Anticlastic Membranes: Correlations of Form, Prestress, Behavior

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SYNOPSIS
The structural behavior, strength, stiffness, and stability, of anticlastic membranes are closely correlated with structural form and prestress. The objective of the presented research was to investigate, by computer analysis, correlation of form, prestress and stability for materials of different elasticity. The term membrane in this study includes fabric membranes as well as cable nets. Previous studies have demonstrated that structural stability of anticlastic membranes is critically reduced Vvilen load conditions reduce prestress to zero in some areas of the membrane. This research investigated the amount of prestress needed to prevent or minimize zero stress and thus instability. It will also be sho'Ml that, under certain conditions, prestress can reduce load induced deformation to half.

INTRODUCION
1. Anticlastic membranes have strong correlation between form, prestress, and structural behavior. For a given span and load, a small radius of cuNature yields smaller membrane forces than a large radius. Further, the anti clastic curvature, combined with prestress, provides stability. A chain or cable hanging between two points will adjust its form for any load condition, but anticlastic membranes resist such deformations by mutually opposing curvature. Two cables that cross each other stabilize their mutual cross point (Fig. 1 A). Similarly, two layers of cables of anticlastic curvature (Fig. 1 B) stabilize a series of cross points and thus an entire surface.

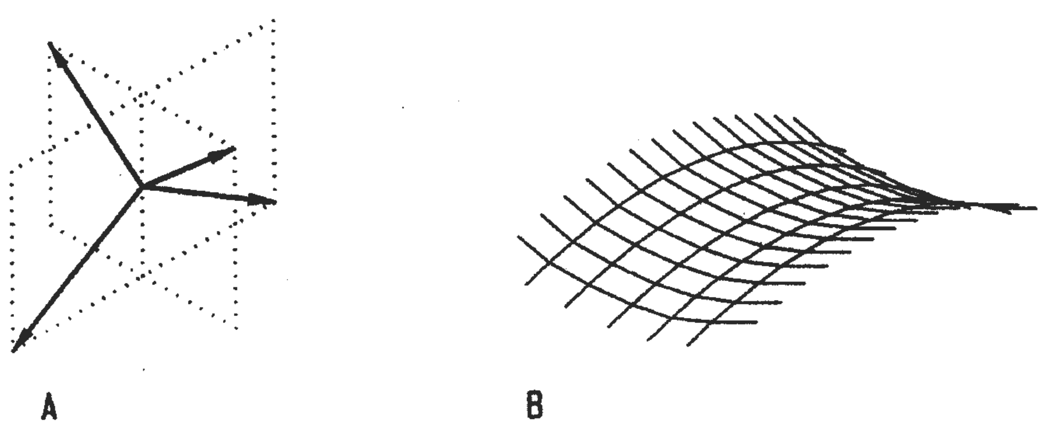

Fig. 1. Effect of curvature to stabilize anticlastic membranes

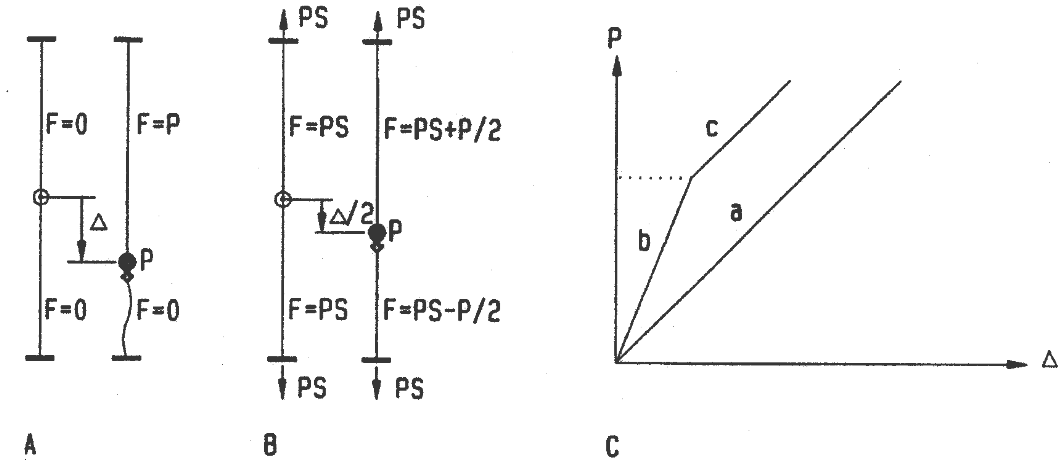

Fig. 2. Effect of prestress to provide stability and reduce deformation

2. The effect of prestress to reduce deformation and provide stability for a membranecan be illustrated with idealized models (Fig. 2) of an elastic string without prestress (A) and with prestress (B); where F=force, PS=prestress, and P=applied load. The load applied at mid-height of the string causes a deformation A in model (A); the upper link carries the entire load while the lower