value for _____   [ ] negative value for _____

[ ] positive value for _____   [ ] negative value for _____

Representations of algebraic expressions:

  Example #1: $2x^2 + 6x - 3$

  Example #2: $1x^2 + 8$

  Combining:

  Example #3: $8 - 9x$

  Example #4: $8x - 2x^2 - 4$

  Combining:

Lesson Plans for Use of Algebra Tiles – week #2

Monday – Visualizing and using symbols for combining integers.

(*notus bene:* Today's and Thursday's lessons require a higher level of tolerance of frustration and greater persistence on the part of the students. A strong focus by the instructor on intentional encouragement and affirmation is critical at this time when students are more vulnerable than in a traditional classroom. These lessons require students to experiment; in their minds this means fail. The class may have more success when framed by the teacher as an exercise in perseverance, effort, and discovery, rather than compliance and correct answers.)

Topics: Review meaning and use of algebra tiles to represent and combine integers and zero.

Explore the use of visualization and symbols to represent and combine integers.

Journal on the learning process.

Materials: Overhead projector with pens, acetate sheets and tile set, algebra tile sets for students,

worksheet #3 (copied), student notebooks and writing instruments.

Process: Outline the three part lesson and materials on the board or overhead. [prepared ahead]
Review students' responses on homework: have students draw their models on the board or overhead sheets to share with the class. While students are working, verbally review the meaning of algebra tiles and record responses on the overhead. Optional: Collect homework for review. [15 min.]
Introduce the symbols involved in the combination integers. Partner students and provide algebra tile examples that they must translate into symbols and combine. Addition and some subtraction examples will be comfortable for students to assess. Of particular interest will be examples that direct students to discover the meaning behind subtracting negatives or positives that don't initially appear, as well as the necessity of applying the zero principle when combing terms of opposite sign. This explorative study may be directed via a handout of examples such as worksheet #3 or through use of the overhead. Encourage students to model their ideas as partners and record their results in tiles – then in symbols. A sufficiently large number of examples must be provided for students to discover the underlying rules of combining integers. [20 min.]
Observe student process and progress by walking throughout the room.
Have students complete 5 examples for homework, recording their ideas in algebra tiles and symbols. Students return materials and journal for last 5 minutes.

Suggested Journal Questions to Pose – choose one or two:
Given a specific problem situation, will all student representations look the same?
Can algebra tiles be used to solve the equation: $3x - 5 = -7 + 4x$?
Write a rule for adding positive and negative numbers and give examples that show your rule is correct.
What did your partner do today that was different from what you did?

For each integer combination below, use algebra tiles to represent each. Manipulate the tiles to

     represent addition or subtraction (using the zero principle) and record the result in tiles.

     Then show your process using mathematical symbols.

You can draw the tiles using colors or shading; be sure to indicate a key for your tiles.

         ☐ positive value   ☐ negative value

+ 7 add -3

- 3 add - 5

+ 4 add -9

+ 12 subtract + 8

- 12 add + 9

Worksheet #3 – on back

- 5 subtract - 3

+ 7 subtract -3

+ 4 subtract + 9

- 2 subtract - 9

+ 11 add – 9

+ 5 subtract + 2

- 7 add + 9

Lesson Plans for Use of Algebra Tiles – week #2

<div align="center">(continued)</div>

Tuesday – Computer Lab

Use of the Cognitive Tutor Math A Year I program – ongoing

Students work at their own pace, establishing understanding through guided practice of math concepts in real life situations. Initial lessons are designed to represent simple linear functions and equation solving via designing a table of values, identifying the independent variable, writing a formula and graphing the resulting data to draw a line.

Wednesday – Computer Lab

same as Tuesday

Thursday – Visualizing and using symbols for combining terms.

Topics: Establish general rules for combining integers – based on student experience.

Explore the use of visualization and symbols to represent and combine terms.

Journal on the learning process.

Materials: Overhead projector with pens, acetate sheets and tile set, algebra tile sets for students,

student notebooks and writing instruments.

Process: Outline the three part lesson and materials on the board or overhead. [prepared ahead]
Establish general rules for combining integers based on students' experience in class
   on Monday and with the homework. Provide new examples that challenge students
   to defend or alter their theory. A review of students' responses on homework will
   facilitate this process; have students draw their models on the board or overhead
   sheets to share with the class. Optional: Collect homework for review. [20 min.]
Have students retrieve worksheet #2 and in the left hand column, use numerical and
   operational symbols to represent the examples. [5 min.]
Have students generate three examples in partnership. They should record their
   results in tiles and then in symbols. [10 min.]
Observe student process and progress by walking throughout the room.
Have students complete 5 examples for homework, recording their ideas in algebra
   tiles and symbols. Students return materials and journal for last 5 minutes of the
   period.