link carries no load and gets slag. The string in model (B) is subject to an initiaJ prestress. When load P is applied, the upper and lower link will each carry half; the latter by reducing its prestress. Since each link carries only half the load, the resulting deflection is Δ/2. Once prestress in the lower link is reduced to zero, any additional load will cause the same deflection as in the un-prestressed model (A) as illustrated in the P/Δ diagram (C) where *a* is the P/Δ curve of model (A); *b* is the P/Δ curve of model (B) with prestress; and *c* is the P/Δ curve of model (B) after the lower link prestress is reduced to zero. When the lower link in either model gets slag, it will be unstable. These observations can be applied to anticlastic membranes, such as the model of Fig. 1B. If the model is without prestress, an applied gravity load would be carried by the concave layer only and the convex layer would get slag and thus unstable. But if the model has the proper amount of prestress, the concave and convex layers would each carry half the load, the latter through reduction of prestress; resulting in only half as much deflection as in the un-prestressed case. Model tests of anticlastic cable nets have demonstrated that when gravity load reduces prestress to zero such structures are highly unstable. Similar observations were made during prototype testing for the membrane structures of Expo'64, Lausanne Switzerland (Fig. 3). A first prototype of natural canvas with edge cables deformed excessively due to creep, resulting in slag fibers and instability. The addition of a supporting cable net resolved the problem. Therefore prestress in anticlastic membrailes and cable nets should always be at a level to prevent zero stress and slag members under any load condition. At least the majority of cables or membrane fibers should retain positive prestress. Thus the objective of this research was to determine by computer analysis the required amount of prestress for anticlastic membranes and cable nets with rigid beam edges and flexible cable edges.

Fig. 3. Anticlastic membrane structures for Expo'64 Lausanne, Switzerland
Architect: M.J. Saugey and G.G. Schierle; Frei Otto, consultant

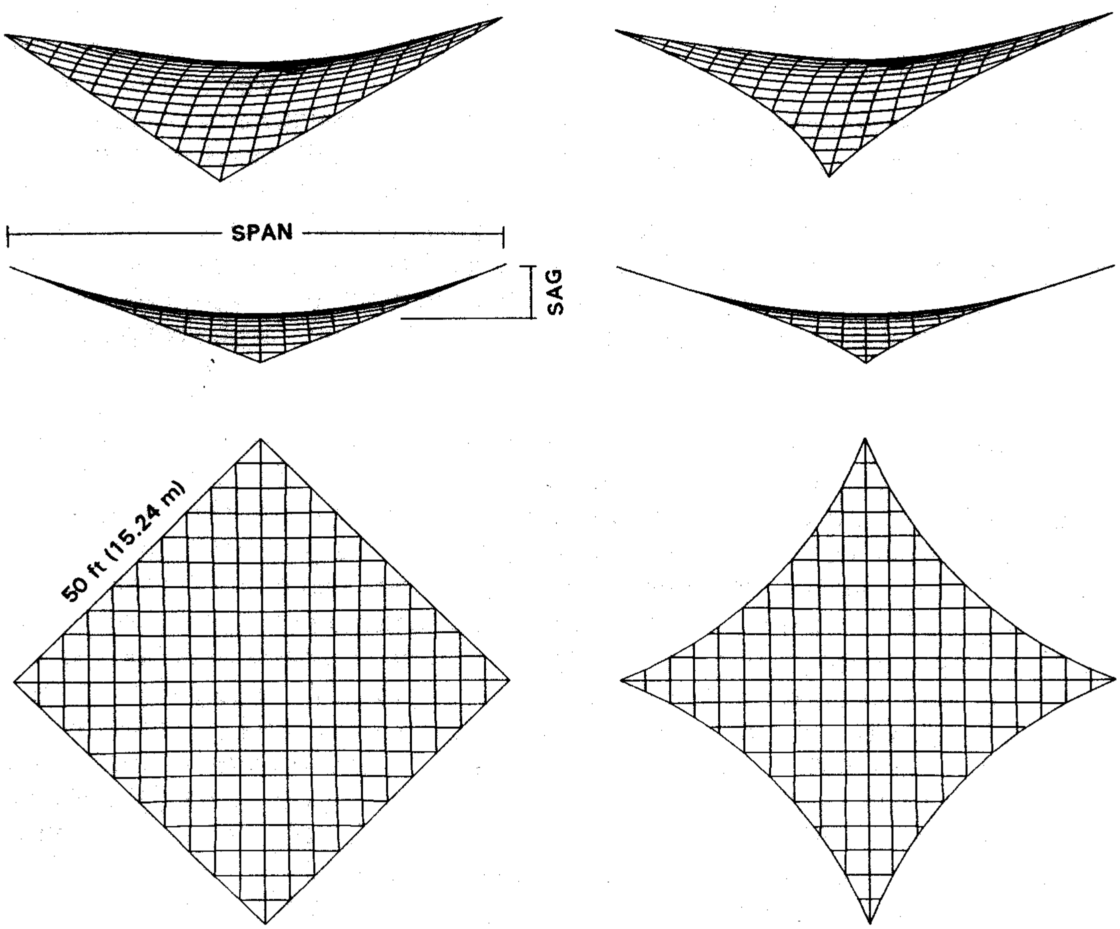

Fig. 4. Analysed prototypes with rigid edge beam and flexible edge cable.

ASSUMPTIONS

2. The computer analysis to determine correlation of form, prestress, and structural behavior was based on the configurations of Fig. 4 and these assumptions: The quadrilateral anticlastic cable nets and membranes had side lengths of 50 ft. (15.24m); with two low and two high points each; with both, rigid edge beams, and flexible edge cables. The high points were positioned to yield sag/span ratios at the lines of principal cuNatures of 1:5; 1:1 0; and 1:15. The cable nets have mesh sizes of 3.5 ft (1.07m) along the edges, reducing toward the center according to lines of least length; providing one of sevral possible mesh configurations. The cable orientation is in direction of the principal curvature for greatest stiffness. Tests have demonstrated that cables arranged parallel to the edges in direction of generating lines and, thus, of least curvature, will have deformations about six times greater than those running in direction of principal curvature (ref. 3, 4). The net cables were of 1 in (25 mm) diameter strands, with metallic areas of 0.6 in^2 (387 mm^2) and elastic modules of 24,000 ksi (165,480 MPa). Edge cables were of 3 in (76.2 mm) diameter strands with metallic areas of 5.4 in^2 (3,484 mm^2) and elastic modules of 22,000 ksi (151,690 MPa).

3. Membranes were assumed of fabric bands of the same ~dth as the cable mesh with elastic modules of 9000/6750 pli (1575/1181 kN/m) in warp and fill directions, respectively.

4. Prestress was applied equal in concave and convex directions, to approximate minimal surface areas. Five levels of prestress were simulated, namely 30%, 40%, 50%, 60%, and 70% of the maximum stress under load. This was achieved by iterative runs, using a trial and error approach for each load case.