Suggested Journal Questions to Pose – choose one or two:

Given a specific problem situation, will all student representations look the same?
Can algebra tiles be used to solve the equation: $3x - 5 = -7 + 4x$?
Write a rule for adding positive and negative numbers and give examples that show
   your rule is correct.
What did your partner do today that was different from what you did?
Reread your last two journal entries. How do you see math differently today than one
   week ago?

Lesson Plans for Use of Algebra Tiles – week #2
(continued)

Friday – Extending knowledge of combining integers and terms to include the use of the
distributive property and division of polynomials..

Topics: Review the general rules for combining integers and terms, and the application of the
distributive property and division of polynomials.

Explore the use of knowledge gained in using algebra tiles to extend students'
experience combining terms by multiplying through distribution and dividing.

Journal on the learning process.

Materials: Overhead projector with pens, acetate sheets and tile set, algebra tile sets for students,
student notebooks and writing instruments.

Process: Outline the three part lesson and materials on the board or overhead. [prepared ahead]
Review: students' responses on homework and pages 66-67 in student text will
facilitate this process. Optional: Collect homework for review. [10 min.]
Have students use page 69 in their texts to begin experimenting as partners with the
application of combining terms when multiplying or dividing polynomials. [5 min.]
Record student responses on the overhead and direct the discussion by posing
questions. Alternate between student exploration and directed practice. [15]
Observe student process and progress by walking throughout the room.
Positively and briefly talk about the results of the two week journey and their efforts
in working with the algebra tiles. Have students record some final thoughts in their
journal and hand in their notebooks for you to review. [10 min.]
Have students complete 10+ examples for homework. [handout on their way out]

Suggested Journal Questions to Pose – choose one or two:

Can algebra tiles be used to solve the equation: $3x - 5 = -7 + 4x$?
What do you know today about combining algebraic terms that you did not know
before?
Are algebra tiles a useful tool in helping you learn mathematics?

Appendix a

itemized list for set of Overhead algebra tiles &

instructions for student sets of algebra tiles

This is a recommended itemized list for transparent algebra tiles that can be used on the overhead and for student sets which can be made by using the following masters to copy and cut colored paper. Two typical sets must be acquired from the Cuisenaire Company of America, Inc. to complete the list for teacher use on the overhead.

quantity	_color_	_description_	_representing_
8	blue	5.3cm x 5.3cm squares	x^2
8	red	5.3cm x 5.3cm squares	$- x^2$
20	green	1cm x 5.3cm rectangles	x
20	red	1cm x 5.3cm rectangles	- x
30	yellow	1cm x 1cm rectangles	constant = 1
30	red	1cm x 1cm rectangles	constant = - 1

The masters which follow this page are drawn for 8.5 x 11 inch colored paper; cardstock is recommended. If construction paper is used, it must be cut down to size or the pattern must be re-formatted. In order to construct 20 students sets, including extra pieces, the recipe below should be followed. Placing pieces in zip-lock bags will assist in order and organization.

For 1x1 constant units: Copy and cut one sheet each of red and yellow
For 1x5 'x' units: Copy and cut 3 sheets each of red and green
For 5x5 'x²' units: Copy and cut 6 sheets each of red and blue

Total paper needed: 10 sheets of red, 1 of yellow, 3 of green, and 6 of blue. Another alternative is to copy these masters on regular white paper and have the students mark the sizes by color.

Resources:

Brown, S. I., & Walter, M. I. (Eds.). (1993). <u>Problem posing: reflections and applications.</u> Hillsdale, NJ: Erlbaum Associates.

Cognitive Tutor Math A Year I. (1999<u>). Computer program, student textbook, student assignments and assessments.</u> Pittsburgh: Carnegie Learning, Inc.

Dewey, J. (1938). <u>Experience and education.</u> New York: Macmillan.

Dewey, J. (1943). <u>The school and society.</u> Chicago: University of Chicago press.

Howden, H. (1994). <u>Algebra tiles for the overhead projector.</u> New York: Cuisenaire Company of America, Inc.

National Council of Teachers of Mathematics. (2000). <u>Principles and standards for school mathematics: E-Standards.</u> Reston, VA: The National Council of Teachers of Mathematics, Inc.