5. Gravity load was assumed 0.5 k (2.2 kN) applied at each joint of the cable nets and equivalent intervals of the membranes. Load was prorated for smaller tributary areas along the edges. The load was applied downward and normal to the ground.

METHODOLOGY

6. The analysis computer program TRITRS (by Dr. Eberhard Haug) was used for form finding and structural analysis. TRTRS is based on a modified direct stiffness method for structural analysis of truss-type structures with geometrically non-linear behavior. The program includes a special prestress element suitable for form-finding. Starting with a user defined approximate geometry, defined by X, Y, Z joint coordinates, connecting elements, and bi-axial prestress, the program finds the exact form for equilibrium, based on an iterative algorithm that converges in usually about 5 to 20 iteration steps. After the initial form is defined, subsequent runs compute member stresses, joint displacements, and support reactions under applied load, also in iterative analysis steps. The program treats cable elements such that if compressive forces reduce prestress to zero or less, the stiffness of such elements will be set to zero to simulate the actual behavior of slag cables wlich have no effective stiffness. However, prestressed cables can absorb compressive forces through reduction of prestress, as described for the idealized model of Fig. 2. But when the prestress is reduced to zero under compressive forces, the stiffness of the member will also be zero.

RESULTS

7. Analysis results are illustrated in Figures 5 and 6 for structures with rigid edge beams and flexible edge cables respectively; both for cable nets and membranes, and for 3 sag/span ratios. Our discussion refers to prestress force as percentage o maximum force under load, as presented on the graphs. Comparing edge beam structures of Fig. 5 with the edge cable structures of Fig. 6 reveals only minor differences. While the edge beam yields slightly higher forces, the differences are not significant, especially for sag/span ratios of 1:10. For sag/span ratios of 1:5 and 1:15 the edge beam structures require somewhat higher prestress to prevent slag members and instability. Hence edge conditions have only minor impact on required prestress in case of the investigated saddle shapes. The following discussion is based on observations of the edge beam cases (Fig. 5).

8. For cable nets, the required prestress to prevent any cable from getting slag is 50% for 1:5 sag/span ratio and 40% for sag/span ratios of 1:10 and 1:15. As expected, a small sag yields higher maximum forces than a large sag.

9. Slag members caused by zero prestress occur initially only in one or two cables and would probably not critically impair the structure's stability. The average force in convex cables is considered a more realistic measure of stability. The graphs show average convex cable forces still above zero at 30% prestress.

10. The initial level of prestressed should be adjusted for expected temperature variations, time dependent creep and possible support displacements. .An increase in temperature would reduce the prestress and so woud creep and support displacements.

11 .For membranes the required prestress to prevent any fibers from getting slag is less than in a cable net: 40% for 1:5 sag/span ratio, 30% for 1:10 sag/span ratio, and less than 30% for 1:15 sag/span ratio. The average stress in convex membrane layers is well above zero even at 30% prestress for all sag/span ratios. Variation of maximum forces'in membranes is minimal due to greater deflection than cable nets.

CONCLUSIONS

The graphs of Figures 5 and 6 and the above discussion of results demonstrate significant correlation between fom, prestress, and structural behavior; more so for membranes than for cable nets, due to their higher elastic module. More research is needed to:

- detemine by dynamic analysis the real impact on structural stability when only one or two cables or a small membrane area gets slag;
- detem1ine the quantitative effect of temperature variation and creep on prestress,
- detemine the non-linear elastic and plastic behavior of membrane materials, to provide better data for more accurate analysis.
- detemine the effect of as'ymmetrical and free-fome shapes on prestress.

Data provided by such research would substantially help to make the design process for membrane and cable net structures more reliable and predictable. This is an essential factor for their wider use. It is hoped that the research presented here will make a contribution toward that end.

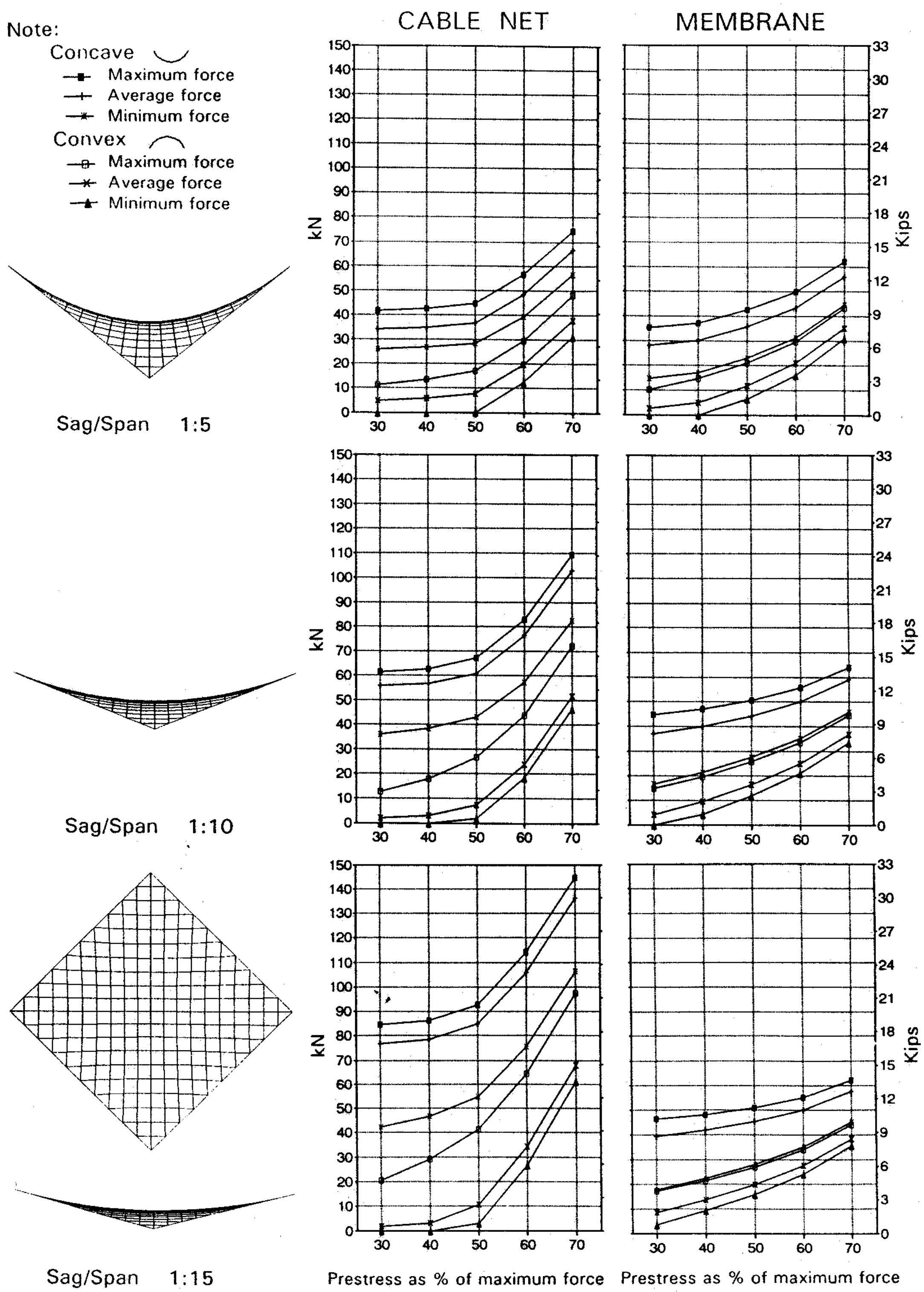

Fig. 5. Graphs correlating form and prestress as percentage of maximum force for anticlastic cable nets and membranes with rigid edge beam

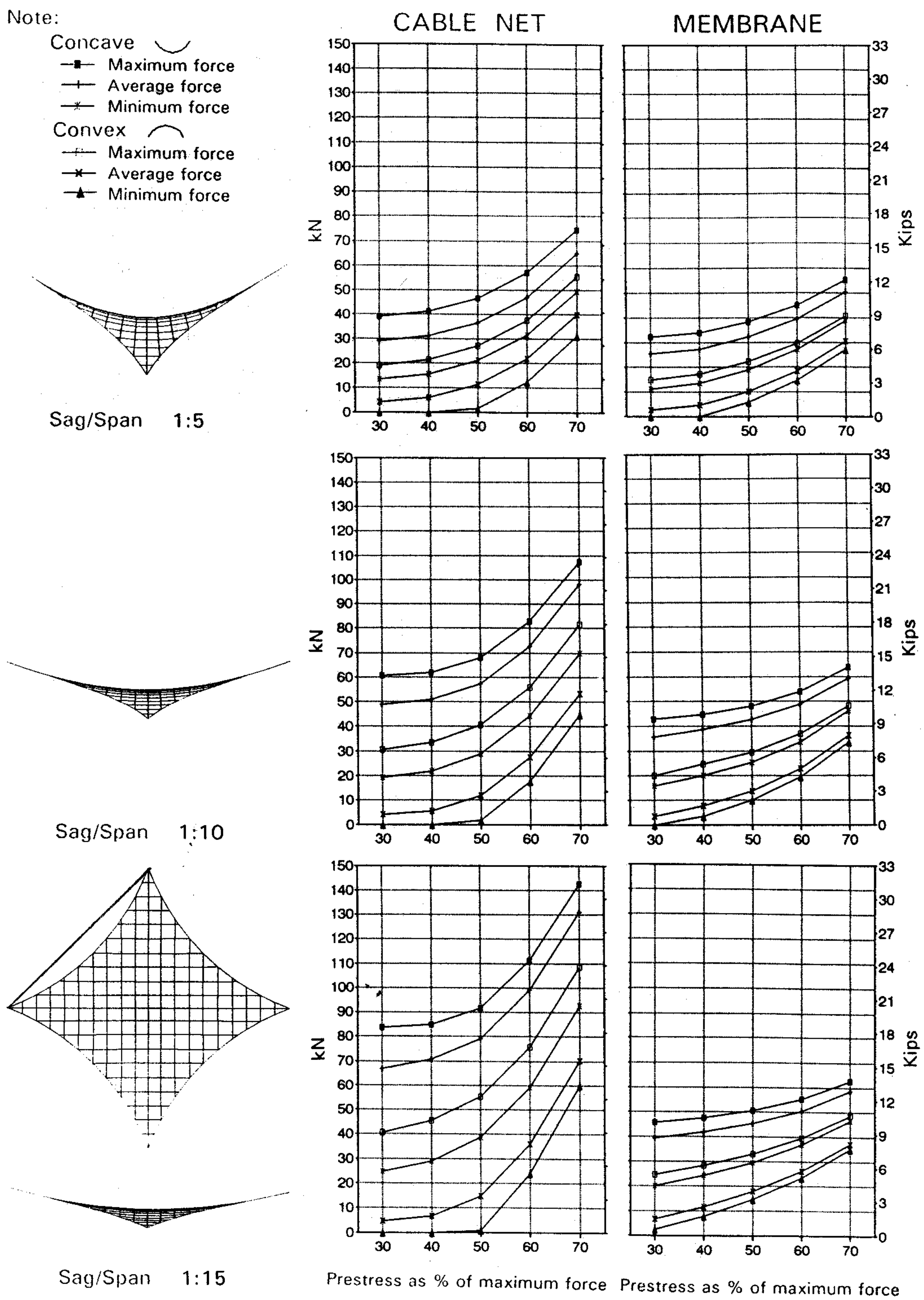

Fig. 6. Graphs correlating form and prestress as percentage of maximum force for anticlastic cable nets and membranes with flexible edge cable